Seismic Analysis of Structures
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Lecture – 05
Seismic Inputs

In the last 4 lectures, we discussed about the seismology dealing with the how earthquake generates how the seismic waves travel from source to site how the ground motions are measured and then we studied about the 2 major ground motion measuring parameter that is the magnitude and intensity of earthquake also. We studied the seismic hazard analysis which deals with the seismic risk analysis of a region which is helpful in finding out the seismic risk analysis of structures also is helpful in obtaining the my presentation map of a region in terms of the probability of occurrence of certain magnitude of earthquake or of certain peak ground acceleration or a certain other earthquake measurement parameters.

Now, in this few lectures, we will be discussing about another important topic which is seismic inputs that is the inputs that we use for finding the response of the structures for earthquake.

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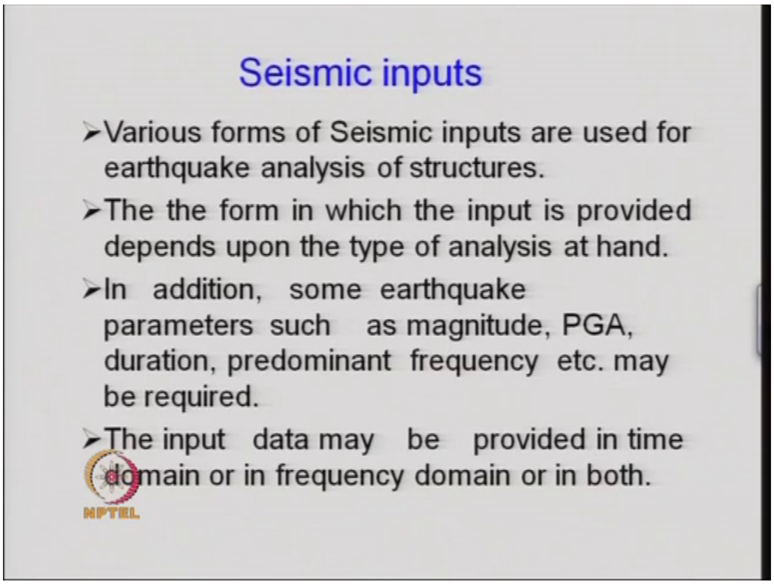

There are many earthquake seismic earthquake input parameters are used out of that the ones that is to be used, depends upon the kind of analysis at hand in addition some earthquake parameters such is such as magnitude of earthquake peak ground acceleration duration predominant frequency etcetera may also be required.

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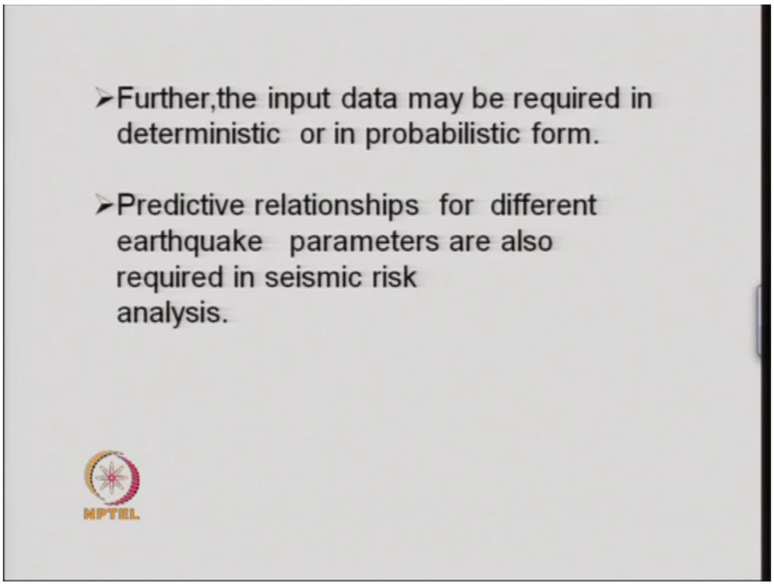

The input data may be provided in time domain as well as in frequency domain or in both.

The data may be required in a deterministic or probabilistic format many times we require also the predictive relationship for different earthquake parameters for seismic risk analysis of structures. So, we shall look into all these kinds of seismic inputs that we just mentioned.

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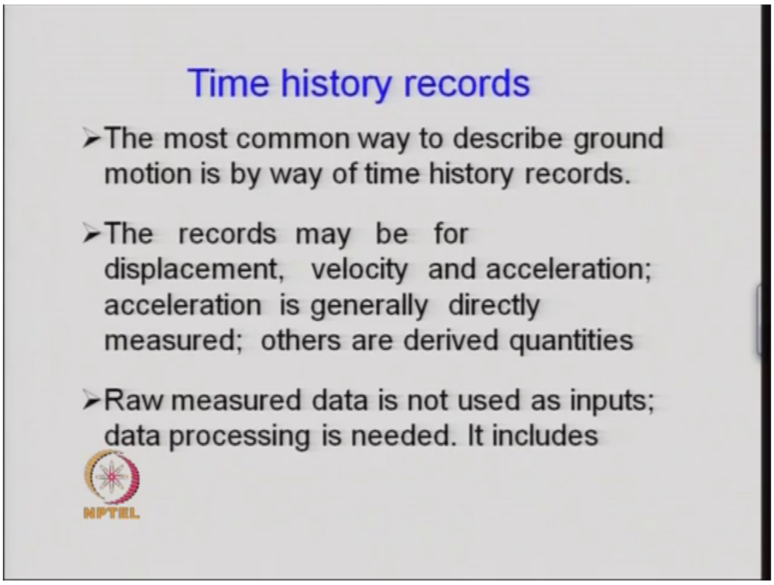

Now, the most direct and simple earthquake input is the time history record the; it is the most common way to describe ground motion using the time history records this records maybe of displacement velocity and acceleration generally acceleration is directly measured the other quantities that is the displacement and velocity; they are derived quantities.

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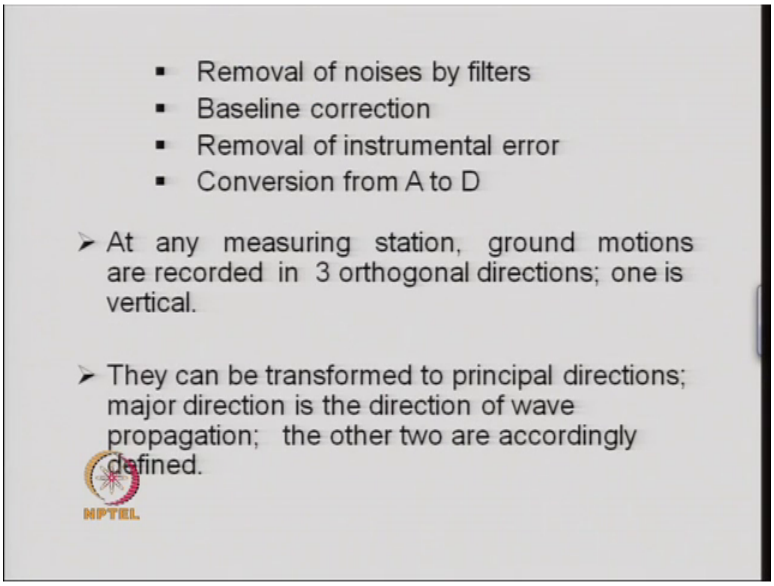

The raw measured data is not straight away used as inputs these data are processed in order to remove the noises by filters then we make a baseline correction in order to provide a proper base line to the earthquake data then we also remove the instrumental error and finally, the one has to get a conversion from analogue to digital data at any measuring station ground motions are recorded in 3 orthogonal directions. One of them of course, is vertical the other 2 could be the 2 horizontal earthquake directions these 3 earthquake records or the measured ground motions in the 3 deductions can be transform to principal directions measure direction is the direction of the wave propagation and the other 2 are accordingly selected .

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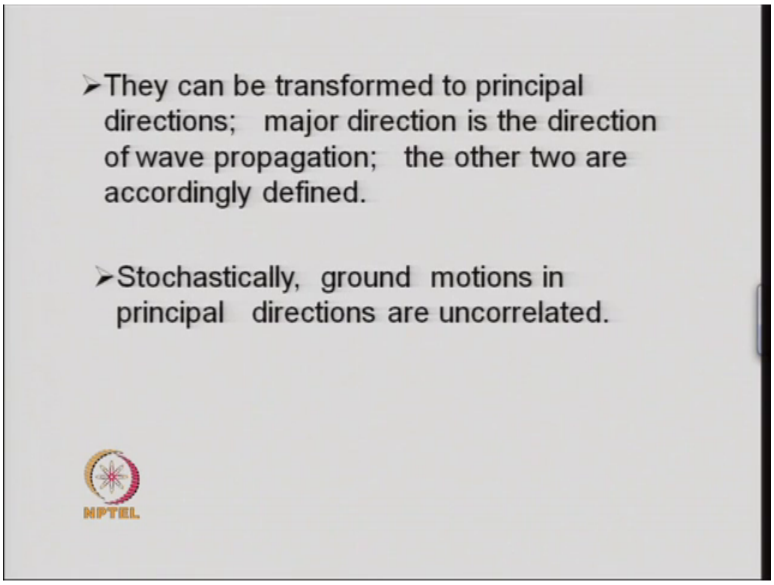

They can be transformed to principal directions by assuming that the ground motion is in principal directions are uncorrelated.

In fact, this is a true for a case when we assume the ground motion or when we describe the ground motion stochastically.

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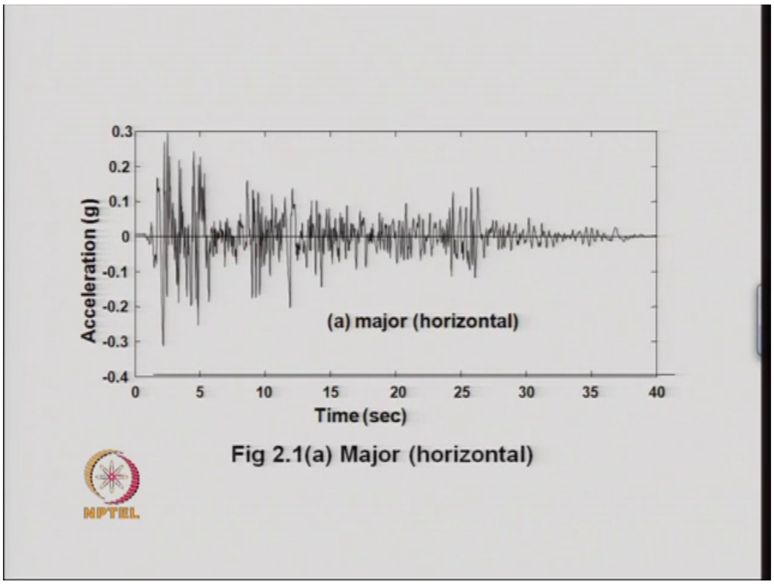

So, this figure shows the measured ground motion in a major horizontal direction and this is a acceleration record.

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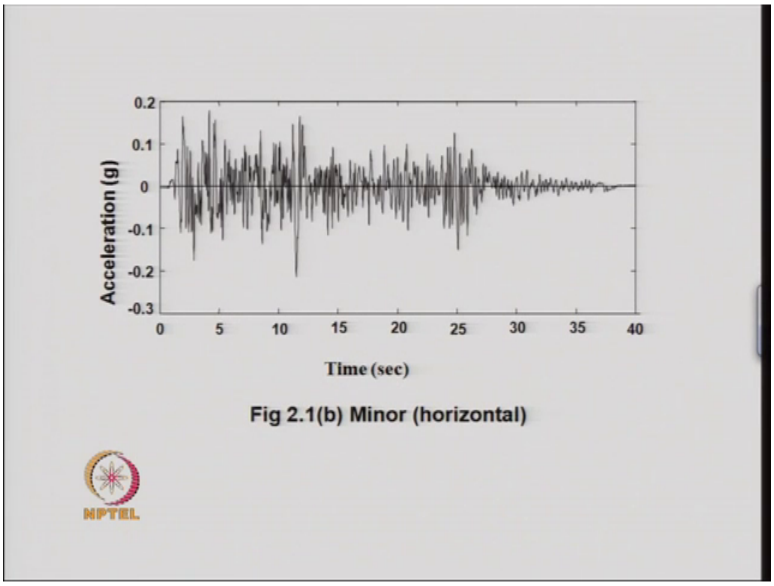

This is the other acceleration record again in the horizontal direction which is a minor or in the minor direction.

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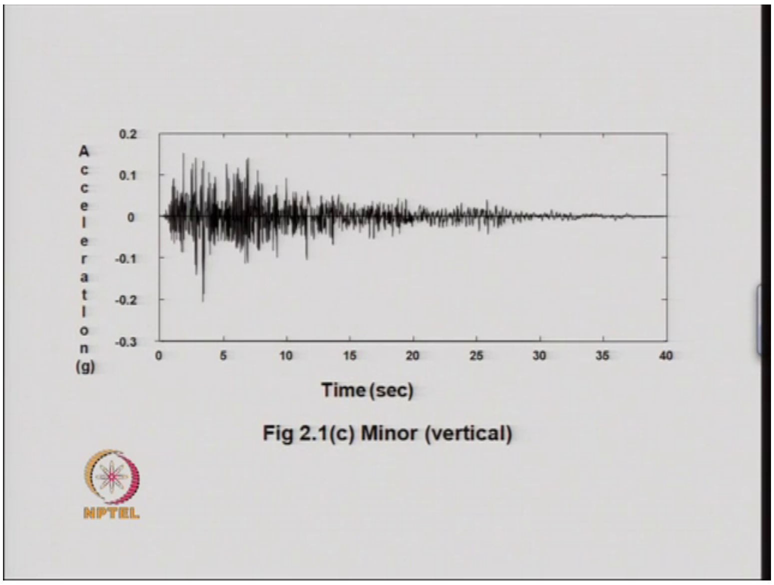

And this is again in the minor direction, but in the vertical direction the ground acceleration record.

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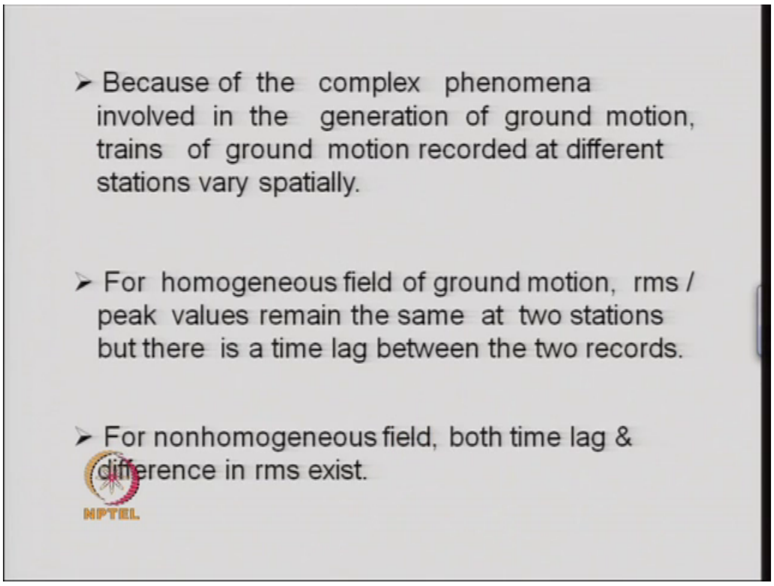

Because of the complex phenomena involved in the generation of ground motion trains of ground motion recorded at different station vary spatially.

For homogenous field of ground motion root mean square or peak values of ground motion remain same at 2 stations, but that is a time lag between the 2 records for homogene; non homogenous field both time lag and difference in the r m s exist.

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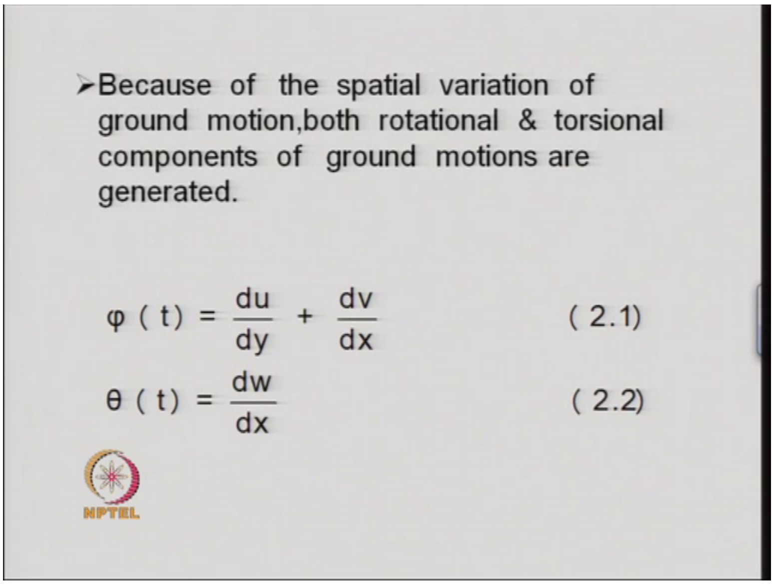

Because of the spatial variation of the ground motion both rotational and torsional components of ground motion are generated equation 2.1 shows how we obtain the torsional ground motion from the horizontal or 2 horizontal ground motions that are measured the since the entire ground acts as a plate. Then if there is a phase lag between the or time lag between the ground motion at 2 points in this direction, say in this direction, then there will be a couple which will be induced or a rotation which will be induced about a vertical axis.

Similarly, for the ground motion in this direction if they vary a spatially or there is a time lag between the 2 ground motions then this will induce again a torsional motion about the vertical axis. So, this is what is reflected in equation 2.1. Similarly if we consider 2 vertical ground motions which has a time lag, then this will induce a rotation in this direction. So, this is shown in equation 2.2; therefore, we have 3 components of ground motion to horizontal ground motion and a vertical ground motion plus we have a torsional ground motion about a vertical axis and a rotational ground motion in the direction of the wave propagation.

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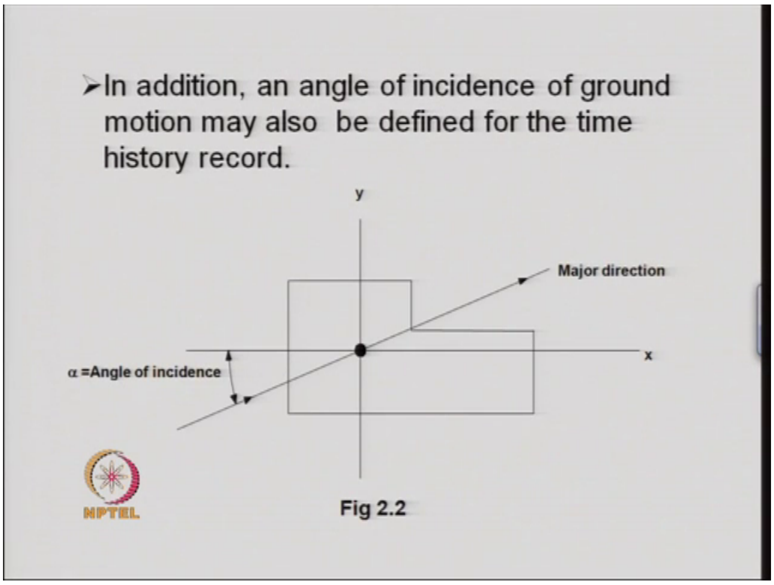

In addition to this, there is an angle of incidence of the ground motion this is defined with respect to the principal direction of the structure. For example, in figure 2.2, we have the principal direction of the one of the principal direction of the structure is lying along x direction and alpha is the angle of incidence that is the major direction of the earthquake ground motion or the seismic wave propagation is at an inclination of alpha with the major axis.

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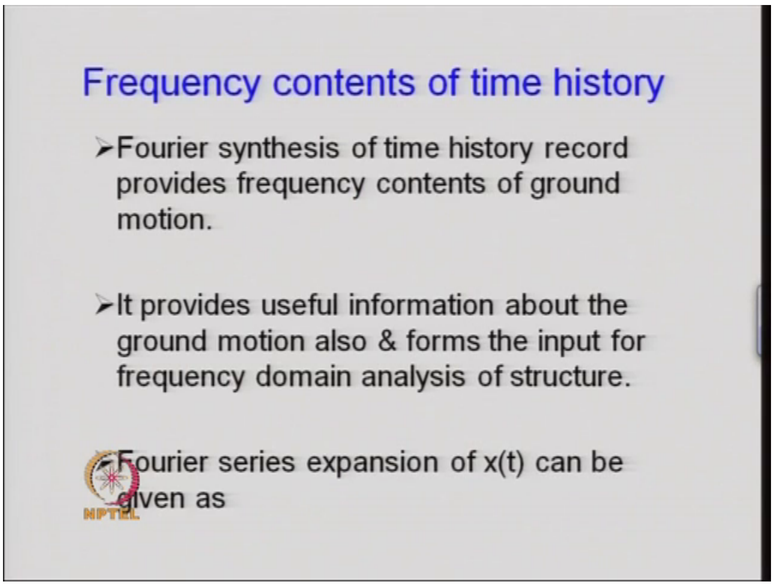

The time history of ground motion although is very simple and easy to understand and gives a direct picture about the earthquake input many a time we require the frequency contents of the ground motion and this frequency contents of the ground motion are used for many purposes. Firstly, to understand what are the kinds of likely predominant frequencies in the ground motion and if those frequencies unknown then one can design structures such that the natural frequencies of the structure can be separated from those predominant ground motions.

Also in the frequency domain analysis of structures for earthquake we need the frequency contents of the ground motion and accordingly one has to device the input in the in terms of the frequency contents the frequency contents of the time history is obtained by the classical Fourier synthesis of time history record it provides useful information about the ground motion also forms the input for frequency domain analysis of structure.

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$$x(t) = a_0 + \sum_{n=1}^{a} a_n \cos\omega_n t + b_n \sin\omega_n t \qquad (2.3)$$

$$a_0 = \frac{1}{T}\int_{-T/2}^{T/2} x(t)\, dt \qquad (2.4)$$

$$a_n = \frac{2}{T}\int_{-T/2}^{T/2} x(t)\cos\omega_n t\, dt \qquad (2.5)$$

$$b_n = \frac{2}{T}\int_{-T/2}^{T/2} x(t)\sin\omega_n t\, dt \qquad (2.6)$$

$$\omega_n = 2\pi n/T \qquad (2.7)$$

Fourier series expansion of any arbitrary function of time t can be written in the form of equation 2.3 where the a 0 is a constant and is defined later and the sum of the sign and cosine terms.

The physical meaning of equation 2.3 is that any arbitrary function of time can be thought to be a sum of a number of harmonics and this number of harmonics has a a phase term that we will see later the constant a 0 is nothing, but the average value of the function x t which is shown in the form of in the in equation 2.4; equation 2.5 and equation 2.6; they describe a n and b n the 2 constants which are associated with equation 2.3 and omega n denotes 2 pi n by T 2 pi by T is the frequency resulting out of the period of period or duration of the ground motion now the in the Fourier synthesis we assume that the duration t which is therefore, the ground motion this as if is repeating after time t and we can expand any function in the form of Fourier series on only when it is periodic in nature.

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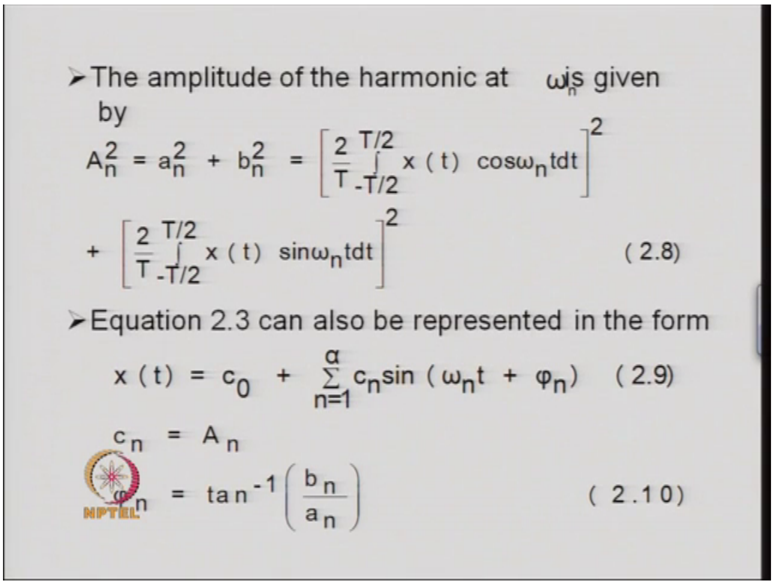

The amplitude of the harmonic at any frequency omega n is given by the expression 2.8 that is A n square is equal to small a n square plus small b n square and this A n and B n has been described before that is by equation 2.5 and 2.6 and they are squared to get the amplitude of the harmonic at omega n the equation 2.3 can also be written in the form of equation 2.9 as I told you before by bringing in a a phase into the equation that is a n cos omega n t plus b n sin omega n t can be written as c n sin omega n t plus phi n.

So, the value of phi and c n can be easily defined c n is same as a n that is computed in equation 2.8 and phi n is tan inverse not b n by a n it is wrong written over here.

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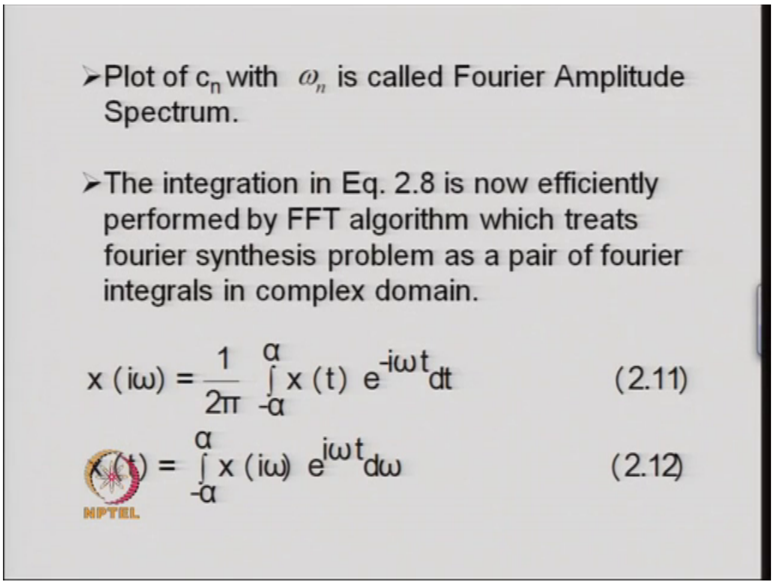

It will be a n by b n is the phi n the plot of c n that is the amplitude of the ground motion at frequency omega n if it is plotted against omega n then we called e to be a Fourier amplitude spectrum or this is known as Fourier amplitude spectrum.

The idea is to obtain the Fourier amplitude spectrum given a time history record this time history record could be a time history record of acceleration in that case we will get a Fourier amplitude spectrum of the ground acceleration and this Fourier amplitude spectrum would show the different kinds of or different compositions of the amplitude of acceleration associated with different frequencies or in other words, we call them as the frequency content of acceleration.

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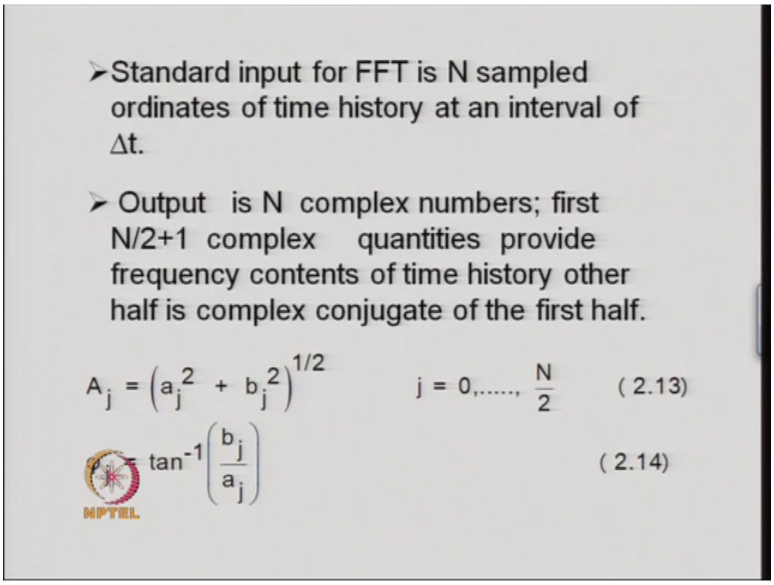

The integration in equation 2.8 the equation that we have shown before these integration now is done very effectively using the FFT algorithm. Now the FFT algorithm transforms the Fourier synthesis into Fourier integral and a pair of Fourier integral define the Fourier synthesis in a comprehensive fashion for example, if x t is the time history of ground motion say acceleration then the first integration would provide the frequency content of the ground motion x t whereas, the second integration would give back the ground motion or the time history of the ground acceleration from the frequency content of the ground motion that is obtained in equation 2.11.

Thus equation 2.11 and 2.12; they form a Fourier transform pair. Now using this Fourier transform pair a analysis of the structure for ground motion can be performed in frequency domain and this technique is known as the FFT analysis of the structure in frequency domain. Now standard input for FFT is n sampled ordinates of time history at an interval of delta t; once these N ordinates or sampled values of the ordinates are provided to the FFT algorithm the FFT algorithm gives back n number of ordinates each ordinate is a complex quantity in the form of a j plus i b j where b is the imaginary part and a is the real part and this provides what is called the x i omega in equation 2.11.

So, given n number of x t values or x values sampled at a interval of delta t n such values if you provide into FFT algorithm, then the FFT algorithm will give as output n coordinates which will be the complex conjugate numbers or the complex numbers and they are nothing, but x i omega sampled at a frequency interval of delta omega the amplitude of the ground motion at frequency omega N is given by equation 2.13 that is A j is written as the real term square plus the imaginary in terms square and then take a square root of that.

So, this is the amplitude associated with frequency omega N and the phase angle phi j is given as tan inverse b j by a j where b j is the imaginary component and a j is the real component the first N by 2 plus 1 values of x i omega they are considered for obtaining the Fourier spectrum because after the N by 2 values the rest of the value that is the other N by 2 values, they are the complex conjugate of the previous N by 2 values therefore, in terms of the amplitude at a particular frequency that N my 2 values do not give any additional information similarly so far as the phase is concerned that also do not give any additional information.

Therefore first N by 2 plus 1 values of the total N values of x i omega that is obtained from FFT that is used for obtaining the Fourier spectrum.

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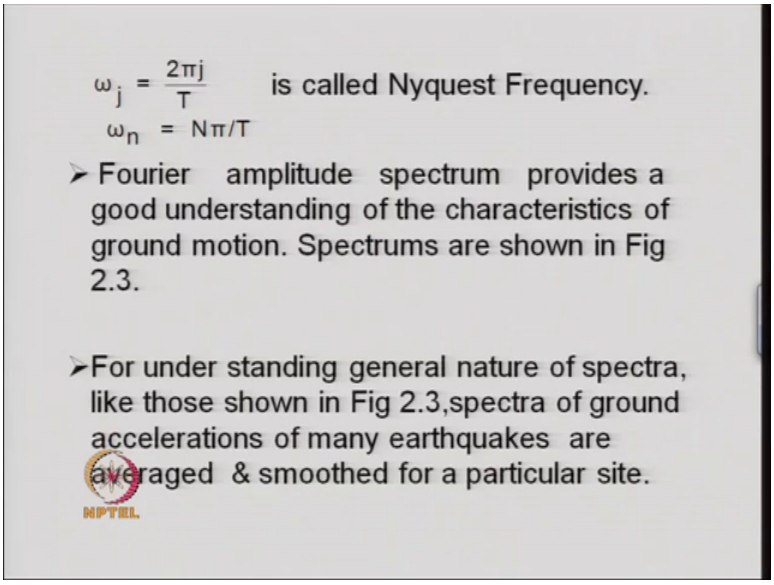

Fourier amplitude for spectrum provides a good understanding of the characteristics of ground motion the spectrums some of the spectrums shown in the figure 2.3.

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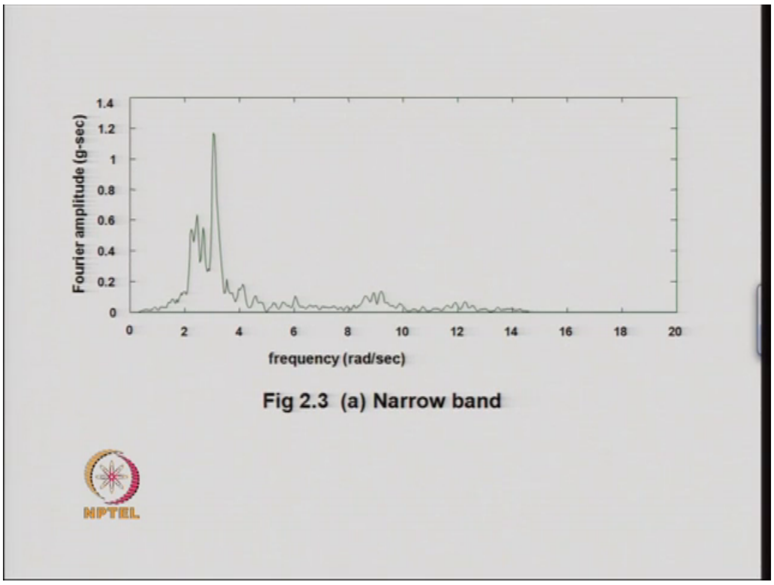

Fig 2.3 (a) Narrow band

So, this is a Fourier spectrum for a narrow band earthquake meaning that the there is a concentration of the frequency within a small band that is within a small band of frequency there is a large amplitude of the acceleration or the ground motion or any earthquake measurement parameters they are concentrated.

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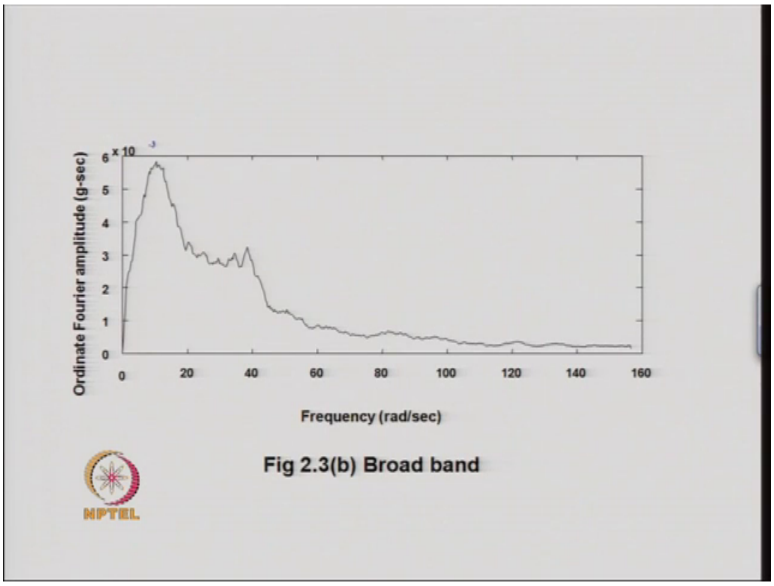

Fig 2.3(b) Broad band

So, this shows the broadband Fourier spectrum where there is not a there is there is not a concentration of the earthquake measurement parameters within a narrow band of frequency.

But it is spread over a broadband generally this broadband of earthquakes that is seen for the hard bedrock or in hard soil whereas, the narrow band ground motions or narrowband time history of ground acceleration, they are observed for the soft soil condition for understanding the general nature of spectra what we generally do is that we find out the Fourier spectrum for the number of earthquakes and then these Fourier spectrums ordinates are averaged and we get a smooth plot of the Fourier spectrum.

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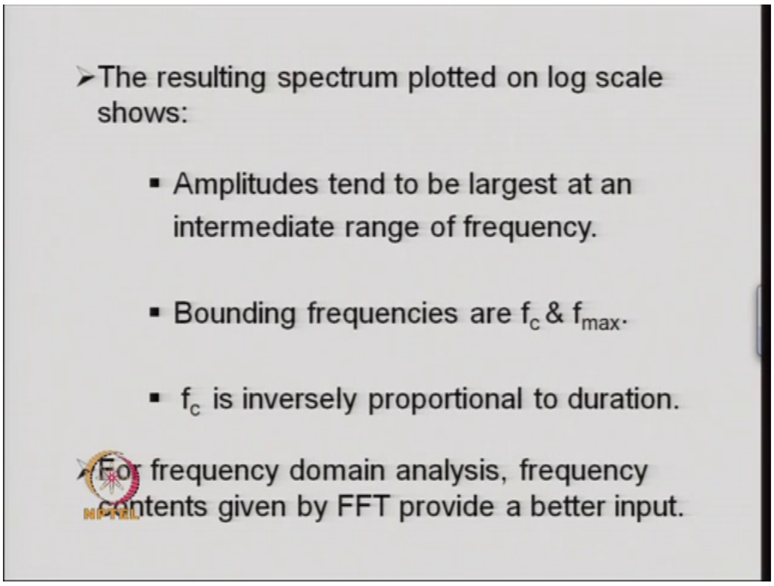

The smooth is plot of the spectrum in log scale shows 3 important quantities that is the amplitudes tend to be largest at an intermediate range of frequency then there are some bonding frequencies which are called f c and f max and f c is found to be inversely proportional to the duration.

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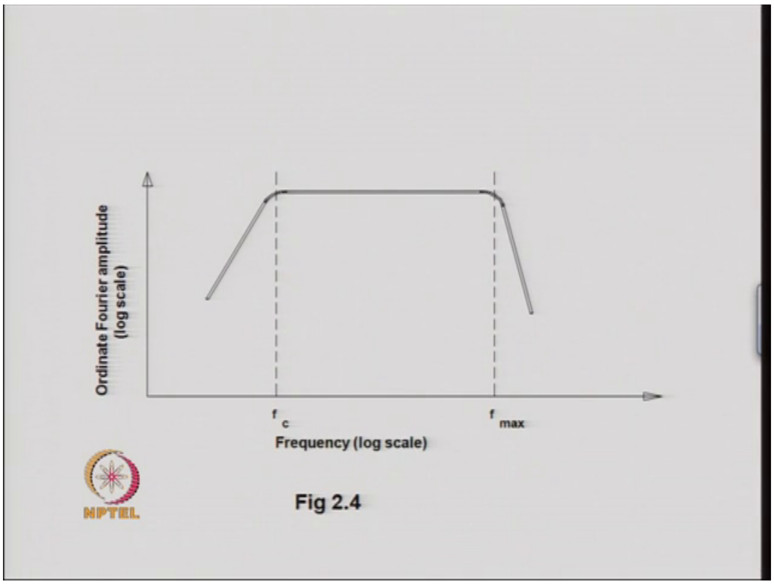

So, this is the figure which illustrates the previous 3 points in the middle region we have the maximum value and this is bounded by 2 frequencies f c and f max and these f c is found to be inversely proportional to the duration of the earthquake.

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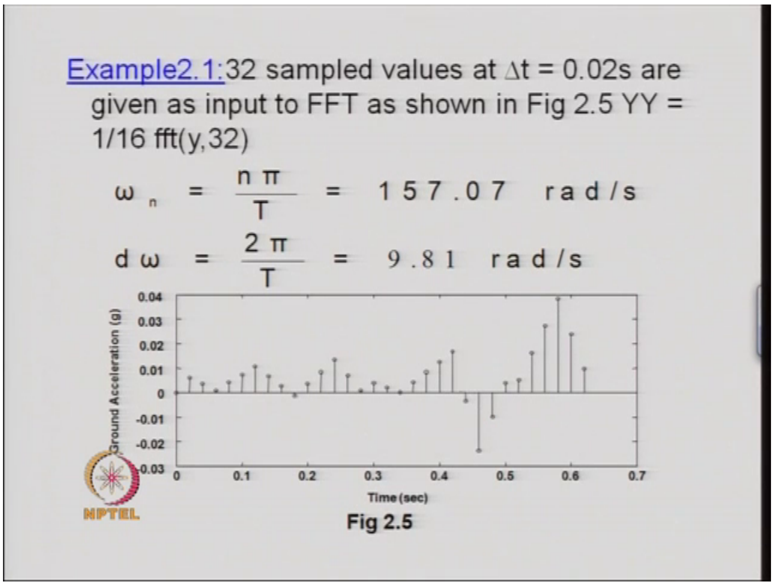

Now, let us look at an example to illustrate how one can obtain the Fourier spectrum for a given earthquake record for making a simplified calculation we considered 32 sample values at a delta t is equal to 0.0 second and the FFT of that is carried out the time duration is T; therefore, the omega n value is equal to 157.07 radian per second and d omega is that is the frequency interval that is equal to 9.81.

The omega n over here denotes the nyquist frequency or the cutoff frequency after this frequency we find that the complex numbers that we obtained from the FFT, it was complex number repeat in the form of complex conjugate . So, therefore, we considered the FFT up to a frequency of omega n that is not for the total frequency that we get in the what we call x i omega plot. Now this figure shows the 32 sampled values at the delta t of 0.02 second.

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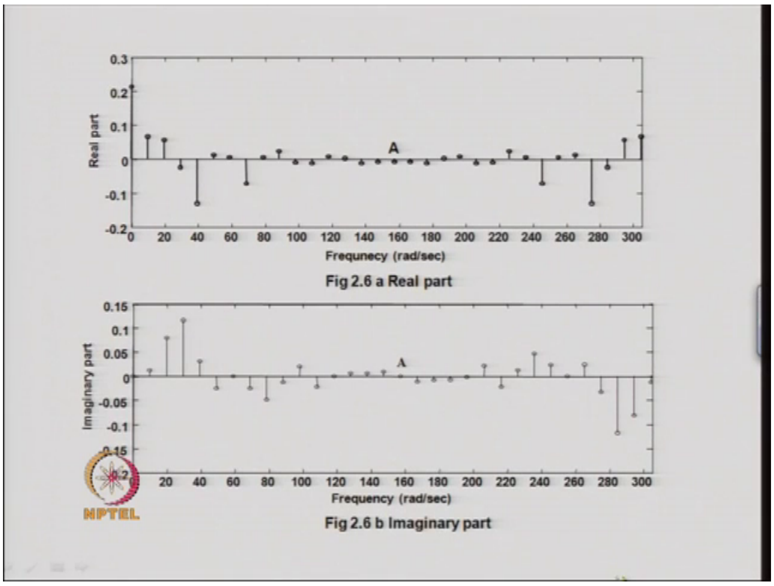

Now, this shows the real part of the x i omega obtained from FFT and we can see that the real part is symmetric about this point; that means, after this point or after this frequency the it repeats whatever we get on to this side the imaginary part is anti symmetric about this point and whatever we get on to this side after this point it is just a mirror image of those points.

Therefore the a square plus b square value or a n square plus b n square values on the left hand side of a and right on the right hand side of a, they are same we do not get any additional information from the right hand side. Similarly the phase that we calculated that is tan inverse b by a b n by a n rather that remains also same for the 2 parts on either side of a. So, we consider only up to this frequency to plot the Fourier amplitude spectrum.

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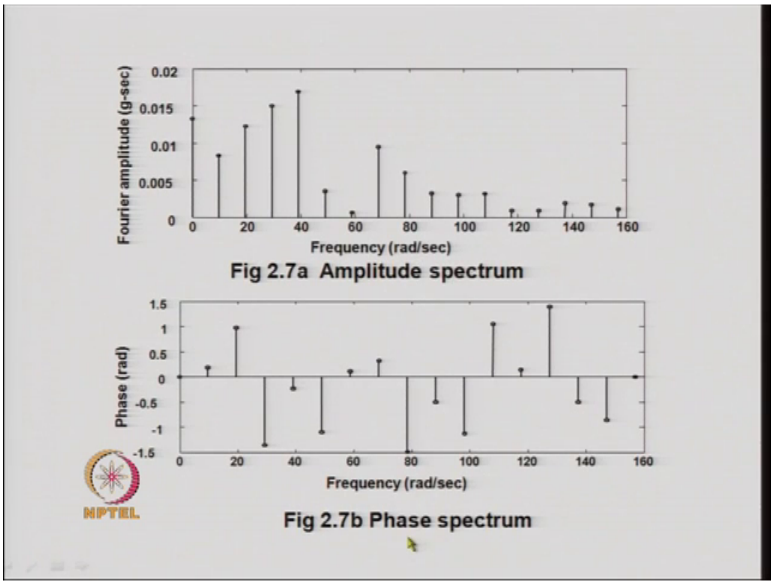

Now, this shows the Fourier amplitude spectrum drawn for the first half that is on the left side of a n; this shows the phase spectrum that is 5 plotted against the frequency .

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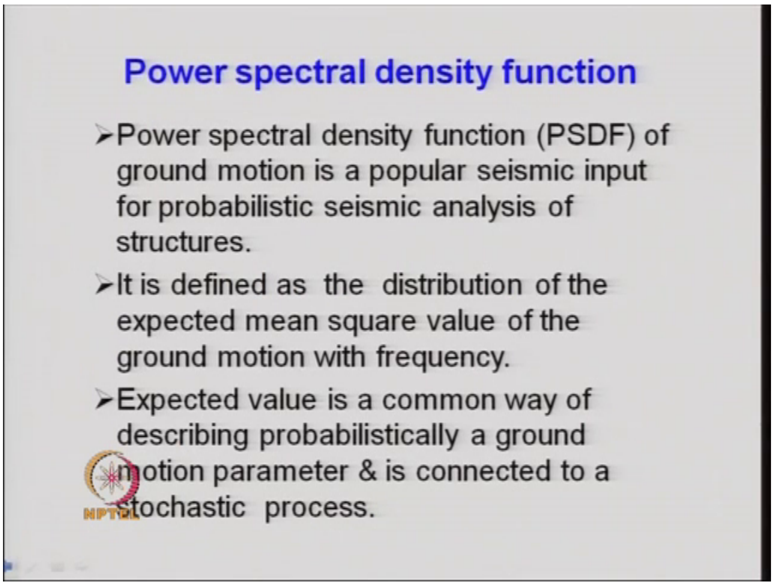

Next we come to another frequency domain input for the structure. Now when you perform the a random vibration analysis of structures for future ground motion that is a ground motions are modeled as a random process not as a deterministic process, then we require power spectral density function the power spectral density function again is a form of input which is given with respect to different frequency or we can say that the different frequencies we have different power spectral density function ordinate showing the frequency; again the frequency content of the ground motion, it is a very popular seismic input for probabilistic seismic analysis of structures.

Now, the definition of the power spectral density function of the ground motion is a very simple definition, but it requires some understanding of the random process now the random process would be discussed later in chapter 4; when will be discussing about the response analysis of structures for future ground motions model as a stochastic process or a random process; right now let me give you a very introductory information about the random process whenever we talk of a random process or whenever we model earthquake as a random process then we do not talk of a single time history.

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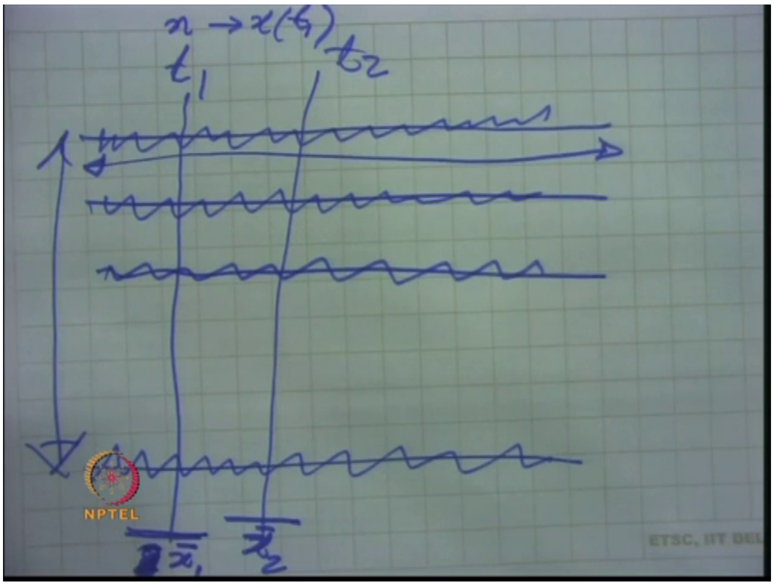

We collected collect at n symbol of time histories like this; this is one time history, then you have another time history that way, we can have a and n symbol of time histories the larger the number of the time histories records better is the prediction ideally one must have an infinite number of records in the n symbol.

Similarly, the duration should be as large as possible for modeling the earthquake has a random process; however, for most of the practical problems we have a duration of earthquake which is of the order of 30 seconds or 35 seconds maximum and we satisfy our self with that amount of duration, but ideally if the duration takes place or the duration is of infinite duration then we have the ideal situation. So, in an ideal situation we can define or distinguish a random process if we have an infinite number of ground motion records of infinite duration.

Now, if we have in reality we have a finite number of ground motion recurs and finite duration. Now if I take any time t 1, then at that particular time t 1, I will get the ordinate from each one of these samples in the n symbol. So, if there are n number of samples in an n symbol, then we will get n values of x t 1 similarly at some other time t 2, we can get n number of values of x t 2 if we take a average of these x t 1 values across the n symbol that is across this sample; let us say the value is x bar one we calculate then x bar 2 that is the average value of x t 2 at time t 2.

If we see that x bar one is approximately equal to x bar 2 and is approximately equal to x bar 3, so on then we can say that across this n symbol the n symbol average is invariant with time. Similarly one can find out the mean square value of the values of x t 1, x t 2, x t 3, so on and if it is found that this mean square values are again more or less the same then we can say that the n symbol mean square value is invariant with respect to time now in any random process. If we find out these criteria or this condition existing then we called that random process as a stationary random process and these stationary random process is uniquely defined with the help of a mean square value and a mean value.

So, the random process can be said to have a unique mean square value the distribution of these expected mean square value of the ground motion with frequency is called the power spectral density function. Now we will look into this power spectral density function more in details later on chapter 4 as I told you, but for the time being with this definition of the power spectral density function, we will go ahead and we will show you how we can construct the power spectral density function the expected value is a common way of describing probabilistically a ground motion parameter expected value means basically and the average value expected value of a random variable means is average value expected mean square value means the squared values are average of the squared values.

And these 2 quantities are closely connected to defining a stochastic process.

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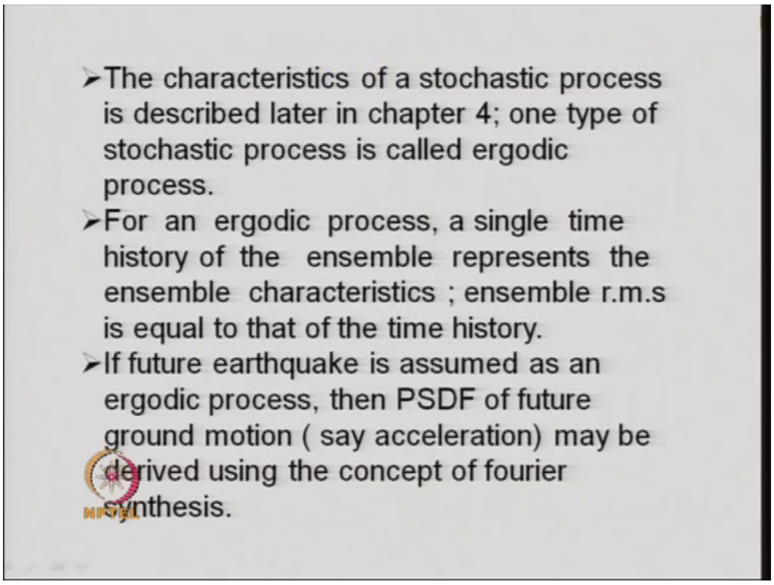

Now, one type of stationary random process is called an ergodic random process many a time the ergodicity or ergodic condition may not be valid in a stationary random process for simplifying the analysis or for simplifying the calculation procedure many a time we assumed ergodicity. Now ergodicity means that if I take a single sample out of the entire n symbol, then this single sample has a mean square value along the time axis t. So, if this mean square value is same for all the samples and is equal to the n symbol mean square value then we call the process to be an ergodic process.

Now, in that assumption it is implicit that a single time history sample taken out of the n symbol represents the mean square characteristics of the entire system. So, therefore, if our intention is to look into the distribution of the mean square value of the process then instead of considering all the samples we can take out any one of sample out of the n symbol and look into its mean square value and then find out the distribution of that mean square value with frequency now this can be easily done with the help of the Fourier series analysis that we discussed before.

So, therefore, at this stage the assumption of a ergodicity helps us in defining the power spectral density function of ground motion with the help of a single time history and using the Fourier series analysis now the rigorous definition of the power spectral density function from the n symbol of time histories will be discussed later now mean square value of an acceleration time history say a t.

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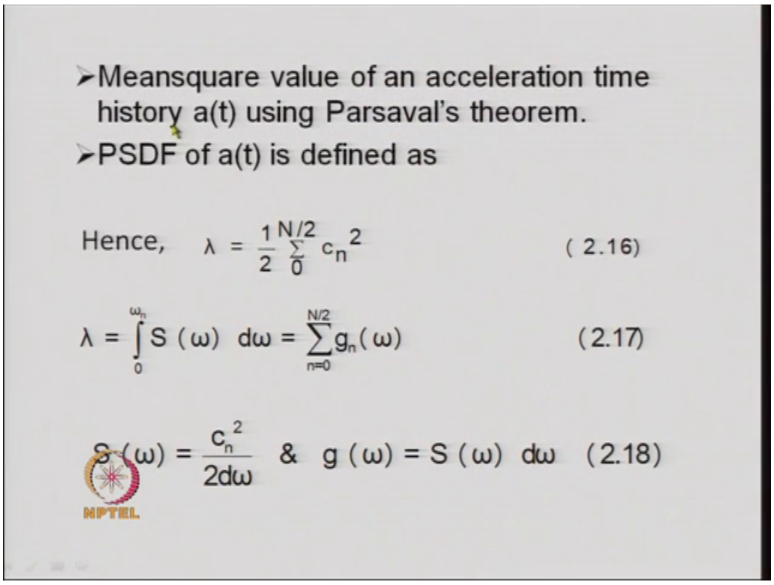

Can be obtained from the time history itself and using Parsaval's theorem which states that the mean square value of a time history is equal to half of the amplitude squares of the Fourier series constants that is Fourier series constants are a n, b n and a 0. So, these are the constants that you had seen in the Fourier series.

So, the Parsaval's theorem says the mean square value of the time history is equal to half of the sum of v a n square and b n square all a n squares and b n square plus the a 0 square now this can be shown to be obtained with the help of the FFT algorithm in this fashion. Now instead of the Fourier series analysis if we carry out the FFT analysis, then from the FFT, we get the amplitude at different frequencies that is whatever shown before and those amplitude squares are taken from 0 frequency to n by 2 that is the first n by 2 plus one values of the FFT that we consider to obtained the value of the c n square. So, c n square is nothing, but the real term square plus the imaginary term square and half of this sum of those squares divided by 2 or half of that sum is equal to the mean square value.

Now, the mean square value again by definition comes to be the integration of these quantity that is s omega says the power spectral density function ordinate at a frequency omega then if we integrate these function s omega from 0 to omega n that is the nyquist frequency that is the up to the point a in the figure that I discussed before. Then that area under the curve you will be the mean square value by definition because by definition the power spectral density function is a distribution of the mean square value with frequency.

Now, this integration can be converted into a summation provided we say that there is a function g n and this g n varies with every frequency and the g n value will be, then equal to nothing, but s omega into d omega; so, the or in other words this s omega d omega if we take together then we can convert this integration into a summation and in that case g n omega is equated to s omega d omega. Now with this definition one can find out s omega to be is equal to c n square divided by 2 d omega thus.

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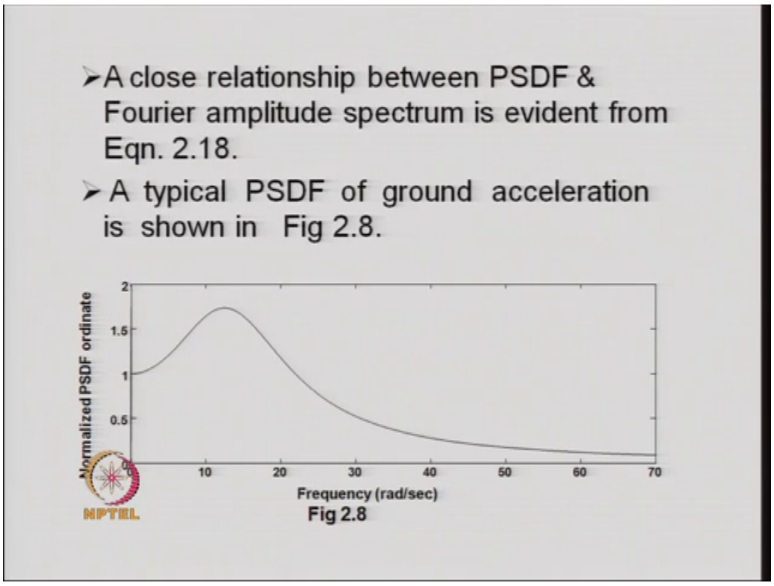

One can obtain the power spectral density function for a ground motion of provided, we have the frequency contents of the ground motion or Fourier amplitude squares or we perform an FFT and from the FFT we can take the real term square plus imaginary term square at every frequency up to the nyquist frequency and with the help of those that information one can obtain the power spectral density function ordinate using this equation that is s omega is equal to c n square divided by 2 d omega.

A typical PSDF of ground acceleration is shown in the figure.

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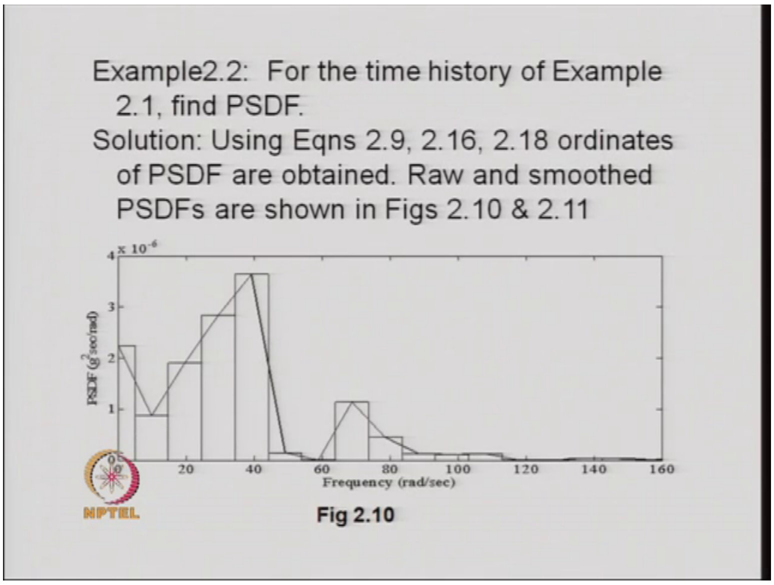

We will then and solve an example to show; how we can obtain the power spectral density function from the time history of a ground motion now the same time history of ground motion that we considered for obtained in the Fourier spectrum that is 32 sampled values of an acceleration time record that was used. And for each frequency we obtained the c n square value that is the real term square plus imaginary terms square that c n square value and then divided it by d omega d omega is equal to 2 pi by T where t is the total duration of the ground motion and that divided again by 2 or in other words s omega is equal to c n square divided by 2 d omega that is what we discussed before.

So, that way we can plot this histograms these histogram is spread over d omega and this value is equal to c n square by 2 d omega. Now know if I join the centre points of these histograms then these shows at the raw spectrum raw power spectral density function of the ground motion now this can be made smooth by some smoothening technique, but if I add up all these histograms the area would be equal to the mean square value.

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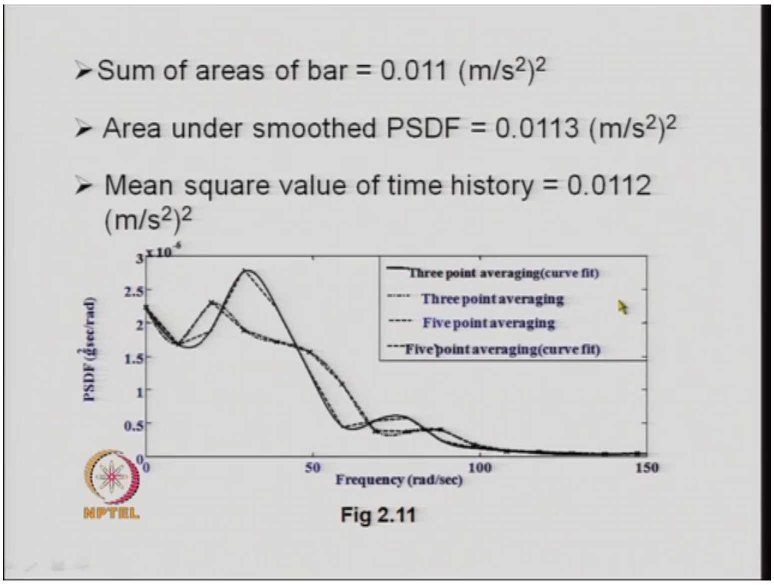

Now, here those the PSDF the row PSDF that we got that has been smoothened by various smoothening technique that is 3 point averaging technique 3, then 5 point averaging technique, then 5 point averaging curve fitting technique and finally, this shows a more or less a smooth response power spectral density function of ground motion obtained for the time history of ground motion having 32 coordinates.

The sum of the areas of those bars that we discussed was found to be 0.011; the area under the smooth PSDF curve was obtained as 0.0113 and the mean square value of the time history that is the by just squaring all the ordinates 32 ordinates and divided by 32 that gave value of 0.0112. So, we can see that these mean square values of the 3 3 mean square values they are matching quite well. So, in this fashion one can obtained the power spectral density function of a ground motion provided we assume the ground motion to be a stationary ergodic process and one single time history of ground motion then can be utilized to obtain the power spectral density function by the use of the FFT algorithm.

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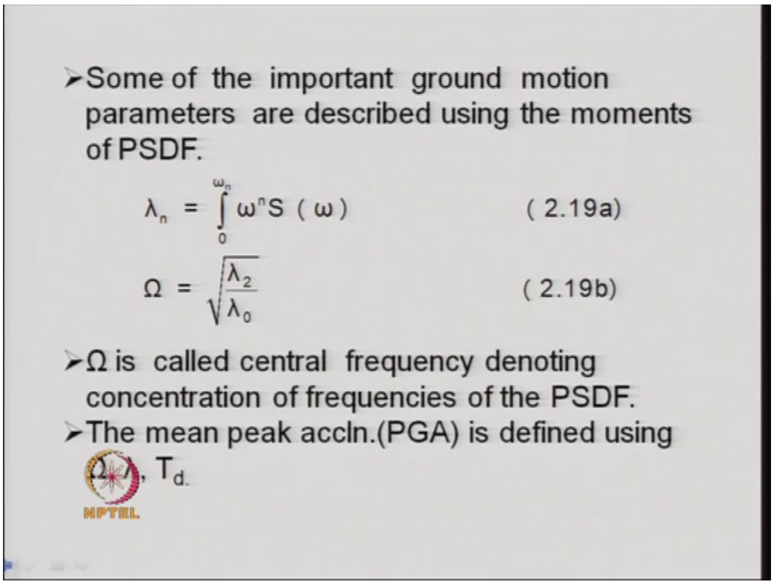

Next for many calculations we require the movements of the power spectral density function of the ground motion. Now the n th moment of the power spectral density function is defined as omega to the power n multiplied by s omega and this d omega is missed over here there will be a d omega now this is integrated again from 0 to the nyquist frequency that is up to the point a that I had shown initially in the figure of the frequency or rather the Fouriers spectrum now the 0 th moment means simply area under the curve. So, the 0 th moment is lambda 0 is nothing, but the mean square value since the area under the power spectral density function curve is the mean square value the second moment will be omega square multiplied by s omega and then you integrate over from 0 to omega n.

So, this quantity called the big omega or capital omega is defined as lambda 2 by lambda 0 that is the second movement divided by the 0 th movement now this capital omega is called central frequency denoting concentration of frequencies of the PSDF. So, or in other words if we wish to find out the predominant frequency content of the ground motion then we go we obtained this value the main peak acceleration that is peak ground acceleration is defined using these 3 quantities that is the value of the capital omega the duration time t and the lambda 0 value.

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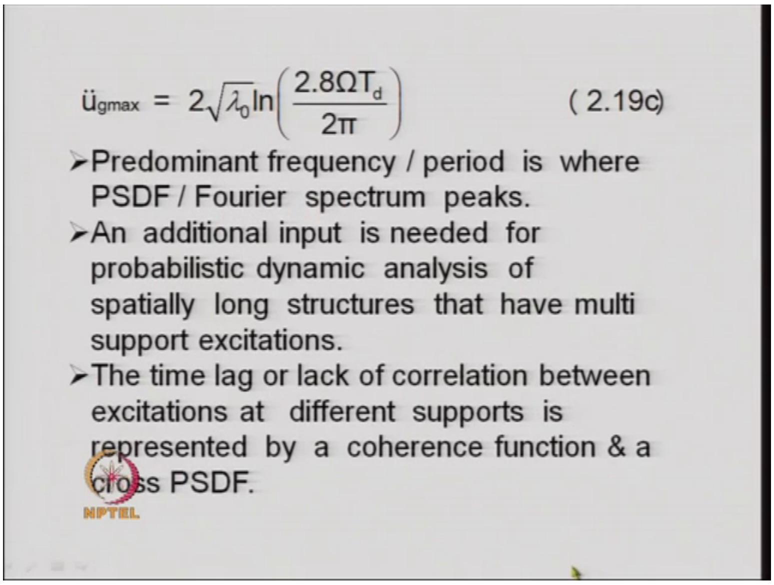

And is defined by this equation and this was derived first by davenport and later on this equation has been improvise somewhat in a better form, but here will be describing the peak ground acceleration using this formula and you can see that this formula requires the square root of the mean square value that is the root mean square value.

Then we require the capital omega the duration and with the help of that one can obtained the peak ground acceleration. So, for obtaining the peak ground acceleration we required the movements of the PSDF curve and the root mean square value of the what we call the ground motion predominant frequency or period is where PSDF and Fourier spectrum peaks and additional input is needed for probabilistic dynamic analysis of specially long structures that have multi support excitation the time lag or lack of correlation between excitations at different support is represented by a coherence function and a cross PSDF function.

In the next lecture, we will look into this coherence function the time lag effect and for specially long structures how do we define the power spectral density function that is a probabilistic description of the ground motion in frequency domain using the PSDF the coherence function and the time lag. So, in today's lecture what we have discussed is that the input for the analysis of the structures for earthquake. So, these inputs could be of several types and the one which we use depends upon the type of problem and the analysis that we are doing the simplest form of the input is the time history records then one can obtained a frequency content of the ground motion using Fourier series analysis of the time history and can obtain the Fourier spectrum and then from the Fourier spectrum one can obtain the power spectral density function of ground motion if it is assumed that the earthquake is a stationary ergodic process.