Seismic Analysis of Structures
Prof. T.K. Datta
Department of Civil Engineering
Indian Institute of Technology, Delhi

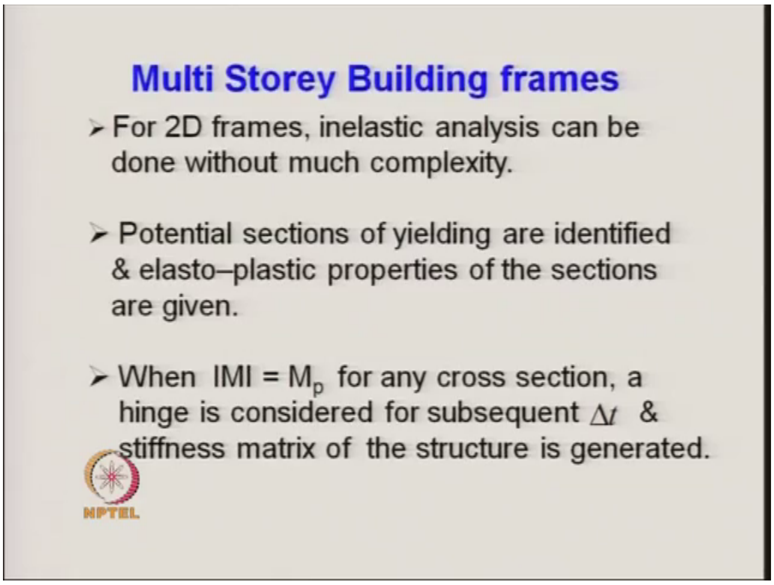
Lecture – 27
Inelastic Seismic Response of Structures (Contd.)

In the previous lecture we are discussing about the inelastic analysis of a three dimensional frame single storey three dimensional frame we say rigid slab on the top of it. The frame was asymmetric as a result of that under the action of earthquake a unidirectional earthquake; there will be a torsional rotation of the frame about a vertical axis leading to the development of the bending moments in the columns in two directions. This requires the consideration of the bidirectional interaction effect on yielding of columns and that was the main point of discussion in the previous lecture and we had seen there that individually the columns may not yield in each direction separately.

But under certain combination of the bending moments in the two directions, the column may yield depending upon the yield criteria that is assumed. And a when a column undergoes yielding then the tangent stiffness matrix or the instantaneous stiffness matrix that gets modified and this modification is done with the help of a stiffness matrix called the plastic stiffness matrix kp, the elements of a kp can be obtained with the use of certain formula which we had discussed in the previous lecture.

And with this modification of the matrix of the columns, the entire stiffness matrix of the system is assembled and then we analyze the system for the subsequent increment of loading or the subsequent interval of time delta t. And after we get the response or the incremental response at time delta t then we add this displacement in to the previous displacement and get the total displacement of the columns in the two directions and also we obtained the rotations, with the help of that the bending moments in the columns can be computed the yield criteria can be checked and we make sure that at any stage of calculation, the conditions of bending moments are such that they should not cross the yield surface or the yield condition is always satisfied.

(Refer Slide Time: 03:37)



If the combination of the bending moments lead to a bending moment greater than the yield moment that is it goes out of the yield curve then the bending moments in the columns are pulled back. So, that the total bending moment becomes equal to the yield moment and it remains on the yield curve itself. And how to do this pulling back that was also explained that requires some iterations and with that the help of that iteration we make sure that the bending combinations of the bending moment in the columns are such that they all the time remain on the yield surface.

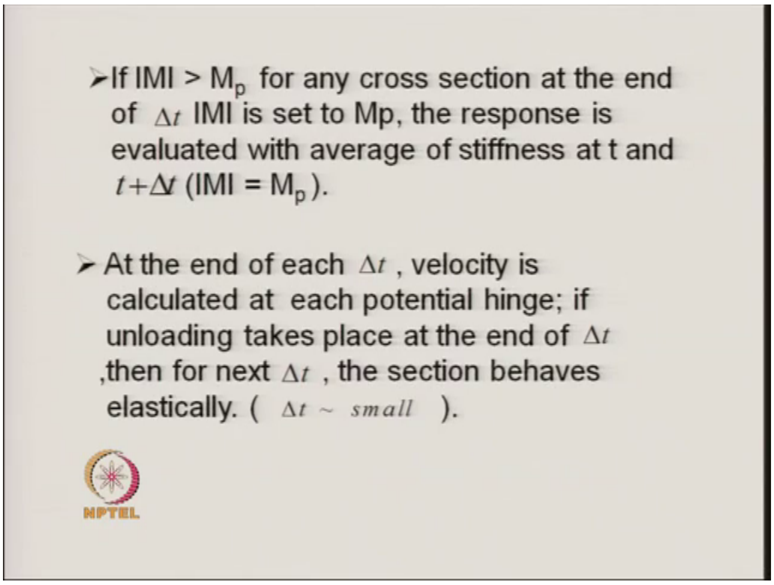
Now, in this lecture today what we will do is that the concepts that we had discussed in the previous lectures, that will be extended to multi storey building frames. In the multi storey building frames the number of stories and the number of base maybe too many and therefore, the idealized conditions that we discussed for a two storey frame or a single storey frame those idealized conditions may be difficult to implement therefore, we do some kind of approximation for extending the concepts to the multi storey building frames. Specially in order to implement those methods for the multi storey building frames we make the delta t to be very small so that the errors which are developed due to the approximation those errors are very little or those errors are minimized as a result of that there is not much accumulation of the error to upset the final a response.

Now, for the 2 d frame what we do is that first we identify the potential sections of yielding at the different cross sections of the frame and for that we isolate two cases; one is the case where the beams are weaker than the column and in the second case where the columns are weaker than the beams. First we take up the case of when the columns are weaker than the beams that is the columns are in yielding, in that case we first increment the loading at small increments and apply on to the structure.

And we calculate the bending moments at the specified sections and where we look for the case when the columns can undergo yielding. Now, therefore, at those cross sections we check the bending moment and when the bending moment at that cross section becomes equal to the plastic moment or the moment carrying capacity of that cross section, we say that that particular cross section has yielded and for subsequent delta t or subsequent increment of loading, we consider an ordinary hinge to be present at that particular cross section and accordingly write down the stiffness matrix for that particular element or the column.

Then we assemble the entire stiffness matrix after considering the ordinary hinge at the plastic hinges. Once you have done that then we use that particular stiffness matrix for finding the response of the system to the next increment of loading.

(Refer Slide Time: 08:34)



And when we get that incremental displacement then we add the incremental moment to the previous moment and see a whether other cross sections are yielding or not. If the m which is computed at the end of any calculation becomes equal or greater than the plastic moment, then at the end of the delta t, we what we do is that we do not modify the calculation on the way we had done for a two storey for a single storey frame, where when the system passes from the elastic state to the plastic state then we find out the point at which the plastification takes place and therefore, a portion the loading incremental loading into two parts, one part of the loading takes the system to the state when it just reaches yield point and then the other part of the loading in which the system behaves plastically and the responses are calculated for these two for separately, and add them together to get the final response.

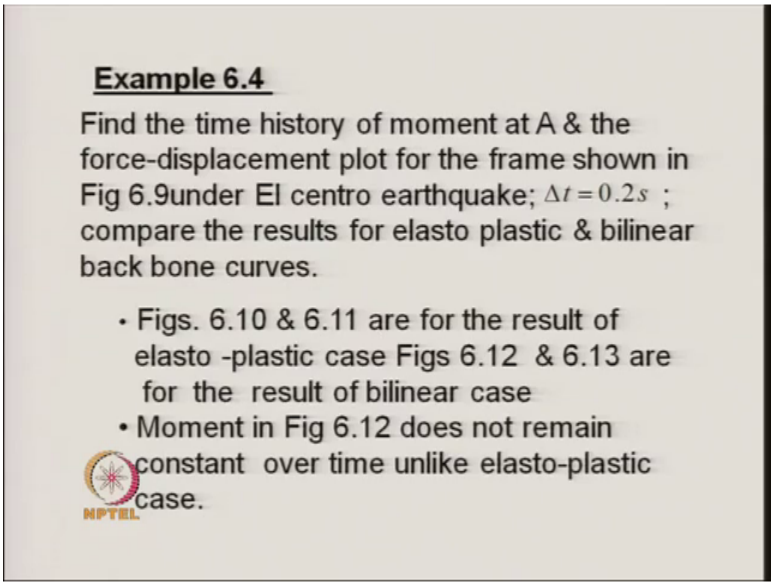
However, when the number of stories or number of ways are too many, these kind of calculation take a lot of computational effort and it becomes somewhat cumbersome therefore, what is done is that if the bending moment at the end of an time interval becomes greater than m at any particular cross section, then what we do that we assume that at that particular cross section there is a yielding and by assuming that there is a yield hinge that has occurred at that particular point we obtained a revised stiffness matrix for the entire system.

So, these revised stiffness matrix and the stiffness matrix that we had calculated in the beginning of the time interval, these two stiffness matrices are taken and averaged and with the help of that averaged stiffness matrix we finally, obtain the incremental displacement or incremental response of the system to the incremental loading and that is how we are proceed multi storey building frames. Similarly when we obtain that at the end of the time interval delta t there is an unloading which is taking place then also we do the same kind of calculation.

We constantly monitor the velocity is at the yield sections these velocities could be the rotational velocity or the displacement velocity depending upon the kind of problem that we are solving, if you are solving a problem in which the columns are only yielding then we constantly monitor the displacement velocities and these displacement velocities when they change their sign; that means, it becomes negative then we assume that the cross section has been unloaded and during that time interval. Then at the end of the time interval we assume a the stiffness at that particular cross section to v is equal to the initial stiffness of the cross section, and with the help of that we obtain a total stiffness matrix of the entire structure, and that stiffness matrix is added to the stiffness matrix of the structure in the beginning of delta t average the stiffness matrix and with the help of that average stiffness matrix we find out the final incremental displacement of the system subjected to the incremental loading.

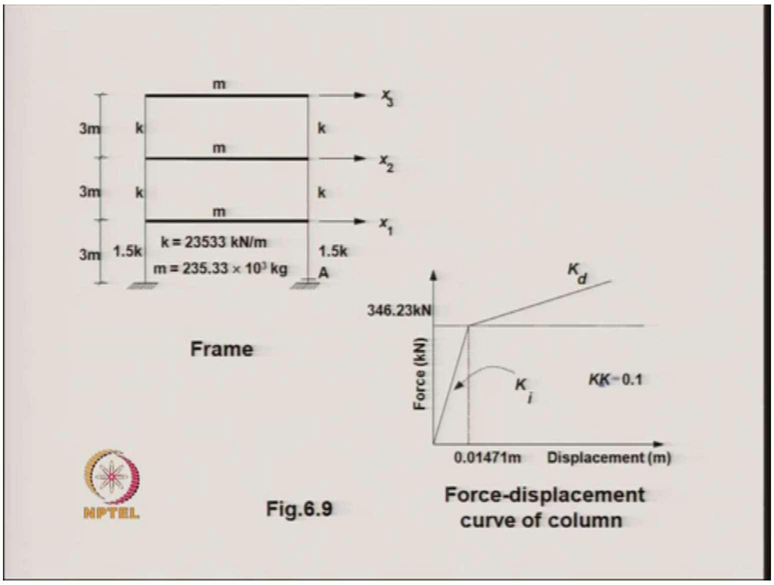
So, for doing this the necessary condition is that the delta t should be small, because the approximation that we are making by averaging the two stiffness that approximation should not lead to a much accumulation of the error.

(Refer Slide Time: 14:04)



The concept is explained with the help of the example 6.4, in which we wish to find out the moment at the base of a particular column of the frame that I am going to show you in the subsequent slide and it is subjected to el centro earthquake, the results are obtained for two cases in one case the remembers that the behave elasto plastically that is the columns behave elasto plastically and in the other case the column behavior is a bilinear back bone curve.

(Refer Slide Time: 14:50)

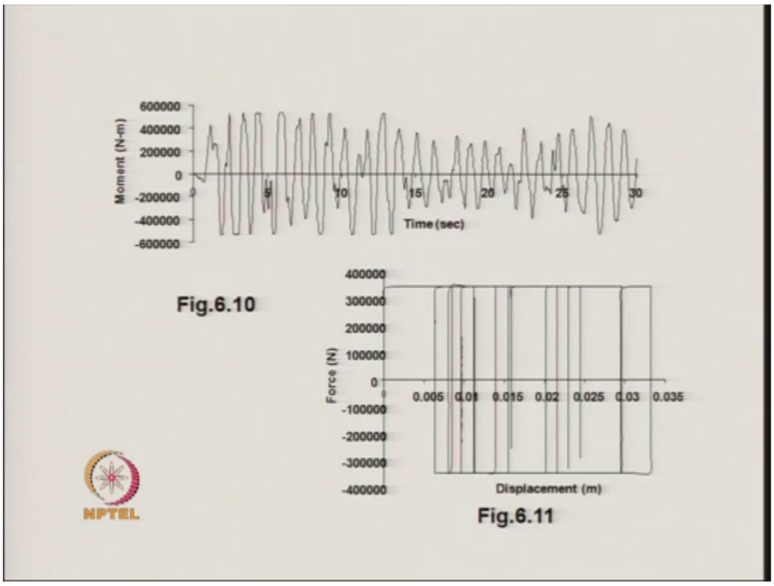


So, the problem is shown over here, this is a shear frame and therefore, the columns are only yielding.

At the bottom columns we have the stiffnesses 1.5 k and the upper two columns we have the stiffness k, the displacements are x 1 x 2 x 3 no rotation is involved the values of mass and the stiffness that is given and at cross section a we wish to find out the time history of the bending moment. The two cases of the behavior of the column is shown with the help of the fort displacement curve of the column, we take two cases in one case we are assume the system to be perfectly elasto plastic, that is at the bending moment of 346.23 kilo newton, we assume that the yielding takes place and after the yielding the system or the column continues to displace under that the same bending moment. So, this is the perfect elasto plastic case and initial stiffness of this elasto plastic system is given by k i. In second case where we considered the column to behave bilinearly in that case the after the yield point that is the yield moment or yield force of 346.23 kilo Newton there is a stiffness of the system that stiffness is called k d and k d I assumed to be is equal to point one times the k i that is the initial stiffness.

So, for these two cases the results were obtained and the method that was discussed before that method was adopted, in the case of bilinear curve whenever we find that the bending moment has exceeded the yield value then the stiffness of the system is taken as k d or stiffness of the column is taken as k d not as k i and unlike the case of perfect elasto plastic case, we do not consider a ordinary hinge to exist at the point of the plastifiction for subsequent calculation. We consider that that particular cross section still can take certain value of the force or that is able to take the shear force or the bending moment under subsequent loading, but with the reduced stiffness of kd. So, in that particular way we perform the calculation the final results.

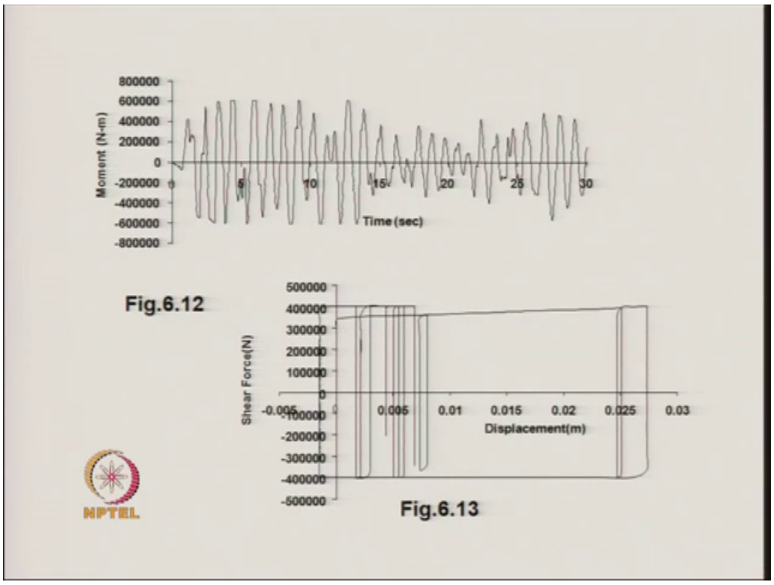
(Refer Slide Time: 18:15)



For the two cases are shown over here that is the plot of the bending moment with the time for the elasto plastic case is shown in figure 6.10 and 6.11, we see that the bending moments are varying in such a way that over the small value of delta t, the values of the bending moment are almost stationary or horizontal.

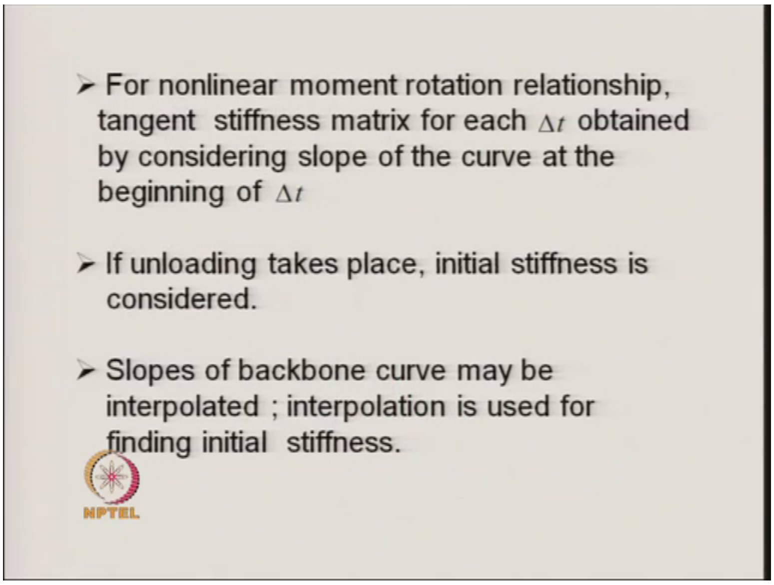
The force displacement relationship is exhibiting on the case of perfectly elasto plastic is state, we can see that the upper line for the ford displacement curve is almost horizontal line and depicting the perfect elasto plastic state.

(Refer Slide Time: 19:19)



For the second case where we had a bilinear curve representing a force displacement relationship, the bending moment plot it shows that at the peaks the values are not remaining stationary over delta t time, it is slightly inclined showing that there is a small variation and because of the bilinear or because of the some stiffness that we find after the yielding or the stiffness that is represented by k d therefore, these the portion at the top is not perfectly horizontal. The force displacement relationship also shows the similar trend the upper curve is not perfectly horizontal it is slightly inclined and this inclination is produced because of the stiffness reduce stiffness k d, after the yielding has taken place.

(Refer Slide Time: 20:33)

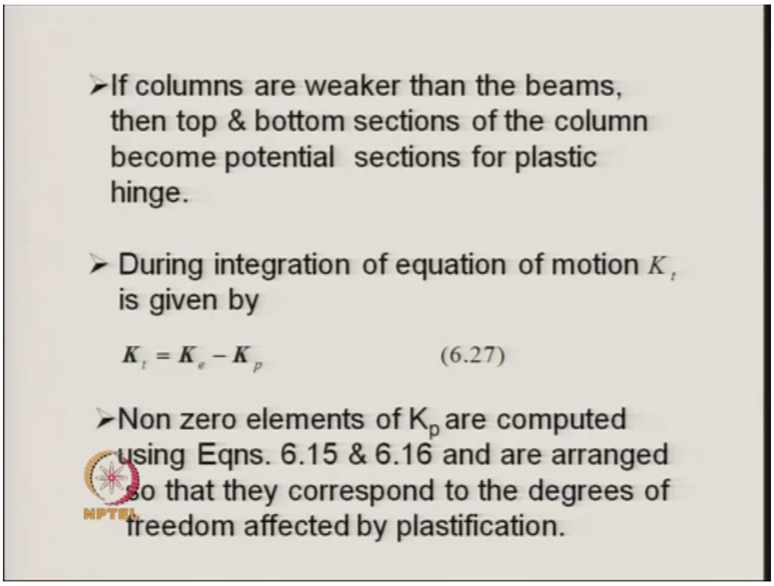


Now, if the moment rotation relationship or the force displacement relationship is not idealized as a elasto plastic perfect elasto plastic or bilinear curve apart by a non-linear curve, then the tangent stiffness matrix for each delta t is obtained. And these tangent stiffness matrix is obtained at the beginning of the time interval delta t with the help of that tangent stiffness, we calculate the entire stiffness matrix of the system and then find out the incremental displacement or incremental rotation of the members and the system for the incremental loading.

If unloading takes place, then at that state we assume that the system is having or the members are having a stiffness which is equal to the initial stiffness and therefore, now as we find that there is an unloading for a particular element at a particular cross section, then the in place of taking the tangent stiffness at that particular for interval we consider the initial stiffness for that particular element and go ahead with the calculation. In order to improve the accuracy we do the same thing, that is a whenever there is a unloading or whenever there is a movement from one state to the other, then what we do is that we find out the stiffness matrix at the end of the time interval and the stiffness matrix at the beginning of the time interval. These two stiffness mattresses are added together averaged and with the help of that averaged stiffness matrix have we find out the incremental final incremental displacement or response over the time of delta t whenever the system is unloaded.

However, whenever the system is not unloaded, but monotonically increasing over the curve linear force displacement relationship then we go on calculating these in tangent stiffness states at every point that is at every interval of time t. Now in order to add that calculation what we do is that the slope of the backbone curve that are obtained at some discrete points and they can be in putted into the program, whenever any point lies within the two such points where the interpolated values are available or the or the values of tangent stiffness matrix are available, then the value for any for value for any value at a particular point in between those two points can be obtained by interpolation.

(Refer Slide Time: 24:25)

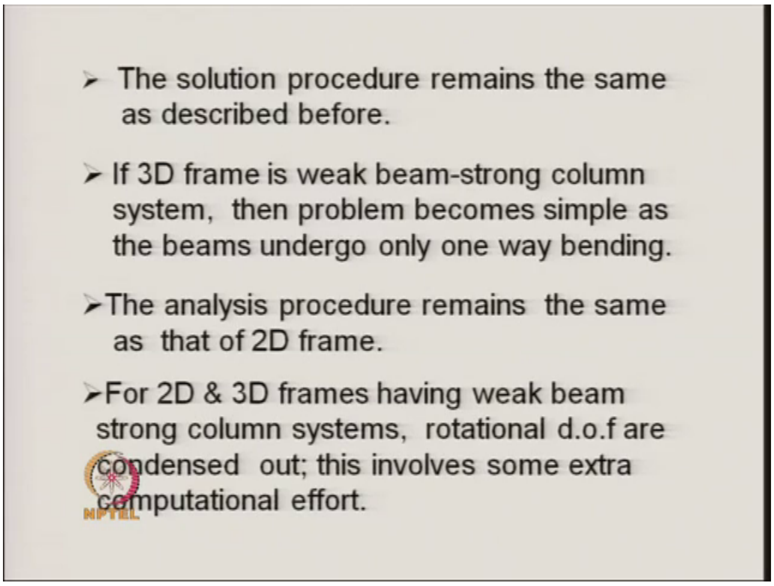


Now, next we come to the case when the system is having a weak column and strong beam, but the system is a 3 d system, in that case the method that we described before for a single storey frame three dimensional frame, the same method is extended and the stiffness matrix which is called the tangent your transient stiffness matrix k t is as obtained k e minus k p, if any section of the column is yielding during the increment of loading.

Now, by considering the yielding of the columns and 3 by the modifications that arise due to this yielding leading to a modified transient stiffness matrix k t for the element, we assemble the total stiffness matrix of the three dimensional frame and with the help of that modified stiffness matrix we find out the incremental displacement of the system for the increment of loading over time delta t. Only thing that one has to take note of is that for individual elements when we make the modification to the stiffness matrix by including the plastic stiffness matrix part that is k p, then we must attach their coefficients to the appropriate degrees of freedom, and in the overall stiffness matrix those modifications should be dually incorporated in the different places denoting the positions for the different degrees of freedom.

Therefore, what is generally done is that the sections which are yielded at those sections whatever is the degree of freedom that are associated, they are arranged accordingly in the entire sequence of the degrees of freedom.

(Refer Slide Time: 27:16)

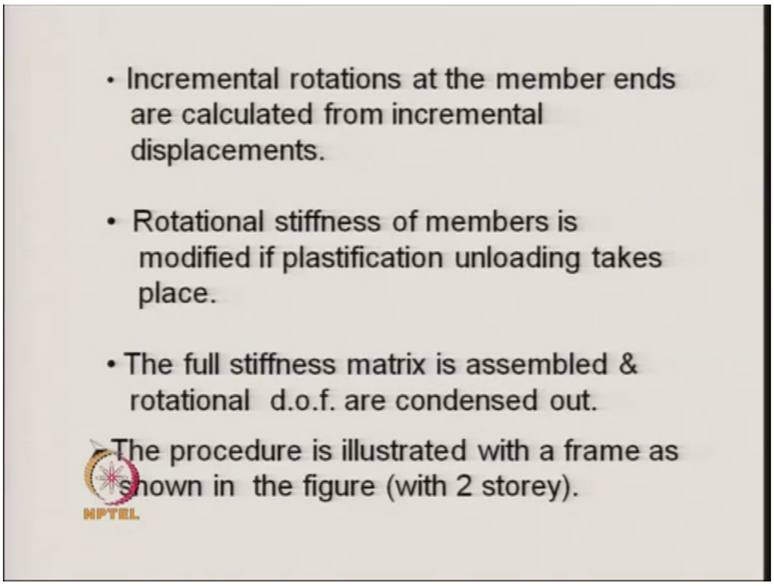


Then the solution procedure remain exactly the same as before that we have described for the case of a single storey three dimensional frame. If the three dimensional frame is a week beam strong column system then the problem becomes a simple as the beams undergo only one way bending, on the case of bidirectional interaction on yielding does not come into picture because if we look at the beams in a three dimensional frame they cannot undergo any lateral bending, because of the high stiffness of the slab.

So, therefore, they only undergo a vertical bending and thus the beams are subjected to only one way bending and one it once it is having a one way bending, then the analysis procedure remains the same as that of the 2D frame. So, for 2D frame or the 3D frame having a week beam strong column system the calculation procedures are remains the same, but some extra precaution has to be taken for condensing out the rotational degrees of freedom in the system because the rotational degrees of freedom do not enter into the problem as a dynamic degree of freedom and in most of the cases we condense out these rotational degree of freedom.

Now, in doing so, these condensation technique can be quite straight forward whenever the system or any cross section has not yielded, but after the yielding has taken place at any particular cross section, then a special note is to be taken care of in the condensation procedure to take into account the effect of the yielding of the cross section. Now these involves some extra computational effort.

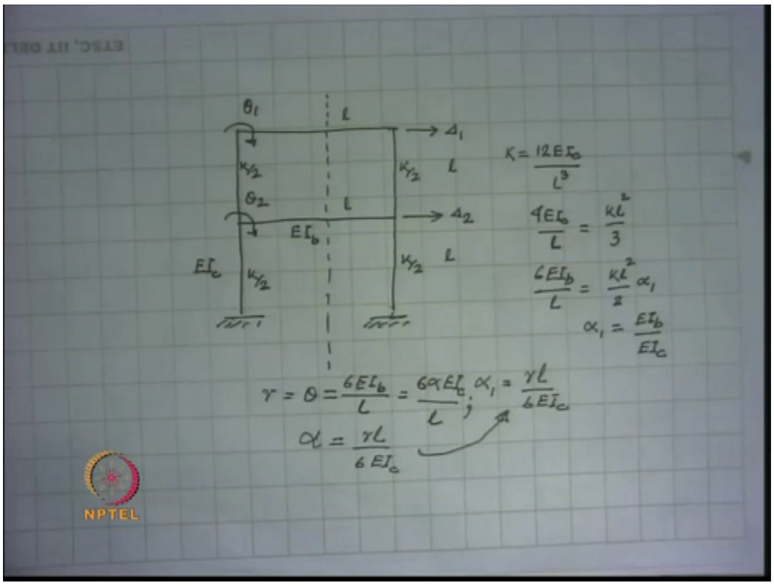
(Refer Slide Time: 29:43)



And that is explained with the help of the example of a two storey frame that I will be explaining shortly, but first what we do is that after we have obtained the incremental displacements. From that incremental displacements we calculate the incremental rotations from the relationship that exist in the process of the condensation and with that relationship we find out the incremental rotations from the incremental displacements and those incremental rotations are then added to the previous rotation to find out the final rotation at the current state and we see that whether the plastification has taken place at a particular cross section.

Similarly we record the not only the rotations at the cross sections, but also record the or tress the rotational velocity at the particular section, where the yielding has taken place and if we find that the rotational velocity is or move from positive value to the negative value, then there is a unloading that has taken place at that particular cross section and we take appropriate measures to take into account these the effect of unloading. Now the procedure here is explained for these two storey frame that is shown over here.

(Refer Slide Time: 31:48)

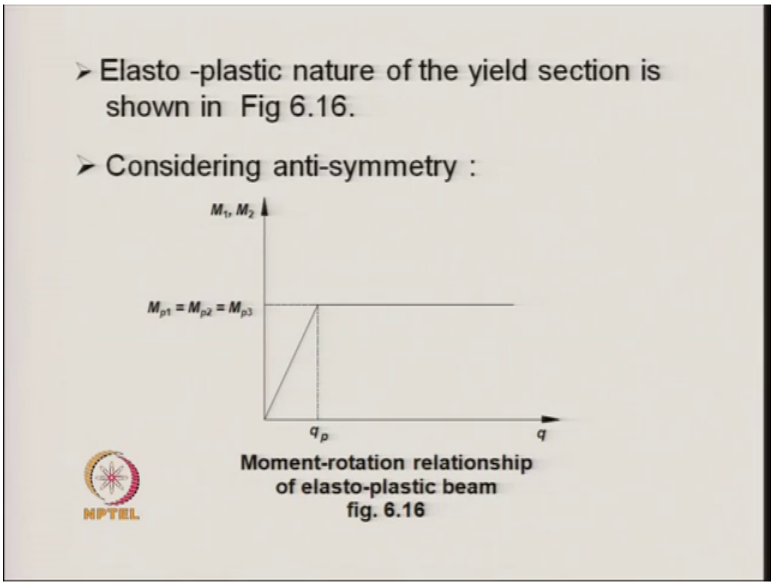


The frame is having a anti symmetric case because under lateral load, the frame will be anti symmetrically as a result of that we consider only delta 1 delta 2 they are the two storey displacements and out of the four rotations we take only two rotations as unknown and we take the line of symmetry over here, and while calculating the stiffness matrix we take help of the anti symmetric case. The stiffnesses of the upper storey of the columns are k by 2 k by 2. So, the total stiffness is k for the bottom also the total stiffness of the columns are k, the beams are having a flexural rigidity of EI b and the flexural rigidity of the columns are EI c the k is equivalent to 12 b i c by L cube where L is this length beams are also having a length of L the 4 EI c by L that turns out to be k L square by 3 from this value of the k which is defined at the top.

Similarly 6 EI by L which will be the stiffness of the beams for the anti symmetric case that can be written as k L square by 2 into alpha where alpha 1 is defined as EI b by EI c. Alpha 1 corresponding corresponds to theta1 and alpha 2 corresponds to theta 2 in general we can call it as alpha this ratio; now the rotation that takes place at the end of the beam in the moment that is developed that can be related with the help of this relationship say r is equal theta, that we can write as 6 EI b by L and that in turn can be written as 6 alpha EI c by L and from that one can write down alpha to be is equal to r L by 6 EI c, and for the top beam it will be alpha 1 that is r 1 L by 6 EI c and for these rotation theta 2 it will be alpha 2. Now as the when the cross section over here they undergo yielding then EI b the flexural rigidity of the beam they basically do not come into picture, and it is very difficult then to define the stiffness of the beam in this particular fashion it is possible. So, long it is in elastic state.

And therefore, the rotation r basically is brought into picture, we calculate the rotations at this particular ends after the yielding and once we get the rotations then the value of alpha can be computed with the help of this relationship or this equation, and once we get the value of alpha then we write down the stiffness matrix of the stiffness matrix of the system in terms of alpha that I show you.

(Refer Slide Time: 35:47)



Now, the beams they have a elasto plastic property or elastic perfect property.

So, the M p 1, M p2 and M p 3 or in other words the plastic moment capacities of the two beams they are the same.

(Refer Slide Time: 36:05)

$$K = \begin{bmatrix} k & -k & -\frac{kl}{2} & -\frac{kl}{2} \\ -k & 2k & \frac{kl}{2} & 0 \\ -\frac{kl}{2} & \frac{kl}{2} & \frac{kl^2}{2}(\alpha_1+0.67) & \frac{kl^2}{6} \\ -\frac{kl}{2} & 0 & \frac{kl^2}{6} & \frac{kl^2}{2}(\alpha_2+1.33) \end{bmatrix} \begin{Bmatrix} \Delta_1 \\ \Delta_2 \\ \theta_1 \\ \theta_2 \end{Bmatrix} \quad (6.28a)$$

$$\bar{K}_\Delta = K_\Delta - K_{\Delta\theta} K_\theta^{-1} K_{\theta\Delta} \quad (6.28b)$$

$$K_\theta^{-1} = \frac{6}{kl^2} \begin{bmatrix} 3(\alpha_1+0.67) & 1 \\ 1 & 3(\alpha_2+1.33) \end{bmatrix}^{-1} \quad (6.29a)$$

$$\theta = \frac{3}{l} \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 3(\alpha_1+0.67) & 1 \\ 1 & 3(\alpha_2+1.33) \end{bmatrix}^{-1} \Delta \quad (6.29b)$$

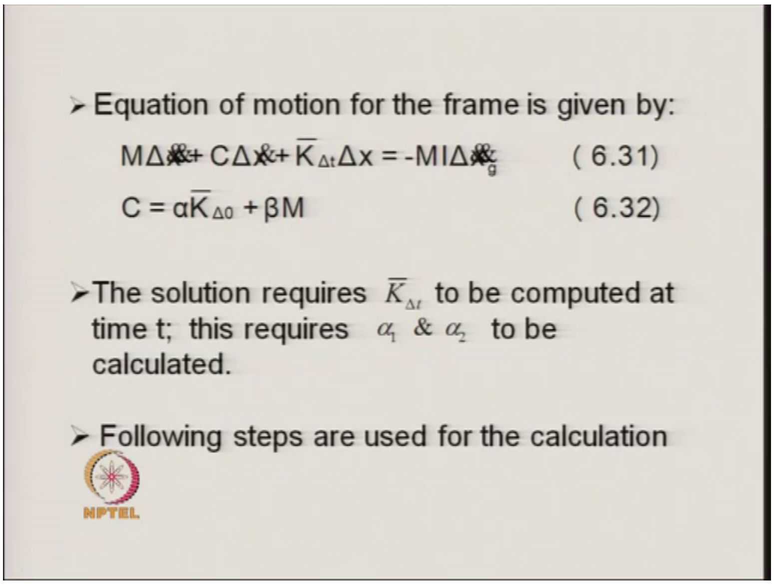
$$\bar{K}_\Delta = k \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} - \frac{3k}{l} \begin{bmatrix} -1 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 3(\alpha_1+0.67) & 1 \\ 1 & 3(\alpha_2+1.33) \end{bmatrix}^{-1} \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix} \quad (6.30)$$

.

And assuming them to be same and considering the anti symmetric property of the frame, we can write down the total stiffness matrix by equation 6.28 a and in which you can see that alpha 1 and alpha 2 are existing this alpha 1 and alpha 2 have been described and it has been brought into picture because after the yielding has taken place. Then simply we will find out the values of alpha 1 and alpha 2 from the current values of moment and rotation. In the beginning of the calculation that is when the system is in the elastic state alpha 1 and alpha 2 they are equal to EI v by c or 6 EI b by 6 EI c.

Now once we write down the stiffness matrix in terms of the 4 degrees of freedom then we condense how the rotational degrees of freedom and after degrees of freedom or conductance out then we get k bar delta that is the condense stiffness matrix with respect to delta. And one can see that the expression of the k bar delta condense alpha 1 and alpha 2 therefore, the alpha 1 and alpha 2 are to be calculated at every step of the incremental loading.

(Refer Slide Time: 37:42)



Now, equation of motion of the frame can be written in this particular way that is m delta x double dot c delta x dot plus k bar delta t delta x k bar delta t denotes the stiffness matrix which remain constant over the increment of incremental time delta t. And in the beginning of the delta t we whatever stiffness matrix we get that is the value of k bar and with the help of that stiffness matrix, we calculate the responses for the incremental loading over a incremental time period of delta t time delta t.

On the right hand side we have got minus m i delta x double dot g that is the usual equation in writing down the c matrix we make an assumption. In fact, c matrix if we assume the proportional stiffness dumping matrix then the value of the c matrix may change as the value of the k matrix changes at every instant, but in that case the system will be having varying dumping matrix. So, in order to simplify the problem what we do is that we assume the c matrix to be is equal to mass and stiffness proportional, where stiffness is the initial stiffness of the system. So, with the help of that we can calculate the c matrix and using that c matrix we go ahead with the further calculation. Solution requires k bar delta to be computed at every instant of time t and this requires alpha 1 and alpha 2 to be calculated.

Now, we use the following steps for calculating the values of are alpha 1 and alpha 2.

(Refer Slide Time: 39:54)

$$x_i = x_{i-1} + \Delta x_{i-1} \; ; \quad \dot{x}_i = \dot{x}_{i-1} + \Delta \dot{x}_{i-1} \qquad (6.33a)$$

$$M_{1i} = M_{1i-1} + \Delta M_{1i-1}; \; M_{2i} = M_{2i-1} + \Delta M_{2i-1} \qquad (6.33b)$$

- $\Delta \theta_{1i-1}$ & $\Delta \theta_{2i-1}$ are obtained using Eqn. 6.29b in which α values are calculated as:

$$\alpha_1 = \frac{r_{1i-1} l}{6 E I_c} \quad \& \quad \alpha_2 = \frac{r_{2i-1} l}{6 E I_c}$$

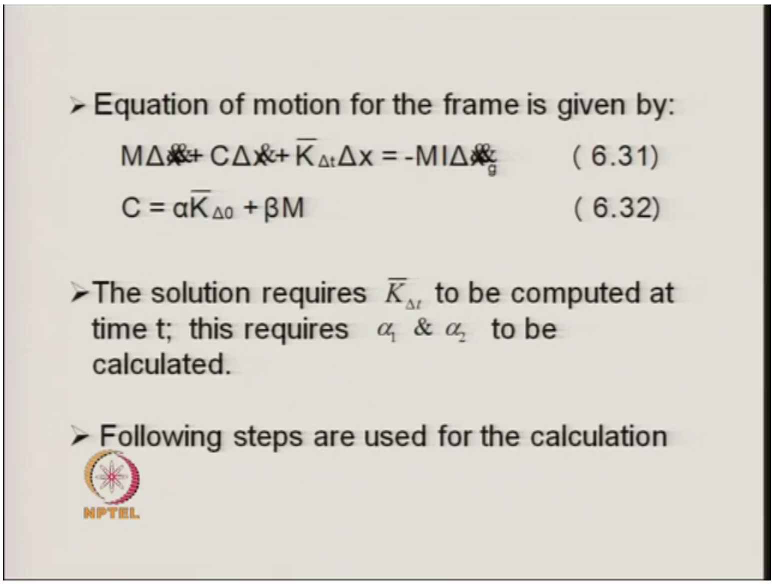
$$r_{1i-1} = \frac{M_{1i-1}}{\theta_{1i-1}} \; ; \quad r_{2i-1} = \frac{M_{2i-1}}{\theta_{2i-1}}$$

From a particular increment of loading we calculate the incremental displacement that is delta x i minus 1 to calculate the final or displacement x i, similarly we record the velocity x dot i that can be obtained by finding out the velocity at the i minus 1 th step that is in the beginning of the delta t time, and after we have calculated the response over the increment the loading then we get the delta x i minus 1 and delta x dot i minus 1 and when we add them together we get the value of the velocity at the ith time station.

Similarly one can calculate the value of the bending moment at the ith time station at a cross at a cross section of the beam and in obtaining the bending moment at the ith time step, we need to know the delta i m minus 1 and delta M 2 i minus 1 that is are the two ends of the beams for that what we have to do is that we you should be able to find out the values of that delta 1 i minus 1 and delta theta 2 i minus 1, that is the incremental rotations at those two sections. These incremental rotations are obtained using the previous equation in which alpha values are calculated as the following.

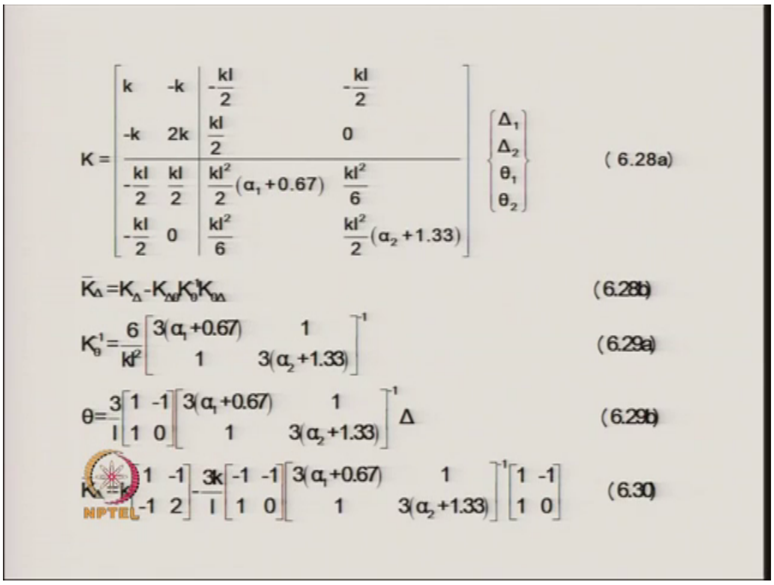
And the equation 6.29 is nothing but.

(Refer Slide Time: 42:05)



This equation.

(Refer Slide Time: 42:06)



That is the equation which is which relates theta and delta and they are alphas 1 and alpha 2 both the things are required.

(Refer Slide Time: 42:16)

$$x_i = x_{i-1} + \Delta x_{i-1} \;;\; \dot{x}_i = \dot{x}_{i-1} + \Delta \dot{x}_{i-1} \qquad (6.33a)$$

$$M_{1i} = M_{1i-1} + \Delta M_{1i-1};\; M_{2i} = M_{2i-1} + \Delta M_{2i-1} \qquad (6.33b)$$

- $\Delta \theta_{1i-1}$ & $\Delta \theta_{2i-1}$ are obtained using Eqn. 6.29b in which α values are calculated as:

$$\alpha_1 = \frac{r_{1i-1} l}{6 E I_c} \quad \& \quad \alpha_2 = \frac{r_{2i-1} l}{6 E I_c}$$

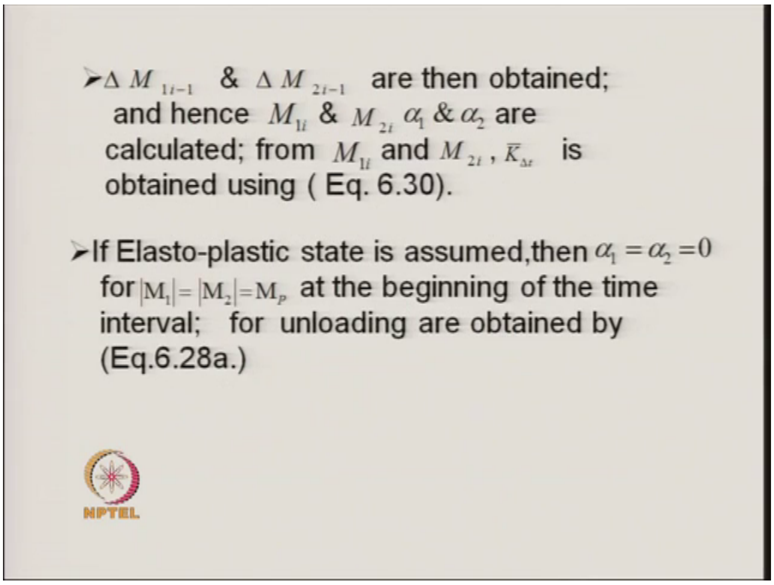
$$r_{1i-1} = \frac{M_{1i-1}}{\theta_{1i-1}} \;;\; r_{2i-1} = \frac{M_{2i-1}}{\theta_{2i-1}}$$

So, these alpha values are calculated using this relationship alpha 1 is equal to r i minus 1 l divided by six EI c and alpha 2 is equal to alpha 2 i minus 1 l divided by 6 EI c that we have derived before and the value of r i minus 1 and r 2 i minus 1 they are obtained from the moment that is existing at i minus 1 th time step and the rotation is also known at i minus 1 th time step. So, by dividing the moment by the rotation we can get the value of r i minus 1.

Similarly, one can calculate the value of r 2 i minus 1 by dividing the moment that is existing at the i minus 1 th time step for the second beam, divided by the rotation that takes place for the second beam and once we get the values of r i minus 1 and r 2 i minus 1 then those values are used to calculate the values of alpha 1 and alpha 2 for calculating the value of delta theta i minus 1 and delta theta 2 i minus 1. And once we get those values then with the help of those rotational values incremental rotational values and the incremental displacement, one can calculate delta M 1 i minus 1 and delta e 2 i minus 1 and after we add it to the existing already existing values of the moment.

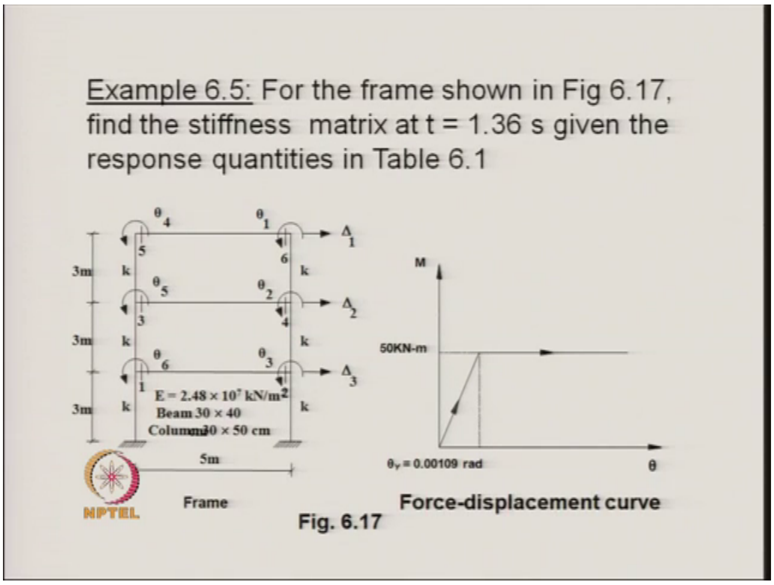
Then we get the final moment at M 1 i and M 2 i.

(Refer Slide Time: 44:14)



And once we get the values f M 1 i and M 2 i then these values are used for calculating the values of alpha 1 and alpha 2 for the subsequent time step and that is how we proceed in the beginning of the problem the values of alpha 1 alpha 2 as I told you before will be equal to EI b by EI c. Now if elasto plastic state is assumed then the values of alpha 1 is equal to alpha 2 is equal to 0, when M 1 M 2 becomes equal to M p at the beginning of a time interval, and for unloading case then we make sure that the initial or the stiffness of the system becomes equal to the initial stiffness. So, in this particular way one can find out the stiffnesses of the system that is the total stiffness of the system at any increment of time delta t and with the help of that stiffness, we calculate the incremental displacement that is displacement and rotations for the system for the incremental loading.

(Refer Slide Time: 45:48)



Now this is further explained with the help of these examples over here, this is an example of a three storey frame and here we have got delta 1 delta 2 delta 3 these are the three. So, a displacement we have the rotations, these rotations are theta 1 theta 2 theta3 on the right hand side of the beam and on the left hand side of the beam we have got theta 4 theta 5 and theta 6. So, they are the rotations on the left hand side the sections are marked as 1 and 2 where the yielding can takes place 3 and 4 where the yielding can take place and 5 and 6 where the yielding can takes place. The elasto plastic property of the rotation and movement that is shown over here.

We call this as the general force displacement curve. In fact, here it is the moment curve at a value of the bending moment of 500 kilo Newton meter we have is equal to the M p value and the yield rotation is given as 0.00109 radiant, this is the value of the e for the material the beams section is this and column section is this. The stiffnesses are given as k k k for the column the we consider the values of the rotations and the displacement and at t is equal to 1.36 second these values are given.

(Refer Slide Time: 48:04)

Table 6.1

Joint	Time Step	x	$\dot{x}$	$\ddot{x}$	θ	$\dot{\theta}$	$\ddot{\theta}$	M
	sec	m	m/s	m/s^2	rad	rad/s	rad/s^2	kNm
1	1.36	0.00293	0.0341	-1.2945	0.00109	0.013	-0.452	50
3	1.36	0.00701	0.0883	-2.8586	0.00095	0.014	-0.297	-23.18
5	1.36	0.00978	0.1339	-3.4814	0.00053	0.009127	-0.098	42.89
2	1.36	0.00293	0.0341	-1.2945	0.00109	0.013	-0.452	-50
4	1.36	0.00701	0.0883	-2.8586	0.00095	0.014	-0.297	23.18
6	1.36	0.00978	0.1339	-3.4814	0.00053	0.009127	-0.098	-42.89

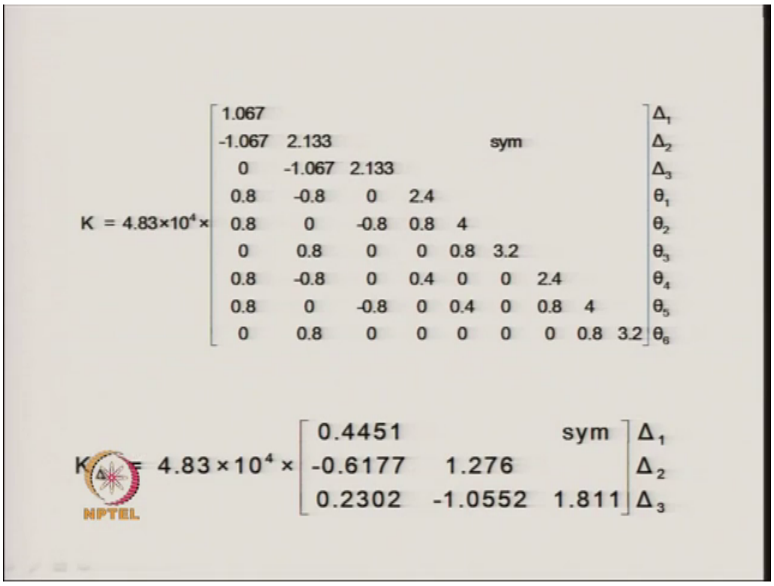
- Table shows that sections 1 & 2 undergo yielding; recognising this, stiffness matrices are given below:

In the following table that is as joint 135 and 246, that is 135 are the left hand joint of the beams and the 246 are the right hand cross sections of the beam and the left hand side is 135. So, they are basically shown over here as joined and at times step 1.36 we have the displacement x as equal to 0.00293 then for the displacement next is 0.00701 and 0.00978.

So, these displacement are the displacement corresponding to the delta 1 delta 2 delta 3 and since in the left hand side and right hand side the displacement this displacement are the same, that is if we assume the members to inextensible then those displacement are repeated over here 0.00293 0.00701 and 0.00978 mind you this is the top displacement this is the middle storey displacement and this is a bottom storey displacement. The velocity is corresponding to this way displacements they are recorded over here and you see again that there is reputation over here, because on both sides we have the same displacement then this is the acceleration which is recorded over here at time 1.36, this is the rotation that is taking place at 135, these rotations are 0.00109 0.3095 and 0.0053.

So, these are the rotations and we can see that at joint one the rotation is maximum that is at this section over here the rotation is maximum that is in the bottom column or bottom beam the rotation here, and the rotation here they should be same because of the anti symmetry, and these two rotations are maximum and then we have the rotation for this beam then we have the rotation for this beam. So, these rotations are listed over here the rotational velocities they are shown over here, again they are of the same nature in both the sides the rotational accelerations are also shown over here with the help of these rotational rotations and the displacement.

(Refer Slide Time: 51:39)



One can calculate the bending moment at the two ends of the lower beam and two ends of the lower beam the bending moment is equal to 50, and next for the next beam it is 23.18 and it is 42.89 for the other beam.

So, we can see that the bending moment at the bottom beam that has reached the yield value, because the yield value is given as 500 k or 50 k n m that is given over here there for the in the table in the first joint and the second joint, which are the two joints of the bottom beam they are the bending moments have released the yield value and for the other two beams the yield values of the moments are not released.

Therefore, a section on one and two undergo yielding recognizing that we write down the stiffness matrix for the system for delta 1 delta 2 delta 3 these are the three displacement and for theta 1 theta 2 theta 3 they are the rotation at one end of the beam and theta 4 theta 5 theta 6 the rotations at the other end of the beam. So, they are shown over here h theta 1 theta 2 theta 3 they are the rotations on the right hand side of the beam and theta 4 theta 5 theta 6 are the rotations on the left hand side of the beam. Now for that we write down the stiffness matrix and we can see that in the stiffness matrix corresponding to delta 1 we have in the first column 1.067.

And then corresponding to delta 2 in the column it will be minus 1.067, then we have for the displacement delta 3. So, for the delta 3 there is it will be 0 then we come to rotations theta 1 theta 2 theta 3 the corresponding to the value of theta 1 that is the rotation on the right hand side rotation of the top beam on the value is coefficient is 0.8, for the next beam the coefficient is 0.8. And note that for the third beam that is the bottom beam the right hand side coefficient is less 0, it is it in 0 because the yielding has taken place over here therefore, for subsequent a time interval and subsequent increment of loading what will be done is that we will replace the that plastic hinge by ordinary hinge and which cannot take any moment therefore, it becomes 0.

Similarly, corresponding to theta 6 we also put 0. In the same fashion the second column third column forth column they have all beam generated in that case we have make sure that at the bottom beam or the first beam at the two ends of the beam there in the moment coefficients will be equal to 0, because we considered there on the ordinary hinge and after we have between down these total stiffness matrix then only we condense out the rotational part of it and land up in the three by three stiffness matrix that is the condense stiffness matrix. So, that is how we proceed with the calculation for a particular frame given the values at a particular time 1.36 and we wish to find out say the values or our responses at 1.38 in that case the stiffness matrix that will be using for finding out the responses at 1.38 or over that incremental loading over the time increment, there these k delta matrix we will be used.

So, let me summaries what we discussed today we have extended the concept of the inelastic analysis of the two dimensional and three dimensional frame systems for the two dimensional and two dimensional frame systems in which the columns are weaker than the beam, then the yielding will take place in the column ends for the case of 2D frame there will be a one way bending. And therefore, the problem becomes easier, we have to only make sure that the bending moment which is computed at any particular cross section whether it is equal to the M p value or not. For the case of the three dimensional frame if the columns undergo yielding then we have to take into account the bi directional interaction on the yielding in order to compute the stiffness matrix of the element by incorporating a plastic component of the stiffness, and then assemble the elementals stiffness matrices to find out the total stiffness matrix and go ahead with the calculation. When the system is a week beam strong column then for both 2D and 3D frame the problem becomes the same because of beams can undergo only one way bending.

Therefore the problem becomes easier, but only thing that is to be taken into account is that whenever there is a hinge that is forming at the beam cross section, then the rotations are to be condensed out and these rotations are in condensing out these rotations. Now, we have to take note of a certain special condition that is the values of r or the values of alpha that we discussed, and with the help of that technique we modify the stiffness matrix whenever any yielding has taken place at a particular cross section of the beam.