

**Seismic Analysis of Structures**  
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**Lecture – 03**  
**Seismology (Contd.)**

In the previous lecture, we discussed about the earth quake measurement parameters, earthquake measuring equipment and the modification of the earthquake waves due to the soil condition. In earthquake measuring parameters we discussed about 2 measure earthquake measuring parameters, that is the magnitude of earthquake and the intensity of earthquake. Both of them denote the size of the earthquake that has occurred. The magnitude of the earthquake, express the size of the earthquake in an objective fashion. That is, it is denoted by some measurement of the ground motion, which is then expressed as a magnitude by certain definition.


Why does the intensity of earthquake is a subjective measure? By looking at the damages and destructions that have taken place at different places. We estimate the size of the earthquake. The intensity of earthquake was prevalent mostly in olden days when we did not have a very good equipments for measuring on the ground motions produced due to earthquake. Therefore, from the visual observation people used to determine the size of earthquake that has taken place. And the observations were categorized under different categories and from that one can understand the size of the earthquake that has taken place. Accordingly a intensity scale was devised and the earthquake used to be measured in that with the help of that particular scale I call the intensity scale.

In the magnitude of the earthquake the thing that is required is to measure the ground motions or the ground motions giving rise to the particle acceleration velocity and displacements. And these measured quantities are utilized in expressing the magnitude of the earthquake. And there are certain fixed definition for the magnitude of earthquake that you have discussed in the previous lecture. So, far as the earthquake measurement equipments are concerned we discussed about the seismographs.

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**Seismic hazard analysis**

- It is a quantitative estimation of most possible ground shaking at a site.
- The estimate can be made using deterministic or probabilistic approaches; they require some/all of the following:
  - Knowledge of earthquake sources, fault activity, fault rupture length.
  - Past earthquake data giving the relationship between rupture length & magnitude.

 Historical & Instrumentally recorded ground motion.

Specially the Wood Anderson seismograph that was the old seismograph or rather the oldest seismograph and the principle of working of the seismograph was described. Then we talked about the long period and the short period seismograph providing the displacement and the acceleration records directly. After that we discussed about the modification of the ground motions due to the soil condition. And mainly we pointed out the kind of modification that takes place for hard soil condition for soft soil condition and what is the kind of energy concentration that takes place when we have a modification of the ground motion due to the presence of the soft soil. In this lecture we will look at the seismic hazard analysis, which is very important to earthquake engineers. The seismic hazard of a particular site it denotes the hazard potential of the site for an earthquake. By hazard potential what you mean is that, the peak maximum peak ground acceleration that can take place in that particular site or the different kind of damages that can occur due to the earthquake at the site.

Mostly the sites for which we perform a seismic hazard analysis at the sites in the vicinity of which we have certain fault lines or earthquake sources. Specially the presence of earthquake sources which are quite active, for those earthquake sources which are our surrounding that particular site we perform a seismic hazard analysis of a particular region or the site. There are quite a number of objectives behind the seismic hazard analysis. Firstly, a seismic hazard analysis enables us to define the maximum

value of the peak ground acceleration and for which the structures should be designed in that particular region.

Also many a time if we can have some systematically recorded earthquake data, then from that one can obtain straight away the future response spectrums for which the structures should be a design for future earthquakes. Secondly, the seismic hazard analysis is very much useful in obtaining the seismic or in determining the seismic risk of structures in that particular region. In that what is done is that the vulnerability of the structure is first assessed or obtained. Then after that the vulnerability of the structures for a particular earthquake is then combined with the vulnerability of the region for different sizes of earthquake.

So, these combined result ultimately provides a seismic reliability analysis of a structure in the region. Also the seismic hazard analysis is important in obtaining the microzonation map of a particular region. This microzonation map is utilized in many ways one of the most important utilization of the microzonation map. One of the major utilization of the microzonation map is in obtaining a systematic planning of a particular region safe against earthquake. Also the microzonation map helps in providing the resource allocation to different parts of the region for earthquake protection.

The seismic hazard analysis is a quantitative estimation of the most possible ground shaking at a site. The estimation can be made using deterministic or probabilistic approaches. They require some or all of the following. Firstly, the knowledge of earthquake sources fault activity and fault rapture length. Now the knowledge of earthquake sources mean that the whether the earthquake source is a line source or it is a area source or it is a point source. Whether is the fault activity means the whether the fault is an active fault. In the sense that there is an earthquake that is frequently occurring from that particular fault. The date faults are the ones in which it is found that for a long period of time there is a no earthquake that has been observed coming from that particular fault line.

So, that is what we understand by fault activity. The other important thing is the fault rapture length that we have discussed before. We have seen that whenever there is an earthquake then around the fault line there is a rapture or the existing for gets modified because of the earthquake. And these fault rapture length is observed from the kind of


damages or destruction that takes place at the epicentral region. And from there one can assess the fault rupture length. Also these days we have data previous data earthquake data available which basically gives a relationship between or empirical relationship between the rupture length and the energy release of a particular earthquake. So, therefore, by measuring the ground displacements or ground accelerations at different places one can assess what is the are energy release of the earthquake and from that energy release one can indirectly obtain the fault rupture length.

So, the first part is to assess the earthquake sources that are existing in the vicinity of the site in terms of fault activity nature of the earthquake source and the fault rupture length. Next is the past earthquake data should be available at the particular site. So, that one can build up a relationship between the rupture length and the magnitude of the earthquake that is what I told you before. The magnitude of for the earthquake that takes place in that particular region should be a systematically collected and I must be available. So, that one can obtain a probability density function for the magnitude of earthquake coming from a particular fault line. And it is a very important and we will see later that this probably density function of the magnitude of earthquake is utilized in obtaining the probabilistic seismic hazard analysis.

Apart from that the historical and instrumentally recorded earthquake and data or the time history records of the earthquake data must be available. And these earthquake data can provide peak ground acceleration peak ground velocity or peak ground displacement. Or in other words the time histories of any one of these quantities must be available. And if the response spectrum of the earthquake or the previous earthquakes they are available then that can also be usefully use employed for obtaining the seismic hazard analysis.

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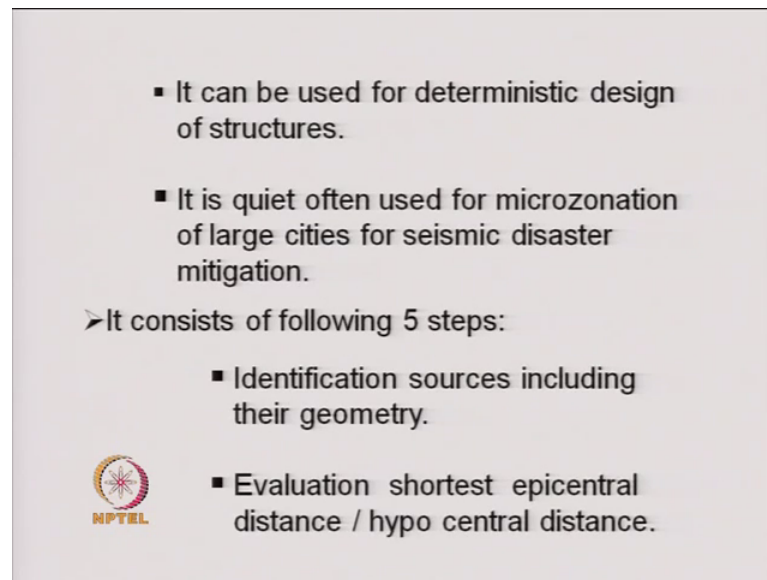
- Possible ground shaking may be represented by PGA, PGV, PGD or response spectrum ordinates.
- Deterministic Hazard Analysis (DSHA):
  - A simple procedure to compute ground motion to be used for safe design of speciality structures.
  - Restricted only when sufficient data is not available to carry out PSHA.
  - It is conservative and does not provide likely hood of failure.



Now as I said before we have the 2 types of seismic hazard analysis. One is the deterministic seismic hazard analysis the other is the probabilistic seismic hazard analysis. In the deterministic seismic hazard analysis, the computations that are to be carried out are quite simple. Therefore, it is quite often used by practicing engineers for finding out the seismic hazard of a particular region. It is a simple procedure to compute ground motion to be used for safe design of structures in general and for specially these structures in particular specially for the kind of the structures like hospital buildings or very big halls or school buildings for that one should have a specifically a peak maximum peak ground acceleration for that particular site for which those structures must be design.

These deterministic seismic hazard analysis is generally performed for cases where the sufficient earthquake data is not available at the site. In the sense that the we cannot carry out the probabilistic seismic hazard analysis for the site. Therefore, we take it close to the deterministic seismic hazard analysis with the limited data that you have it is also conservative and does not provide likelihood of failure. That is the probability of the accidents of certain and level of the ground acceleration is not available from the deterministic seismic hazard analysis. In the deterministic seismic hazard analysis one only specifies the maximum earthquake parameter which should be taken into consideration for the design analysis of structures in the region.

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▪ It can be used for deterministic design of structures.

▪ It is quite often used for microzonation of large cities for seismic disaster mitigation.

➤ It consists of following 5 steps:

- Identification sources including their geometry.
- Evaluation shortest epicentral distance / hypo central distance.




it can be used specifically for the deterministic design of structures probabilistic design of structure is not possible from the results of the deterministic seismic hazard analysis.

The deterministic seismic hazard analysis is quite often used for microzonation of large cities for seismic disaster mitigation. In that the entire city is microzon into smaller regions and for each region one can specify the maximum level of the ground acceleration for which the structures should be design in that particular sub zone or sub area. The deterministic seismic hazard analysis consist of the following not 5 steps 4 steps. First one is the identification of sources including they are geometry. As I told you before these sources could be a point source, could be a line source, could be an area source. So, one has to identify what kind of sources of earthquake that are existing surrounding the site. Then the evaluation of the shortest epicentral distance or hypo central distance; that means, if it is a line source or in an earthquake source, then one has to find out what is the shortest distance between that line source or the area source to the site in question.

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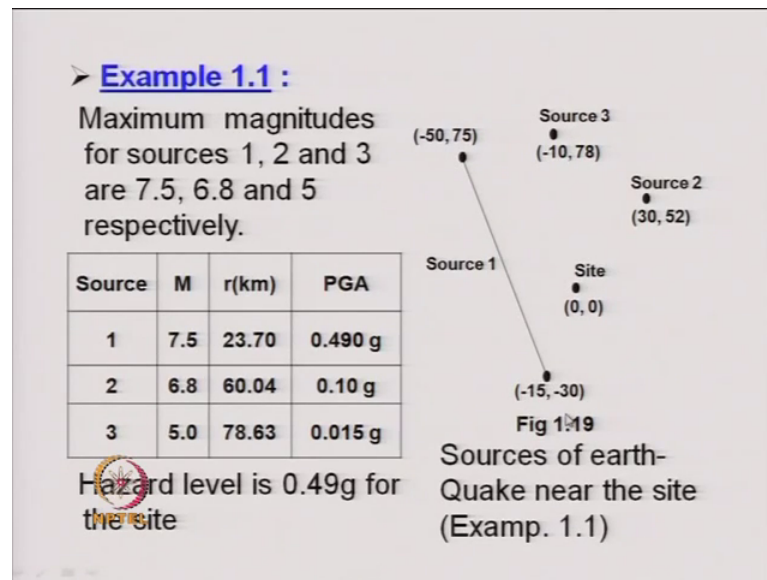
- Identification of maximum likely magnitude at each source.
- Selection of the predictive relationship valid for the region like,  
$$\ln \text{PGA (gals)} = 6.74 + 0.0859m - 1.80 \ln(r+25)$$
- The method is demonstrated with the help of an example to find the design PGA in a region surrounded by several faults.



So, that is another important thing that is to be carried out for those kinds of sources. Then identification of maximum likely magnitude at each source. So, this is generally obtained from the past earthquake recorded data. And from that data one can identify what is the maximum level of the magnitude of earthquake and that is expected at the site. And then finally, one has to select an appropriate predictive relationship valid for the region. And this predictive relationship is one of the predictive relationship I shown over here. These predictive relationship gives or predicts the peak ground acceleration at a particular place given the magnitude of earthquake and the epicentral distance of that particular place.

So, this is an empirical equation and these kind of empirical equation is required in the seismic hazard analysis of a particular site. In fact, these predictive relationship constitutes the most important equation that is used in the seismic hazard analysis of a structure. Therefore, one has to find out the appropriate predictive relationship that is valid for the region. If it is not available as such then one has to see that what is the most similar site that is site for which the this kind of predictive relationship is existing and then one can use that predictive relationship for this for the deterministic or progressive seismic hazard analysis of that particular site.

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The method of the deterministic seismic hazard analysis is demonstrated with the help of these example. This is the site in question and surrounding the site we have got 3 sources of earthquake these 2 sources of earthquake. At the point sources, this is a line source. And the sources or the epicentral distance for the different sources that can be computed from the coordinates that are given over here. For finding out the epicentral distance for the site with respect to the source we simply joined and these 2 points. For this source we simply join this 2 points and get the distances. So, far as finding out the epicentral distance for the line source, we take the shortest distance of the site from the source and for that we drop a perpendicular from the site to this line and that distance becomes the epicentral distance we take the shortest distance because the if you look at the equation then we find that for the shortest distance would provide the maximum effect on the particular site coming from this source.

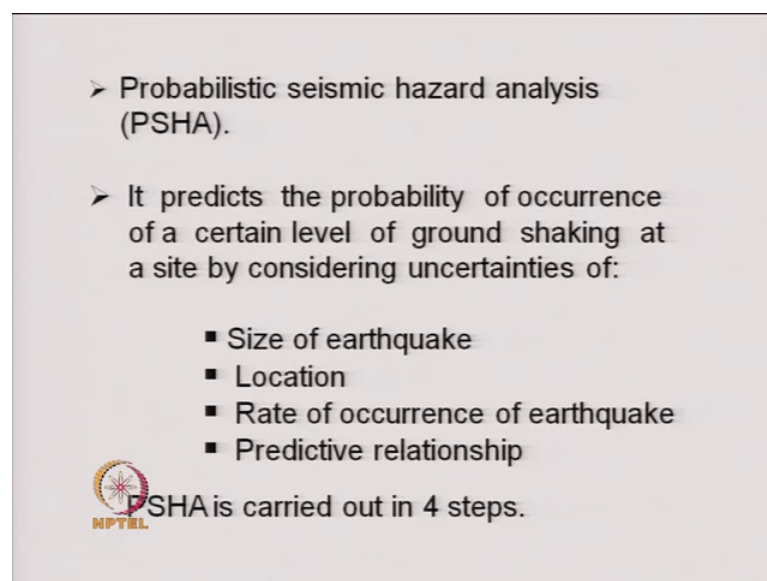
The maximum magnitudes which are specified for these 3 sources are 7.5 for source 1. 6.8 for source 2, and for source 3 it is 5. So, they are put in a tabular form over here for source 1 2 3, we have the magnitudes recorded as 7.5 6.8 and 5.0. The epicentral distances that were calculated, 60.04 70.63 for source says 2 and 3, and for source 1, 23.70 that is the perpendicular distance of the site to a the line source. Then once we have the epicentral distance, then we put in the values of the magnitude maximum magnitude of the earthquake and epicentral distance into this equation. And we get the value of peak ground acceleration in gals that is centimetre per second square.



So, those values of the peak ground accelerations are computed and expressed in g units and they are shown over here. For the source 1, we have a peak ground acceleration of 0.49g. For source 2 it is 0.1g. And source 3 it is 0.015g. Thus the maximum peak ground acceleration that is expected at the particular site is 0.490g. And we say that the hazard level and for the particular site is represented by a peak ground acceleration of 0.49 g or in other words in designing any structure in that particular site we should take a peak ground acceleration of 0.49g.


Now, for this particular problem even without performing this kind of calculation one can see that it is easily or it is very apparent that line source or the source one we will have or dictate the hazard level for the site because the distance epicentral distance for this particular source to site is the least amongst them and also the magnitude of earthquake or expected magnitude of earthquake in that particular source is the largest of the 3. Therefore, it is quite obvious that this particular source would determined or dictate the hazard level for the site; however, there could be problems in which even if a particular source is very close to the site, but the magnitude of earthquake that is expected in that particular source maybe small compared to others. In that case one has to calcu perform this kind of calculations and put the quantities in a tabular form in order to identify the maximum hazard level.

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- Probabilistic seismic hazard analysis (PSHA).
- It predicts the probability of occurrence of a certain level of ground shaking at a site by considering uncertainties of:
  - Size of earthquake
  - Location
  - Rate of occurrence of earthquake
  - Predictive relationship

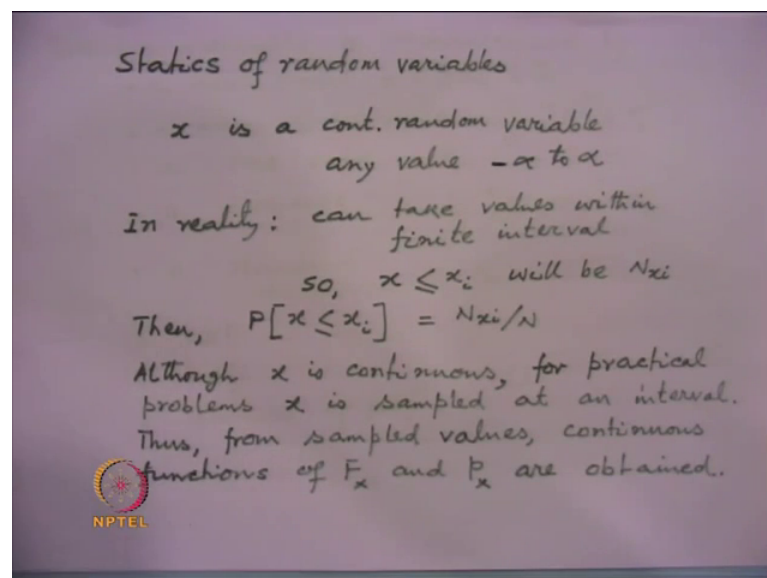
PSHA is carried out in 4 steps.



Now, we come to the probabilistic seismic hazard analysis. This probabilistic seismic hazard analysis is carried out when we have sufficient recorded data for that particular site coming from different sources of earthquake. And the data that are recorded will permit to obtain the different quantities of interest that is required in performing the probabilistic seismic hazard analysis. Now since the probabilistic seismic hazard analysis requires some elementary knowledge of the statistics of random variable, we must have an idea about them. It is presumed that you have some all already some knowledge or the elementary knowledge of the statistics of random variable.

however, for the completeness and for recapitulation of those statistics let me first describe in connection with the probabilistic seismic hazard analysis.

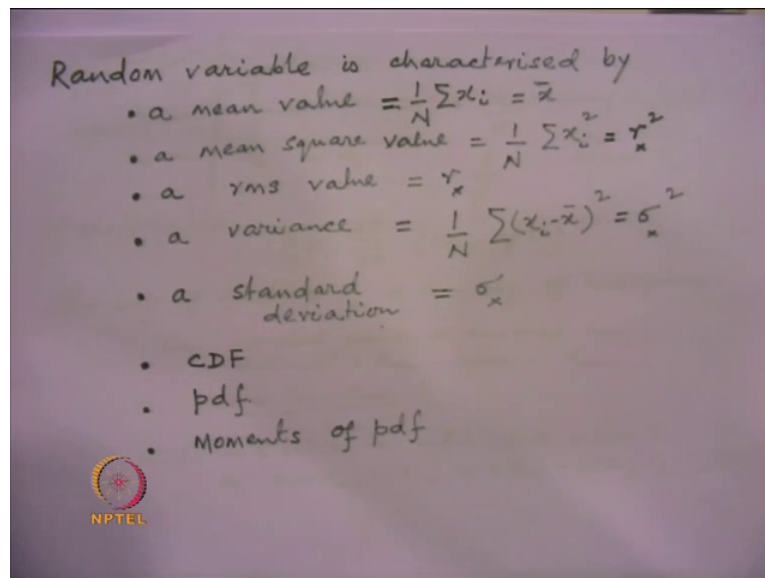
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The first is the statistics of the random variable is obtained with the help of certain quantities which are called the statistical quantities of a random variable. Let us consider  $x$  to be a continuous random variable. Means that  $x$  can take any value between minus infinity to plus infinity; however, in a reality the  $x$  can take values only within some finite interval and the  $x$  values are also discretized in the sense that the  $x$  values are sampled at certain interval therefore, making it a the quantity which is in fact, not a continuous random variable, but a random variable which can take a number of discrete values between the finite interval.

So, now let us say we are interested in finding out the probability of  $x$  being less than equal to a specified value  $x_i$ . Then we can count the number of values of  $x$  and that are less than or equal to the value of  $x_i$ . And let that number be  $n_{x_i}$ . Then we say that the probability of  $x$  being less than equal to  $x_i$  is equal to  $n_{x_i}$  by  $n$ . Although we said that  $x$  is a continuous for practical problems, we get the values of  $x$  as sampled values at an interval. That is what I told you before; however, from that sampled values we can obtained the probability of the particular value being less than a certain level in this particular fashion. And then from the plot of then we can obtain a continuous car by joining the points and express the probability density function or the community distribution function as a continuous a curve.

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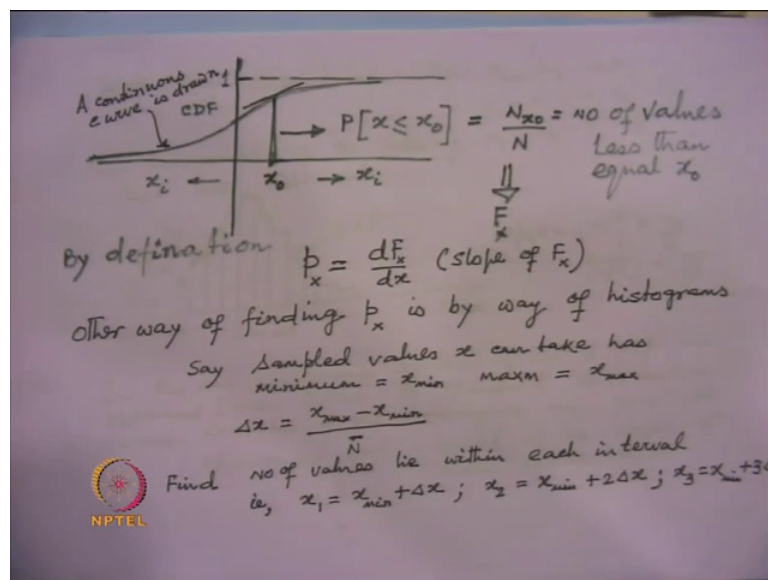
So, that is a the first part of the random variable  $x$ . Now these random variable  $x$  is characterized by certain quantities which we called as the statistics of the random variable. Firstly, we express the random variable by a mean value. And the mean value is described by this a particular equation, that is 1 by  $n$  and summation of the all the possible all values that the  $x$  can take. And that gives the mean value  $\bar{x}$ . Next week represent it by a mean square value and the mean square value is defined by this that is 1 by  $n$  summation of the square of all the possible values that  $x$  can take.

Then we take sometime represent also it by RMS value, which is a simply a square root of the mean square value. We can define also the variance of the random variable. In the

variance is defined as  $\frac{1}{n}$  multiplied by summation of  $x_i - \bar{x}$  whole square. That is for all the values that  $x$  can take from those values we deduct the mean value and then take a square of that and add them and divide by  $n$  and that gives the variance of the random variable. And the standard deviation is simply the square root of the variance. Then we try to describe the cumulative distribution function, probability density function and movements of the probability density function.

So, these are some of the elementary statistics of a random variable that we must know in order to deal with a random variable and carry out the probabilistic seismic hazard analysis.

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if we talk of the cumulative distribution function, then the cumulative distribution function is drawn in this particular fashion. Say there is a sampled number of sampled values of  $x$ . Then we can obtain the probability of  $x$  being less than a particular value  $x_0$  that will be given by  $n \times x_0$  on that is the number of values of  $x$ , that are below or equal to  $x_0$  and that divided by  $n$  and that we put as an ordinate over here.

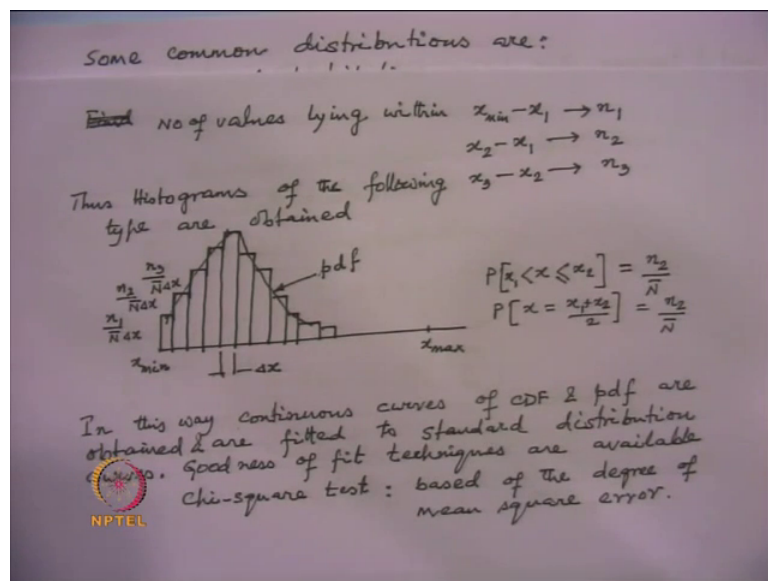
Similarly, for different levels of  $x_0$ , one can have such ordinates. And if we join the points of those ordinates, then we can get a curve and this curve is it becomes a continuous curve and we say that the this denotes the cumulative distribution function for the curve. Now if a it is a possible to give a mathematical expression for this continuous curve, then we have an expression mathematical expression for the cumulative

distribution function in terms of the variable  $x$  and some other statistical quantity that will see later. Next is the probability density function. The probability function is defined as the slope of the cdf curve at a particular point and this is denoted by  $dF_x/dx$ . So, therefore, if we have a mathematical expression for  $F_x$  or the cumulative distribution function in terms of  $x$ , then one can differentiate that with respect to  $x$  for every point and then can obtain an expression for the probability density function.

However, we can straight away find out the probability density function from the sampled values itself the way we have obtained the cumulative distribution function. Say the sample values of  $x$  can take can have some maximum value, that is the  $x_{max}$  and a minimum value which is  $x_{min}$ . Then we divide the interval between the  $x_{max}$  and  $x_{min}$  by a number of discrete value or by a number. That gives an interval or the sampled interval  $\Delta x$ . The larger the value of  $n$  smaller becomes the value of  $\Delta x$  and better becomes the estimate of the probability density function.

Now, we find out the number of values which lie within each interval. That is we say  $x_1$  to be is equal to  $x_{min}$  plus  $\Delta x$ . And  $x_2$  as  $x_{min}$  plus twice  $\Delta x$  then  $x_3$  as  $x_{min}$  plus 3  $\Delta x$ . In this way we can have the values of  $x_1, x_2, x_3, x_4$ . So, on till we come to the value of  $x_{max}$ . So, the values between  $x_{min}$  and  $x_{max}$  now is represented by a set of values which is varying between  $x_1$ , oh  $x_{min}$  to  $x_{max}$  and these values are first obtained.

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after we have obtained those values. Then we count the number of values that are lying between  $x_{\min}$  and  $x_1$ , let us say this is equal to  $n_1$ . Then between  $x_2$  and  $x_1$  let it be  $n_2$  and so on. That is  $x_3$  minus  $x_2$  within this interval let the values be  $n_3$ .

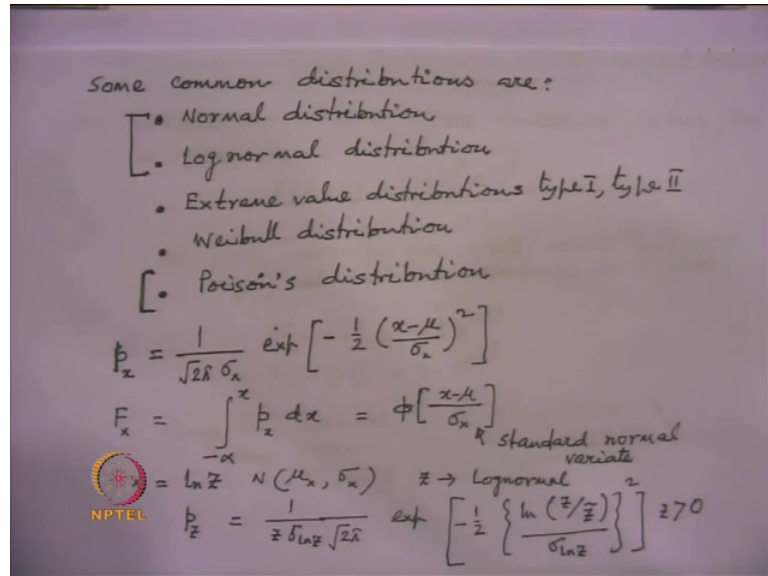
Now, once we have that then, one can obtain these quantities that is probability of the  $x$  line between  $x_2$  and  $x_1$  will be; obviously, equal to  $n_2$  by  $n$ , where  $n$  is the total number of values that exist between  $x_{\min}$  and  $x_{\max}$  including  $x_{\min}$  and  $x_{\max}$ . Similarly, the value of the  $x$  probability of  $x$  being equal to  $x_1$  plus  $x_2$  by 2 will be given by  $n_2$  by  $n$ . So, the centre of the interval that is denoted by  $x_1$  plus  $x_2$  by 2. And we say that what is the probability that  $x$  will be equal to this value will be also given by  $n_2$  by  $n$ . Or in other words we assumed that within the pocket interval of  $x_1$  and  $x_2$ , whatever values we obtain those values are the values from those values you construct what is the probability of  $x$  being equal to the average ordinate between the pocket interval  $x_1$  and  $x_2$ .

So, utilizing that one can draw this histogram it starts with  $x_{\min}$  over here. And this particular value is equal to  $n_1$  by  $n$  and we divide it by  $\Delta x$ . Similarly, the next histogram is equal to  $n_2$  by  $n$  into  $\Delta x$  and next one is  $n_3$  by  $n$   $\Delta x$  and so on. Now if we join the centres of these histograms and plot a curve then this curve would give you the probability density function of the random variable  $x$ . And if you are able to give a mathematical expression to this probability density function then the pdf can be expressed as a mathematical function. So, that is how one obtains the in real in real situation obtains the cdf and pdf expressions or mathematical expressions for any random values which are sampled.

Now, this continuous functions that we are continuous curves that we have obtained for a cdf and pdf. They are fitted to some standard distribution curves. And if we find that those the curves that you have obtained that is the pdf and cdf curve or confirming to some standard distribution curves then we say that the probability density function or the cumulative distribution function of the random variable is confirming to a standard distribution. And for that we carry out what is known as a goodness of fit test in which we have a many kinds of test that are possible one of the common kind of type of test that is carried out is the chi square test. And this chi square test is based on the degree of mean square error and from the degree of the mean square error we denote the goodness

of the feet between the target distribution and a distribution and that we have obtained from the numerical data.

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Next is the some common distributions that are utilized in the earthquake engineering and structural engineering and in civil engineering. These a distributions common distributions are the normal distribution, lognormal distribution, extreme value distribution type one and type 2 Weibull distribution and a Poisson distribution. Now out of these distributions we find that normal distributions and a lognormal distributions, they are quite often used in many engineering problems and specially for the cases which are which are the natural processes. And it is generally seen that any natural process if we consider it as a random quantity or then is distribution that is the cumulative distribution function or the probability distribution function or density function conform to the normal distribution and lognormal distribution also is quite common.

The extreme value distributions are the distributions where we talk of the distribution of the maxima. That is for example, if we wish to find out the distribution of the maximum wind speed in a particular year. Then for each day one can calculate what is the maximum value of the wind speed and so that way we can have 365 values of the maximum wind speed and if we find out a distribution using those values then that will be called a extreme value distribution. Similarly, maximum early wind speed can be expressed also as a extreme value distribution that is if we have a collected data for 50

years of the maximum wind speed that has taken place at a particular site in a year, then from those 50 values one can construct a distribution and that distribution will be called an extreme value distribution.

Why will distribution is a specific kind of extreme value distribution. Now these distributions require 2 quantities that is the standard deviation of the random variable and the mean value of the random variable. And how to find out the mean value when the standard deviation of the random variable that we have discussed before. So, knowing these 2 quantities one can express or give a mathematical expression to the probability density function and the cumulative distribution function on for a different kind of distributions and the data that we collect from the field say in the case of earthquake the past earthquake data, in terms of peak ground acceleration or in terms of the magnitude of earthquake those data can be analyzed one can construct a cdf and a pdf from the numerical data and after we have obtained the cdf and pdf as a continuous curve then we can go for a curve fitting technique to examine whether the distributions confirmed to normal distribution all lognormal distribution.

The expressions for the probability density function for the normal variate that is given by the first exp this expression over here. In here  $\sigma$  is the value of the standard deviation of the random variable  $\mu$  is the mean value and  $f(x)$ ; obviously, is an integration of the probability density function because the probability density function is defined as is a derivative of the first derivative of the cdf function  $F(x)$ . And in many standard textbooks in statistics we get a standard normal variate and a value of the probability density function for the standard normal variate. These standard normal variate is defined as  $x$  minus  $\mu$  divided by  $\sigma$ .

Utilizing that standard tables one can find out for the case of normal distribution what is the probability of accidents of certain value or probability of some of the random variable being less than equal to some value now. When we come to the lognormal distribution, then the sake the value is  $x$  becomes is equal to  $\ln z$  is the values sampled values that we have obtained and we take a logarithm natural log of that and that is  $x$  then we say that the  $x$  is a you of all rather follows a normal distribution designated by a  $\sigma$  value and a  $\mu$  value that is the mean value and a standard deviation.



So, log of the original variable  $z$  becomes normal. So, in that case we say that it is a lognormal  $z$  is a lognormal. This quantity. And the probability density function for the lognormal variable is given by and this expression. Here, this is the  $z$  not bar it is a curved line indicating it is not the mean value of the  $z$  bar, but it is a it is a median of  $z$ . And there exist a relationship between the mean value and the median.

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Distribution of Eq. occurrences with respect to time is modeled as Poisson process:  
 Temporal distribution of Eq occurrence for Mag./PGA is given by

$$P(N=n) = \frac{(\lambda t)^n e^{-\lambda t}}{n!}$$

For  $n=1$ , exactly one event occurs in  $t$  years  
 $n \geq 1$ , at least one exceedance in  $t$  years

$$P(N=1) = \lambda t e^{-\lambda t}$$

$$P(N \geq 1) = 1 - e^{-\lambda t}$$

Using this, any Probability of exceedance of seismic parameter  $\gamma$  in time  $T$   
 $P[X_T > \gamma] = 1 - e^{-\lambda_\gamma T}$

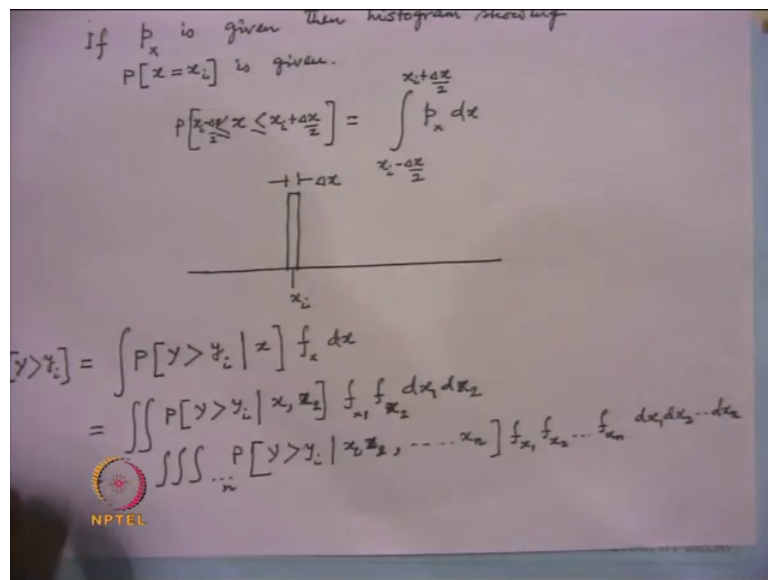
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Finally, for the case of the earthquake we have another distribution which is very important. That is called the Poisson distribution which is associated with the Poisson process. If we considered the distribution of earthquake occurrences with respect to time, then it has been found that this temporal distribution follows a Poisson process or in other words we say that temporal distribution of earthquake occurrence for certain magnitude or a peak ground acceleration is given by this expression, and this expression is known as the Poisson distribution.

What it says is that probability of  $n$  being equal to  $n$ ; that means, probability of sum you went in to be is equal to a value  $n$  is given by  $\lambda t$  to the power  $n$   $e$  to the power minus  $\lambda t$  divided by factorial  $n$ . Now the  $\lambda$  basically is known as the average rate of occurrence of that particular event. And over the time period  $t$ . And  $n$  basically is a the number of events that has taken place. Therefore, the  $p$  probability of  $n$  being equal to  $n$  is a given by this expression in which we must know the value of  $\lambda$ . Now for  $n$  is equal to 1 exactly one event occurring in  $t$  years that is one quantity that is

of interest other is that  $n$  greater than equal to 1 means at least one accidents that has taken place in  $t$  years this is another quantity of interest. And both of them can be straight away obtained from this particular equation and they take this particular form. And out of this this one is very important because using this particular expression, one can find out the probability of accidents of any seismic parameter  $y$  bar, say  $y$  bar is peak ground acceleration in a time interval of capital  $t$  will be simply given by this expression  $1 - e^{-\lambda y \bar{t}}$  that is where  $\lambda y \bar{t}$  is the average occurrence rate of that particular seismic parameter. Now if we know that then in  $t$  years of time what is the probability of accidents of that particular seismic parameter can be worked out.

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now with the help of these knowledge of the statistics of random variables. Let us now a look at the different steps that we consider for obtaining a probabilistic seismic hazard analysis. Now it predicts the probability of occurrence of a certain level of ground shaking at a site by considering the uncertainties. These uncertainties are size of earthquake location a rate of occurrence of earthquake and the predictive relationships.

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➤ Step 1 consists of following:

- Identification & characterization of source probabilistically.
- Assumes uniform distribution of point of earthquake in the source zone
- Computation of distribution of  $r$  considering all points of earthquake as potential source.



The we have talked about all these quantities before probabilistic seismic hazard analysis is carried out in 4 steps. First step is the identification and characterization of the source probabilistically. Here we not only talk of the maximum magnitude of earthquake that can occur at a particular source, but we also want to find out or we must know also the probability density function of the magnitude of earthquake. Then it assumes a uniform distribution of the point of earthquake in the source zone; that means, if we consider a line source or an area source, then we assume that in that particular line source or the area source the possibility of the earthquakes occurring at every point is uniform.

Then the computation of the distribution of  $r$  considering all points of earthquake as a potential source. So, this part I will see how we can obtained from the data that we obtain about the earthquake sources considering every point in the source zone to be a potential point of earthquake.


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➤ 2 step consists of following:

- Determination of the average rate at which an earthquake of a particular size will be exceeded using G-R recurrence law.

$$\lambda_m = 10^{a-bm} = \exp(\alpha - \beta m) \quad (1.23a)$$

- Using the above recurrence law & specifying maximum & minimum values of  $M$ , following pdf of  $M$  can be derived (ref. book)




The second step consists of the determination of the hazard rate at which an earthquake of a particular size will be exceeded using Gutenberg recurrence law. That is given by equation 1.23a. In 1.23a the lambda m denotes the occurrence of earthquake greater than certain magnitude value or the occurrence a average rate of occurrence of earthquake above certain level of magnitude you know of earthquake m that is called lambda m is given by this equation 10 to the power a minus b m. And this can be also written in the exponential form as exponential alpha minus beta m.

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$$f_M(m) = \frac{\beta \exp[-\beta(m - m_0)]}{1 - \exp[-\beta(m_{max} - m_0)]} \quad (1.26)$$

➤ 3rd step consists of the following:

- A predictive relationship is used to obtain seismic parameter of interest (say PGA) for given values of  $m, r$ .
- Uncertainty of the relationship is considered by assuming PGA to be log normally distributed; the relationship provides the mean value; a standard deviation is specified.



Using the above recurrence law and specifying maximum and minimum values of  $m$  the following probability density function of  $m$  can be obtained that is given by equation 1.26. And here we can see that there is a  $m_0$  and  $m_u$   $m_0$  and  $m_u$  denotes or the minimum level of the magnitude of earthquake that is considered in the analysis and the maximum magnitude of earthquake that is coming from the source. And beta is a parameter that has been explained before the third step consists of the following we obtain a predictive relationship and these predictive relationship could be of the similar type that I have shown before in data mystic seismic hazard analysis. And the idea of having a predictive relationship is to obtain a peak ground acceleration given a set of magnitude and a value of  $r$  from a particular site.

Then the uncertainty of the relationship in the predictive relationship is considered by assuming peak ground acceleration to be lognormally distributed. That is the relationship that the predictive relationship gives that gives a mean value of the variable and a mean value of the variable, say is a peak ground acceleration then there will be a standard deviation which will be attached to this particular random variable which is a peak ground acceleration. Now what it means is that for a given set of magnitude of earthquake and an epicentral distance the peak ground acceleration that is computed at the site is a random variable and these random variable is found to be lognormally distributed.


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> 4th step consists of the following:

- Combines uncertainties of location, size & predictive relationship by

$$\lambda_{\bar{y}} = \sum_{i=1}^{N_s} \lambda_i \iint P[Y > \bar{y} | m, r] f_{M_i}(m) f_{R_i}(r) dm dr \quad (1.27)$$

- A seismic hazard curve is plotted as  $\lambda_{\bar{y}}$  vs  $\bar{y}$  (say  $\bar{y}$  is PGA level).



The fourth step consists of the following: it combines all the uncertainties of location, size, and predictive relationship by this combination rule, which is expressed by equation 1.27. It represents that  $\lambda y$  represents the average rate of occurrence of a seismic diameter say peak ground acceleration being greater than equal to  $y$  is equal to this summation that is summation over all the sources.

$\nu_i$  is the average rate of occurrence of earthquake, which is greater than the magnitude of  $m_0$  that is specified for each source. Then this double integration within this integrand we have probability of  $y$  being greater than  $y$  conditional upon a set of value of  $m$  and  $r$ . So, given a  $m$  and  $r$  value one can obtain a value of the probability of the accidents of the peak ground acceleration being greater than a value of  $y$ . This we can obtain from the predictive relationship. And then we multiply it by the probability density function of the magnitude and the probability density function of the epicentral distance and integrate over the entire values of magnitude and epicentral distances that you have considered for a particular source. And sum of these integrand weighted by the  $\nu_i$  values for each source to obtain the value of  $\lambda y$ . And once we get the value of  $\lambda y$  then one can obtain a seismic hazard curve that is a  $\lambda y$  versus  $y$ .