Seismic Analysis of Structures
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Lecture – 06
Seismic Inputs (Contd.)

In the previous lecture, we started discussing on the seismic inputs that is the inputs which are necessary for analyzing the structure for earthquakes. Now there are various forms of seismic inputs and the one which is to be used depends upon the problem at hand out of the different inputs the time history records are the most direct and the easy seismic inputs. There are 3 components of ground motions which can be given as an input to a structure.

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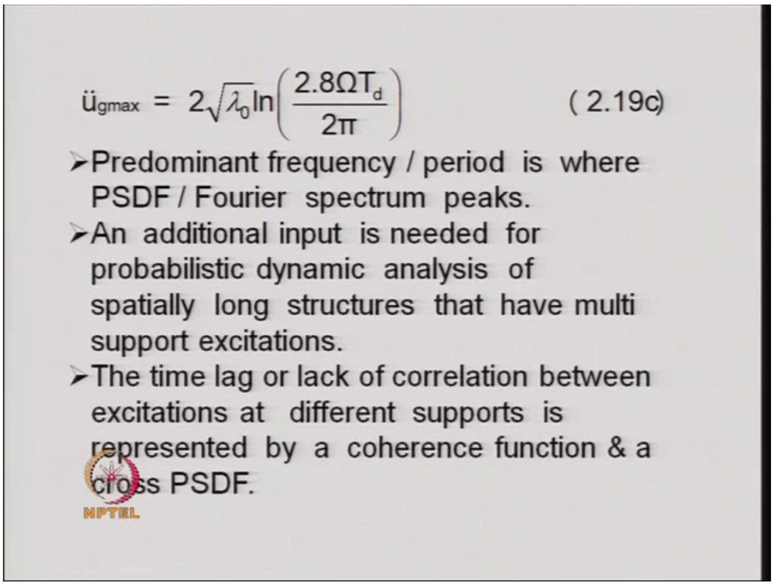

Apart from that there is a torsional component of ground motion and the rotational component of ground motion which can be obtained from the ground motions measured in the 3 directions.

The frequency contents of the ground motion is an extremely important thing in the earthquake literature looking at the frequency contents of the ground motion one can understand the characteristics of the earthquake and can take a decision about what kind of structure which is dynamic properties should be design the frequency contents of ground motion is also very important for frequency domain analysis of structures for earthquake the frequency contents are obtained using the Fourier synthesis technique that we described in the previous lecture.

Using the FFT algorithm, these days the frequency contents can be very easily obtained and a Fourier spectrum can be constructed for given ground motion then we looked at the power spectral density function of ground motion the description of the power spectral density function of ground motion is required for the random vibration analysis of structures for earthquake forces. In fact, in that case the ground motions are modeled as a stochastic process and the power spectral density function forms one of the important inputs for analyzing the structure are considering the earthquake as a random process and we continued with this and see the definition of power spectral density function assuming the ground motion to be a stationary ergodic process.

The power from the power spectral density function of the down motion, one can obtained the peak ground acceleration using the relationship that is shown over here that is in the equation 2.19 c and this requires the knowledge of the duration of the ground motion the capital omega that is this symbol and lambda 0; lambda 0 is the mean square value of the ground motion or in other words, it is the area under the power spectral density function curve and the capital omega is equal to the square root of the second moment of the power spectral density function.

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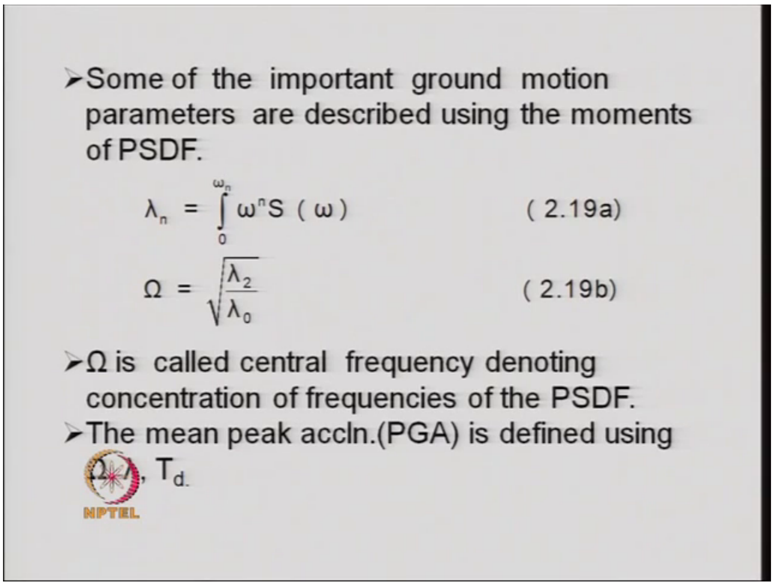

And divided by the 0 th moment or the mean square value of the ground motion and taking a square root of that.

Now, once we have the peak ground acceleration that is a knowledge of the peak ground acceleration then this can be utilized for analyzing the structure are for different levels of the peak ground acceleration the predominant frequency of the period is where the PSDF or Fourier spectrum peaks another input is needed for probabilistic dynamic analysis of specially long structures that have multi support excitations it exact results. So, in order to take into account the effect of this phase lag the cross power spectral density function or cross correlation function is inputted a along with the power spectral density function of the ground motion. Now this time lag or lack of correlation between excitation at different supports is represented generally by a coherence function and they cross power spectral density function.

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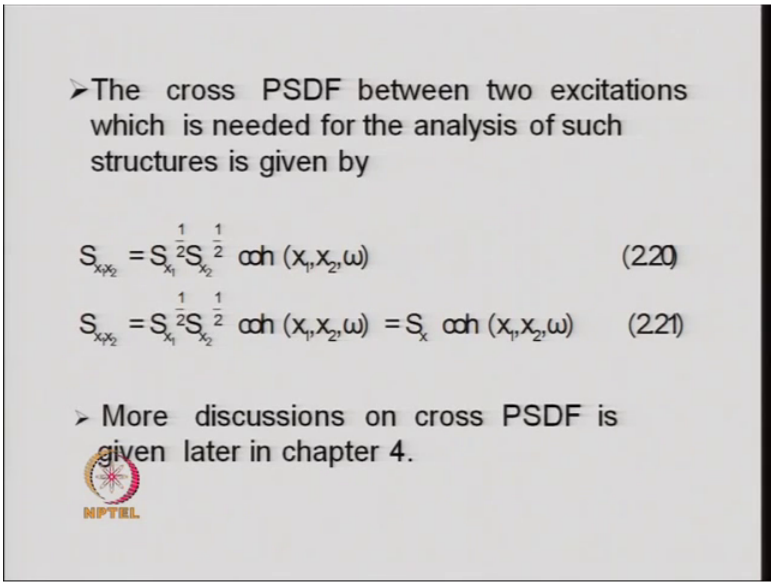

The cross power spectral density function between 2 excitation which is needed for the analysis of such structures is given by these 2 equations that is if x 1 and x 2 are the 2 points, then we say that there exists a cross power spectral density function between on the ground excitation at these 2 points and it is represented by a S x 1, S x 2. And that can be written as S x 1 to the power half that is the power spectral density function at point x 1 S x 2 to the power half that is the power spectral density function to the power half at point 2 and there multiplied by a coherence function which is a function of x 1, x

2 and omega. In fact, this coherence function is the correlation represent the correlation between the 2 ground excitations at points x 1 and x 2.

For the case of a ground motion which is moving in the particular direction and if we the points are aligned in that then the power spectral density function at x 1 and power spectral density function at x 2, they are the same or in other words, in the mean the sense of mean square value these 2 ground excitations are the same. And therefore, this product turns out to be the same as S x that is the unique value of the power spectral density function of the ground motion and it gets multiplied by this coherence function coherence function generally is a exponentially decaying function with the distance r n the value omega.

The greater the distance between the 2 points we expect that there will be less correlation between the 2 ground motion and therefore, a function is chosen which is exponentially decaying. So, this kind of function is multiplied with the S x to get a cross power spectral density function which represent the correlation between the 2 ground motion in frequency domain for a very small value of the distance between x 1 and x 2 and for small value of frequency we can see that the coherence function approaches towards unity that is we get a case of perfect correlation in that case the cross power spectral density function between the 2 points become same as the power spectral density function of the ground motion.

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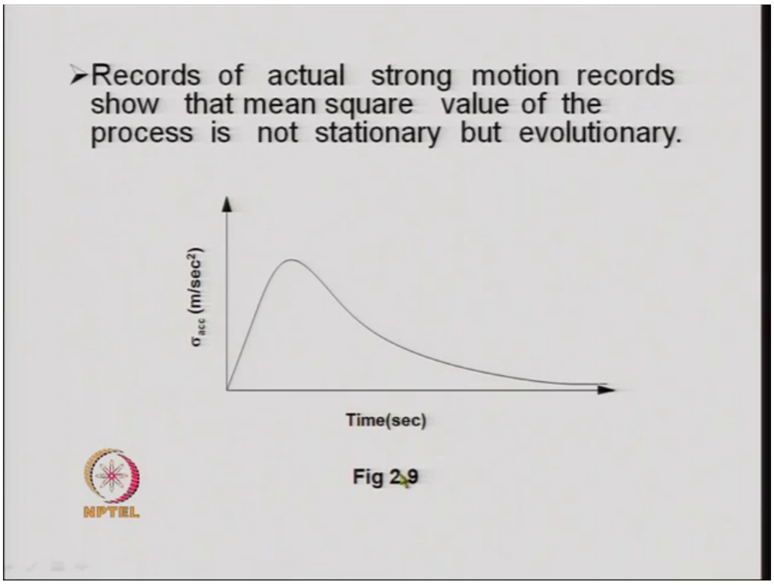

Fig 2.9

Now, more discussion on the cross power spectral density function will be taken up in chapter four which will discuss later, then another interesting thing was observed for the ground motion although we have assumed that the ground motion is a stationary random process in which the mean square value along the ensemble does not change with time or is in variant with time, but in reality it is not. So, it is found that the mean square value does change; change with time and therefore, we have the account for this particular pattern of the ground motion in modeling the ground motion as a random process. So, this is usually done by considering the earthquake to be a evolutionary stationary process meaning that the mean square value of the process changes with time in a particular fashion and the weight changes is a function called the modulating function modulating function which modulates the stationary process with time there is the mean square value changing with time.

One such modulating function is shown in this figure later on we will see at the end of this lecture series on inputs that we have several empirical relationship or empirical equations representing different types of modulating function that are used in modeling the earthquake as a evolutionary stationary process.

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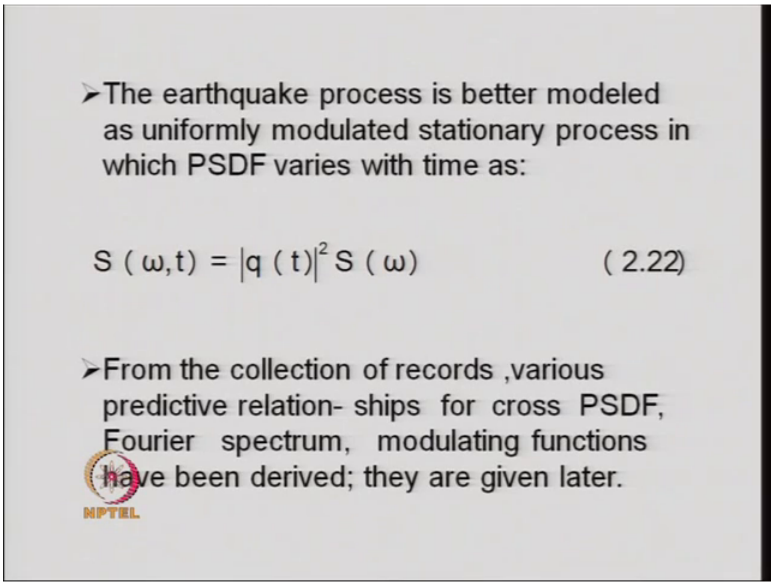

In the case where we consider the earthquake as a evolutionary stationary process the equation 2 point 2 to represents the evolutionary power spectral density function of the ground motion here the parameter t comes into picture because the power spectral density function changes with time and we obtain it by simply in multiply the modulating functions where we the power spectral density function which we obtained assuming the earthquake to be a stationary random process.

So, in order to describe the evolutionary power spectral density function of ground motion we must know the modulating function as well as the power spectral density function of the ground motion. So, these 2 inputs are provided for analyzing the structure now from the collection of records various predictive relationships for cross power spectral density function Fourier spectrum modulating functions have been derived and they are given later or rather the at the end of this lecture series on seismic input.

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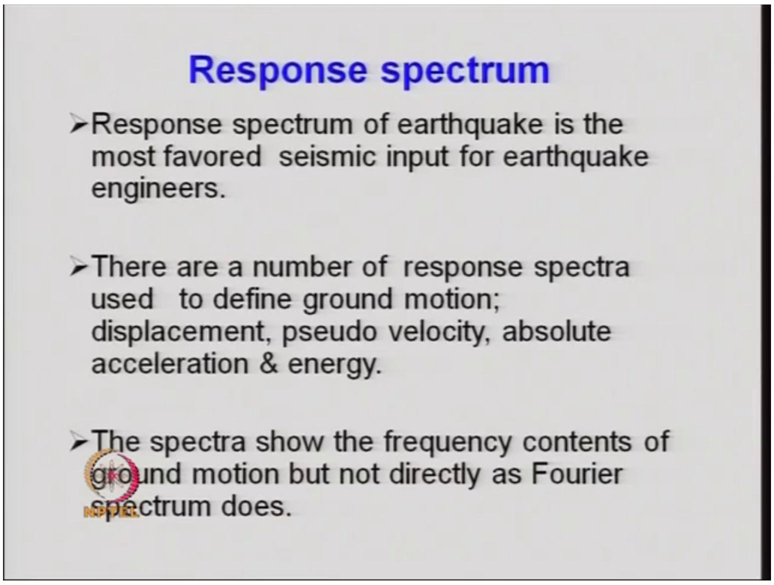

Now, let us come to the next seismic input which is the most favored seismic input for earthquake engineers and this input has been could you find and people use for the seismic design of structures for the design response spectrum and the basis of the response spectrum and the design response spectrum are the subject of discussion here.

Response spectrum of earthquake is most favored seismic input for earthquake engineers there are a number of response spectra used define ground motion displacement should the velocity absolute acceleration and energy. So, no one can obtained the response spectrum for each one of this parameter the spectra show the frequency contents of ground motion, but not directly as Fourier spectrum does.

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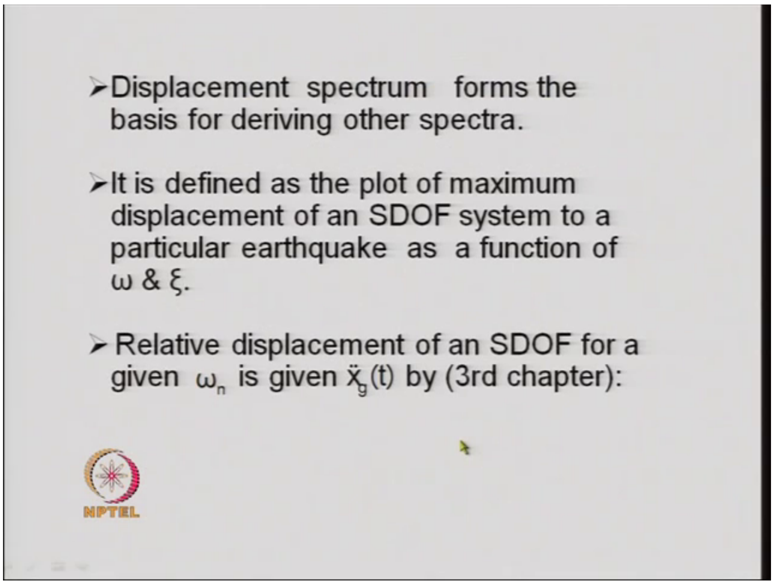

Displacement spectrum out of them forms the basis for deriving other spectrum the displacement response spectrum is defined as the plot of maximum displacement of an SDOF system to a particular earthquake as a function of the frequency and damping ratio.

I assume that all of you have a preliminary knowledge about structure dynamics they are you have solved a single degree freedom system having a natural period and a damping ration to support excitation and for the support excitation you can obtained the relative displacement of the ground motion and that can be obtained by various method. For example, one of the method that is widely used special in earthquake engineering to obtain the response of the single degree of freedom system to were support excitation is using the Duhamel integral; this is the Duhamel integral which is shown in equation 2.23 where we obtain the relative displacement of ground motion using the recorded ground acceleration and using this equation.

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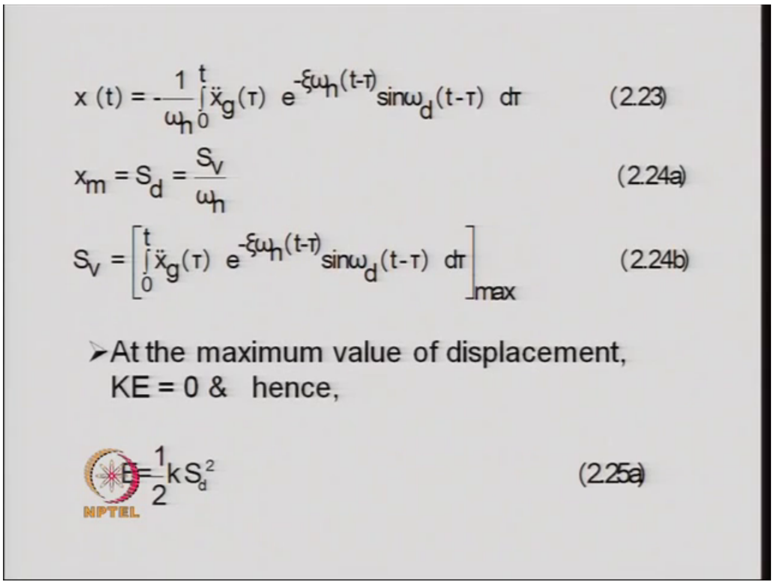

Now, the we can see that this equation contains a term which is sin omega d t minus tau and other term is a exponentially decaying function which gets multiplied with this the basis of the Duhamel integral I just want to mention. So, that you can recall it that if you give a impulse of x double dot g tau into d tau then the response at a time t will be equal to the free vibration that x plus due to the impulse the impulse. In fact, provides a initial condition to the single degree of freedom system that initial condition is that the system is at 0 displacement at that point if we assume, but there is some velocity which is important at that particular point of time because of the impulse then for the elapse time that is t minus tau elapse time the single degree of freedom system vibrates as a few vibration problem or vibrates as a damped free system.

Now, using that concept we are able to obtain the x t using this integration the maximum value of the displacement we can call it as S d is written as S v a parameter divided by omega n it results from this equation where S v is nothing, but this equation that is this integration and the maximum value of that; that means, you perform this integration at every time t and take a maximum value of that that comes to the value of S v and that S v divided by omega n and from that we get the value of the maximum displacement. Now this maximum displacement for different values of omega can be plotted for a given damping ratio at the maximum value of displacement all of us know that the kinetic energy would be equal to 0 because the maximum displacement condition is equivalent to the 0 velocity condition and therefore, the entire energy is the static energy that is half into k into S d square.

S d is the maximum displacement. So, the other part of the energy that is a kinetic energy that turns out to be 0 at the position of the maximum displacement.

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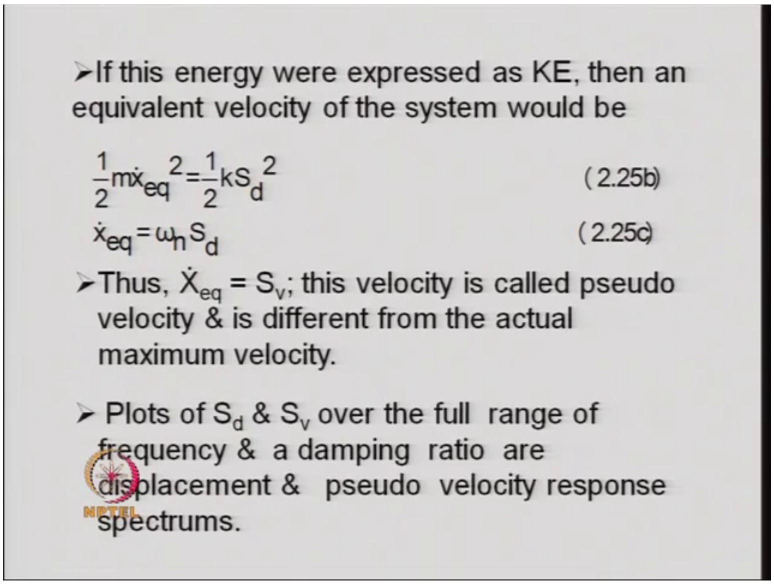

Now, if you say that we want to express this energy as an equivalent velocity of the system then the equivalent velocity for the system can be obtained by equating this energy with half m v square where m v is the equivalent velocity square and from this one can get an expression for the equivalent velocity which is equal to omega n times S d that is the frequency of the oscillator multiplied by maximum displacement.

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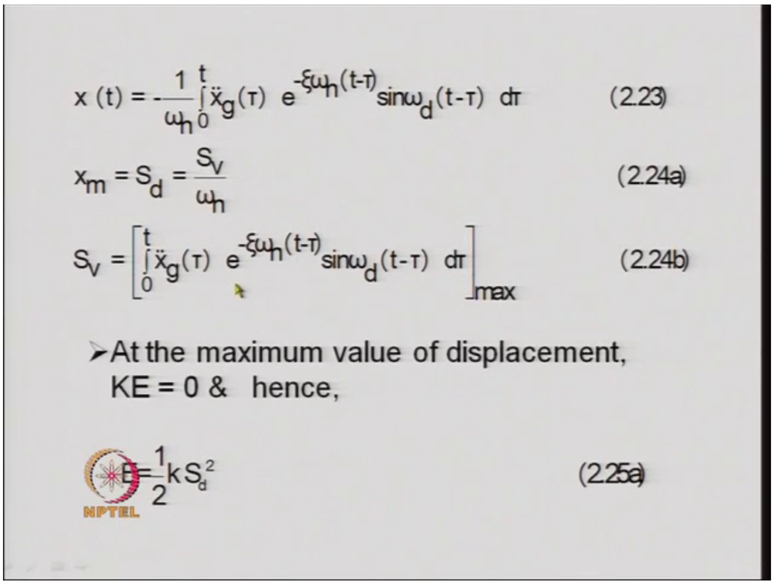

Now, if we look at the previous relationship that we have written x m to be is equal to S v by omega n then we see that this x dot equivalent is nothing, but S v itself.

So, the we can write down x dot equivalent is equal to S v this velocity is called pseudo velocity and is different from the actual maximum velocity the plots of the S d that is the maximum displacement and the S v that is a pseudo velocity over the full range of frequency and a damping ratio are called the displacement and pseudo velocity response spectrum.

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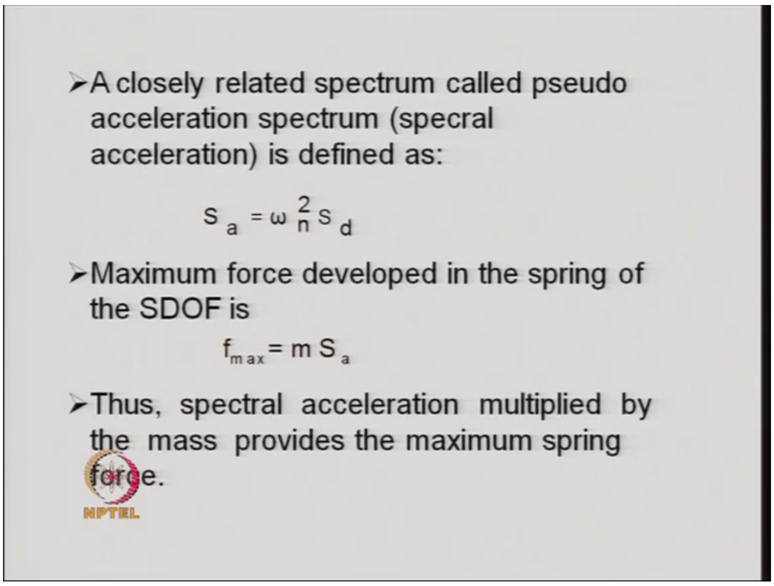

A closely related spectrum called pseudo acceleration spectrum also called spectral acceleration is defined as S a equal to omega n square S d now one can see that the omega n multiplied by S d that is called the S v and if it is further multiplied by omega n that is then we get the definition of the spectral acceleration.

Now, the spectral acceleration is the most important quantity in determining the maximum force that comes on to the structure during earthquake and the definition of spectral acceleration is consistent with that. So, one can plot now the S a versus omega n for given damping ratio. So, we can have we can plot 3 response spectrum that is a displacement response spectrum velocity response spectrum and the spectral acceleration response spectrum or called spectral acceleration for different values omega; omega n. In fact, for the plots that are available they are the omega it is not plotted again is omega n, but they are plotted again is the time period that is 2 pi by omega n giving the value of t n. So, the plots are available against the period t n.

Now, if you look at the definition of the maximum force developed in the spring then one can show that the maximum force in the spring is equal to mass times the spectral acceleration that is obtain now this can be proved in this particular fashion that is k by m is equal to omega n square that is known to everyone then k becomes equal to m into omega n square.

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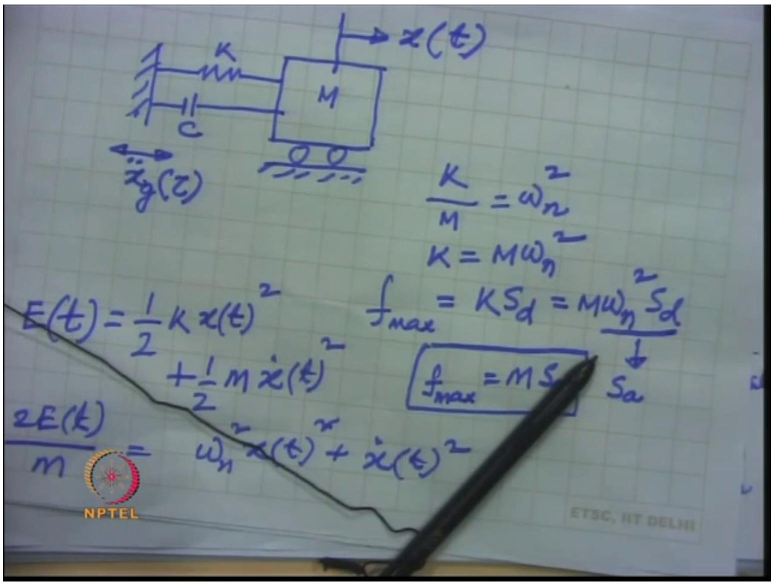

Now, f max that is the maximum force in the spring that will be given by the spring constant k multiplied by S d and if you substitute for k over here that is m omega n square into S d that becomes the value of the maximum force now omega n square multiplied by S d that is nothing, but the spectral acceleration S a that we had seen here from this equation S a is nothing, but omega n square into S d.

So, therefore, the f max becomes equal to mass time the spectral acceleration. Now that is a very interesting thing and the spectral acceleration; therefore, had become a very useful quantity for defining the maximum force in the structure what we need to know for finding out the maximum force in the structure developed due to earthquake is the quantity mass of the structure which are easier to obtain then the stiffness.

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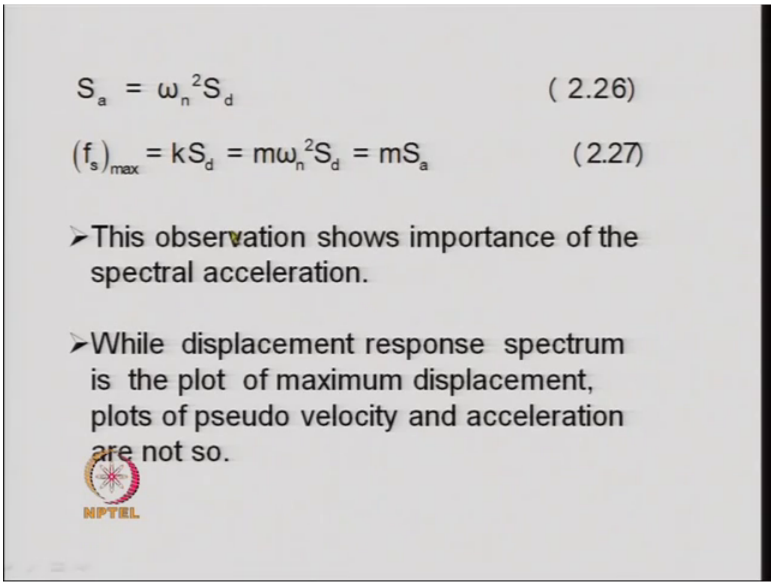

Now, this observation shows the importance of the spectral acceleration while displacement response spectrum is the plot of maximum displacement plots of pseudo velocity and acceleration spectrums are not.

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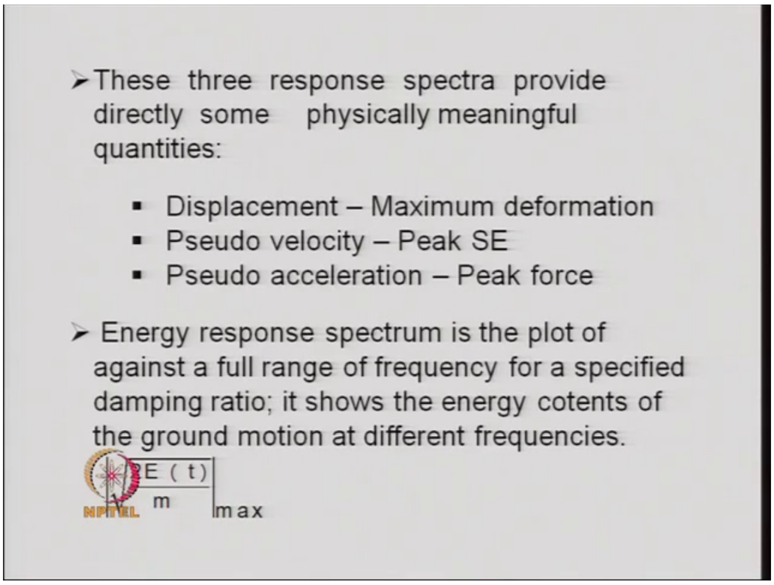

So that is the pseudo velocity is not equal to the maximum velocity the pseudo acceleration or spectral acceleration is also not the maximum relative acceleration these 3 response spectra provide directly some physically meaningful quantities namely displacement response spectrum provides the information about the maximum deformation that takes place in the single degree freedom system pseudo velocity spectrum provides the peak value of strain energy in the system and pseudo velocity or pseudo acceleration spectrum provides the peak force in the system. Now let us come to energy spectrum energy spectrum is the plot of the energy of the system over a full range of frequency for a specified damping ratio. So, the energy is expressed in this particular fashion that is written as square root of 2 times E t divided by m and the maximum value of this is obtained at every instant of time t.

For example, if we look at the definition of the energy at any time t this is equal to half into k x t square plus half into m x dot t square and this dividing the entire thing by m and taking this 2 on to this side then we can write down twice E t divided by m is equal to omega n square k by m will be equal to omega n square into x t square plus x dot t square. So, therefore, for a given single degree freedom system at every instant of time t if we know the displacement end of velocity then easily we can obtain this quantity twice E t by m and the square root of that is obtained for every instant of time t and the maximum value of that is obtained and that maximum value is plotted against omega n and for a given damping ratio is i.

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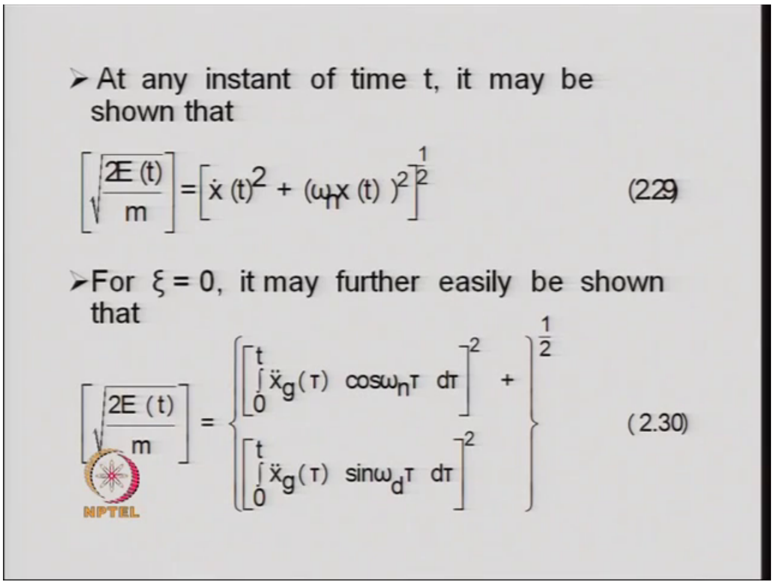

The plot, in fact, also can be obtained for or the time period rather than omega n now for the damping ratio to be 0 it can be easily shown that the value of this maximum or this quantity root over twice E t by m turns out to be of this form. And if we compare this equation with the amplitude of the earthquake; amplitude or Fourier amplitude of the earthquake record then we can see that there is a similarity between this equation and the equation that is given over here.

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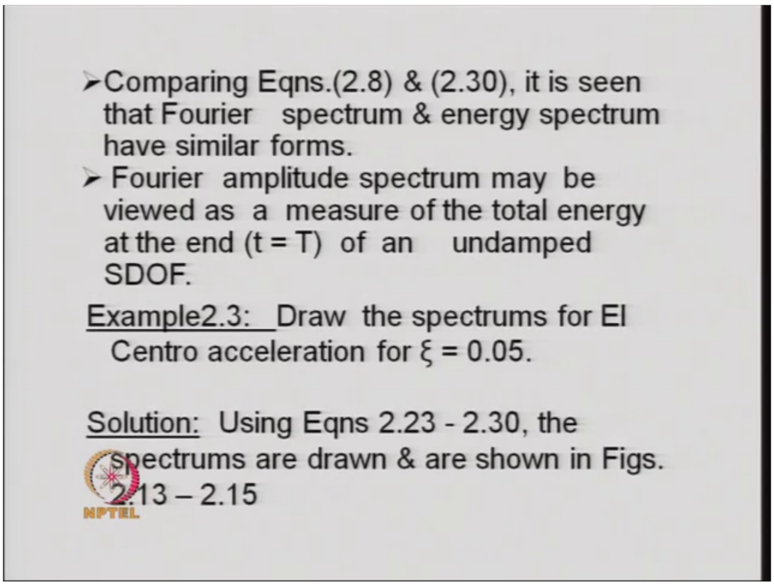

So, we generally say that the Fourier spectrum and the energy spectrum have similar form.

However Fourier amplitude spectrum may be viewed as a measure of the total energy at the end of t is equal to t of an un damped single degree of freedom system now we taken example and draw the spectrums for the El Centro acceleration for xi is equal to 0.05. Now the equations that had been shown earlier using those equation one can obtained the values of S d, S v, S a and the maximum energy and can plot them against not omega n, but 2 pi by omega n that is the period for a damping ratio of 0.05.

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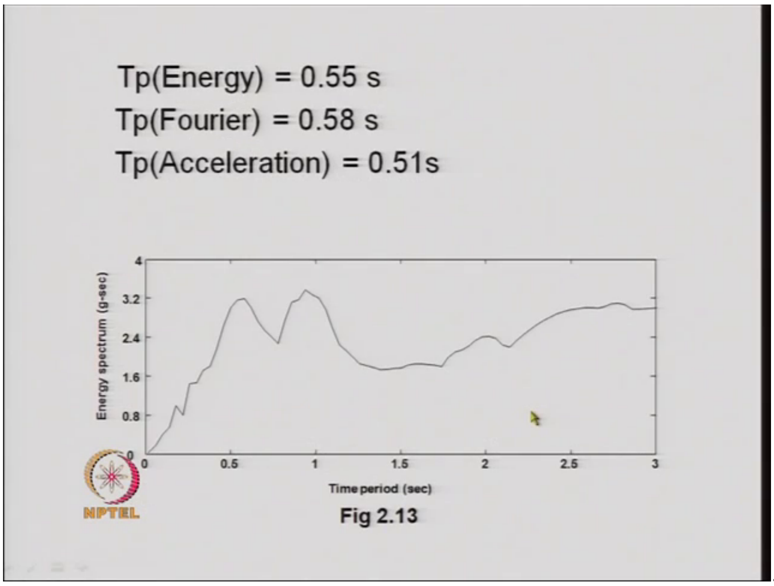

Fig 2.13

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And these plots are shown in this figures this is the energy spectrum that is the maximum energy against the time period; that means, we change the oscillator time period every time we calculate the maximum value of the energy and that is how by varying the time period of the oscillator, we can obtain the value different values of the maximum energy and plot them for a particular damping ratio.

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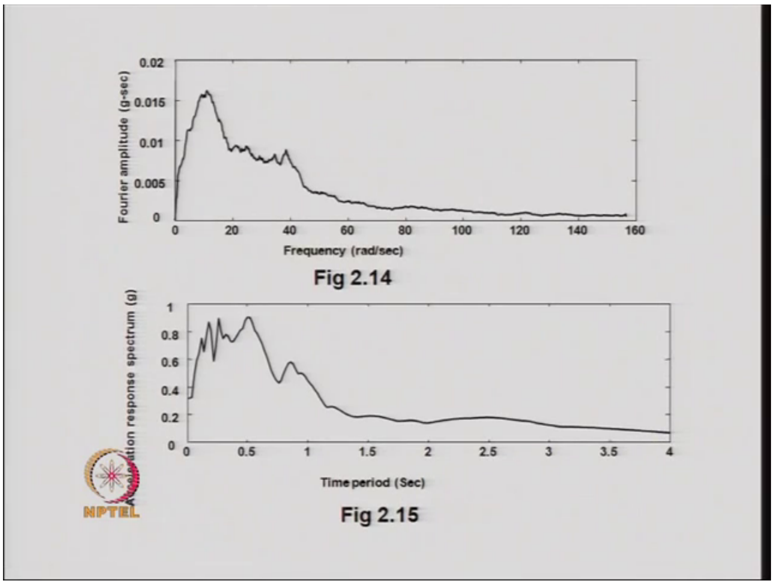

The same thing is done for the Fourier amplitude spectrum where the Fourier amplitude can be obtained against frequency or we can obtain also in the form of time period whereas, the response spectrum that is the acceleration response spectrum which is plotted over here that is plotted against time period. Now if we compare the peaks of these energy spectrum Fourier spectrum and the acceleration spectrum the if this where the peaks occur if we find out then we see that the energy spectrum peaks around 0.55 second Fourier spectrum peaks around 0.58 second where as the acceleration spectrum peaks around 0.51 second; that means, the ground motion has a predominant frequency content around a period of point say 52.6 within this range of the time period, we get the maximum frequency content of the ground motion.

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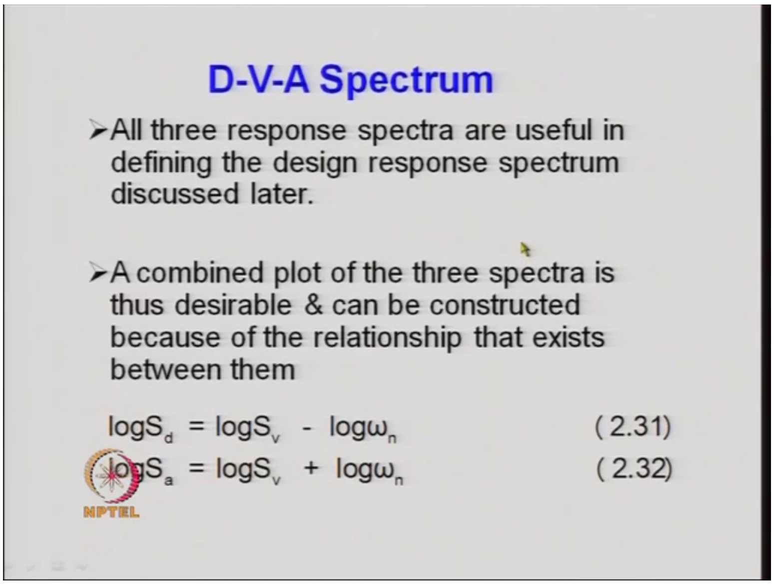

Now, we come to what is known as the D-V-Aspectrum or the combine displacement velocity acceleration spectrums mind you displacement response spectrum is denotes the maximum displacement, but the velocity and acceleration spectrum or actually the pseudo velocity and pseudo acceleration spectrum they do not represent the maximum value. So, of those quantities. Now D-V-S spectrum forms the basis for defining the shape of the design spectrum that is used in different courts all over the world now all the 3 spectrums are useful in defining the design and response spectrum which will discuss later a combined plot of the 3 spectra is desirable and be constructed because of the relationship that exist between the 3 spectrums for example, here.

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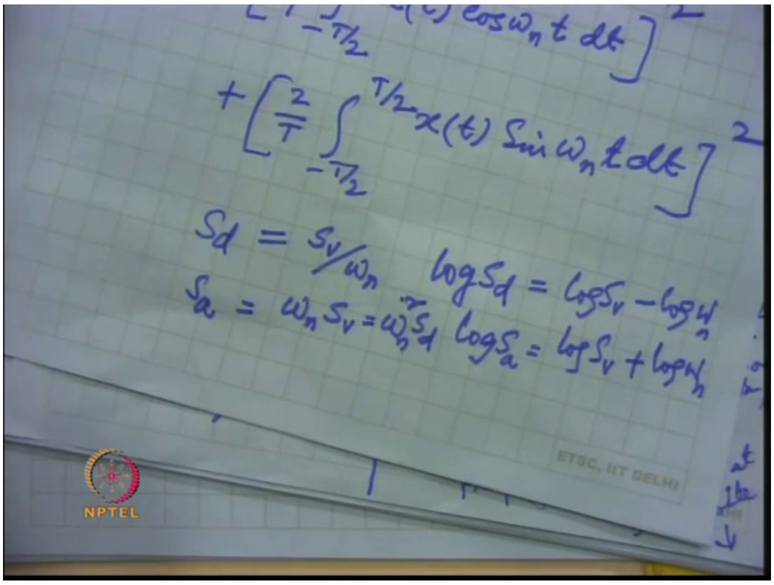

We define the maximum displacement is equal to S v divided by omega n and therefore, by taking log we can write that log S d is equal to log S v minus log omega n.

S a the spectral acceleration can be written as omega n square into S d equal to omega n into S v. So, if you take a log of these then we get log S a is equal to log S v plus log omega n now these 2 equations are shown over here and by looking at these 2 equation one can see there is; these 2 equation represent 2 straight lines on a log paper and these 2 straight lines will be orthogonal to each other because 1 is minus other is plus and these 2 lines will be an inclined lines in a plot where log S v is the vertical axis and log of omega n will be the horizontal axis.

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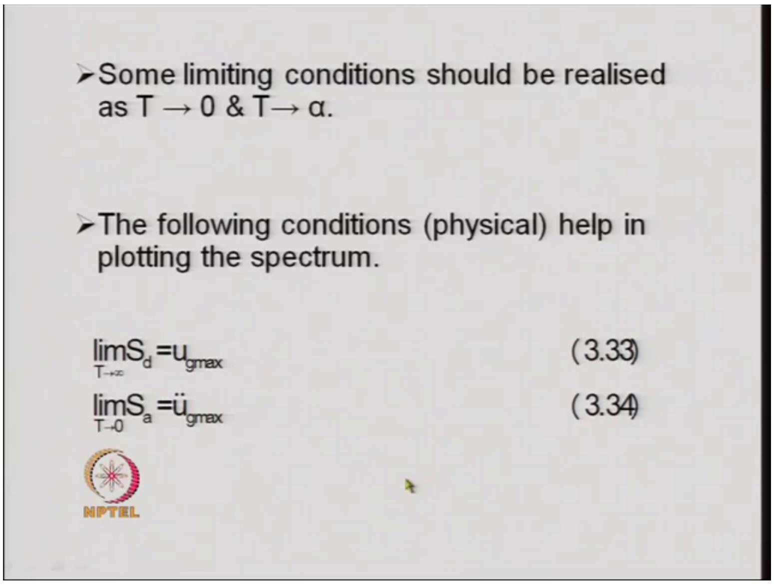

So, taking advantage of this 2 relationship, we are able to plot all the 3 quantities that is the S d, S v and S a in a single plot having the logarithmic relationship or in other words a special kind of log or a graph.

In which we have four ordinates one is the horizontal while axis in the log scale other is a vertical axis in the log scale and 2 other scales that is which will be inclined lines perpendicular to each other and also defined in a log scale now these four axis define the quantities that is S v, S d and S a for a given omega n value the plot also can be obtained not against omega n, but against t n that is the period in that case the 2 inclined lines we get interchanged that is the S a line would become the S d line and S d line will become S a line, if we plot it against t n. Now while plotting the response all the 3 response spectrum in this particular graph papers and which is known as a tripartite plot.

We need some limiting condition in order to define the spectrum. So, these limiting conditions are obtained by setting t tending to 0 in one case and in other case, t tending to infinity. In fact, it can be easily shown that limit of the maximum displacement S t tends to infinity becomes the maximum ground displacement itself similarly the limit of the spectral acceleration as t tends to 0 becomes the maximum ground acceleration now this can be easily realized from these 2 figures when t tends to infinity the frequency of the system becomes or tends to 0 and there would those kinds of systems are called very flexible system.

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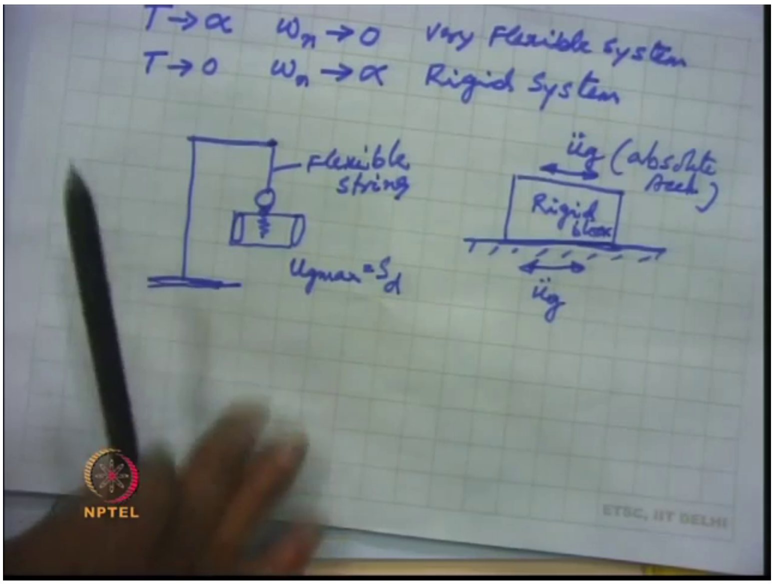

Once of system were shown for the earthquake measurement equipment that is there was a bracket and from that bracket a bob mass hanks with the help of a flexible string and there was a drum and as the ground motion takes place in this direction then the bob goes on tracing the ground motion on this chart paper it was possible because the flexible string cannot pass on the vibration from this point to this point as a result of this the bob remain stationary whereas, this entire bracket started moving like this. So, we can see that and that plot was this relative displacement plot came over here therefore, we can see that for a very flexible system the relative displacement of the system becomes equal to the ground displacement itself because the plot which you got is a plot of the ground displacement on the chart paper.

On the other hand, when t tends to 0 the frequency of the system tends to infinity or this is called what as a digit system the example of this is that we put a digit block on a on the ground and if there is a ground acceleration here then the acceleration that is passed on to the top of the digit block is the ground acceleration itself because the this entire rigid block moves along with the what to call the ground motion. And there is no relative displacement of the digit block with respect to the ground because it is very rigid and since there is no relative displacement and there is no relative acceleration and therefore, the acceleration which is experienced at the top of this is rigid block is the ground acceleration itself.

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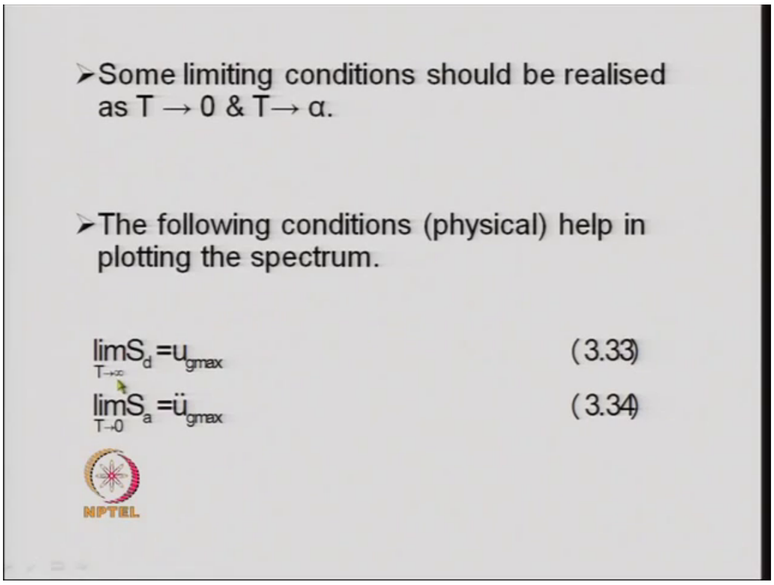

So, therefore, the following physical conditions emerge that is the limit t tending to infinity for S d becomes equal to u g max and limit t tendings to 0 S a becomes equal to the ground acceleration.

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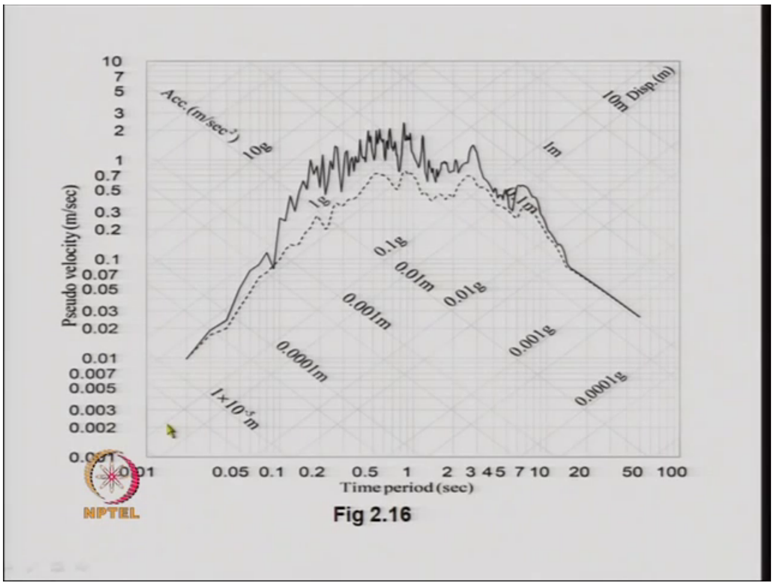

Fig 2.16

So, now we the show the plot this is the log S v that is the S v is in the log scale in the vertical axis and this is the log of not omega n, but log of t n which is shown in the horizontal axis and these inclined line is one axis and this inclined line is another axis now these axis corresponds to the acceleration in the logarithmic scale and this axis shows the displacement in the logarithmic scale again. Now when we can plot the S v versus t n in the logarithmic scale and if we make a plot; let us say this plot looks like this for a particular ground record.

In this figure the ground record taken is the El Centro earthquake and the dotted line is the average; that means, we are average that zigzag portion and more or less is smooth curve has been obtained for the El Centro earthquake now this plot which is the plot of log S v versus log t provides also the information about S a and S d that is if we measure; that means, if I take a point like this then for this particular point the S v value will be read from this axis S a value will be read from this axis and S d value will be read from a line parallel to this axis. So, any particular point reviles 3 quantities; S a, S v and S d therefore, in one single plot we get all the 3 response spectrum. So, this is called the tripartite plot of the response spectrum in a special log graph paper.

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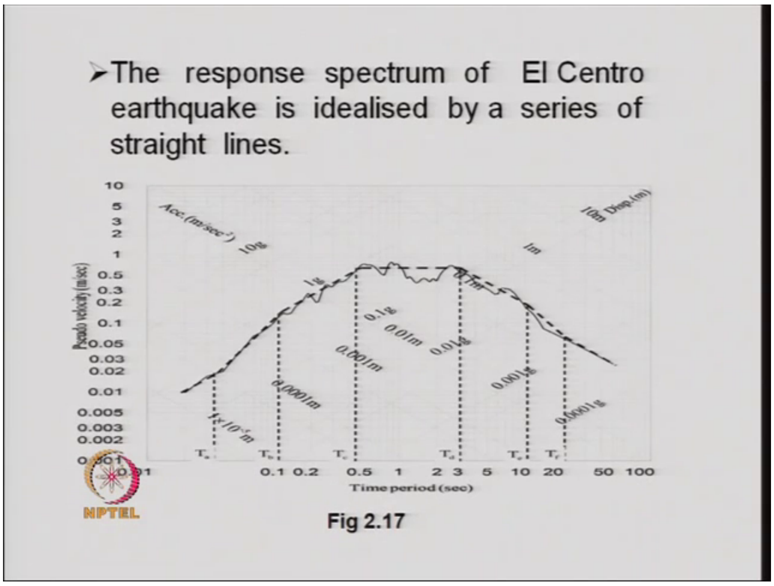

Fig 2.17

Now, what has been done is that the graph that we get or that is the response spectrum that the smooth response spectrum that we get for the particular earthquake in this case is El Centro earthquake this has been further idealized by a number of straight lines which are represented by the dotted lines over here and we obtain the response spectrum for a number of earthquakes and plotted them in the tripartite plot.

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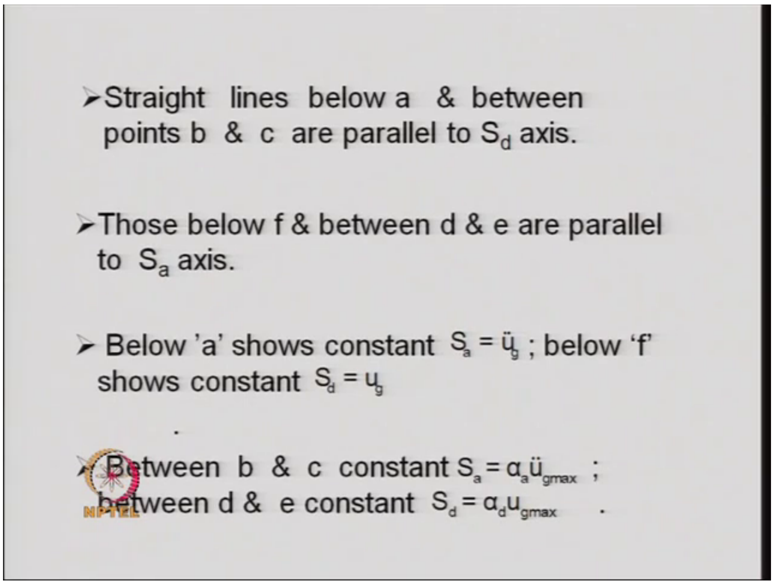

And those response spectrums show similar kind of trend and to understand the trend let us look at the different segments for example, this is a segment which is designated by t a time period t a that is be below this time period t a we have a segment then we define an another time period t b between t a to t b, we have another segment then from t b to t c we have got another segment then from t c to t d we have another segment and from t d to t e, we have another segment and from t e to t f we have got another segment and beyond t f we have another segment.

So, these segments are represented with the help of these time periods t a, t b, t c, t d, t e, and t f. Now if we plot to different earthquake spectrums in the tripartite plot and idealize them by straight line then we get similar kind of form only the t a value, t b value, t c value, these values would be different for different earthquakes now each one of these segment represent certain thing that that is what we are going to discuss now the straight lines below a and between points b and c are parallel to S d axis that is this line below a and between this point and this point if you look at these 2 lines these 2 lines are parallel that is what observed parallel to the line the line which is called the S d line. So, that is parallel to S d.

Those below f and between d and E are parallel to S a axis that is this line and this line these 2 lines are parallel to the this acceleration or S a line below a shows constant value of S a which is equal to the ground acceleration and below f shows constant S d which is equal to the ground displacement that is what we discussed before that is in they give the limiting conditions that is here the limiting condition that value is shown as the acceleration; that means, the we measure along this axis. Therefore, it shows a constant acceleration and these shows a constant and displacement which is equal to the ground displacement that is how the specified ground acceleration and specified ground displacement they are maximum values are utilized in plotting this curve.

In between these between b and c constants S a is equal to alpha times u double dot g max and between d 2 e the S d is equal to alpha d times u double dot or u g max that is this one this line and this line if we considered since this line and this line and parallel which is now logical is that these value. That means, this parallel line this will indicate a particular value of acceleration constant acceleration this constant acceleration will be equal to the constant acceleration or the ground acceleration that is shown over here the maximum ground acceleration which is represent this constant portion this multiplied by some factor gives the constants acceleration over this inclined line.

Similarly, the maximum ground displacement which is represented by this segment and this portion of the segment which is parallel to this shows a displacement or maximum displacement which is equal to sum multiplication factor multiplied by the maximum ground displacement. So, that is what is written over here the S a in that segment is equal to sum multiplication factor alpha a into the maximum ground acceleration and on the other side the it is the maximum displacement is shown or the S d shown as the sum multiplication factor multiplied by the maximum ground acceleration.

Now, the left of c is directly related to maximum acceleration because in this plot we can see that on the left of c everything that is there that is parallel to the axis the S d axis representing the acceleration and beyond this point everything that is there that represent the displacement because there these lines are parallel to the acceleration axis representing the displacement.

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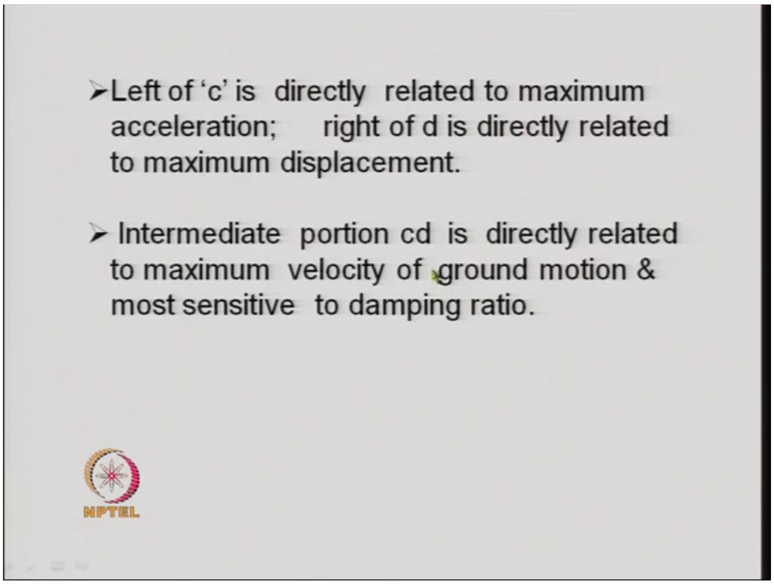

So, the intermediate portion c d is directly related to maximum velocity of ground motion and most sensitive to damping ratio that is the horizontal portion it is; obviously, is most related to the velocity and their and this is also found to be very sensitive to the damping ratio; that means, if you change the damping constant will find that in this range the response spectrum is varying widely.

So, now let me summarize over here what we discussed we discussed the cross power spectral density function that exist between 2 points along the direction of the wave propagation these cross power spectral density function represents the lag time lag or the lack of correlation between the ground motion said these 2 points and this cross power spectral density function is defined with the help of a coherence function which is a exponentially decaying function of the distance between the 2 points and frequency then we obtained the then we started discussing on the response spectrum.

So, we have seen that there are 3 types of response spectrum acceleration response spectrum velocity response spectrum displacement response spectrum and also there is energy spectrum all of them can be plotted using a single degree freedom system and for a particular earthquake we can go on changing the period of the single degree freedom system and obtained this quantities and can plot it. Then we discussed about the D-V-A spectrum that is the tripartite plot where all the 3 spectrums can be plotted and can be seen with the help of one particular graph and the characteristics of the idealized D-V-S spectrum.