Seismic Analysis of Structures
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Lecture - 28
Inelastic Seismic Response of Structures (Contd.)

In the previous lecture, we discussed about the inelastic seismic response of 2D and 3D multi storey frames. For both we can have two situations: one is a strong column weak beam, the other is a strong beam weak column. When we consider the case of strong beam weak column, we generally take a shear frame model so that the hinges conformed only in the columns.

In the other case, when the columns are strong and beams are weak, then the hinges form in the beams. So, for both types of the cases, we described how to perform the analysis. When the curve beam is weak, and the columns are strong, then even for the case of 3-dimensional frame, the analysis is not a complex. Because we do not have to take into consideration the bi-directional interaction, because the hinges formed in the beams and the beams undergo only one directional bending.

Therefore, one calculates easily, the bending moment at the end sections of the beam and check whether that moment is equal to M p or not. The same case is with the 2D frame for the case, when the hinge is forming in the beam. Only problem that is encountered in solving the cases, where the beam is weak, and column is strong and the hinges are forming on the beams. In that case one has to find out the rotations for finding out the bending moment in the beams. And that rotation is to be calculated using the condensation relationship. There we obtain the relationship between the theta and delta, after finding out the delta for an incremental time interval of delta t. Then we also find out the incremental rotation that takes place at the cross section.

Then at this incremental rotation with the rotation at the previous time step find out the total value of the rotation, also the total value of the deflections, so a deflection. Find out then the bending moment at the cross section. In doing so we had brought in a factor called alpha 1 and alpha 2 which are the ratio between the beam rigidity to the column rigidity. So, when the system is in the elastic state, the ratio E I b by E I c can be easily calculated and one can found the stiffness matrix.

However, when the system goes into the plastic state, or in other words in the plastic hinges have formed in the beams, then one has to be cautious in finding out the values of alpha 1 and alpha 2. The formulation that we discussed was with respect to a general condition; that is the behaviour the material behaviour or the force deformation behaviour is not idealized at elastic perfectly plastic or a bilinear model but by a non-linear histidity system in which the stiffness changes at every point.

So, in that case, in the beginning of the solution when does we draw a tangent to the initial point in the curve, and that gives us the initial stiffness tangent stiffness of the system. And then we can calculate E I b and E I c based on that. And find out the values of alpha 1.

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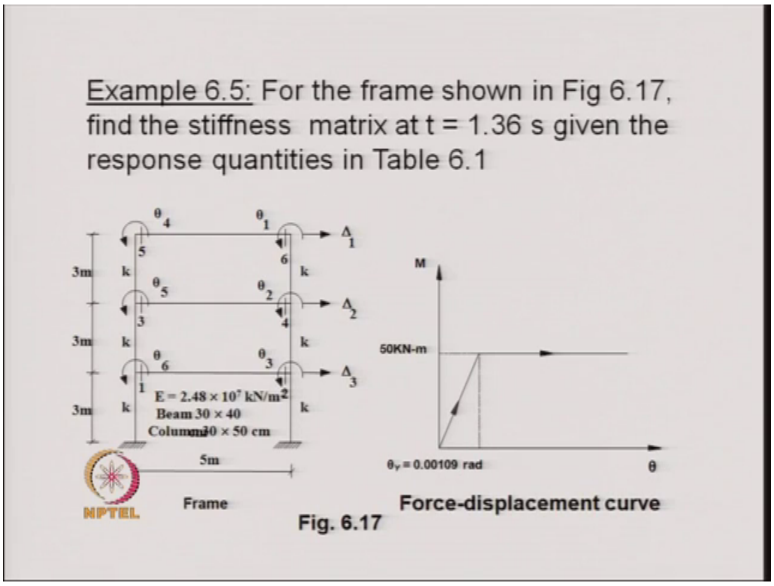

But subsequently as we proceed with the integration, then over a time of delta t, it is difficult to find out the values of E I b by E I c. In that case the weight is to be calculated was explained in the previous slide.

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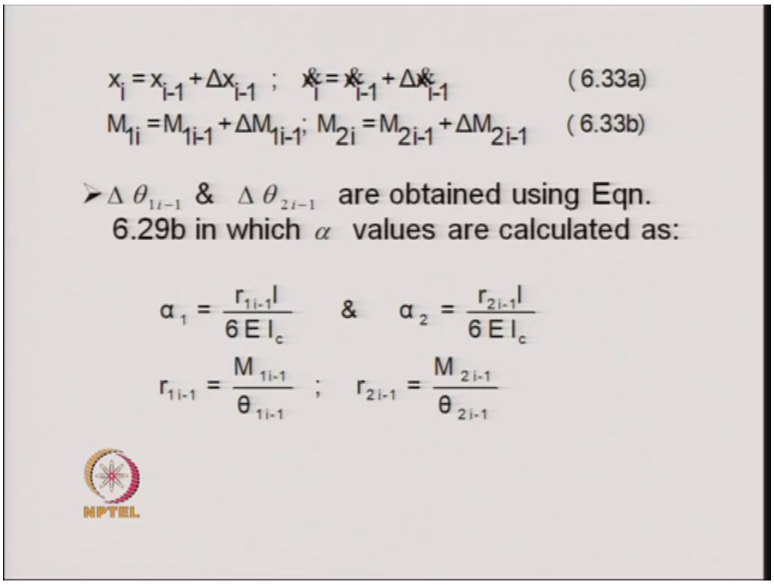

That is, we bring in the concept of r. And this r is a can be calculated iteratively, and using the value of r one can find out the values of alpha 1 and alpha 2 to be used for generating the stiffness matrix at the time t plus delta t.

So, with that background, the entire scheme was illustrated with the help of a very simple example; that is example 6.5, we had got a 6 rotational degrees of freedom, and 3 translational degrees of freedom. These rotational degrees of freedom are condensed out. We get a stiffness matrix with respect to the degrees of freedom delta.

So, with the help of the delta we carry out our analysis, and for every incremental step we calculate delta theta. And add it to the previous value of theta to get the value of theta at the current time step.

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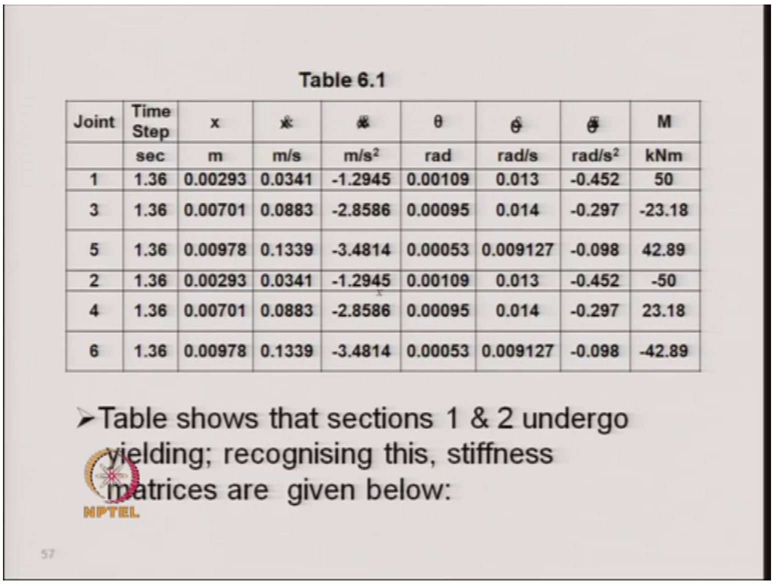

Table 6.1

Joint	Time Step	x	$\dot{x}$	$\ddot{x}$	θ	$\dot{\theta}$	$\ddot{\theta}$	M
	sec	m	m/s	m/s^2	rad	rad/s	rad/s^2	kNm
1	1.36	0.00293	0.0341	-1.2945	0.00109	0.013	-0.452	50
3	1.36	0.00701	0.0883	-2.8586	0.00095	0.014	-0.297	-23.18
5	1.36	0.00978	0.1339	-3.4814	0.00053	0.009127	-0.098	42.89
2	1.36	0.00293	0.0341	-1.2945	0.00109	0.013	-0.452	-50
4	1.36	0.00701	0.0883	-2.8586	0.00095	0.014	-0.297	23.18
6	1.36	0.00978	0.1339	-3.4814	0.00053	0.009127	-0.098	-42.89

So, for this problem at 1.36 second, the values of the displacements, acceleration, velocity, rotation, velocity; that is or the yield rotation and the maximum permissible rotation they had given in this table. And from that one can see that they are the at joint 1 and joint 2 in the beam the moment is equal to the M p value; that is here the adjoint 1 and 2, here at for these beam at joint 1 and joint 2 we have the values of 50 KN M p.

So, these 2 sections are yielding. Therefore, this beam does not contribute to the overall stiffness of the structure. Therefore, we said that to 0 in obtaining the stiffness matrix of the system.

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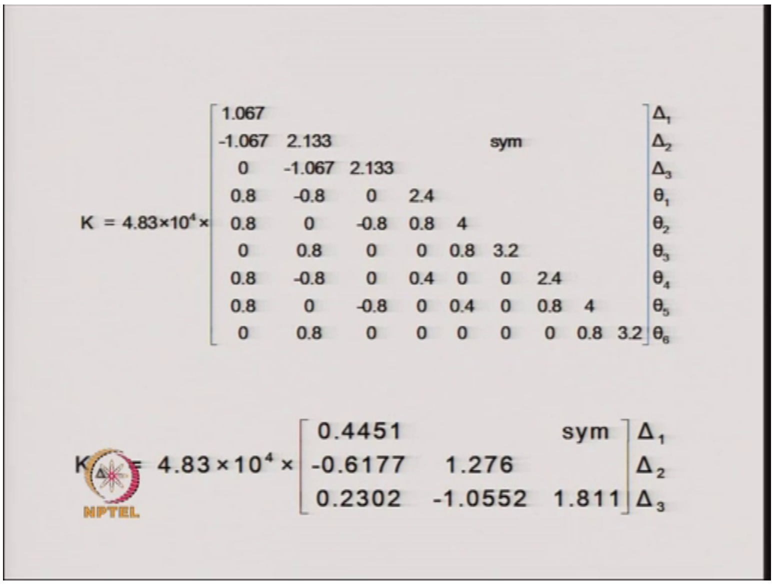

And that is how we calculated the total stiffness matrix of the system, which is shown here. And from that the stiffness matrix, we calculated the condensed stiffness matrix which is a 3 by 3 stiffness matrix. With that stiffness matrix k delta k delta, we now calculate the incremental displacement for the next increment of the delta t.

So, this was the scheme and for the calculation, and if therefore, we see that at every instant of time t, we have to check in the beam members whether the there is a plastic moment is there at a particular cross section. And if there is a plastic moment coming at a particular cross section then we assume for the next analysis a simple hinge at that particular cross section, and carry out our a solution with a modified stiffness matrix K t.

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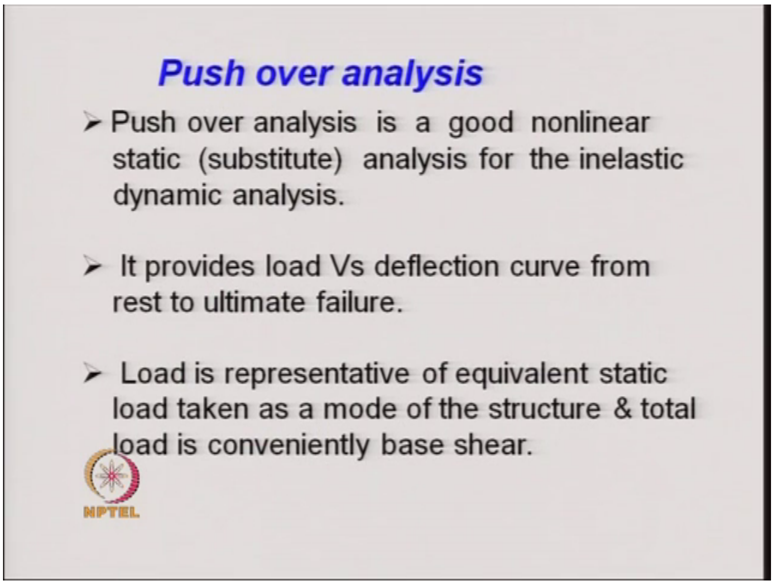

And how to take care of the case when the bending moment at a particular time exceeds the value of M p? Then how to you know rectify that? That was also discussed, the same concept will be subsequently discussed in connection with the pushover analysis; that is the subject of what discussion today.

Now, pushover analysis is one of the very popular methods of non-linear analysis or rather static equivalent non-linear analysis; that is performed for earth quake loading. In fact, the way the response spectrum the method of analysis is a very good equivalent static load analysis for earth quake forces courses. In the same fashion the push over analysis is a very good non-linear static analysis for the inelastic dynamic analysis for earth quake forces. The push over analysis is carried out for many purposes, the cheap among them is the push over analysis for finding out the behavior of the structure, at the inelastic stage during the earth quake; that is the inelastic analysis that you carry out for the earth quake forces, that inelastic dynamic analysis is replaced by a equivalent static non-linear analysis that I mentioned before.

Second thing is that for finding out the performance criteria of the structures in in the inner inelastic range that also is obtained with the help of a pushover analysis or equivalent static pushover analysis. That helps designers to understand the different states inelastic states of the system, when the structure goes in to the inelastic zone during earthquake. And one can specify a certain criterion like immediate occupancy criteria or failure criteria etcetera. Therefore, attaching some kind of performance level of the structure in the inelastic state, this is also known as the performance based analysis and design, and it is routinely carried out for most of the degree of freedom systems especially the multi storey building frames, in order to find out or assess the performance level of the structure in the inelastic state.

The pushover analysis is incorporated in most of the standard softwares that are available these days, I like sat 2000 or the ansis or abacus for all these softwares we have pushover analysis. And these pushover analysis required some kind of data in the beginning, and we will described what are the kind of data that is needed for performing the pushover analysis. The pushover analysis provides a load deflection curve, a single load deflection curve, and that single load deflection curve gives us an information about the state of the system after yielding, but the system is then idealized as if as a single degree of freedom system.

So, in an overall sense the behaviour of the structure in the inelastic range is understood using the push over analysis. So, h the main thing that the pushover analysis provides is a single load deflection curve from 0 loading to the ultimate failure state. Load is representative of equivalent static load, taken as a mode of the structure and total loss is conveniently taken as the base shear.

So, the important thing over here is that; the loads that is acting in the frame, that load is considered to be distributed in any fashion or any in an reasonable passion; however, one prefers for the earthquake analysis; that the load should be in accordance with the mode shape. Or in other words according to the shape of the first mode the loads are distributed.

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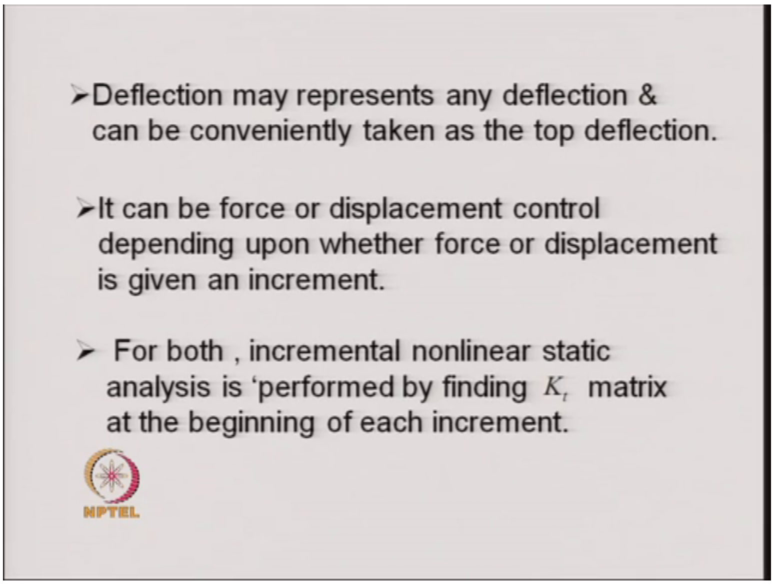

Now, once we do that, then the sum of the load gives us the base shear, and typically by load we mean the base shear in this case. The deflection may be represented by any deflection; however, it can be conveniently taken as the top deflection of the structure. So, the pushover analysis finally, provides us a load deflection curve where load is the base shear, and the deflection is the top displacement of the mass frame. Now the pushover analysis can be force or displacement control depending upon whether we increment the force or increment the displacement.

So, what we do is that either we gradually increase the displacement in the structure, and they look into the behaviour of the structure or we increment the force, and then we study the behaviour of the structure. For both incremental non-linear static analysis is performed, and for that what we require is the k matrix which is a transient k matrix that is what; that means, these k matrix goes on changing with the deflection, but over an incremental displacement we assume that the stiffness or the transient stiffness matrix that does not change. So, with the help of that we perform a linear analysis within the small in incremental displacement, and find out the values of incremental responses.

The matrix at the beginning of each increment is obtained to find out the response for the over that increment; that is if the increment is at a deflection level of delta 1, then the for the response at delta 1 plus delta we use the stiffness matrix that is developed at a deflection stage of delta 1; however, these may have to be modified in certain cases, those modifications we will describe little later.

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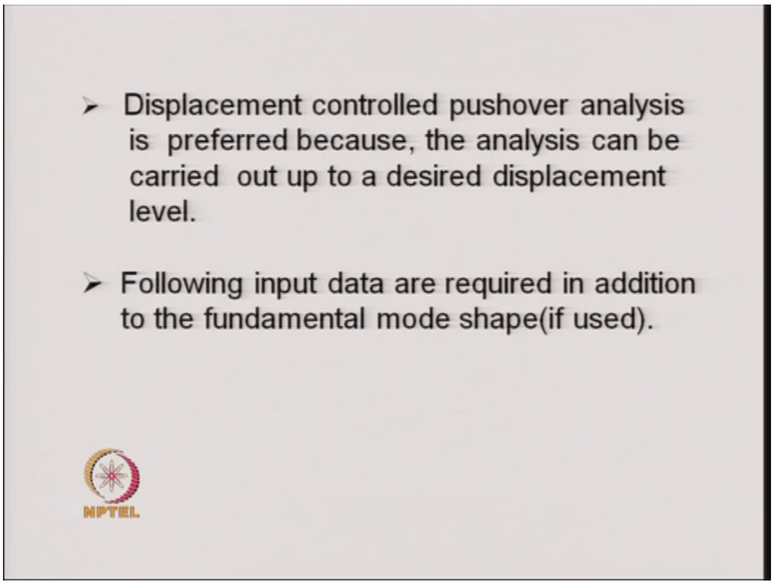

The displacement control pushover analysis is preferred in most of the cases, because the analysis can be carried out up to a desired displacement level. So, one can stop the pushover analysis at any displacement required displacement stage, and saying that we are interested to find out the behaviour of the structure up to this displacement level. So, from that point of view the displacement control a pushover analysis is preferred.

The analysis can be carried out to any desired level, and these desired level; obviously, depends upon as I told you the displacement level, or the force level that we wish to impose on to the structure as the final displacement or final load. Following input data are required in addition to the fundamental mode shape.

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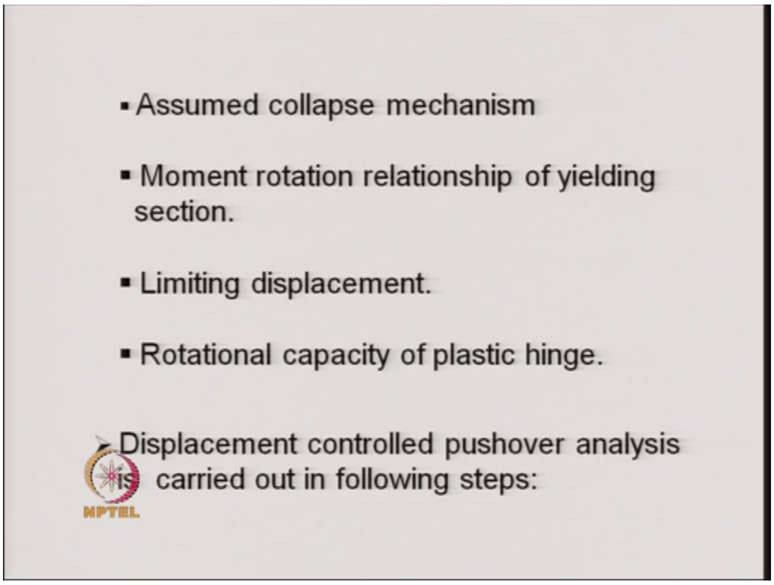

So, as I told you that the loads are distributed along the height of the building according to the mode shape of the structure; however, it is not necessary that one should consider the mode shape or the first mode shape of the structure one can assume any reasonable distribution also.

The other information that is required is that one has to assume a collapse mechanism for the structure, and this collapse mechanism is somewhat difficult to assess for a multi storey frame structure; however, one can assume any kind collapse mechanism, in the sense that in order to stop the analysis, we can say that we will perform analysis up to this particular state when certain numbers of plastic hinges have occurred into the structure and that we considered as the collapse state.

The structure may not have actually collapsed. In many cases the entire analysis is performed unless, we find that the there is a singularity in the matrix. Or in other words the solution cannot be carried out further. At that stage we have abounded and say the structure has collapsed. And these kind of collapse which are not again a desirable kind of collapses that may occur even before the complete collapse.

So, these collapse mechanism that we are defining to stop our calculation is an important thing or sometimes a premature or collapse state can come into picture because of the singularity of the matrix and at that stage one has to stop. And the mechanism that we get a we say that is the failure mechanism. Next important information that is to be provided is the moment rotation relationship of yielding cross section.

Now, this moment rotation relationship of yielding sections that has to be obtained from the cross sectional properties of the beams and the columns, including the reinforcement. By knowing the percentage of reinforcement in the beams and columns, one can find out a moment rotation curve. Once we get the moment rotation curve for the cross sections, then those the moment rotation curves are provided as an input for the analysis.

So, the whatever be the number of cross sections that we consider, where the plastic hinges are assumed to form, for those cross sections we provide the moment rotation relationship. Next input that is a necessary is the limiting displacement. And now the limiting displacement may be provided. So, that either we cutoff our analysis before the complete collapse. In that case we can give a limiting displacement which is much smaller displacement compared to the displacement that takes place at the time of complete collapse.

So, in order to stress the complete load deformation or load displacement curve that is up to the collapse state we assume some kind of a displacement which is generally a large displacement and give it as an input to the structure so that the solution automatically stops before all, the actual displacement that you have provided is achieved. Next very important information that is required is the rotational capacity of plastic hinge. That is also very important.

In many cases we say that if the rotation at a plastic hinge exceeds certain value, then we say that there is a rotational failure at that plastic hinge. Now this kind of a failure is generally achieved many a time, and once we can say that a particular plastic hinge has exceeded the value of the permissible rotation, and then we marked that particular cross section and say that that particular cross section has completely failed.

Although the structure has not collapsed completely and we identify in the process of our calculation, the plastic hinges which have exceeded their rotational capacity, and the plastic hinges which have not cross their permissible limit. Displacement control pushover analysis is carried out in the following steps.

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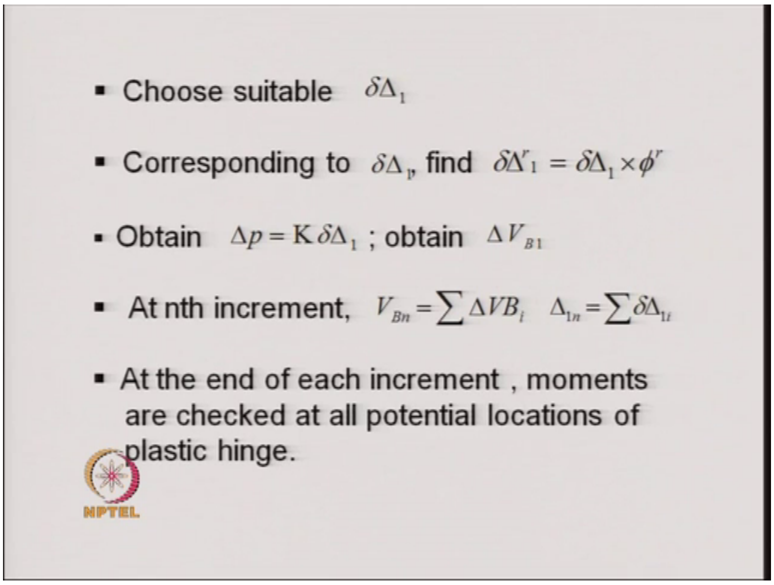

We choose a suitable displacement interval; that is a delta delta 1 for the top a story of the frame. Corresponding to this delta delta 1, we find out the delta delta 1 at different levels of the frame; that is for the r-th level the value of delta delta 1 r is equal to delta delta 1 multiplied by the mod shape coefficient for that particular or r-th floor. So, that is how one can get the values of the displacements at different floor levels, once we have assumed some value of incremental displacement at the top a story of the frame. This gives a vector of displacement the index one denotes that this is the first increment of a displacement.

Once we can obtain the displacement vector, then or the incremental displacement vector then we multiply it with the stiffness matrix k. Now these stiffness matrix in the beginning or the for the first increment is an elastic stiffness matrix k; however, as the calculation proceeds, these k becomes a transient stiffness matrix, and it goes on changing depending upon the displacement a level, or depending upon on the plastifications that take place in the frame at different cross sections.

At any time that is a at the n-th increment of the load, the total base shear that can be calculated by adding up the total load, or total load will be is equal to summation of the delta p over the n increments. Similarly, the base shear can be calculated by summing up the base shear for each instrument of loading up to the n-th increment. The deflection delta 1 in that again is equal to the displacement increments that are given to the top floor up to the n-th increment. So, that is how one can get delta 1 in and delta v n plot vn and these plot of v B n versus delta 1; in that is continuously obtained as we go on incrementing the displacement.

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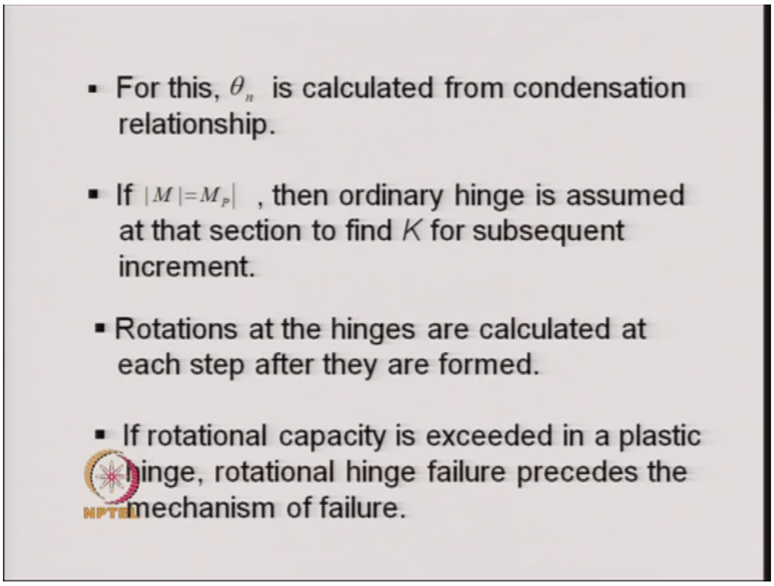

At the end of each increment moments are checked at all potentials plus trans locations of the plastic hinge and that part is little complicated; in the sense that first one has to find out the value of theta n at the cross sections from the condensation relationship. And as I described before, we first find out the delta theta at all the sections were the yielding may takes place from the incremental displacement.

Once we get the incremental displacement those incremental rotations those incremental rotations are added to the previous rotation to find out the current value of the rotations. Knowing the value of rotation, and the story displacement, one can find out the bending moment at the desired cross sections of the structure. If it is found that at any particular section, the moment value is equal to the M p value. Then for subsequent increment what we can do is that we assume a ordinary hinge at that particular cross section, and find out the total stiffness matrix of the structure; that is for that element we assume a hinge ordinary hinge at the section where the plastification has taken place.

Also during the calculation, we go on calculating the rotations at the hinges rotations at the plastic hinges. Also the even if we assume a ordinary hinge at the time of the analysis for the cases where are the section has undergone on yielding and there is a plastic moment or the moment is equal to plastic moment. After the analysis is performed we find out the rotation, incremental rotation also for the ordinary hinge.

Now, once we get that rotations incremental rotation, then we add this to the previous a rotation to find the final rotation. So, therefore, the we can find out the rotations at the hinges or the plastic hinges, and we keep a record of that in order to check whether the permissible rotation capacity is exceeded in a plastic hinge or not. If we find that there is a case where the rotational capacities in sufficient number of hinges have taken place, then it may so happened that a rotational failure may precede the actual collapse mechanism.

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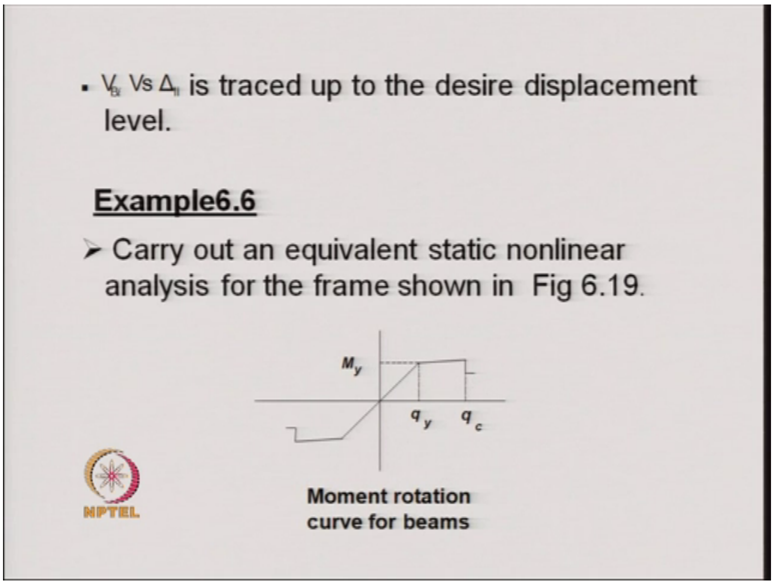

The vv versus delta 1 is stressed up to the desired displacement level or the collapse state. Now during these calculation procedure, the kind of iterations that are involved of that let me explain. First iteration is that as we go on incrementing the displacements At a particular level, then say we take delta 1.

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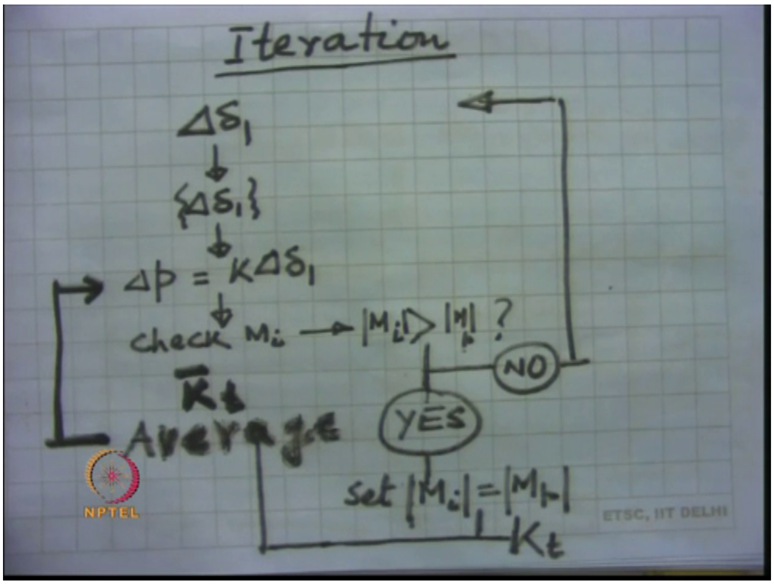

And from these deltas 1 we construct the delta delta 1 vector. With that delta delta 1 vector we get the delta p vector. Then check for the M i; that is the moment at the cross sections, where the desired cross sections where the plastic hinges are likely to form.

And check whether the moment there is a greater than M p or not. The moment being just equal to M p is a rare case. Therefore, in most of the cases we find that the moment is may be greater than a value of M p, which is less than value M p then of course, that particular cross section is (Refer Time: 34:42) in the elastic state.

Now, if the this is not greater than M p; that means, it is elastic or if it just is equal to M p, then to handle that is very simple we simply put a ordinary hinge at that particular plastic hinge and obtain the stiffness matrix and carry out the solution as before; that means, you go for the next increment. If it is greater then what we do is that for those cross sections we set M i to be is equal to M p that; obviously, in today in introduced some kind of imbalance into the overall equilibrium equation.

So, what we do is that we can have 2 alternative, 2 options. One is that after setting those values of the moment 2 is equal to M p, then we calculate a revised value of the stiffness matrix; that means, for those cross sections where the moment has exceeded the value of M p at those cross sections we introduced a ordinary hinge, and calculate a revised stiffness matrix. And after that we find out an average stiffness matrix; that is the stiffness matrix before the increment that plus these modified or the updated stiffness matrix. This we add together and divide it by 2 to obtain a average stiffness. And with that average stiffness we now calculate the delta p; that is the load increment for the value of delta delta 1 that is the incremental displacement that we have considered.

So, therefore, at the end with this calculation, we get different values of delta p vector. And after the solution; that means, once we have this solution, it is automatically assume that the cross sections where we have a set M i to be is equal to M p. At those cross sections yielding have taken place and for next iteration or next displacement increment we use these value of K t.

So, in this way one can perform an iteration or an iteration may be required, whenever the bending moment is exceeding the value of M p at any particular cross section.

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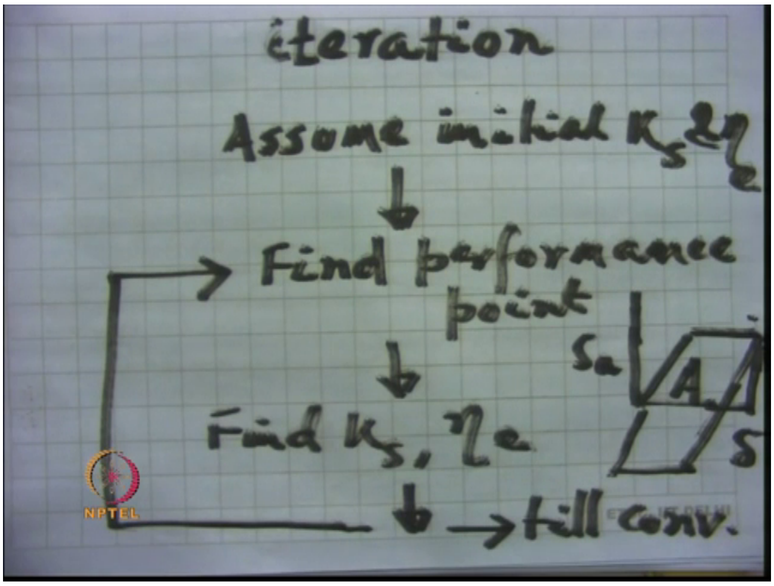

The other we, is to proportion as I told you before in the class in elastic analysis; that is it value of the M p's that is obtained after the solution, those out of that we take the value of M p which is the largest one.

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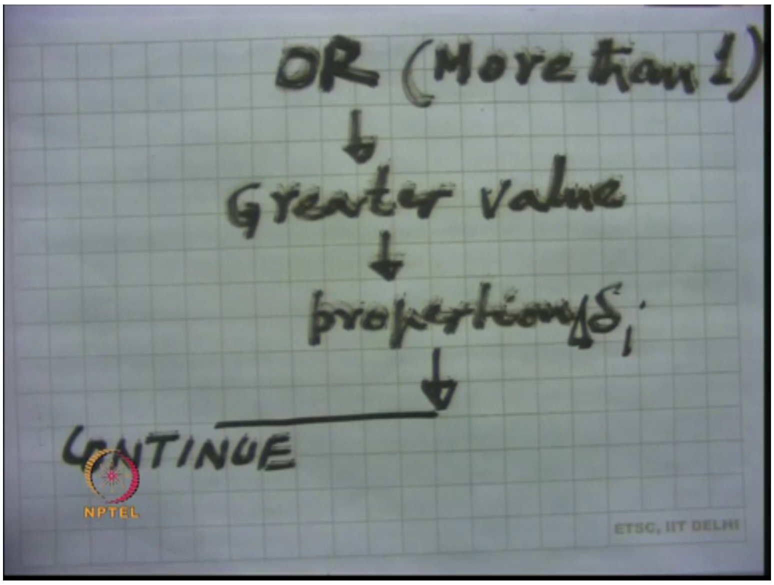

And this is done for the case when more than one sections have moments greater than the value of M p. So, there we take the greatest value of the M p. And then with proportion the incremental displacement in such a way that the bending moment at that particular cross section just becomes equal to M p.

In that case what will happen for that particular cross section, M i will be simply is equal to M p. For other cross sections; obviously, the values will not be equal to the value of M p in most of the hinges the, or plastic cross sections, the values would be would become less than the value of M p. Because we have proportion the value of delta delta i; however, these proportioning may not always lead to all sections. Having a value of a m less than M p; however, with the little bit of you know expertise, one can proportion the value of delta delta 1 or the incremental displacement in such a way so that we get a plastification only at only at one cross section.

In that case the incremental displacement becomes non-uniform. And, but that does not affect the solution and one can continue in this particular fashion, on that is we can go for the next increment of displacement with a plastic hinge performing at one cross section only at a time. This kind of a calculation may require little more time to trace the load deflection. May be an it is greater that one goes for an l average stiffness technique to take care of or this kind of situation.

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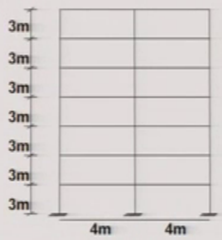

Frame

Fig.6.19

Cross section	Location	b (mm)	d (mm)	(kNm)	(rad)	(rad)
C1	G,1st, 2nd	400	400	168.9	9.025E-3	0.0271
C2	3rd ,4th, 5th & 6th	300	300	119.15	0.0133	0.0399
B1	G,1st, 2nd	400	500	205.22	6.097E-3	0.0183
B2	3rd ,4th, 5th & 6th	300	300	153.88	8.397E-3	0.0252

Table 6.2

The problem of a frame is solved here pushover analysis. This is the moment rotation curve for a particular beam, and we can see that we are specifying it will not be q y and qc it will be theta y and theta c. Theta y indicates the yield rotation, and correspondingly we define a yield moment. And theta c is the rotational capacity or the maximum rotation that we allowed at the plastic hinge. The properties of the frame are given over here. We see that we are we have grouped ag the cross sections in c 1 c 2 and b 1 b 2 c 1 c 2 are the 2 groups of columns. And b 1 and b 2 are the 2 groups of the beams. So, from the ground first floor and second floor level, we have one kind of cross section.

And then their m y value is given, and the corresponding yield rotation is given. And the last column shows the rotational capacity for that particular cross. Second group is at third 4th fifth and 6th, storey levels there we have a reduced b and d. And therefore, the yield moment is also reduced the rotation permissible rotation is given, that is a yield rotation is given and the maximum rotation that is allowed or that is also provided. In the same way the b 1 and b 2 that these 2 groups of the beams for that the dimensions and the other required values are given over here. One can see that the there the again there is a change in cross section for b 1 and b 2 level.

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➢ Solution is obtained by SAP2000.

D (m)	Base shear (KN)	Plastic Hinges at section
0.110891	316.825	1
0.118891	317.866	1,2
0.134891	319.457	1,2,3
0.142891	320.006	1,2,3,4
0.150891	320.555	1,2,3,4,5
0.174891	322.201	1,2,3,4,5,6
0.190891	323.299	1,2,3,4,5,6,7
0.206891	324.397	1,2,3,4,5,6,7,8
0.310891	331.498	1,2,3,4,5,6,7,8,9
0.318891	332.035	1,2,3,4,5,6,7,8,9,10
0.334891	333.11	1,2,3,4,5,6,7,8,9,10,11
0.350891	334.185	1,2,3,4,5,6,7,8,9,10,11,12
0.518891	342.546	1,2,3,4,5,6,7,8,9,10,11,12,13
0.534891	343.207	1,2,3,4,5,6,7,8,9,10,11,12,13,14
0.622891	346.843	1,2,3,4,5,6,7,8,9,10,11,12,13,14,15
1.448699	307.822	1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16
1.456699	308.225	1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17

NPTEL

Table 6.3

The solution provides us a result like the one which is shown here; are the different displacements and the corresponding base shear that are shown in the tabular form over here. And the plastic hinges that has taken place at a different levels of displacement, that has been noted down on; that is 1 1 2 1 2 3 1 2 3 4 1 2 3 4 5 and so on.

And when we come to the last value, then we see that there is there are sufficient number of sections where plastic hinges have taken place; the if we see look into the base shear, we can see that the base shear is increasing up to 346. That is the last but one row, not last but one row, above 2 this is the a value which is has the maximum value of the base shear. And after that the base shear drops down to 307. And then it is slightly increases 308. The base shear versus displacement plot is shown over here.

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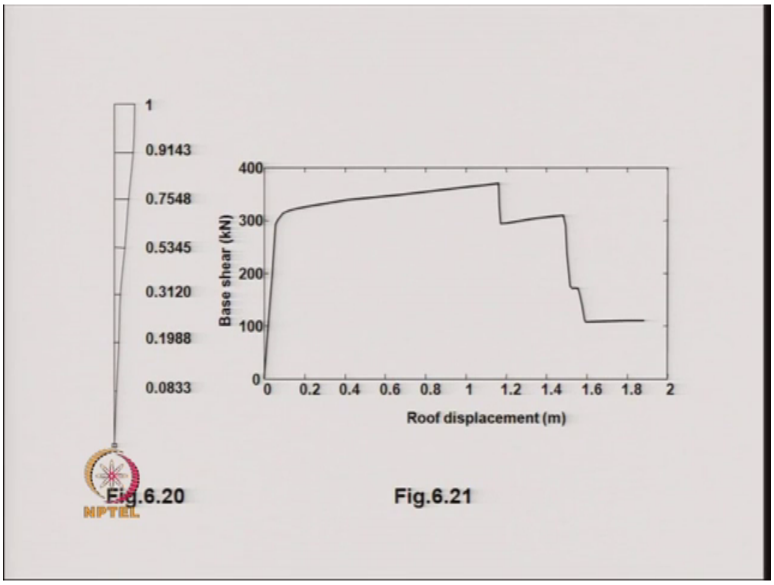

Fig.6.20 Fig.6.21

And this goes on increasing, and this value is about 346. And then there is a drop in the displacement, or in the displacement or the base shear. This is the distribution of the lateral forces that is considered along the height of the building; this or these kind of mode shape is assumed for the analysis; so one can get a plot like this from the pushover analysis; and if we go up to the state of collapse.

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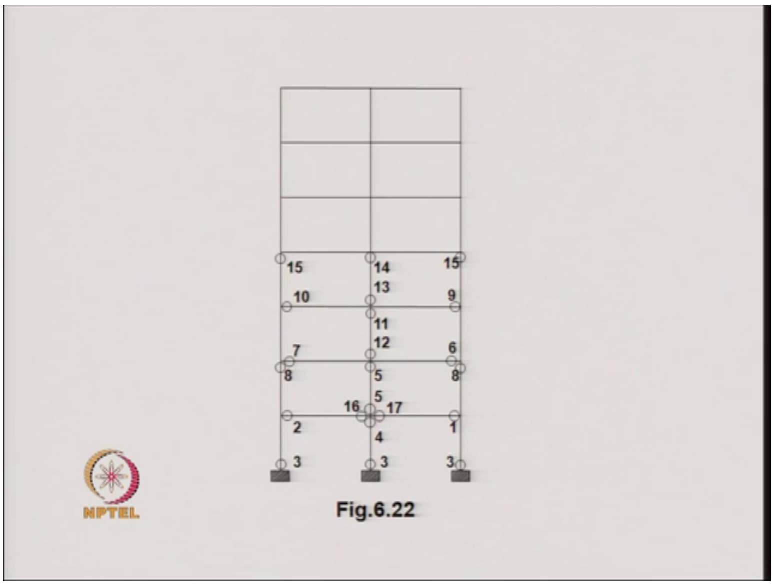

Fig.6.22

The plastic hinges that were stressed and shown in the table they are shown over here. And one can see that the 17 plastic hinges have occurred, and at that stage the solution had to be a stopped or the calculation had to be stopped. The reason for this is that the if you look at this particular joint, we find that the plastic hinges are forming at all the 4-cross section surrounding this join. As a result of that there is a rotational failure that has taken place at this particular or joint. And there because of this rotational failure, or the stiffness matrix became yield conditioned. And therefore, the calculation had to be stopped.

However, this also corresponded to a situation, where there is a certain drop of the base shear. And one can say that this is almost a state of failure. Not necessarily that one may have to stop the solution or the calculation procedure at such a premature stage, unless we have this kind of situation. If this kind of situation does not take place, then perhaps more number of plastic hinges can firm into the structure, and one can have a calculation calculations may be continued further or the in incremental displacements can be provided and the solution procedure can be carried out till a desired collapse state or collapse mechanism is formed by way of the plastic hinges performing at different cross sections. And the plastic hinges thus will not provide a situation of premature or failure like this.

So, it depends upon the problem to problem one can pursue or continue the incremental displacement up to the state of a desired collapse mechanism or the collapse mechanism that they were assumed. And one can get that particular collapse mechanism or achieve that collapse mechanism by way of implementing the displacement. It may so happened that one may not go, or may not be able to go up to that state and before that a premature collapse state can occur. Now with the help of the pushover analysis some important things extracted as I told you in the beginning.

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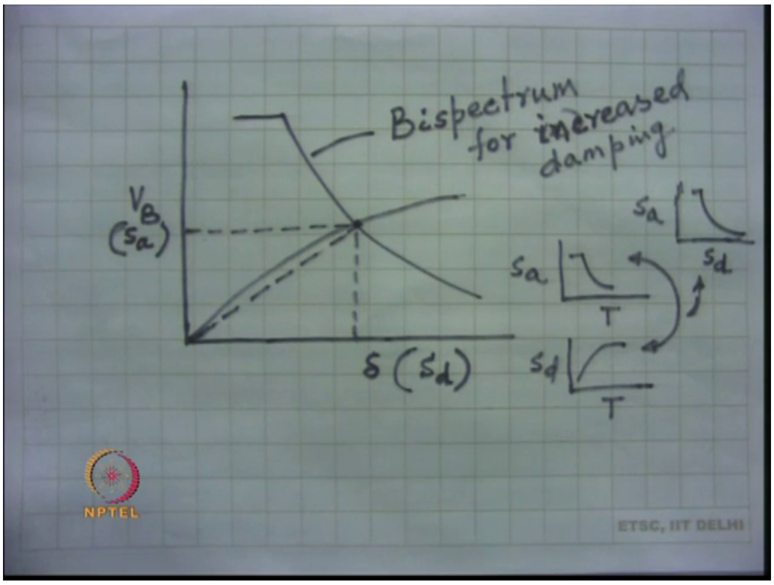

That is a first thing is that one can see the different states of the system of the building; that is how the plastic hinges are forming and different cross sections, and what are the nature of those plastic hinges repair the plastic at the plastic hinges the rotational capacity. I have accident or not and the plastic hinges where are still some rotation can take place, all those information can be achieved during the process of the calculation. And from that one can define some state of the system after yielding or the performance level of the structure at different states.

That is a very important in so far as the performance based analysis is concerned; however, one of the important thing that we try to obtain, in the case of by doing a pushover analysis is that to find out an overall ductility of the system. In that case the basic philosophy with which the pushover analysis is carried out is that we convert the entire system into a single degree of freedom system. And try to draw the load versus the deflection curve. The load is given by vb and a displacement is given by delta. So, one can find out a performance point like this; that means, with this is a load deflection curve.

And one can have a bispectrum drawn from the response spectrum the displacement response spectrum and the acceleration response spectrum. And from that one can construct a bispectrum by eliminating the time period scale. And that bi spectrum can be obtained with an assumed value of the increased damping, and the tentative equivalent stiffness for the entire system considered as a single degree of freedom system that is when it is vibrating only in a first mode.

Now with that one can get a performance point, but these performance points can be updated. By successive iterations that is once we get a particular performance point, then one can find out a device value of the equivalent stiffness. And the equivalent damping can be obtained from the loop hysteresis loop are giving these value of maximum value of displacement. And get a revised value of the equivalent stiffness and damping, and find out and the bi spectrum curve.

So, that way we can go on doing this particular iteration, till we find and that the performance point that is achieved in two successive iterations are the same. And from there one can obtain the value of the ductility required ductility, that is the maximum displacement that we obtained divided by the elastic displacement; that is when the structure first gets into the into the inelastic state or the first plastic hinge is found out, that is taken as the yield displacement. And the delta becomes the final displacement ratio between the two gives a overall ductility for this system.