Seismic Analysis of Structures
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Lecture – 26
Inelastic Seismic Response of Structures (Contd.)

In the previous lecture, we discussed about the solution of the single degree of freedom system subjected to ground excitation and the single degree of freedom system was taken such that it goes into an inelastic range. The problem was that for a particular time station the displacement velocity acceleration of the single degree freedom system was were given and for subsequent time station we are supposed to find out the displacement velocity and the acceleration of the system.

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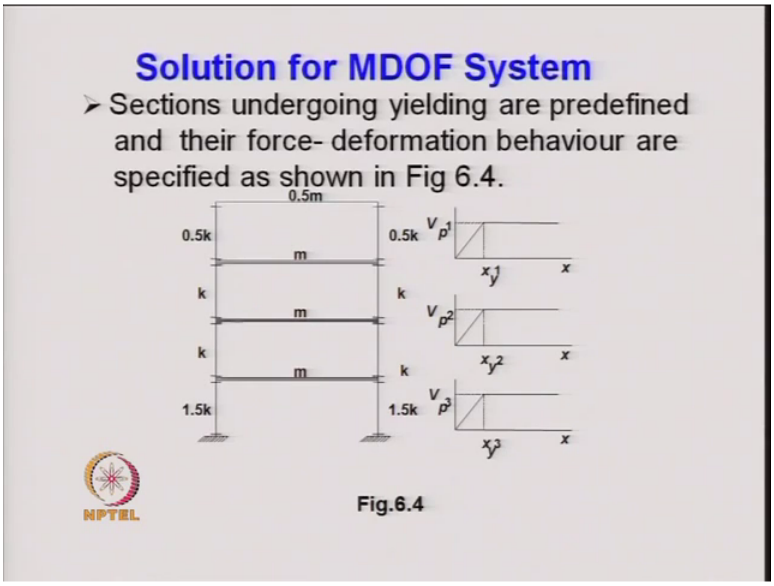

The time steps was chosen such that there was a transition point when it was passing from one state to the other and therefore, all the theoretical background that were discussed when the state of the system moves from elastic to plastic; plastic to plastic and plastic to elastic for those state changes are explained with the help of that example.

Now, we take up the problem for a multi degree of freedom system this is the frame that is considered and in this particular frame, we have the different stiff nesses for example, the bottom story has a stiffness of 1.5 k the top storey has a stiffness of 0.5 k and the in between stories have the stiffness of k the plastic shear capacity of the columns for the bottom column it is V p 3 for the top column it is V p 1 and V p 2 and these are the elasto plastic behavior of the column in shear. So, it is designated by a yield force sensor which is called V p 1, V p 2, V p 3, etcetera and in yield displacement that is designated by x y 1, x y 2, x y 3.

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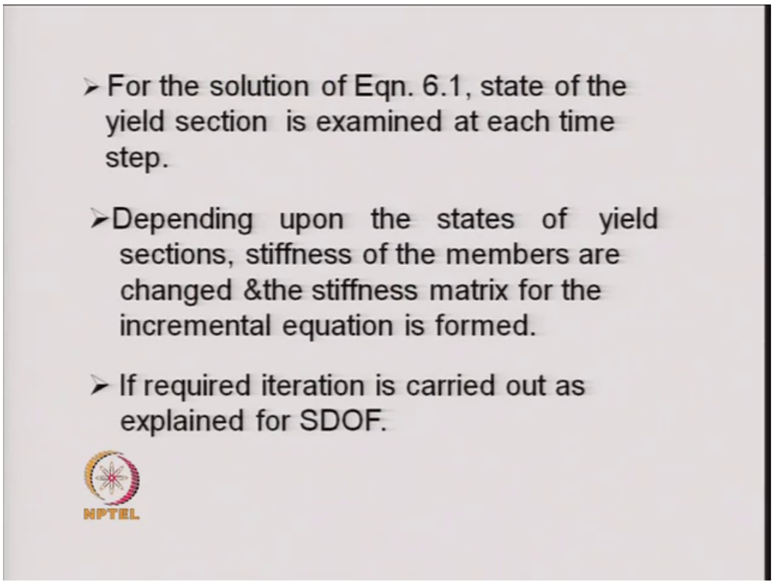

The equation that we had discussed in the beginning that was written for a multi degree of freedom system that is m x double m m delta x double dot plus c delta x double dot plus k delta x is equal to minus m into delta x double dot g. So, in that the k is the k matrix c is the c matrix and m is the m matrix.

Now, depending upon the state of the system the stiffness of the members are changed the members which undergo yielding for those members do not contribute to the overall stiffness matrix of the system and k is set to 0 for the m and if the system is in the elastic or the member is in the elastic state then the full value of the stiffness that is k 1.5 k or point five k for the columns that you have shown that are taken into consideration.

Now, in order to solve the problem we required iteration as we have carried out in the case of single degree of freedom system.

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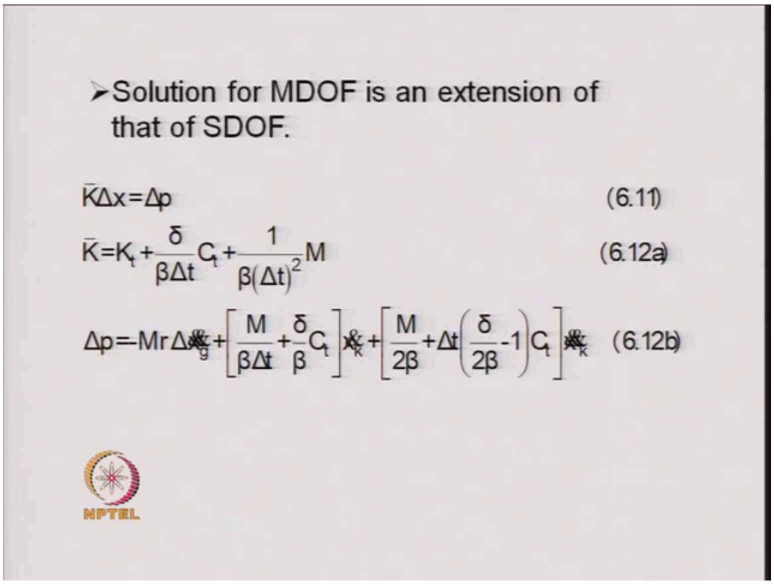

The solution for the multi degree of freedom system is just an extension of the single degree of freedom system and we have discussed how we developed the equation 6.11 that is K bar into delta x is equal to delta p using Newmark beta method. Here K bar is a matrix instead of a simple one quantity that was there for single degree of freedom system and delta p is also a vector delta s is a vector the K bar matrix is given equation 6.1 to a that if it is consisting of K t then C t and n. So, K t is the matrix which is the transition matrix or the stiffness matrix at time t C t K similarly is a dumping matrix and the M matrix is the mass matrix.

In obtaining the K t matrix we take care of the yielding of the elements if the elements if any element has yielded, then for that for construction of the K t matrix, we set K to be is equal to 0. Similarly delta p vector that consists of minus a naught double dot delta x double dot g where r is the coefficient vector and these M is a matrix it is again matrix. So, whatever equation we had in the case of single degree freedom system they are M C and K they are replaced by the matrices M C and K rest of the definitions of other parameters remain the same.

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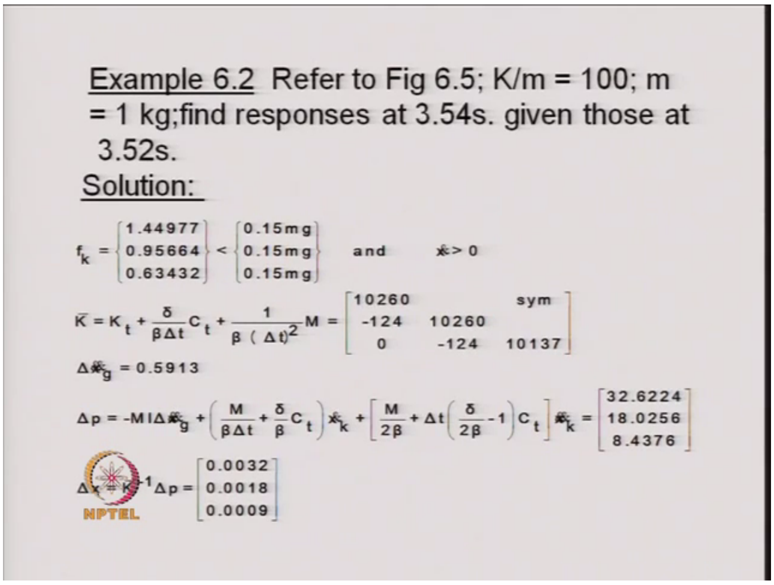

Now, we explain how we go ahead for the problem which is shown in the figure 6.5.

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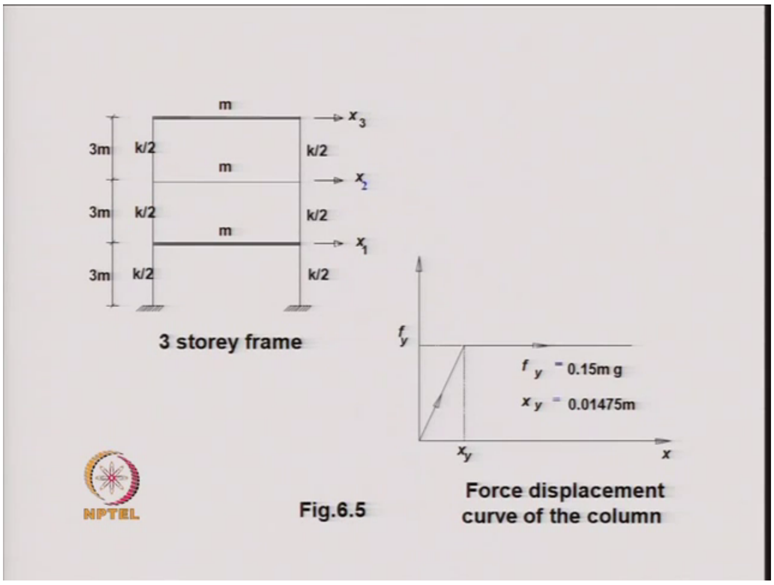

So, this is the problem that we are trying to solve its a 3 study frame having a stiffness of k by 2 k by 2 k by 2 for each column and the f y the yield shear for the column is f y and x y is the yield displacement f y s given as 0.15 m g and where m is the mass of the system or this, this mass and x y that is the yield displacement is 0.01475 meter.

Now for this frame we are trying to solve the problem at 3.54 a second we have to find out the responses given those at the time at 3.52 seconds the quantity is that are given is that f k that is at time 3.52 second the forces shear forces in the columns are 1.4, 4.956, 6.6433. So, this is for the bottom column this is for the middle column this is for the top column and all of them happens to be less than equal to the permissible yield shear for the column therefore, none of the columns as yielded x dot is given as greater than 0, therefore, the it is in a elastic state and it is increasing in the elastic state.

The k bar matrix is given by this and finding out the value of K t since none of the columns has yielded therefore, one can construct the k metrics and this k matrix is given.

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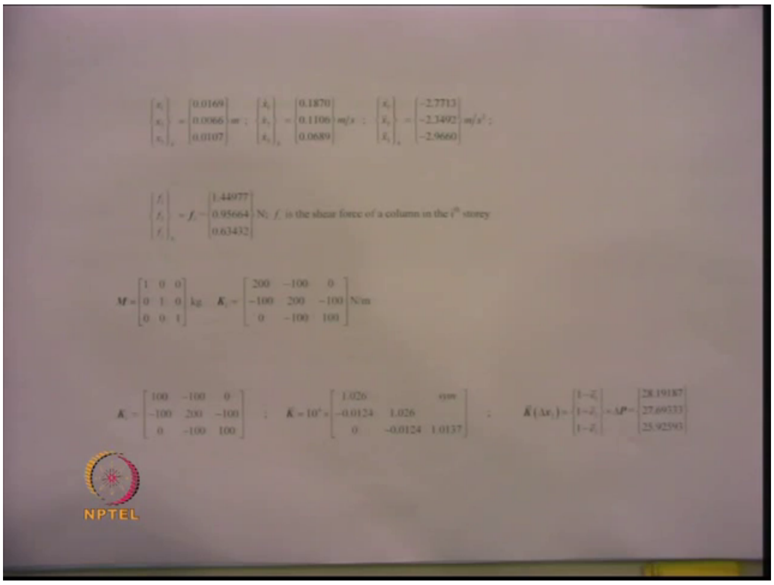

Over here; it is not very clear, but this is the K matrix which is computed and this is the m matrix which is computed and the values of the displacement velocity and acceleration at the time 3.52 seconds are given as this. Now with the help of these values of the displacement velocity and acceleration the k bar matrix is determined using this equation and the k bar matrix turn out to be this.

And using these the value of delta x double dot g as 0.5913 and the r matrix as the identity matrix that is of a vector of 1, 1, 1, 1 can calculate this quantity and then knowing the values of x double dot k x dot k and x k the values of delta p is calculated as this one and once we get the value of delta p then one can find out the displacement delta x as k bar inverse delta p and that turns out to be this.

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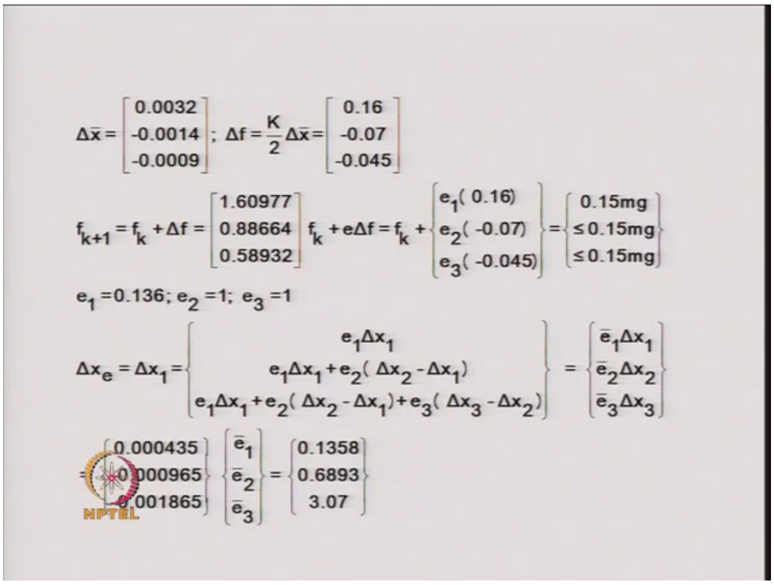

Now, from the displacement that is incremental displacement that is calculated from that we find out the drift between the different floors and drift is a given shown by delta x bar the drift for the first floor is same as the displacement itself 0.00032 the drift for the second floor will be the displacement of the second floor minus the displacement of the first floor. Similarly the drift for the top floor will be top displacement minus the displacement at the second floor. So, that gives us the drift.

And the drift multiplied by the k by 2 that is the stiffness of the column that gives the incremental force in each one of the columns at k th time station f k the values where given. So, to that values delta f is added to find out f k plus 1 that is for the current time station and it is found that the these value of the force shear force in the bottom column that exceeds the 0.15 m g that is the yield shear therefore, the bottom column is yielding whereas, the other 2 columns that is the middle column and the top columns, they remain in the elastic state.

So, what is done is that we factor as before what we discussed in this for single degree of freedom system factor the delta f in such a fashion. So, that the shear force in the bottom column that becomes equal to 0.15 m g. So, we multiply this value of delta f by e 1 by a factor of e 1 then a factor of e 2 for this factor and a factor of e 3 to the third column and find out the values such that the yield shear remains less than equal to 0.15 m g. So, in that way we calculated the values of e 1, e 2 and e 3 since the column 2 and 3 or second floor column one and the third floor column they are in the elastic therefore, e 2 and e 3 are is equal to 1.

Next what we do is that once w know these values of e 1, e 2, e 3 which are the scaling factor for the drift; that means, a drift is scaled . So, we wish to obtain now the scaling factor for the displacement and. So, that the displacements that we have got that displacements are scaled such that the yield shear in the bottom column becomes just equal to the or the shear becomes equal to the yield shear for the first column so and that is what is called the first part of the displacement vector or the scale displacement vector delta x e or what we calling as delta x 1 and this is written as that that is e 1 into delta x 1 that is for the that is for the drift whatever scaling factor is for the drift the same scaling factor used for delta x 1 and for the second column it would be e 1 into delta x 1 plus e 2 into delta x 2 minus delta x 1 that is the drift multiplied by e 2. So, that is what we have turned over here we know e 2.

So, similarly we can find out the displacement for the top floor that is e 1 delta x 1 plus e 2 into the drift plus e 3 into the drift that can be written as a revised scaling factor of e bar 1, e bar 2 and e bar 3 into delta x 1 delta x 2 and delta x 3 and this these can be worked out what are the values of delta x 1, delta x 2 and delta x 3 that is known to us because this quantities are known delta x 1, delta x 2, etcetera are known e 1, e 2, etcetera are known. So, this entire vector can be calculated and that is equal to this, this vector and once you know this vector from there one can find out the values of e bar 1, e bar 2 and e bar 3.

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$$\Delta x_2 = \begin{Bmatrix} 0.0028 \\ 0.0027 \\ 0.0026 \end{Bmatrix}; \quad \Delta x = \Delta x_1 + \Delta x_2 = \begin{Bmatrix} 0.00324 \\ 0.0018 \\ 0.00074 \end{Bmatrix}; \quad x_{k+1} = x_k + \Delta x = \begin{Bmatrix} 0.02009 \\ 0.00833 \\ 0.0114 \end{Bmatrix}$$

$$\Delta \dot{x} = \frac{\delta}{\beta \Delta t}\Delta x - \frac{\delta}{\beta}\dot{x}_k + \Delta t\left(1 - \frac{\delta}{2\beta}\right)\ddot{x}_k = \begin{bmatrix} 0.-0.0509 \\ -0.0406 \\ -0.0524 \end{bmatrix}; \quad \dot{x}_{k+1} = \dot{x}_k + \Delta \dot{x} = \begin{bmatrix} 0.1361 \\ 0.07 \\ 0.0165 \end{bmatrix}$$

$$\Delta f = \begin{Bmatrix} e_1(0.16) \\ e_2(-0.07) \\ e_3(-0.045) \end{Bmatrix} + \begin{Bmatrix} 0.00 \\ -0.005 \\ -0.005 \end{Bmatrix} = \begin{Bmatrix} 0.0218 \\ -0.075 \\ -0.05 \end{Bmatrix}; \quad f_{k+1} = f_k + \Delta f = \begin{Bmatrix} 1.4715 \\ 0.882 \\ 0.584 \end{Bmatrix}$$

$$\ddot{x}_{k+1} = M^{-1}\left(P_{k+1} - C_t \dot{x}_{k+1} - F_{k+1}\right) = \begin{bmatrix} -2.289 \\ -1.7018 \\ -2.2825 \end{bmatrix}$$

So, these are the values of the scaling factors and with the help of this scanning factors we get the value of the delta x 1 or the scaled displacement now next part is the once we have scaled the displacement to bring the bottom column forces within or equal to the yield shear, then we obtain the next part of the displacement that is the plastic displacement part. So, this plastic displacement part delta x 2 that is calculated using a revised stiffness matrix and the device stiffness matrix is given over here, you can see that in place of 200. Now it is 100 and this remains equal to minus 100. So, the matrix is changed because of the yielding of the bottom column.

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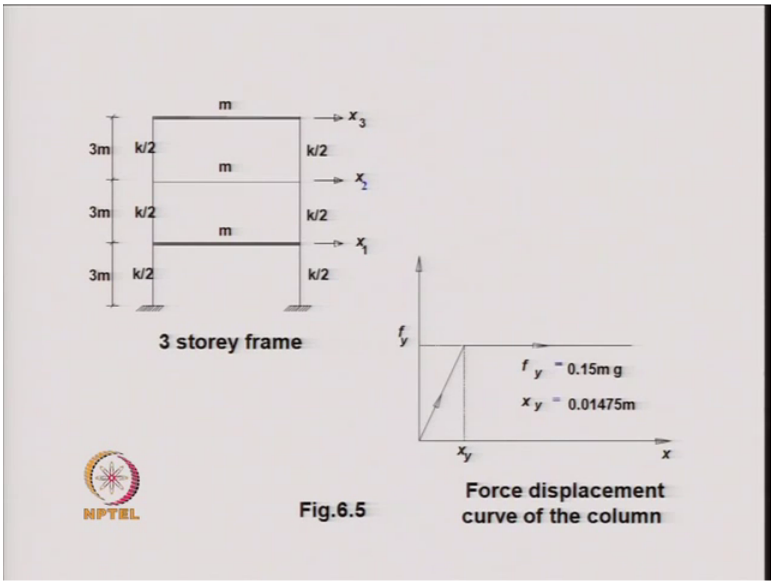

Fig.6.5

So, you can see here the same thing in this figure that once the bottom column is has yielded then there will be no contribution or the k value for that column would be equal to 0 and the force to produce a unit displacement at the level of x 1 that will be equal to only k by 2 plus k by 2 from the bottom portion no contribution will come and k by 2 plus k by 2 is equal to k and that is equal to hundred and that is how we are getting over here the value of 100.

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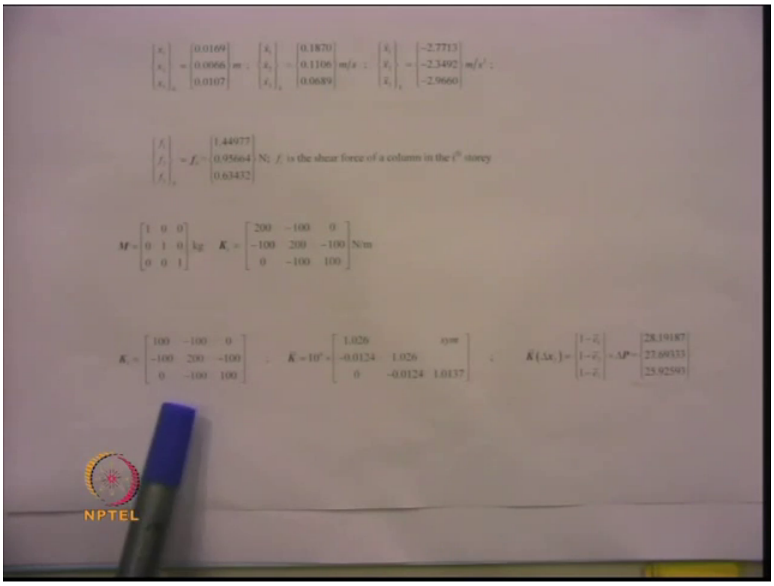

So, the K t is revised from this K t we get the value of k bar and once we get the value of k bar then we obtain k bar into delta x 2 is equal to the 1 minus e bar 1, e bar 2, e bar 3 into delta p, this e bar 1, e bar 2 and e bar 3 that we have obtained before that is the scaling factors.

So, delta p vector is multiplied by these scaling to get the value of delta p for which we will find out the plastic component of displacement that is delta x 2. So, once we get the value of delta x 2, then the we get the value of total displacement delta x delta x is equal to delta x 1 plus delta x 2 delta x 1 is the scaled displacement and delta x 2 is the displacement which is the coming due to the plasticization part or we call this as a plastic part of the displacement this we call as the elastic part of the displacement.

And once we add them together we get the total displacement delta x and vector and delta x vector comes out to be this. Now with this delta x vector the x k plus 1 that is the displacement at the current time step is obtained as a x k plus delta x. Similarly one can find out delta x dot knowing the value of delta x and from these delta x dot is computed this way and once we obtain the value of delta x dot we can find out x dot k plus 1 that is the velocity at the current time station vector and after we obtained x dot x k plus 1 x dot k plus 1. Then we try to find out the total force vector at the current time station for that we get a delta f and with the delta f, once we get the value of delta f, then we add delta f with f k to get the values of the f k plus 1.

Now, the f k plus 1 is the yield shear the shear at 3 floor levels and for a particular column and we can see that the bottom column for that the shear is equal to the yield shear for the other 2 it remains less than the yield shear because they have not yielded the 2 times these value is the total force. So, this f k plus 1 vector is 2 times this because you have got 2 sets of columns in a particular floor.

So, you use the equation of motion at k plus 1 station to find out the values of the acceleration vector that is what we discussed because if we find out the acceleration by satisfying the equilibrium equation then the error that is introduced into the calculation that is compensated and as a result of that the there is no growth of error as the computation is carried over the time steps.

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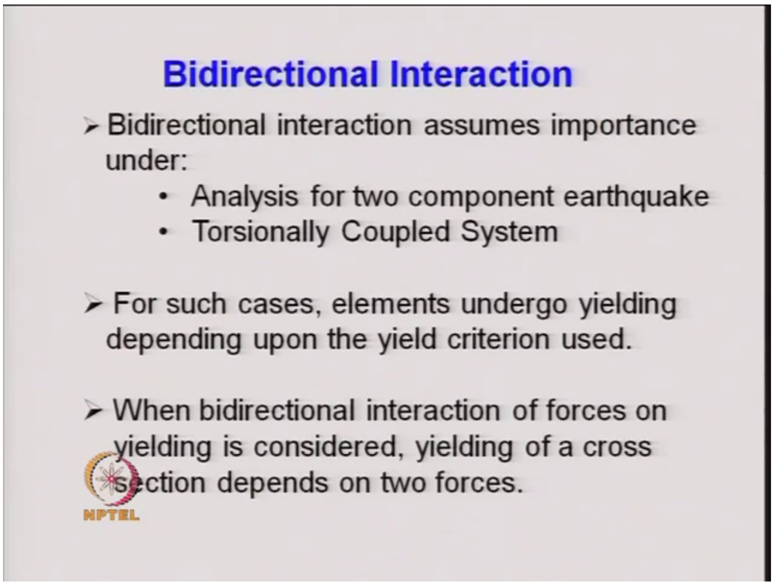

Next we come to bi directional interaction in the previous in the 2 dimensional frame we had only one directional yielding in the sense that in the direction of the earth quick the entire frame the; it is a plane frame was undergoing a bending and accordingly there was a yield shear and whenever the column reaches the yield shear it was assumed to be yielding.

Now, in the case of bi directional interaction we consider the 2 component earthquake or it could a torsionally coupled system in which the columns will develop shear forces in 2 direction that is x and y direction and the yielding that takes place will be having an interaction between the 2 shears that are developed n the column for such cases elements undergo yielding depending upon the yield criteria that is used. So, whenever we have this kind of situation that is a yielding is due to more than one forces then we have an e criteria and one can adopt different kinds of yield criteria.

In the case of a bidirectional interaction of forces that is the forces developed in the x and y directions of the column, if we consider the yielding, then the 2 forces that are developed in the 2 directions.

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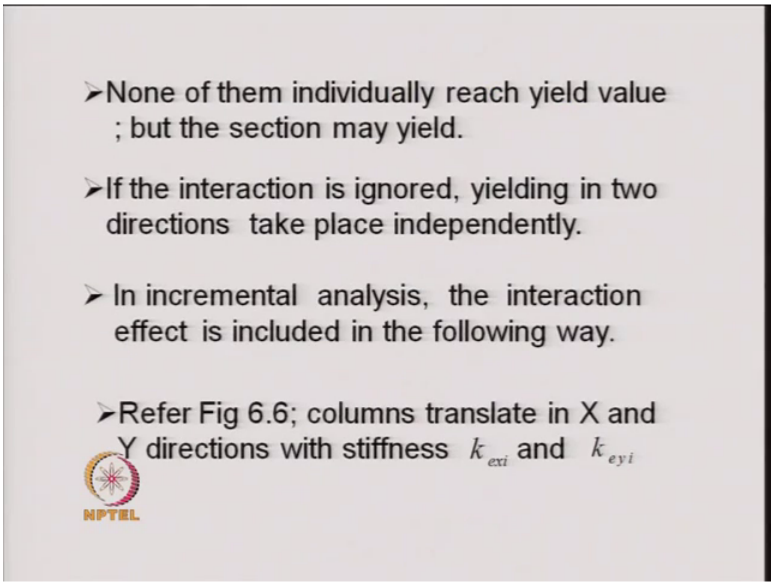

They are basically they are to be considered and it is seen that individually in each direction the column may not reach the yield value yet the section may yield because of the yield criteria that is assumed. Now if the interaction is ignored, then the yielding in the 2 direction takes place independently that is whenever the force in the x direction exceeds the yield shear in the x direction or whenever the force in the y direction of the column exceeds the yield shear in that direction then we say that the columns have yielded. So, they take place independently, but whenever we consider the interaction effect then the yielding is considered best on certain yield criteria that is assumed.

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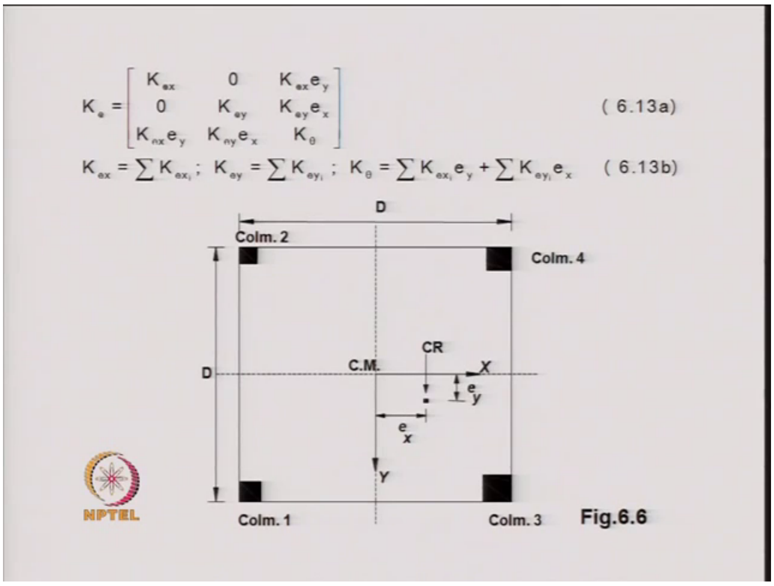

Fig.6.6

In the incremental analysis the interaction effect is included in the following way that is explained over here this say this is the this is a frame in which you have got 4 columns it is a one story frame the center of mass is at the centre the columns 1, 2, 3 and 4, they have varying dimensions as a result of that center of resistance is such that there is a 2 eccentricity e x and e y the y and x directions are this. Now the k matrix that is the elastic stiffness matrix for the system can be written as K e x, K e y and k theta K e x should be the sum of the stiffness of the columns in the x direction similarly K e y will be the sum of the stifnesses of the columns in the y direction and k theta will be equal to K e x i into e y plus K e y into e x summation for all the columns that is how we can find out these stiffness matrix for the entire system.

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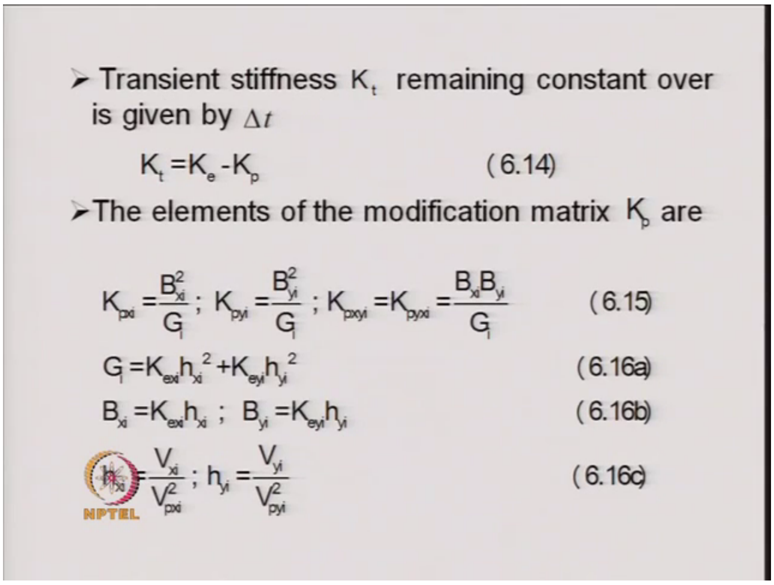

The transient stiffness matrix K t which remains constant over the period delta t for which you are writing down the incremental equation of motion. So, that K t transient stiffness matrix is written as K e minus K p that is the K p is the modification factor this modification factor is coming in to picture because of the yielding of any element that can take place during the time merging scheme. So, at any interval of time delta t or in the beginning of that time interval delta t the sum of the elements might have yielded and for that we should account for the effect into the stiffness matrix of the system and thus the K t matrix is modified and the modification is represented by this K p matrix.

Now, the elements of the K p matrix that can be calculated using this formulas these formulas are proved in some of the papers and these available in many of the papers dealing with the plasticization of the 3 dimensional frame system in which bi directional introduction comes into picture. Now the K p x i, K p y i and K p x y i and K p y x i, they are given by these formulas where b x i and b y i are defined by this K e x i and K e y i multiplied by h i and h x i where h x i and h y i are related to V x i and V p x i in this particular form.

So, knowing V x i, V y y and the permissible yield shear for those columns; if they are known, then one can calculate h x y and h y i and knowing the values of K e y and K e x y one can calculate b x i and b y i and once we know that then K p x i and K p x x y i etcetera can be found out where g basically i is given by this here again h x i and h y i, they are known K e x and K e y i are known.

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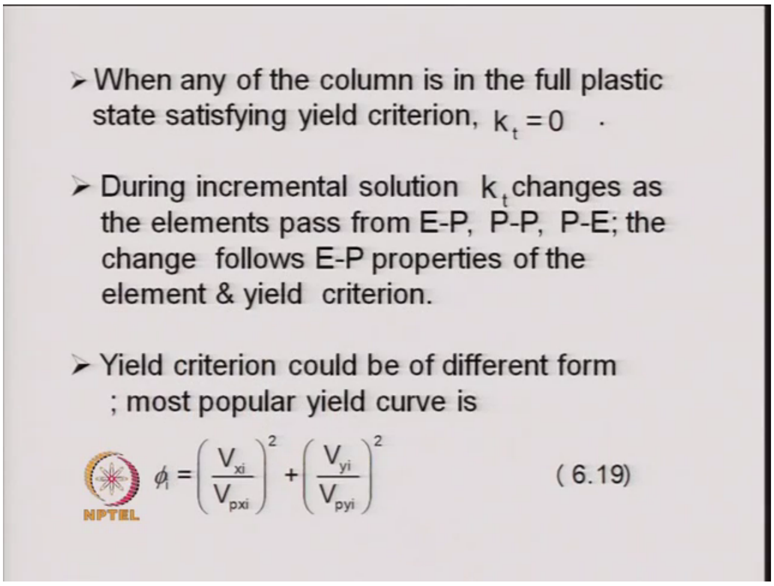

So, at any given state knowing the values of V x i and V y i one can calculate the components of the K p matrix and with the help of this K p matrix one can generate the K p matrix itself and get the transient stiffness matrix of the system at any instant of time t.

Now, if any column happens to have gone into the plastic state satisfying the yield criteria, then for that particular element we considered K t to be equal to 0. Now during the incremental solution the K t changes that is for a particular element the stiffness changes as the elements pass from elastic to plastic, plastic to plastic or plastic to elastic the way we discussed in the case of single degree of freedom system when it is passing from elastic to plastic then there is a transition point and we should take care of that transition point into the solution. So, that the displacement that takes place by incremental that takes place consists of 2 parts one is the scale displacement we call that as the elastic displacement and another displacement part in which the K t becomes equal to 0 and we get a plastic displacement. So, the scale displacement plus the plastic incremental displacement that becomes equal to total displacement.

When the system changes from p to p that we remains in the plastic state itself then we check the yield condition that will and describe shortly and whenever the system goes from plastic to elastic state; that means, any element goes from a plastic state to elastic state then we look into the plastic work done or this sign of the plastic work done from that we did decide about the transition point the change follows a EP properties of the element and the yield criteria. So, the elasto plastic properties must be given for all the elements.

And the yield criteria is given by many kinds of equation here we take one of the very popular yield equation which is given in this form V x i divided by Vpxi, Vxi is the shear in the column in the x direction of for the i th column and Vpxi is the plastic, plastic yield of the i th collection in the x direction or the yield shear of the column in the x direction. Similarly Vyi and Vpyi can defined that is the quantities n the y direction and these square of these 2 together is called phi i.

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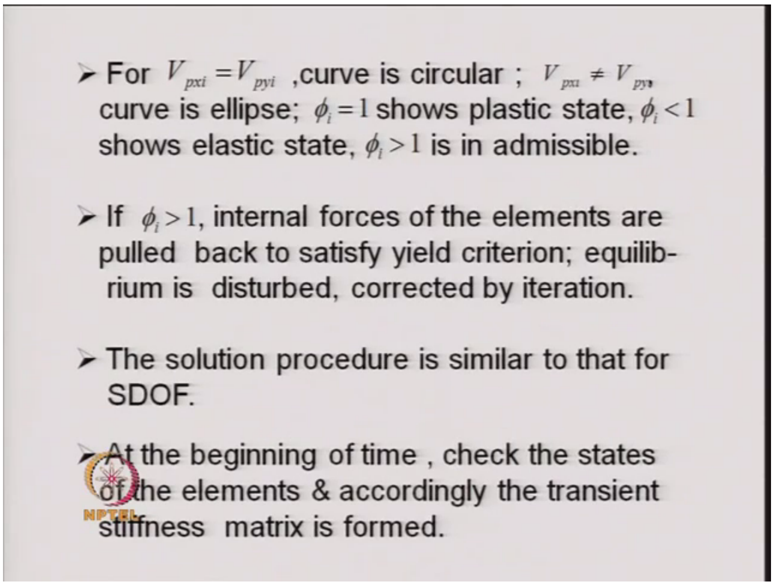

Now, if Vpxi and Vpyi are the same then the curve turns out to be a circular curve if Vpxi is not equal to Vpyi then the curve is an ellipse phi i is equal to 1 that is when the sum of these 2 quantities become equal to 1, then we say the plastic plastification has taken place or we say that it is the column is in the plastic state phi i less than one shows the column is in the elastic state and phi i greater than one is admissible. And this is a check that we can do basically when the system move from one plastic state to other plastic state that is the for each column, we see examine or tress what is the values of phi i and if the phi i happens to be greater than one when it is moving from one plastic state to the other plastic state then that is inadmissible and in that case one has to pull down the forces. So, that is give me the next step that is if phi i is greater than one the internal forces of the elements are pulled back to satisfy yield criteria; that means, we pull back the forces. So, that phi i becomes equal to 1.

So, as a result of that the equilibrium is disturbed the moment the forces are pulled back and this disturbance in the equilibrium is corrected by an iteration technique that will be described with the help of an example problem the solutions process otherwise is similar to that what we adopted for single degree of freedom system at the beginning of the time we check the states of the elements and accordingly obtain the transient stiffness matrix.

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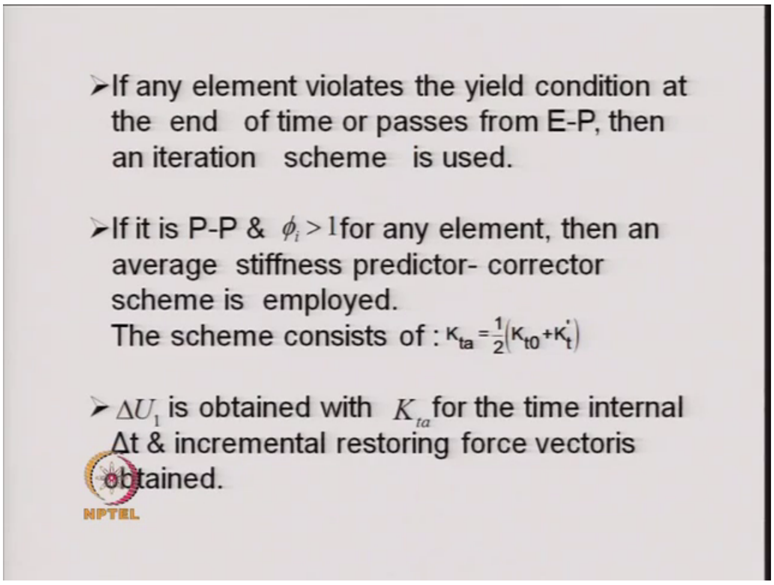

And once we get the transient stiffness matrix, then we find out the solution for that and check the stresses in the elements and if it violates the yield condition at the end of the time or passes from elastic to plastic, then an iteration is utilized to correct the state of the equation or to make sure that the yield condition is satisfied.

Now, in the case when the these any column passes from plastic to plastic state and phi i is computed as greater than 1, then an average stiffness predictor corrector scheme is employed and this scheme consists of finding out a tangent or the transient stiffness of the element and this transient stiffness is an average transient stiffness consisting of the initial transient stiffness of the element that is at the beginning of the time step and k dash t is the tar trans transit stiffness at the end of the time step and from this we work out an average and that average transient stiffness is used for that particular column.

In order to find out this k dash t, what is done we make some guess in the beginning and with the help of that we find out a K t a and with that K t a, we go ahead with the solution and the solution gives a new value of a displacements or incremental displacements and from there one can calculate what is the revised value of the k dash i that is at the end of the increment what is the value of the transient stiffness for that element.

So, now that one is used to calculate now K t a and the process continues like that did some convergence is achieved.

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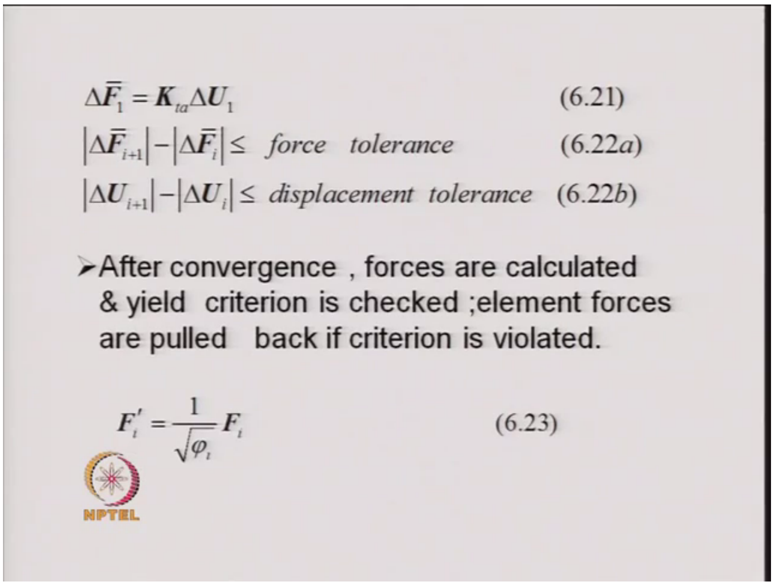

So, this convergence is examined for both the force in the column and the displacement in the total displacement in the system. So, the incremental displacement or incremental forces that we get that incremental displacement are examined in successive iterations and between 2 successive values or the difference between the 2 successive values if they remain within the some tolerance level then we say that the convergence has occur.

Now, once the convergence is achieved then the forces are calculated in the columns again and the yield criteria for each one of the column is checked and the if it is found that phi i is greater than one then the element forces are pulled back and the this pulling back is done by this equation that is the new force in the element that is a phi dash is equal to the previous force; F i divided by the square root of phi i and the say phi i, here will be greater than one; obviously, whenever it is greater than one it is not admissible. So, therefore, we divide the force by root over phi i to get the a revised value of the force.

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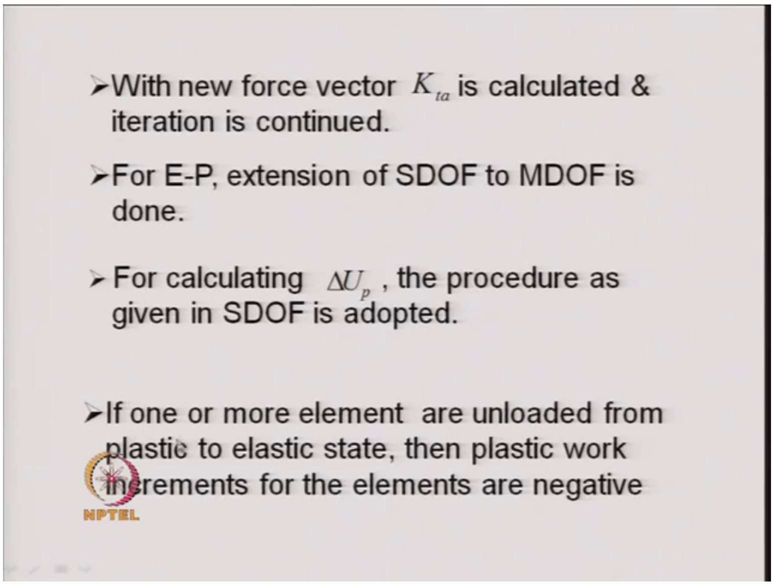

Now, with the new force vector again we calculate the K t a is calculated and the iteration is a continued for the case when the column is passing from elastic state to plastic state that is a transition point then the it is a straight forward extension of the single degree of freedom to multi degree of freedom system that is we scale the displacement in such a fashion. So, that the yield condition is satisfied and the total displacement consists of 2 parts one is the scaled displacement other is the plastic a part of the incremental displacement. So, we add this to find out the total value of the delta x.

If one or more element are unloaded from plastic to elastic state then the plastic work increments for the elements are calculated and if they are negative; that means, those elements are unloaded.

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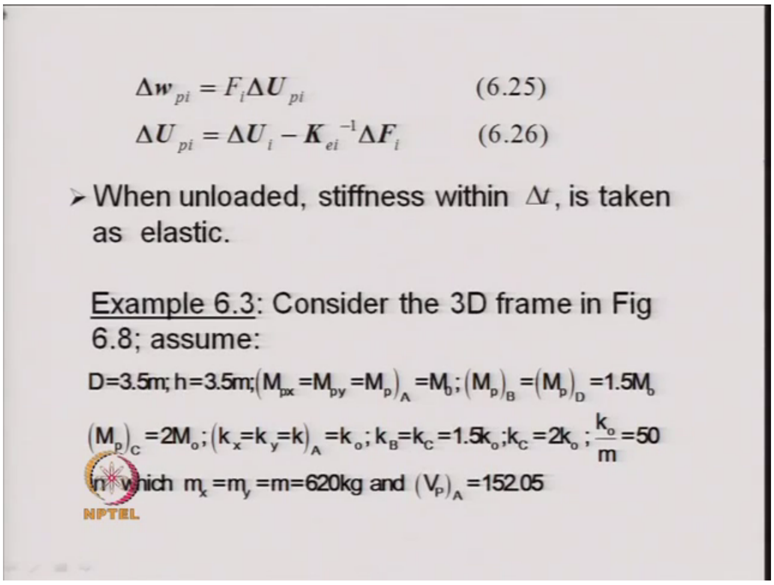

And the plastic increment a work increment is given by this equation that is F i that is a force in the element multiplied by incremental plastic displacement and incremental plastic displacement is obtained like this is equal to the total incremental displacement minus the elastic part of the displacement that is K e i it is a simply the elastic stiffness matrix of the element into delta F i.

So, that is how we calculate delta U p i and once we know that one can find out these plastic work increment for that element if it happens to be a negative quantity, then we say that the element has is unloaded and then for the calculation over that interval of time we consider that element to be an elastic element and the stiffness elastic stiffness of that element is considered in constructing the overall stiffness of the structure.

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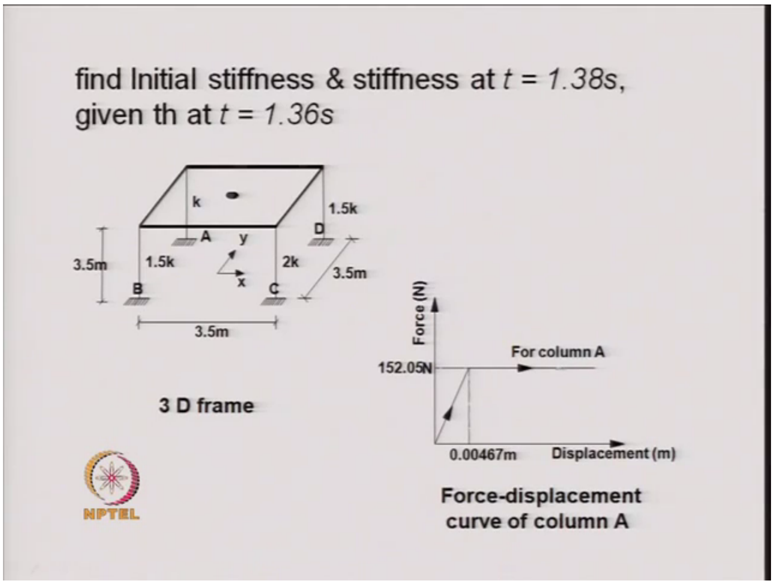

The method is explained with the help of a 3 dimensional frame which is shown in this figure the this is a 3 dimensional frame a having different columns with different stiff nesses as a result of that we have an centricity both in the x and y direction the yield shear for the column a that is given as 152.05 Newton and by knowing the yield shear in column a the yield shear for other columns can be obtained from this particular condition that is the N p x, N p y is equal to M p for column a is equal to M 0 and N p b and N p d, they are equal that is equal to 1.5 M 0 and N p c is twice M 0. So, since the moment are in this proportion they are therefore, the yield shears also can be obtained in that particular proportion.

So, we know for all the columns the yield shear and the ones you know that then you go ahead.

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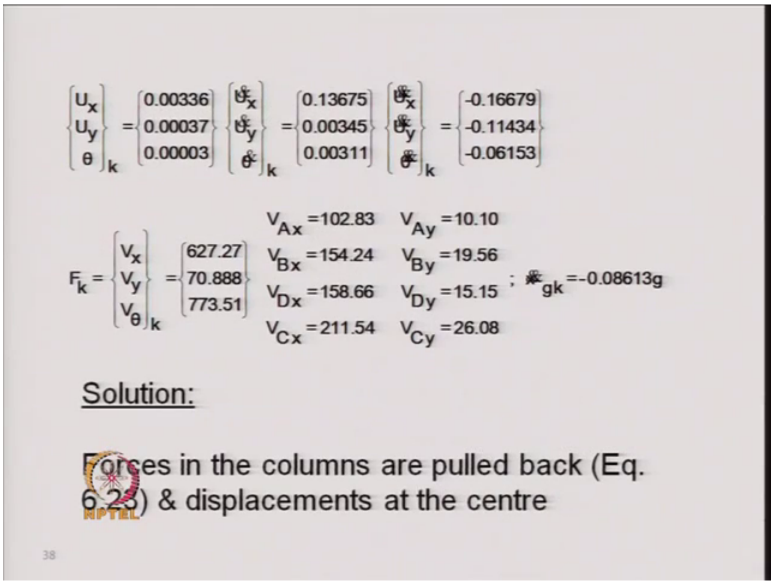

With the calculation that is the values of U x, U y and U theta are given in these at k th time station this is the velocity this is the acceleration the f k that is the V x, V y and V theta is given that is at the centre of mass the total shear in the x direction total shear in the y direction and the rotational that is a given as this at the k th time station, the individual shears in different columns in the x and y directions are given like this and this is the incremental ground acceleration at a given for that particular delta t.

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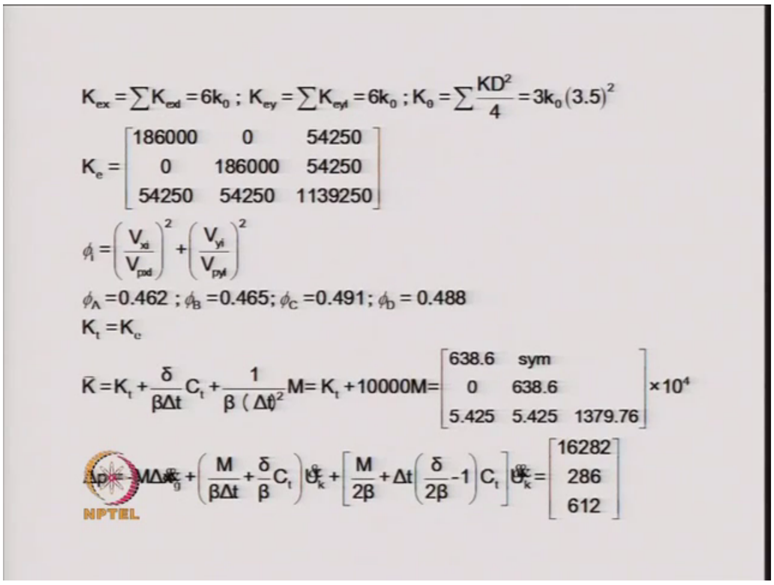

Now, the forces in the columns that are pulled back later, but in the beginning we compute the value the K e matrix and once we obtain the value of the k th matrix, then using that K e matrix one can check find out the solution for the problem. And we see that for each one of the columns the phi value is less than equal to 1 therefore, K t is simply is equal to K e that is the term K e minus K p that correction term K p does not come into picture. So, that is the initial state of the system.

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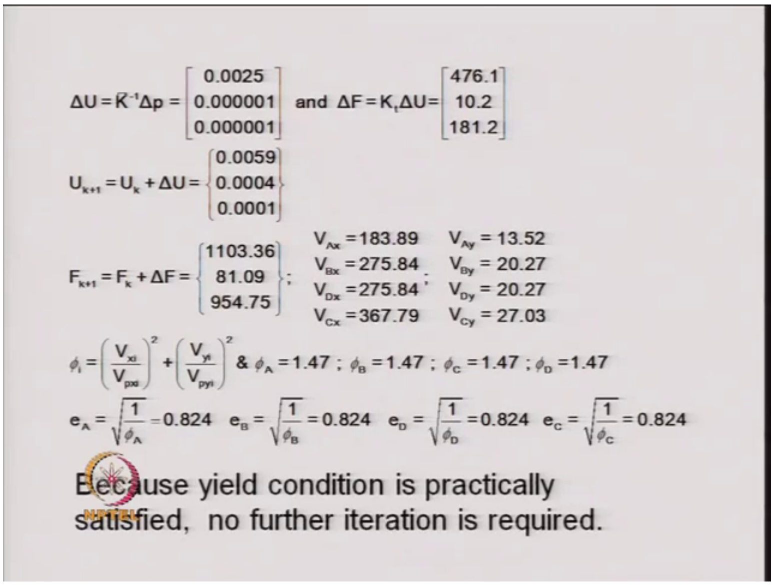

Now, for that initial state of the system we compute the value of k bar and with the help of this k bar and delta p we get the value of delta U that is delta U is equal to k bar inverse delta p and we find out the value of delta F that is K t into delta U and we get the value of U k plus 1 that is the next time step the values of the 3 displacement that is the 2 displacement in the x and y direction and a rotation.

The F k plus 1 that we obtained that is the total shear in the x and y direction and the rotational are force over here that you obtain at the end of the time step and the shear forces in each of the columns are calculated they are shown over here and then we check for the yield condition by utilizing this equation and we find that for column A, B, C and D for all the columns the yield condition is violated that is it has become more than 1.

So, therefore, the forces are to be pulled back. So, that the yield condition is satisfied. So, for that we calculate the e A, e B, e D and e C as this one that is one by phi a square root of that 1 by phi b square root of that and so on and with the help of these factors. Now we go to these values that is the delta U v is calculated and the value of e bar is calculated.

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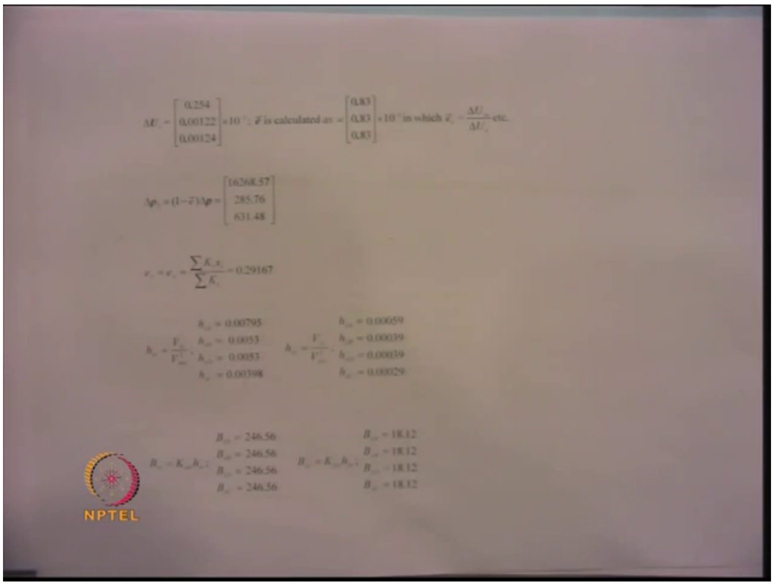

As 0.83 for all the columns where e bar x is given by this, then delta p 2 is calculated that as one minus e bar into delta p. So, we get the value of delta p then we calculate e x and e y for the new state of stresses in the column then from there we get the values of h x i h i h y i for each one of the column because we know the revised value of the shear and they differ and the yield shear are from that ratio one can calculate these h values and then one can calculate the values of Vxi and V y i and once they are and then we calculate the g i value for all the columns.

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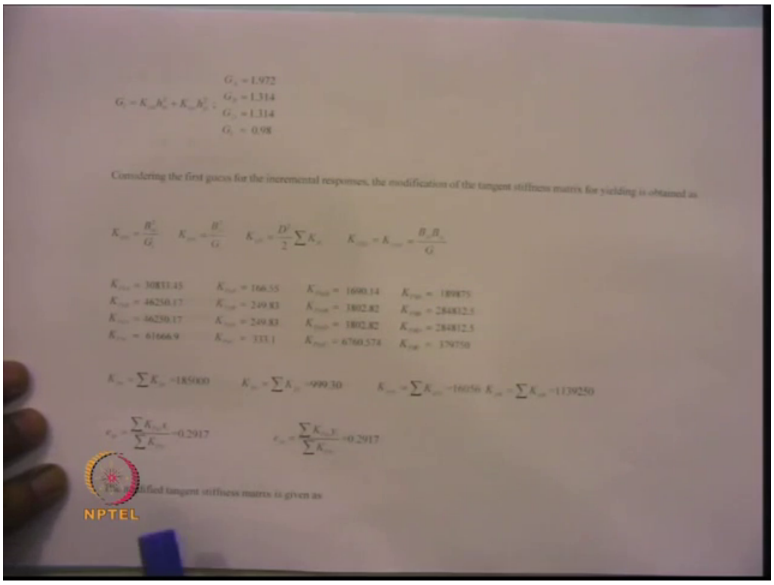

And after you have obtain that then the components of the K t matrix that is calculated using this formula and the components of the K t matrix are given over here and there then one calculates the Kpx, Kpy, Kp theta.

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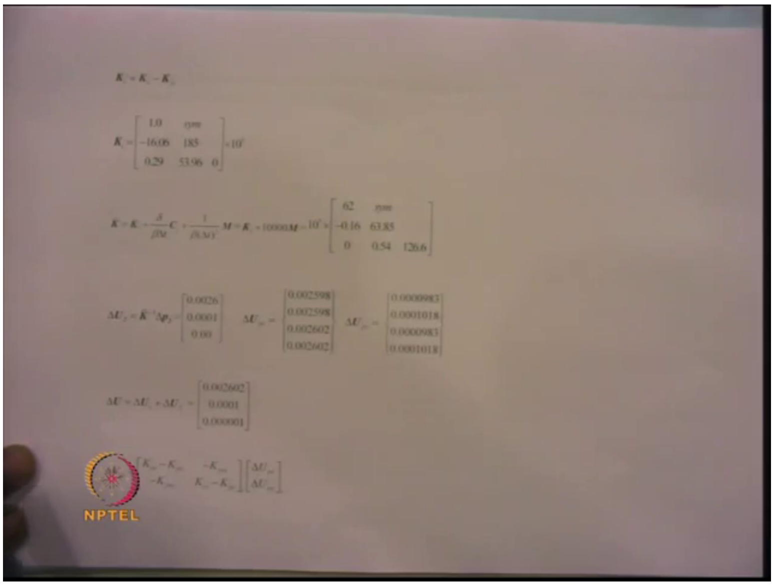

So, that the modified tangent stiffness matrix that comes out to b is equal to K t is equal to K e minus K p, we know all the elements of the K p matrix now and that is how we get the value of the K p matrix and once we get the value of K t matrix and then from there we get the value of k bar and using the value of k bar we get delta U 2 that is the part of the displacement which is the plastic component of the displacement and from there we can find out delta U p x i and delta U p p y y for all the columns and find out finally, that total displacement at the centre of mass that is by summing of the scaled displacement delta U 1. And the plastic displacement part incremental displacement delta U 2 and delta V b p i for each one of the elements can be obtained and form this delta U p x i multiplied by the corresponding to modified stiffness matrix in that the you can see that the both K e and K p, they are present elements of K e and K p are present.

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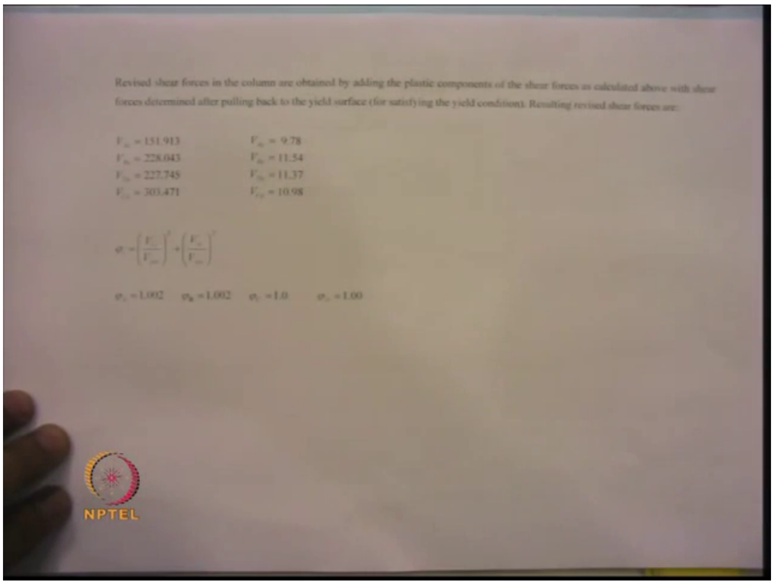

And finally, using then we get the shear values new shear values in the columns and once we get the new shear values of the columns, then we apply this yield criteria and check whether the yield condition is violated or not and for this particular problem, it is found that once we make this particular modification then the phi value for columns A, B, C, D; they remain more or less s equal to 1 that is no further iteration is required.

So, what is been done over here is that in the beginning we have obtained the values of the shear at different columns and checked that the columns where not yielded or not are not yielded. So, therefore, everything was in the elastic state. So, using now K t matrix which does not have any K t component we obtain the value of delta U that is the displacement at the centre of mass and added that displacement with the displacement at the beginning of the time step to find out the total displacement and then we found out the revised shear forces in each one of the columns in x and y directions and then we checked the yield condition we found that the yield conditions were violated and therefore, the forces in the columns were pulled back. So, that they the yield condition is satisfied that is phi becomes become equal to unity.

So, with the help of that we found out revised value of the displacement delta U 1 and delta U 2 that is the 2 components plastic component the elastic component of the displacements and finally, get a revised value of delta U and that when we added with the previous displacement we found that the yield conditions for each one of the columns were satisfied therefore, no further iteration was required.