Seismic Analysis of Structures
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Lecture - 29
Inelastic Response Analysis of Structures (Contd.)

In the last few lectures we discussed about the Inelastic Seismic Response Analysis of Structures. We started with the inelastic seismic response analysis for single degree of freedom system. And then how it is extended for a multi degree freedom system was discussed. After that we discussed the effect of the bilinear introduction on yielding and how it is included in the inelastic seismic response analysis of structures.

The concepts were then utilized for the analysis of 2D frame and 3 D frames. In 2D and 3D frames were considered again two cases: in the first case the beams were strong and columns were weak. Therefore, the plastic hinges that occur they are in the ends or at the ends of the columns. And we generally assume the beam to be rigid, in the sense that we make the frame a shear frame. For the case when the beam is weak and the columns are strong then the plastic hinges are formed in the beams to treat that case is relatively easier compared to the case when the columns are yielding in the case of a 3 D frame, because there one has to take into consideration the effect of the bilinear interaction on yielding.

Only thing that is to be considered for the case of weak beam a strong column system is that after the yield has taken place at a particular cross section in the beam, then from the incremental displacement we have to compute also the incremental rotation. With the help of that only we can find out the bending moment at a particular cross section, and check whether it is exceeding the value of empty or not.

If it exceeds the value, then another thing that you have to compute is that; what is the rotational velocity at the plastic hinges and also the amount of rotation?

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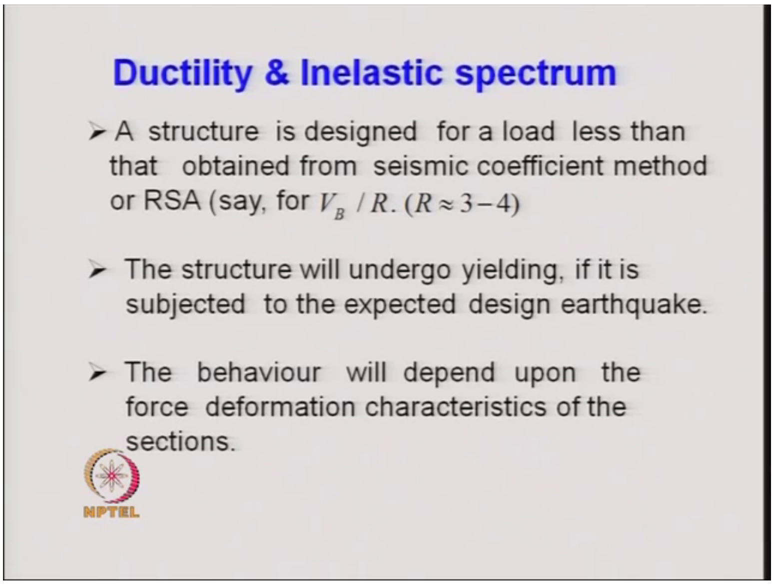

After that we discussed a very important topic called the pushover analysis in earthquake engineering. The pushover analysis is a is a very good equivalent static non-linear analysis of structures for earthquake with the help of pushover analysis we try to understand the elastic behaviour of the structure after the yielding during the earthquake, and also try to see the performance levels of the structure at different at stages of the earthquake loading.

And pushover analysis is extremely used for most of the cases, where we have it is the situation the where the structure is subjected to a lateral a dynamic loading and because of that the structure goes into other inelastic zone. After this we want to discuss now two very important aspects in earthquake analysis and design they are the ductility and inelastic spectrum I have defined what ductility is before. So, let me define it again ductility is a for a single degree freedom system is defined as the maximum displacement that the structure undergoes during earthquake divided by the yield displacement. So, that is the definition of the ductility or in other word after the yielding, how much the structure can deform is a measure of the ductility and in aseismic design of structures we design the structures to have enough ductility I mentioned this in connection with the design philosophy that we have.

In the seismic design philosophy we have got three very important criteria, that is the stiffness strength and ductility. So, stiffness provides the resistance to the earthquake forces in the elastic zone the strength denoted by the yield of the yield resistance of the element that decides when the element goes into the inelastic range and ductility denotes how much the element can deform beyond yielding. Ductility detailing of reinforcement is a very crucial issue in the structural drawings, at every joint structure these structures must be a detailed properly for earthquake and so far as the reinforced concrete structure is concerned there are guidelines given in all codes.

How one can provide the adequate ductility by way of reinforcement detail. So, we will look into this aspect of ductility in earthquake resistant design of structures, along with that will discuss another important aspect in earthquake resistant design, that is the inelastic response spectrum and we will see how inelastic response spectrum differs from the elastic response spectrum that we have discussed already, how we if one can construct an elastic response spectrum from the elastic response spectrum and what are the uses of inelastic response spectrum.

As I told you a structure is designed for a load less than the load which the structure experiences during the design earthquake, and how the equivalent static lateral load is calculated using seismic coefficient method or response spectrum method of analysis the for both cases that is computed or the base shear that is computed is divided by a reduction factor R. So, that the effective loading on the structure is reduced.

The typical values of R that are considered is a between 3 to 4 for ordinary kinds of structures, for a different kinds of structures. However, the code specifies the values of the reduction factor that is to be taken. Because of this reduction factor a reduced load is a acting on the structure and we analyze the structures for that reduced load as a result of that in the case of the actual earthquake the all the structures that we design they undergo inelastic excursion that is it goes beyond the yield limit.

Different elements and go into the yield limit to different extent and therefore, there is a specific ductility demand which is imposed on the structure or not as a overall ductility demand, but a ductility bend which varies from element to element. To cater to all the ductility demands imposed by the earthquake on different elements is very difficult task, but we try to assess what is the ductility meant as such on the structure and for that pushover analysis is one of the very important analysis technique by way of which we convert a structure into a an equivalent single degree freedom system and look into the ductility demand imposed on that equivalent single degree freedom system. This is done by way of considering that the structure as if is vibrating only in the first mode.

But that gives us a an idea about the ductility demand and that is imposed on the structure as a whole or because of the earthquake. For finding out the ductility demand for each element one has to carry out a non-linear dynamic analysis or non-linear seismic analysis of the structure find out the maximum displacements or deformations that take place in individual members divide them by the elast or the or the yield deformation or yield displacement and find out the ductility for each member. So, we want to see now how this ductility is obtained for a structure element wise and as well as a single degree freedom system or what we call as a overall ductility of the structure.

The structure will undergo yielding if it is subjected to the expected design of earthquake that is what I told you before, the behaviour will depend upon the force deformation characteristics of the section. So, that is one of the again important input for the analysis, we have to provide the load deformation behaviour for each of the elements of the structures that is the cross sections, where the where expect the plastic hinges to form for those cross sections, we provide load deformation characteristics or moment rotation characteristics from the designed cross section.

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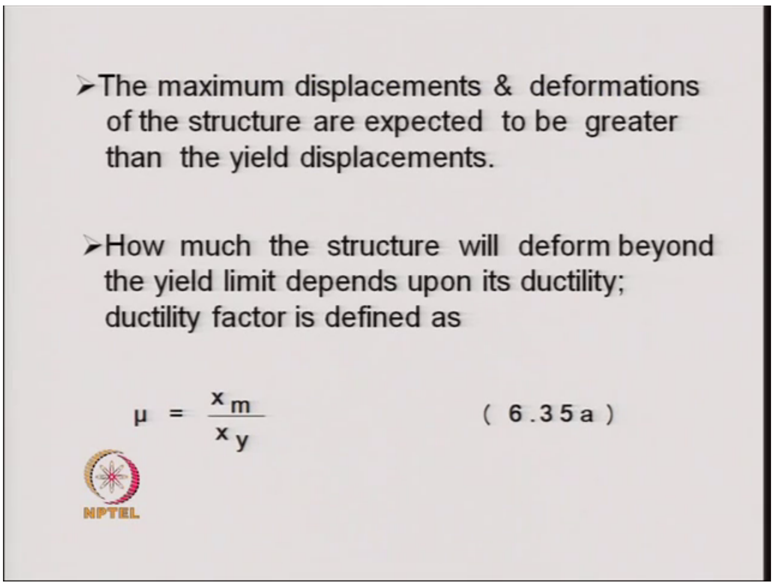

The maximum displacements and deformations of the structure are expected to be greater than the yield displacements, and that is that gives rise to the concept of ductility.

However, there could be situation where the maximum displacement or deformation in a structure may become less than the yield displacement that kind of scenario can also happen. How much the structure will deform beyond the yield limit depends upon the ductility. Ductility factor is defined as the equation 6.35 a that is mu is equal to x m by x y where x m is the maximum displacement and x y is the yield displacement, and this is defined for a single degree of freedom system.

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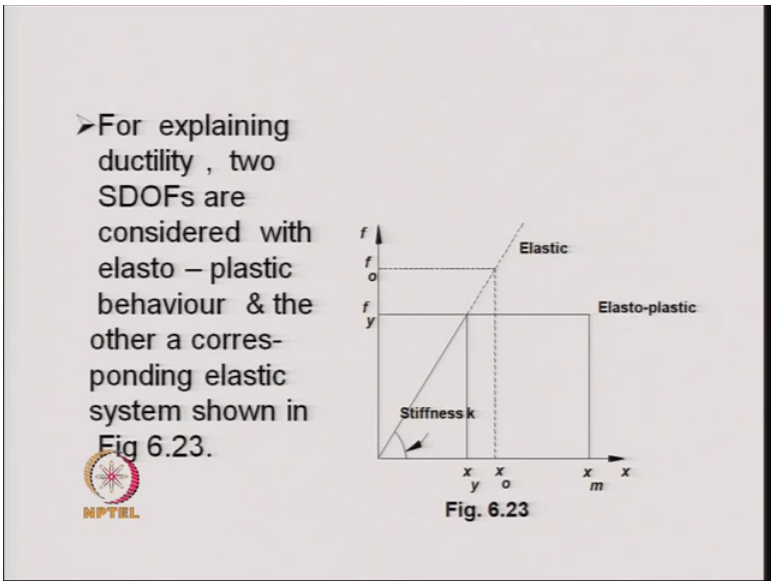

For explaining the concept of ductility and understanding how one can understand the effect of ductility on the structure, we consider two single degree of freedom system one is the elasto-plastic system other is a corresponding elastic system. So, that is shown in this figure. So, this is one system that is elasto-plastic system the corresponding elastic system is this one; that means, in the elastic range both of them has the same stiffness, but for the elasto-plastic system after the yield force the system moves on these horizontal line where as in the elastic system it continues to move on this same inclined line representing the stiffness of the system.

We have these definitions of the displacement here say x m is the maximum displacement that can take place in the single degree freedom system, x y is the yield displacement that is that is well defined with the help of the yield force and the stiffness of the system, and x 0 is a displacement for a force of level f 0 applied to the structure and all of the single degree freedom system and it is assumed that thus a single degree of freedom system is behaving as a elastic system.

So, beyond yielding in an elastic equivalent or a corresponding elastic single degree of freedom system, the value of x 0 will be obtained through the stiffness of the system and we will have for a particular value of f 0 the displacement as x 0.

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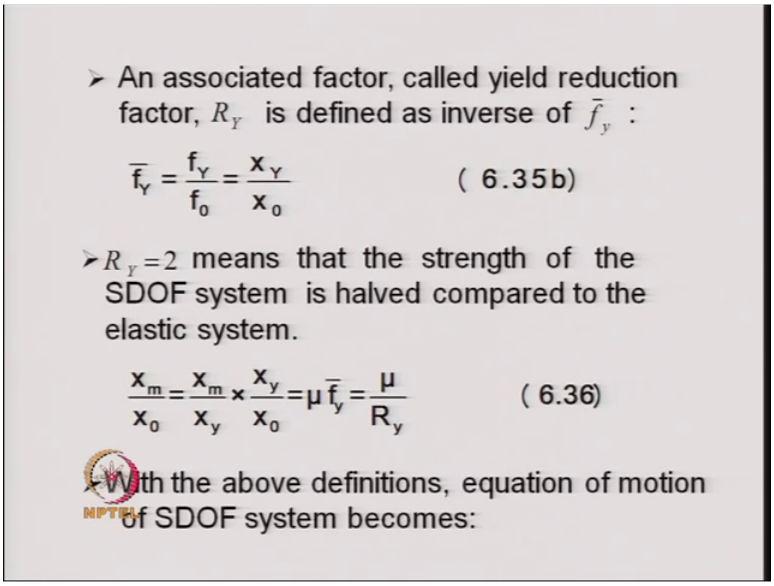

In order to make the formulations and the necessary curves for understanding ductility, we make certain parameters or introduce certain parameters into the analysis procedure. First parameter is f bar y; f bar y is a non dimensional yield force defined by f y divided by f 0 that is f y is the maximum force that can occur in the elasto-plastic system.

And f 0 is the maximum force that is developed if that particular is 0 system where behaving as an elastic a system and this is equal to x y by x 0 which is shown in earlier figure. So, this f y by f 0 will be equal to a x y by x 0. Now inverse of f bar y is called a factor which may be called as a reduction factor and these reduction factor R y is simply the inverse of f bar y, that is R y is equal to two means that the strength of the SDOF system is happed compared to the elastic system that means, these ratio between f y by f 0 will be equal to half.

Now with the these equations 6.35 b one can write down another equation which is given in 6.36 that is x bar x m by x 0 that is the maximum displacement that can take place in the elasto-plastic system divided by the maximum displacement that can take place in the corresponding elastic system, that can be written as mu times f bar y using this manipulation and this can be finally, written as mu divided by R y.

So, this particular relationship is very useful and will be later on used for drawing certain curves and understanding the meanings of those curves. We can see here that the maximum displacement that takes place for the elastoplastic system by corresponding maximum displacement in the elastic system that is finally, written as mu divided by R y or the ductility divided by the reduction factor.

Therefore, if one knows the value of say mu y mu is given and x m is known x 0 is known, then one can find out what is the corresponding reduction factor. Similarly if reduction factor is given and x m and x 0 has given then one can find; what is the ductility.

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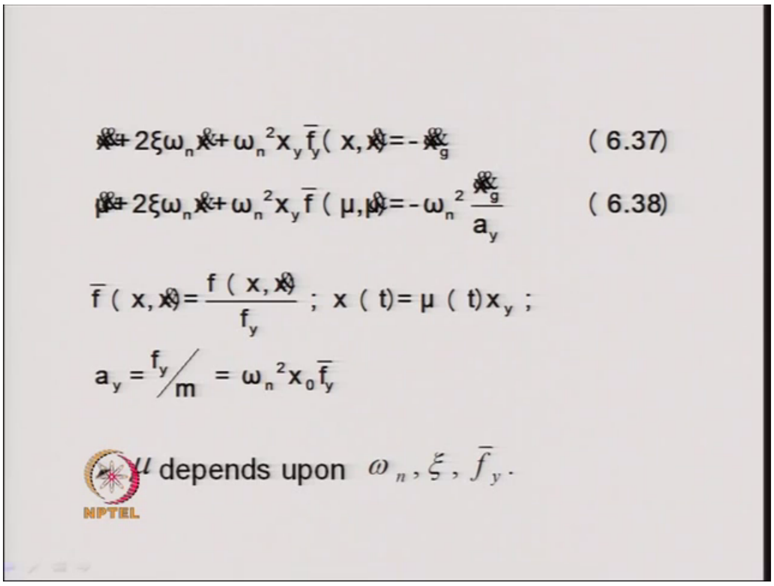

Now with the above definitions the equation motion of motion of single degree of freedom system is given by equation 6.37, these they are x double dot this is double dot this is a single dot and we write down here in since it is not a non-linear system, it is not linear system we have this expression and this expression means that f bar y x comma x that is x dot that is f bar y is a function of both displacement and velocity multiplied by omega n square multiply it by x y. Where f bar x, x dot is written as simple f x dot divided by f y. The entire thing is written in this particular fashion because of a certain reason which will be clear little later.

But here in fact, one should have retained straight away if x comma x dot that is we have the acceleration, we have the damping term and then we have here a function; that means, instead of writing k x for the linear system one should write down a function of x and x dot with a general function showing the resistance of the structure at different stages of loading including the elastic phase and the inelastic phase, this is equal to minus x double dot g.

Now, f x y, f x y dot is the general function one can show that the thing that is written over here is nothing, but f x comma x dot. So, that is shown over here with the help of this derivation.

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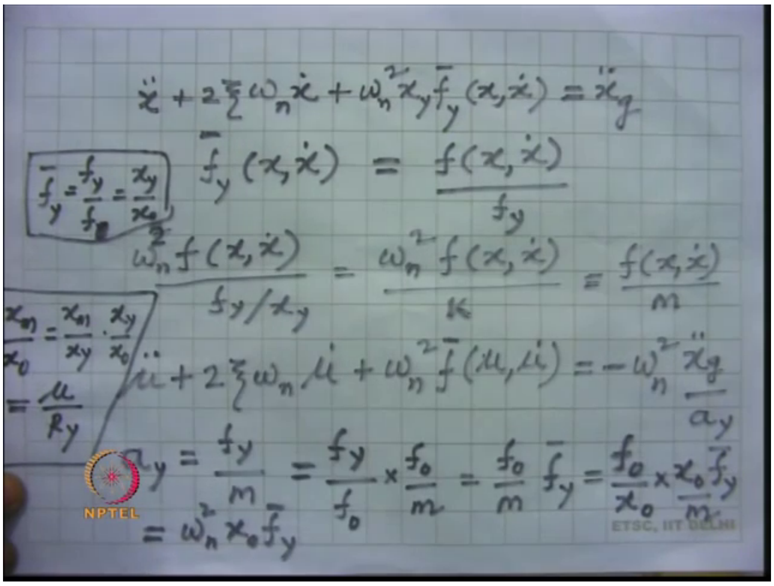

If we look at the equation we are writing down the equation in this particular form, that is f bar y x x dot is written as f x dot divided by f y. Now if I take omega n square if x comma x dot divided by f y that is the definition of your f bar y.

So, in place of f bar y, we are substituting f x comma x dot divided by f y and then divided it by x y. Now if I divide it by x y then the x y automatically goes up over here. So, in the denominator we have f y by x y, since f y by x y will be the stiffness that is how the elastic stiffness is defined f y is the yield displacement and x y is the yield force and x y is the x yield displacement, then f y by x y would give the value of k.

So, this is replaced by a value of k. So, one can write down omega n square if x comma x dot divided by k. Since k by n is equal to omega n square then this turns out to be f x comma x dot divided by n. So, this entire expression that we have taken (Refer Time: 25:04) in to expression is simply is equal to f x comma x dot divided by m, that is what we should have written over here because we have divided all through by m and m on this side therefore, cancels out.

And the generalized force resistive force restoring force is given by a function general function of x and x dot divided by m. So, we deliberately replace this by these particular terms and there is a reason for that. Next week define the value of the mu that is x t over here x t can be written as mu t divided by x y. So, that is the definition of the ductility at any instant of time t that is how much the displacement can take place over and above the yield displacements.

So, therefore, x t in this particular equation can be written as mu t in to x y. So, once we have this relationship, then this can be substituted into the top equation so that the second equation can be simply written in terms of the not x variable, but mu as a variable that is a ductility at every instant of time t. So, these becomes mu double dot t plus twice zeta omega n mu dot t plus this particular term x y term will not be there, it will be omega n square f bar mu comma mu dot that will be there, and is equal to we write down minus omega n square x double dot g by a y.

Now, this ay is defined again in this particular form, a y is equal to f y by m or in other words m multiplied by a y that gives the value of f y or one can say this if it as an yield acceleration a y, yield acceleration multiplied by m gives you the yield force and that can be shown to be is equal to omega n square x 0 divided by f bar y. So, this proof again is shown over here in this derivation, that is we write down first a y to be is equal to f y by m and this is then manipulated like this f y divided by f 0 multiplied by f 0 by m. So, this is equivalent to f 0 by m multiply it by f bar y. Further we write down f 0 by x 0 into x 0 into f bar y divided by m. So, we multiply here by x 0 and divide by x 0, then it turns out to be now f 0 by x 0 is the stiffness of the elastic system corresponding elastic system. So, this becomes k and this is m.

So, k by m becomes omega n square. So, a y can be written as omega n square x 0 multiply it by f bar y. So, that is what is shown here, that is a y is equal to mega n square x 0 f bar y. So, the equation 6.38 which is written in terms of the ductility as a variable, that is now is a this equation now has the following variables omega n psi and f bar y.

So, if we wish to now solve this problem that is equation 6.38 we see that mu depends upon three factors omega n then psi and f bar y because a y will contain and f bar y. So, that is what we have shown over here; if we now want to find out the value of mu by solving this equation 6.38 the value of mu. Obviously, would depend upon not only frequency and the damping of the system, but also by f bar y that is the inverse of the reduction factor.

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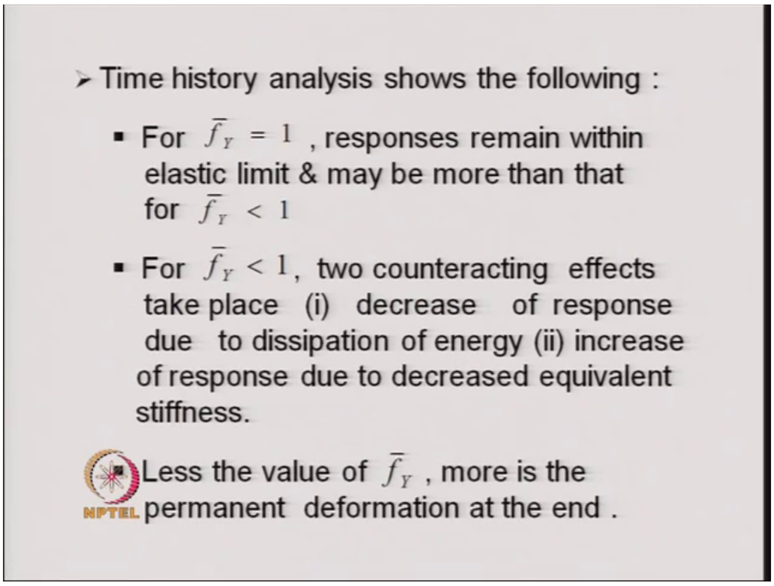

The time history analysis for the second equation shows the following important things. First for f bar y is equal to 1 that is the elastic system responses remains within the elastic limit and maybe more than that for f bar y less than one f bar y less than one means the elasto plastic system. For the elasto plastic system that is f bar y to counter acting effects takes place, first is the degrees of response due to dissipation of energy and increase of response due to decreased equivalent stiffness.

As the single degree of freedom system goes into the inelastic range, there is a dissipation of energy because of the hysteresis loop that is formed or because of the force deformation behaviour of the system and second is the since the system goes into the plastic state or inelastic state is equivalent stiffness is decreased. Thus one can see that the first effect tends to reduce the response of the system.

The second effect tends to increase the response of the system therefore, it is very difficult to say whether in the inelastic range, the oscillation will be a greater or less than the corresponding elastic system less the value of f bar y more is the permanent deformation at the end that is obvious, because of the more value of a say f bar y means that the system is having more inelastic effect. Therefore, at the end of the earthquake episode we observed that their system has undergone some permanent deformation element wise as well as overall structure.

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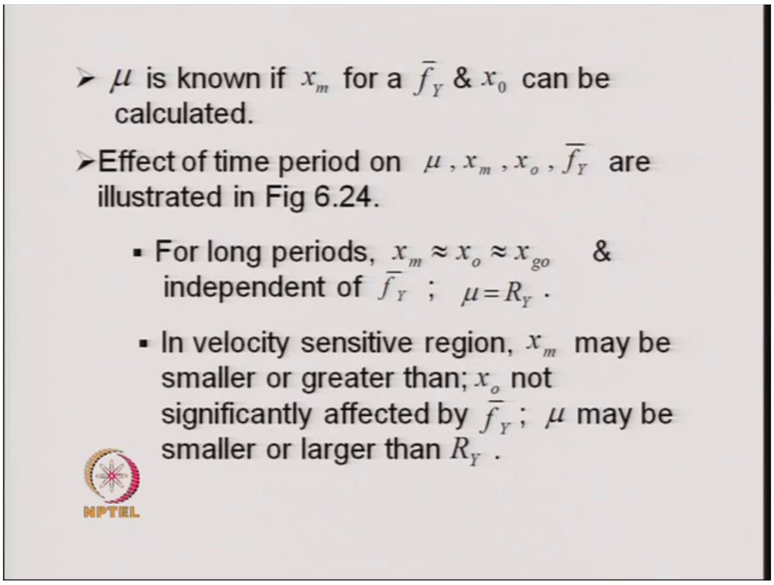

Next mu is known, if x n for a f bar y and x 0 can be calculated. So, that is that follows from the relationship that we have shown before that means this important relationship given by 6.36 equations.

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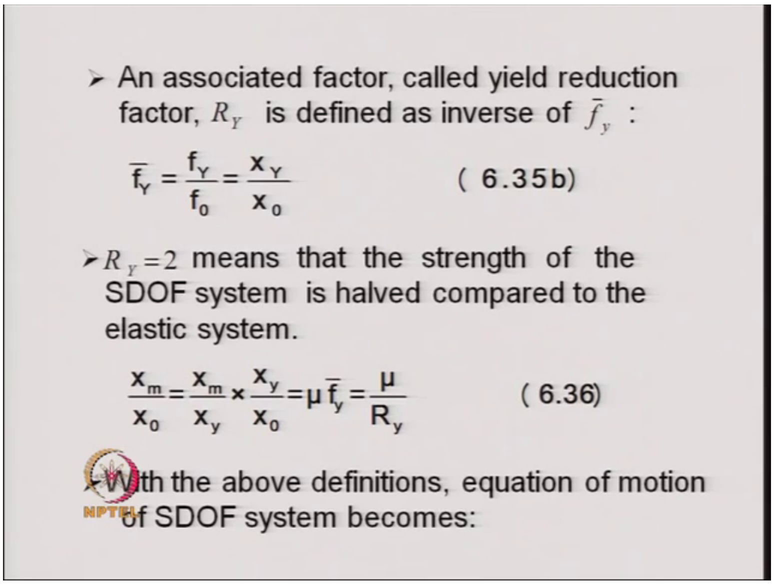

So, it from these equation one can say that provided we know x n f bar y at x 0 one can calculate the value of a mu, that is one of the equation merging out of that equal six point or relationship we merging out of equation 6.36 is that, x my by x 0 is equal to mu divided by R y and R y is a reduction factor it is nothing, but inverse of f bar y.

So, this particular relationship is very important and we will use this relationship later for explaining many figures. Effect of time period on mu x m x 0 and f bar y are illustrated.

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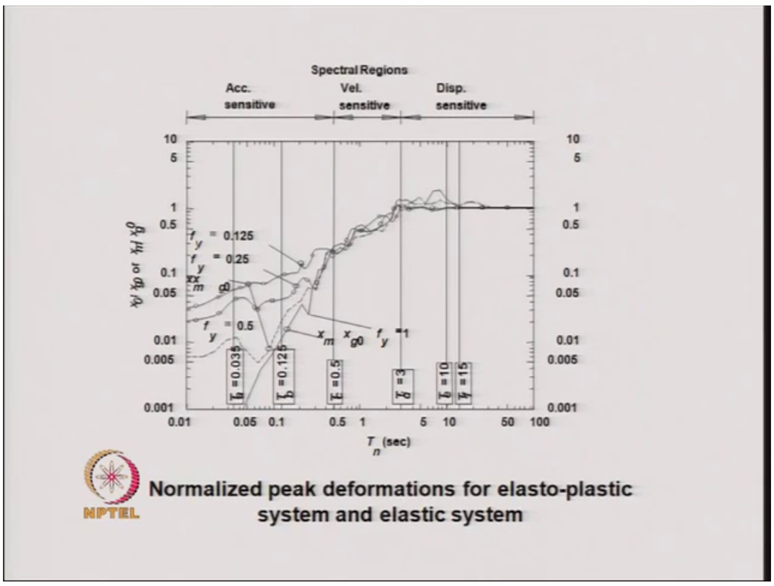

Normalized peak deformations for elasto-plastic system and elastic system

We on this figure we can see that we have the on this ordinate we are plotting x dot 0 divided by psi 0 and divided by x 0 divided by x g 0 and this is x m divided by x g 0.

So, any one of them are plotted over here that is the normalized displacement elastic displacement and the normalized value of the maximum displacement for the elasto plastic case. And this is a on this scale we have got a time period the scales are logarithmic scales. The curves are plotted for different values of f bar y or in other words for different reduction factors R y, f bar y is equal to 1 that denotes the elastic case and all other denote the inelastic k sorry elasto plastic case.

We divide this entire range into three ranges that is the displacement sensitive for range this is the acceleration sensitive range and in between we have the velocity sensitive range that is for large value of t. We call this is a displacement sensitive range, for the low value of the t we call this as the acceleration sensitive range and in between time periods we call as the velocity sensitive range these definitions and the meaning of them are discussed when we are discussing the tripartite I plot for the response spectrum.

Now, from the figure the following observations can be made, first one for long periods that is in the displacement sensitive zone we find that x m is almost is equal to x 0 or is equal to x g 0 and this the value is independent of f bar y, we can see that the here in this particular zone irrespective of the value of f bar y the all the values are more or less the same and here the this is equal to 1. That means, x m is equal to x 0 is equal to x g 0. And therefore, in this range the mu simply is equal to Ry; in velocity sensitive region x m maybe smaller or greater than x 0 or greater than the x 0 value and is not significantly affected by f bar y mu may be smaller or larger than R y.

So, we can see that again from the curve, this is the velocity sensitive range that is the in between region from here to here and in this range you we can see that the value it is not very sensitive to again to f bar y, and ductility can be less or more for different values of f bar y.

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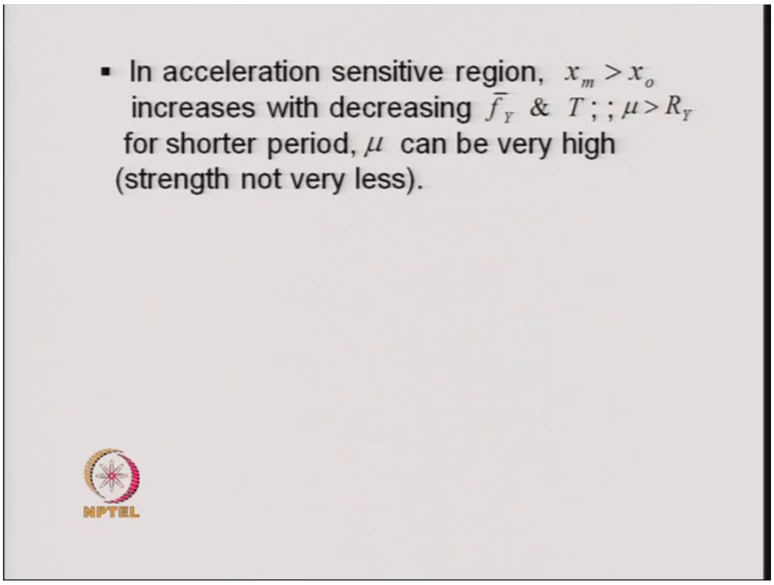

And then is the acceleration sensitive range here we see some interesting things that is in the acceleration sensitive region, x m is always greater than x 0.

Increase with decreasing f bar y and T mu is always greater than R y for shorter period and mu can be also very high. So, this is seen here one can see that in this particular range the values of x 0 by divided by x g 0 or x m divided by x g 0 they are very sensitive to the values of f bar y for and we can see that different values for different values of f bar y these ratios of the non-dimensional displacement ratios they are widely different and specially these difference increases for shorter time period, mu value is greater than R y value over year that is one of the important observations.

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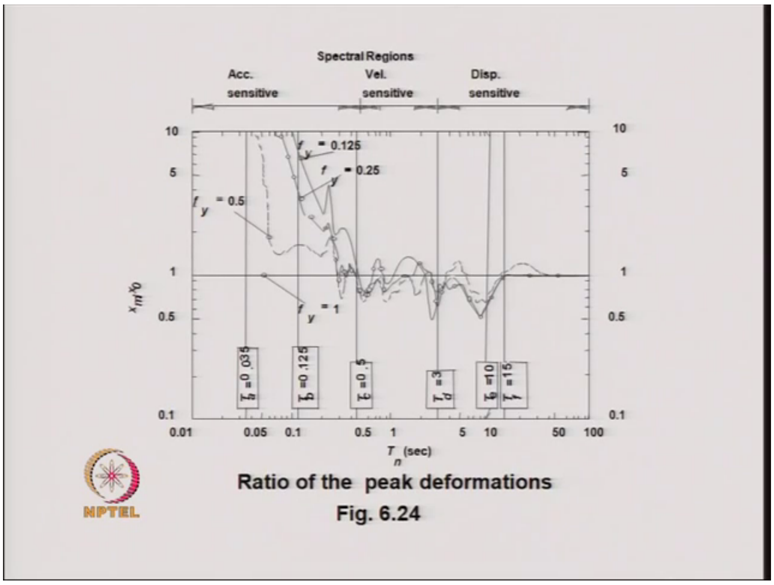

The x m by x g 0 plot is separately drawn over here. So, that we have discussed already in the previous curve.

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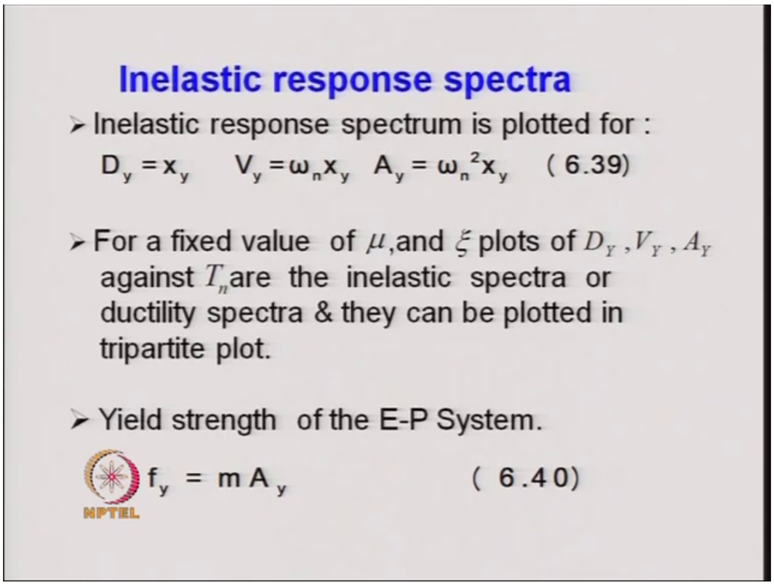

Next, we come to with this understanding of the ductility, we come to the inelastic response spectrum or the definition of the inelastic response spectrum and how one can construct the inelastic response spectrum.

We will see that the inelastic response spectrum and the ductility factor are close related. In fact, the in elastic response spectrum are drawn for specified values of mu, which is a difficult, but we will see that how we can obtain this in elastic spectrums for a given earthquake. Let us first try to define the inelastic response spectrum d y that is the displacement response spectrum or displacement inelastic response spectrum is simply is equal to x y that is the if we plot x y that this is the yield displacement over entire time period then we get the displacement in elastic response spectrum.

Similarly, the velocity in elastic response spectrum or pseudo velocity inelastic response spectrum is defined as omega n times x y the way we did for pseudo velocity spectrum in the case of the elastic response spectrum and the acceleration inelastic response spectrum is equal to omega n square into x y, that is the displacement inelastic response spectrum is multiplied by omega n square like we have done in the case of the elastic response spectrum.

So, plot of these quantities can be made against d n and that would give us the inelastic response spectrum. We can see that because of the relationship that holds good between A y by and A y they can be plotted in a tripartite plot like the elastic response spectrum. So, that is a very advantageous thing because we will see later that if we know the elastic response of a single degree of freedom system, drawn on a tripartite plot then from that one can obtain inelastic response spectrum for different values of the ductility factor mu.

For a fixed value of mu and xi lots of D y V y and A y against t m are the inelastic spectra or ductility spectra and they can be plotted in a tripartite plot that is what I told you. So, here the parameters are that the mu and psi these are two parameters, which control the inelastic response spectrum as against the elastic response spectrum where we do not have the concept of mu a task to the spectrum, only we have the damping ratio xi attest to the spectrum.

However, mu is equal to 1 and for that if we plot the spectrum then it will automatically become the elastic response spectrum. The yield strength of the elasto-plastic system can be written as f y is equal to m in to A y that is the yield acceleration multiplied by mass that we have described before yield strength for a specified mu is difficult to obtain, but reverse is possible by interpolation technique.

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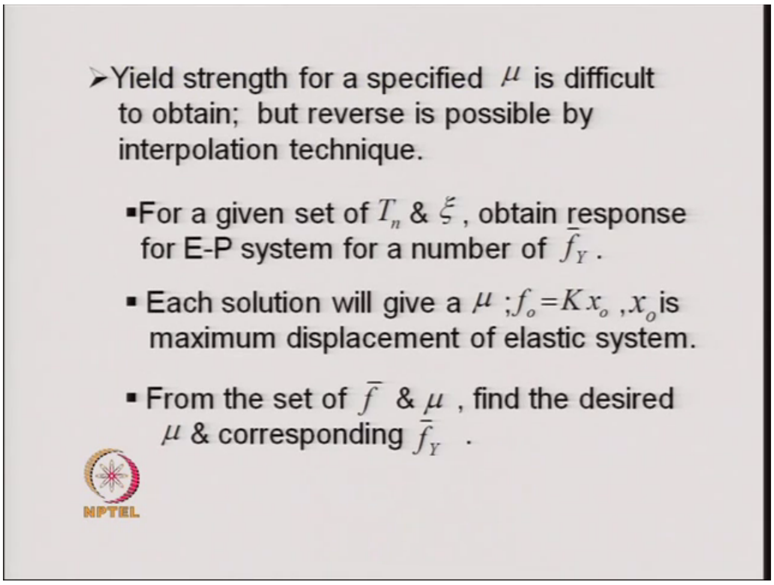

So, that is a very important statement, we want to define the response spectrum for a specified value of mu.

However, it is very difficult to find out what is the yield strength for a specified value of mu, but the reverse thing can be done because of the equation that we had performed before and that was the intent of writing down 6.38 equation, that is converting the x x dot variables into u mu mu dot variables. If we solve this equation for different values of a y or different values of f bar y then these for different values of f bar y we can get different values of mu. So, that is why we have converted 6.37 equation into the form of equation 6.38.

So, for the yield strength is known provided f bar y is known, because f bar y is defined as f f 0 divided by f y we have seen before therefore, the yield strength can be immediately obtained provided the value of the mu is known. So, what we do is that we go for interpolation technique and an iteration for given set of T n and psi we obtain the responses for the elastoplastic system for a number of f bar y values that is we solve the equation 6.38 for different values of f bar y and find out the values of the mu the each solution gives the value of mu and f 0 is given is equal to k in to x 0 x 0 is the maximum displacement of the elastic system; that means, the same equation can be solved also considering it to be a absolutely elastic system.

So, the for the same earthquake one can solve the single degree of freedom system considering as if it is a elastic system. So, that will give us the value of x 0 and correspondingly one can find out f 0. From the state of the values of f bar and mu find the desired mu and the corresponding f bar y. So, if you have a number of combinations of f bar and mu from one can find out the desired value of mu say for mu is equal to two or 3or 4 what will be the corresponding interpolated values of f bar y.

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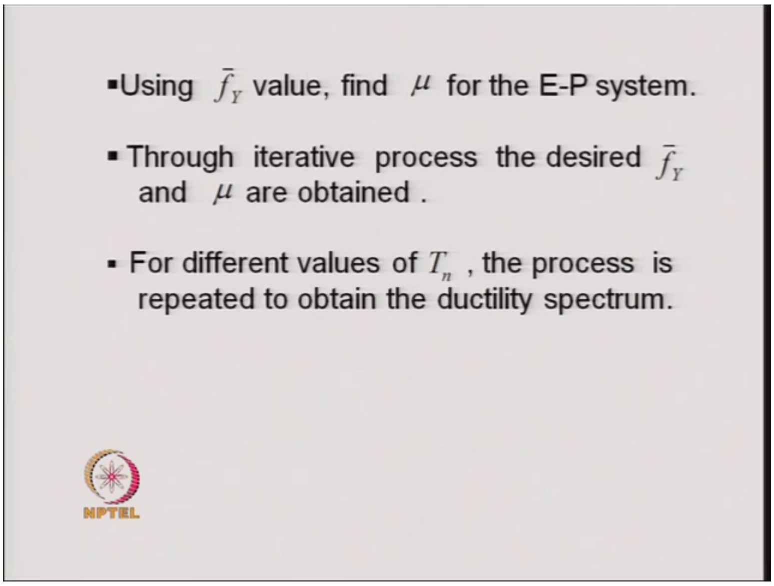

Then what we do that that interpolated value of f bar y and the mu this interpolated value of f bar y now is used in the equation 6.38 and it is solved, and value of mu is obtained and if you find that the value of mu that we have interpolated and the value of mu that we have obtaining from equation 6.38, if they are same then we say that a convergence is taken place otherwise what we do we through iterative process we find out the values of f bar y and mu.

So, through some iteration process one can get value of f bar y corresponding to a given mu value. So, once you know that then it is possible for us to obtain the values of the x y then a y and the d y and the v y or v. So, the in elastic acceleration inelastic displacement inelastic velocity, they can be obtained provided we know the value of yield displacement and to know the yield displacement we must know the yield strength value and to yield strength we must know the value of f bar y.

So, what we do is that for different values of the time period T n, we repeat these entire exercise and we plot the ductility spectrum.

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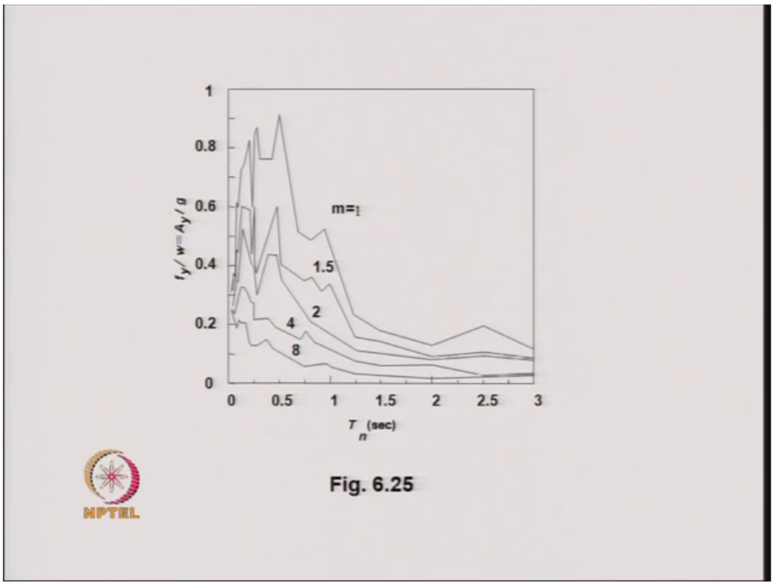

Fig. 6.25

So, a ductility spectrum which is the acceleration ductility spectrum is shown over here it is not m it will b is equal to mu. So, for mu is equal to 1 that means, which is the elastic response spectrum. So, a mu is equal to 1.5, mu is equal to 2, mu is equal to 4, mu is equal to 8 we can see the accelerations or inelastic accelerations and these inelastic accelerations for different values of the mu value is plotted here and one can see that for higher values of mu the value of the spectral acceleration is drastically reduced compared to the elastic response spectrum.

So, as the system goes into the inelastic range, the effective force that is coming on the system gets reduced.

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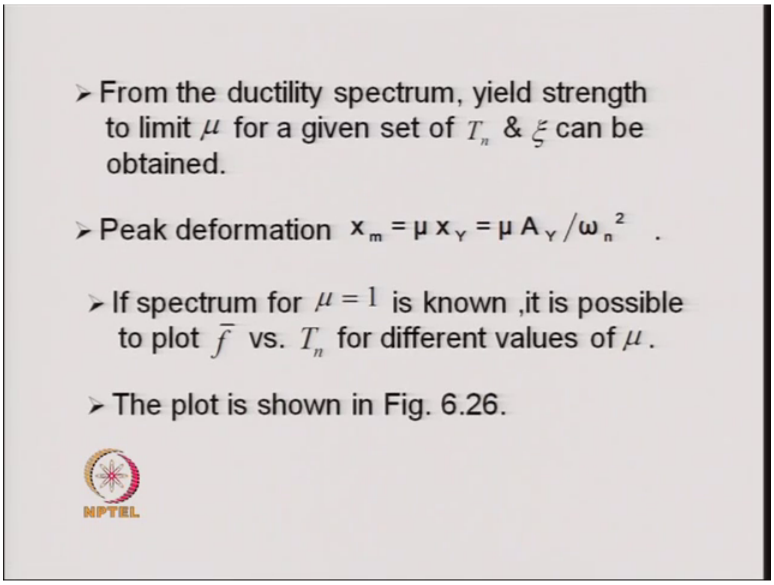

From the ductility spectrum yield strength to limit mu for a given set of T n, T n and psi can be obtained.

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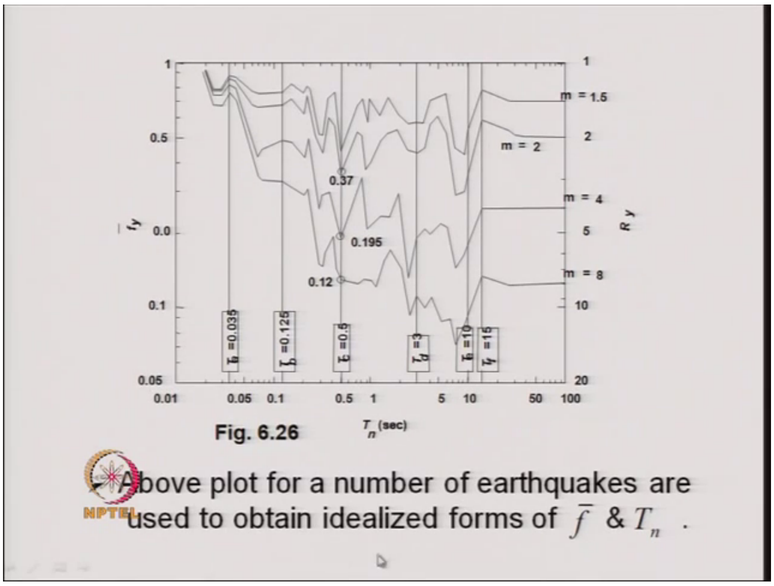

So, that we can see over here in this particular curve the f bar y and T n that is plotted over here for different values of the mu. So, there mu is equal to 1.5, mu is equal to 2, this is not m. So, there will be or all mu equal to 1.5 or 2, 4, 6, 10 so on and for these we have curves plotted these curves are nothing, but f bar y against a T n and that is the aim that we had.

So, for a given value of mu, we can find out what is the value of f bar y. So, once we do that then you take any time period and for a specified value of mu, one can immediately get the value of f bar y and once we get the value of f bar y, then from there we can get the value of f y and once we know the value of f y we can get the value of x y. And the once you know the value of x y then the inelastic response spectrums are defined, because the displacement inelastic response spectrum will be equal to x y simply, the velocity inelastic spectrum will be x y multiplied by omega n and the inelastic acceleration response spectrum will be equal to omega n square multiply it by x y or d y.

So, that is how we finally, obtain the value of the or finally, brought the spectrums inelastic spectrums for given values of mu. So, let me summarize here what we do and then we will proceed with it for constructing the values or construct and in elastic response spectrum. So, what we have done over here is that we have essentially solved equation 6.38 for different values of f bar y assumed, and each f bar y value provided a value of mu and that is how one can have a collection or a collection of the combination of f bar y and mu one for a particular time period T n and psi.

Once we have that then from there say we are wanting to construct the response spectrum in elastic response for spectrum mu is equal to 2. So, we find out from interpolation what is the value of f bar y corresponding to the value of mu is equal to 2, from the set of values that we have got after the analysis.

Once we know the value of f bar y then that f bar from that f bar y, one can find out the value of f y and once we know the value of f y then one can value know the value of x y and inelastic response spectrum ordinate for that particular T n and psi combination is known. So, that is how the inelastic response spectrums are obtained.

Thank you.