

**Seismic Analysis of Structures**  
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**Lecture – 07**  
**Seismic Inputs (Contd.)**

In the previous lecture we were discussing about (Refer Time: 00:28) spectrum and we had seen that the response spectrum can be plotted in a tripartite plot and can be idealized as a series of straight lines representing all the 3 spectrums that is the displacement, velocity and acceleration spectrums. We continue with this, response spectrum for different earthquakes that were recorded all over the world for though for then the response spectrums were obtained and plotted in a tripartite plot and similar exercise was done to a represent it by a series of straight lines and it was seen that the most of the earthquakes represented the same kind of pattern only difference was between the values of  $T_a$ ,  $T_b$ ,  $T_c$ ,  $T_d$ ,  $T_e$  and  $T_f$  that were obtained and they were different for different earthquakes.

So, this observation led to a general shape of the response spectrum for earthquake and therefore, that shape was utilized for constructing what is known as the design response spectrum the design response spectrum; obviously, is the response spectrum that for using which we design all the structure for future earthquake.

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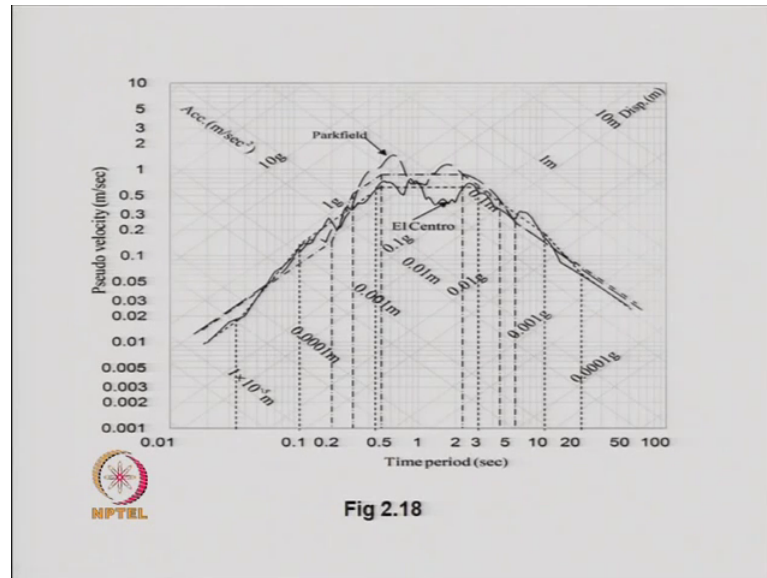
**Example 2.4:** Draw the RSP for Park field earthquake for & compare it with El Centro earthquake

**Solution:** Using Eqns. 2.23-2.26, the spectra are  $\xi=5\%$  obtained & drawn in tripartite plot; it is idealized by straight lines; Fig 2.18 shows Parkfields & El Centro RSPs. Comparison of  $T_a$  to  $T_f$  between the two is shown in the book.



Now, an exercise was carried out to show that the different earthquake records provide the same kind of idealized response spectrum on a tripartite plot for Parkfield earthquake and for EL Centro earthquake this exercise where carried out.

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And one can see that the idealized response spectrum for the two earthquake this is the EL Centro earthquake and this was the Parkfield earthquake when they were idealized with the help of straight line they nearly at the same kind of feature and only thing that differ was the  $T_a$ ,  $T_b$  values,  $T_c$  values at different points.

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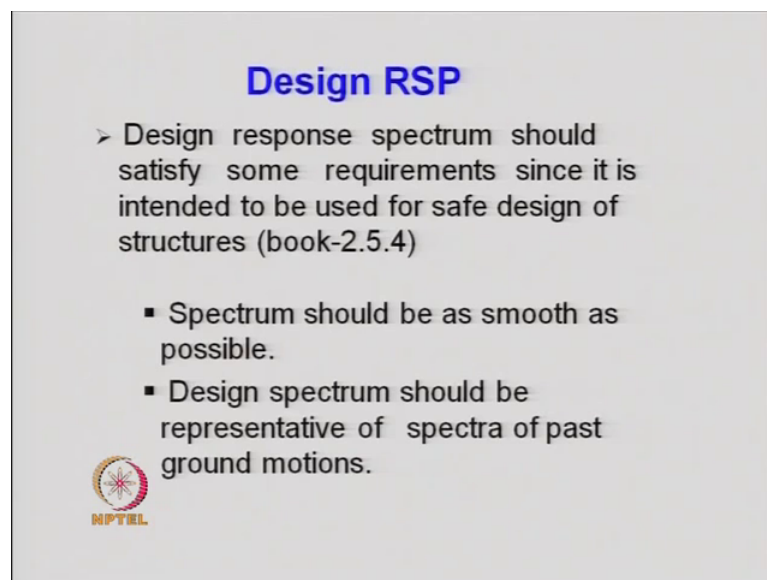
Earthquake	$T_a$ (s)	$T_b$ (s)	$T_c$ (s)	$T_d$ (s)	$T_e$ (s)	$T_f$ (s)
Park field	0.041	0.134	0.436	4.120	12.0	32.0
El Centro	0.030	0.125	0.349	3.135	10.0	33.0

**Table 2.1 Comparison of periods between Parkfield and El Centro earthquakes**

And these T values were compared in this table for the Parkfield earthquake and for the EL Centro earthquake one can see that T a value that basically was obtained for Parkfield and for the EL Centro it was 0.03 and this was 0.041. For Parkfield it was 0.134 that is T b and it was 0.125. For EL cento for T c it was 0.436 and 0.349 for the EL Centro and 4.12 was for T d for Parkfield and for EL Centro it was 3.135. T e and T f they were more or less the same for the two earthquakes especially the T f.


So, thus the these two earthquake records revealed that there could be difference between the values of T a, T b, T c, T d, T e, T f, but the nature of the response spectrum can be approximated by a series of straight lines having the characteristics that we discussed before.

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**Design RSP**

- Design response spectrum should satisfy some requirements since it is intended to be used for safe design of structures (book-2.5.4)
  - Spectrum should be as smooth as possible.
  - Design spectrum should be representative of spectra of past ground motions.



Now, let us come to the design response spectrum. The design response spectrum is the response spectrum which is used for designing all the structures for future earthquakes which are known not known therefore, it is a difficult task what generally is done for obtaining the quantities for future earthquake is that we try to see the trend of the parameter that we are looking for of that parameter in the previous earthquakes.

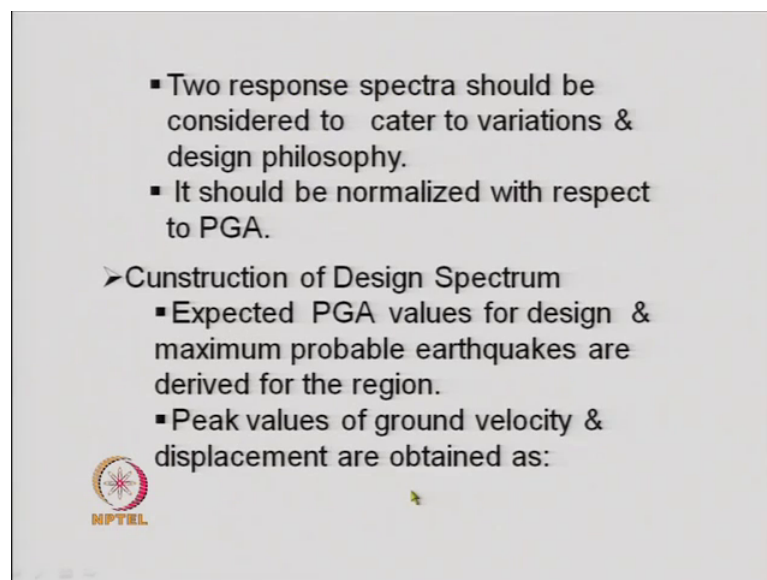
So, therefore, ones we are convinced from the results of the different response spectrums drawn on the tripartite plot that the shape of the response spectrum can be idealized by a series of state lines between T a T b T c T d T e etcetera. We said that the future earthquake also we will show up similar kind of trend and therefore, design response

spectrum was developed on that basis. However, the design response spectrum should satisfy some other requirements for example, the spectrum should be as smooth as possible and therefore, the idealizing the response spectrum by a series of straight line is quite rational. If the response spectrum or the design response spectrum is not smooth enough and it changes rapidly with the time period then there could be a sudden change in the value of spectral acceleration for a small difference of time period.

Now, it is quite expected that the time period of a structure in reality could be different than the time period that we theoretically calculate therefore, if there is a large change in the spectral value or the spectral acceleration for a small change in time period. Then there could be a lot of difference between the theoretical spectral acceleration that is considered in the design and the actual spectral acceleration that the structure experiences at the time of earthquake. So, in order to eliminate that the design response spectrum where are considered to be as smooth as possible.

Then design spectrum should be representative of spectra of past grounded motions that we explained before.


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- Two response spectra should be considered to cater to variations & design philosophy.
- It should be normalized with respect to PGA.

➤ Construction of Design Spectrum

- Expected PGA values for design & maximum probable earthquakes are derived for the region.
- Peak values of ground velocity & displacement are obtained as:



Next two response spectra should be considered to cater to variations and design philosophy. Now, the design philosophy of the earthquake resistant design is that we design the structure for a design response spectrum and see that that structure can survive in the extreme earthquake condition. That is there will not be a complete collapse of that

structure in the severe earthquake it can have a huge amount of damage. So, this is the current design philosophy and therefore, we have to have two level the response spectrum - one is the design response spectrum other is the extreme response spectrum.

And next is that the response spectrum should be normalized with respect to peak ground acceleration because if you are wanting to provide or prescribe a design response spectrum that should be prescribed in the form of the shape. But the actual response spectrum would be obtained after we multiply that shape with the peak ground acceleration which may differ from place to place.


Next let us see how we can construct a design spectrum. So, for that we follow certain states steps first the expected peak ground acceleration values for design and maximum probable earthquakes are derived for the region then peak values of ground velocity and displacements are obtained using this empirical relationship. That is if any one of the quantities generate the ground acceleration maximum ground acceleration if it is given then with the help of that one can find out the peak ground displacement and peak ground velocity using these values of this constants  $c$  and  $c_1$  and  $c_2$  has this.

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$$\dot{u}_{gmax} = c_1 \frac{\ddot{u}_{gmax}}{g}; u_{gmax} = c_2 \frac{\dot{u}_{gmax}^2}{\ddot{u}_{gmax}}$$

$c_1 = 1.22 \text{ to } 0.92 \text{ m/s} \quad c_2 = 6$

- Plot baseline in four way log paper.
- Obtain bc, de & cd by using  
 $\alpha_a \ddot{u}_{gmax}; \alpha_d u_{gmax}; \alpha_v \dot{u}_{gmax}$
- b & d points are fixed; so  $T_c$  is known.



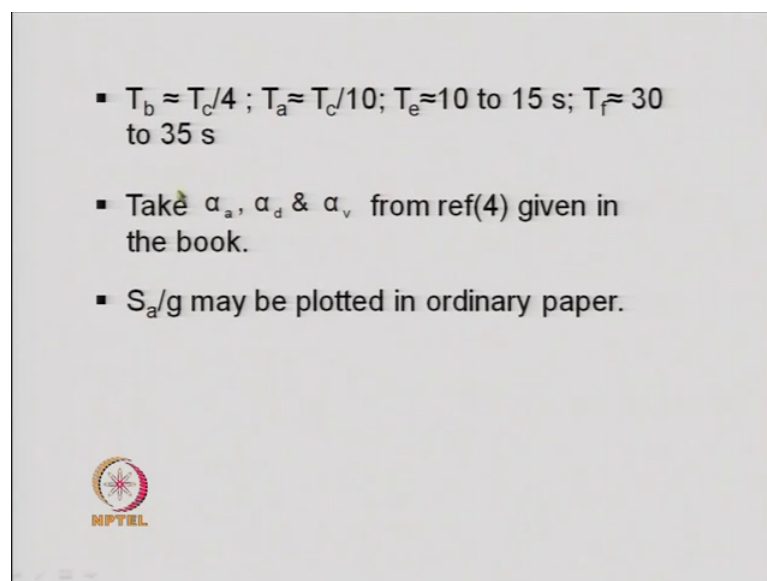
So, once we have the specified values of the maximum ground acceleration and from that once we have obtained the maximum ground displacement and maximum ground velocity then in a tripartite plot we can have a baseline plot in the four way log paper.

That means, that baseline plot we will simply draw or show the peak ground acceleration peak ground velocity and peak ground displacement.

Then as we discussed before obtain the value obtained the b c d and c d segments using these multipliers that is the maximum ground acceleration is multiplied by alpha a then maximum ground displacement is multiplied by alpha d and maximum ground velocity is multiplied by alpha v. These alpha a, alpha d and alpha v they are available for different soil condition and also for the design earthquake level and the extreme earthquake level in various literature and they are obtained from the past earthquake records.

Since c and d points are fixed the  $T_c$  is known and once  $T_c$  is known then with respect to  $T_c$  other values  $T_a$ ,  $T_b$ ,  $T_d$ ,  $T_e$ ,  $T_f$  etcetera can be obtained from the again the experience that people have at from the earth previous earthquake records.


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▪  $T_b \approx T_c/4$  ;  $T_a \approx T_c/10$ ;  $T_e \approx 10$  to 15 s;  $T_f \approx 30$  to 35 s

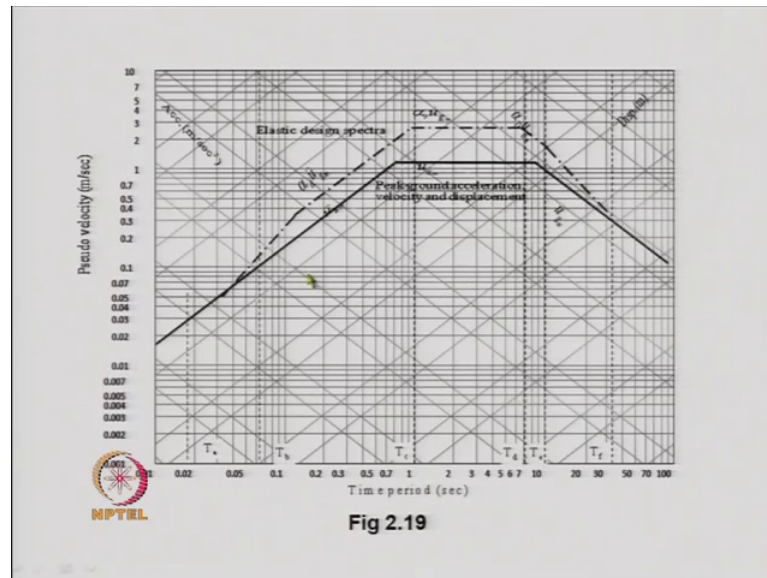
▪ Take  $\alpha_a$ ,  $\alpha_d$  &  $\alpha_v$  from ref(4) given in the book.

▪  $S_a/g$  may be plotted in ordinary paper.



Now, generally the this empirical relationship is used in obtaining the values of  $T_b$   $T_a$   $T_e$  etcetera for example, they are all define in terms of  $T_c$  and  $T_e$  and  $T_f$  it has been seen that  $T_f$  is generally in the ranges in this range that is 30 to 35 second and  $T_e$  generally ranges between 10 to 15 second.

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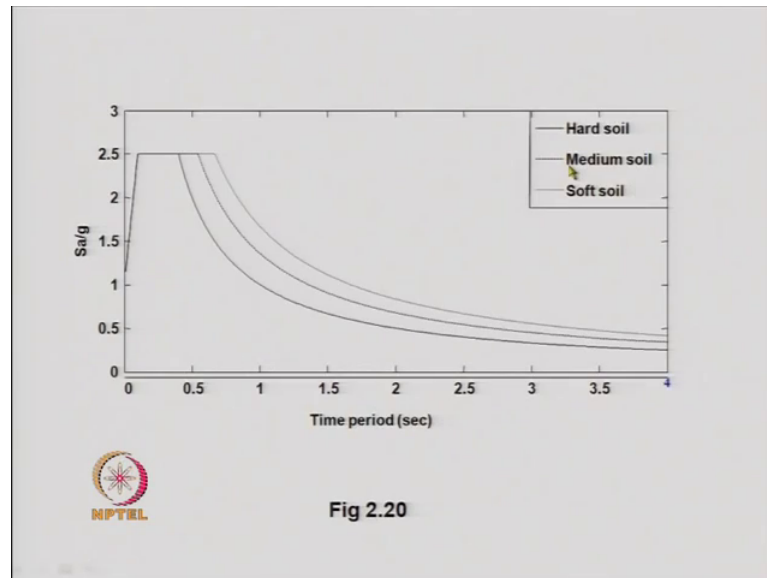


So, now let us look at how we can obtain using the steps the response spectrum in a tripartite plot. This is the baseline, baseline means the ground maximum ground acceleration that is plotted over here and it is parallel to the  $s d$  axis and we measure the acceleration along this line.

Then this is the maximum ground displacement line which is parallel to acceleration line and the ground displacement of the ground displacement  $s d$  not ground displacement  $s d$  is measured along this line. So, this is the line showing the maximum ground displacement this is the line showing the ground maximum acceleration and obviously, this is the line which shows the maximum ground velocity. So, this is the base line that we obtained. Then as we have described before this segment that is from  $c$  to  $b$ , between  $b$  to  $c$  this segment basically is parallel to this line as we discussed before and they that that is equal to  $\alpha a$  times the ground acceleration. So, therefore, we draw a line this line parallel to this line and by this, this value being equal to  $\alpha a$  multiplied by  $u \ddot{x} g$ .

Similarly this line is parallel to this line and this is equal to  $\alpha d$  times the maximum ground displacement and this line is parallel to this line and this value is equal to  $\alpha v$  times the maximum ground velocity. So, once you have these lines plotted then one can join these lines to get, that is idealized response spectrum this described with the help of a series of straight lines.

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Now, if we wish to plot the spectral acceleration  $y$  versus time period in a ordinary the paper; that means, they are not log scale, but ordinary scale and normalized with respect to the gravity that is  $g$  that  $S_a$  by  $g$  plot can be shown to be of this time. That means, these shape has emerged from the shape of the response spectra that was observed in a tripartite plot. So, in almost all codes we find the spectral acceleration plot versus time is of this particular shape.

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**Example 2.5:** Construct design spectra for the 50th percentile & 84.1 percentile in Tripartite plot.

**Solution:**  $T_a = 1/33s$ ;  $T_b = 1/8s$ ;  $T_e = 10s$ ;  
 $T_f = 33s$   
 $\alpha_A = 2.17(2.71)$  ;  $\alpha_V = 1.65(2.30)$   
 $\alpha_D = 1.39(2.01)$   
For 5 % damping;






Now, a design spectrum can be obtained for 50th percentile and 84.1 percentile in tripartite plot 50th percentile means the design spectrum and 84.1 percentile that is the mean value plus 1 standard deviation that gives a 84.1 percentile and that is taken as the response spectrum for the extreme earthquake. So, we can have these two earthquakes and for these are earthquakes one can construct a design spectra given the following quantities.

For example, the T a was given as 1 by 33 second, T b was 1 by 8 second, T e was 10 second and T f for 33 seconds and the alpha A, alpha D and alpha V values that this multiplying factors they can be taken from a standard text books on earthquake engineering for example, in this case alpha A was 2.7, alpha V was 1.65 and alpha D was 1.39 against the these values in the bracket did you note for the 84.1 percentile and these 1 for the 50th percentile and the damping was considered 5 percent.

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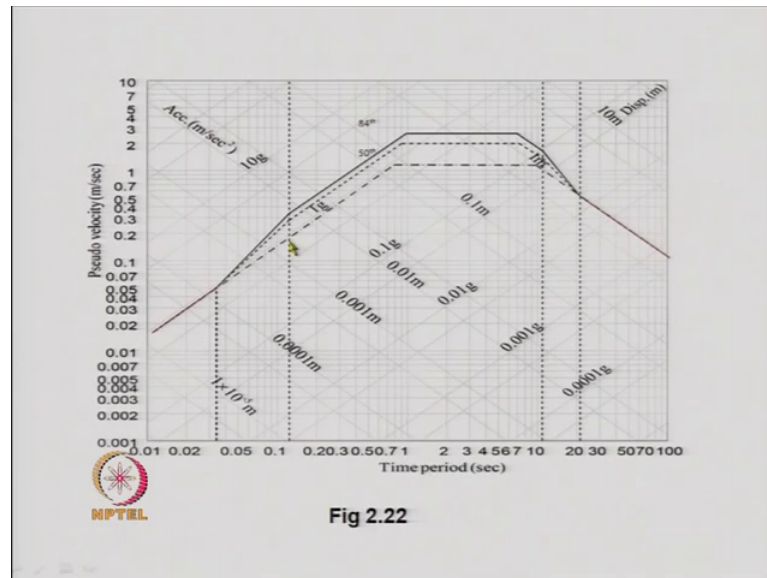
$$\dot{u}_g = \frac{1.22}{g} \quad \ddot{u}_g = 0.732 \text{ ms}^{-1} \quad \ddot{u}_g = 0.6g$$
$$u_g = \frac{(0.732)^2}{0.6g} = 0.546\text{m}$$

Values within bracket are for 84.1 percentile spectrum. Plots are shown in Fig 2.22.



So, what we did is that this ground acceleration peak ground acceleration was giving and with the help of that we obtained the peak ground velocity and peak ground displacement using the empirical relationship that we are I have shown before.

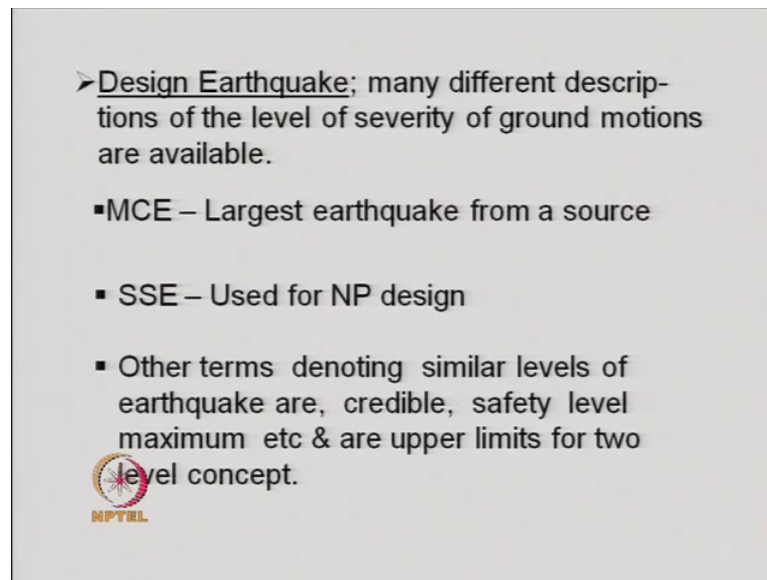
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And then we plotted the two spectrum this was the base line that is showing the peak round velocity peak ground acceleration and peak ground displacement and then multiplying by alpha A, alpha D, alpha V values we obtained this segments of the straight line and join and obtained the response spectrum in the tripartite plot. And the  $T_a$   $T_b$   $T_c$   $T_d$   $T_e$  value was where are specified. Therefore, once we have the  $T_a$   $T_b$   $T_c$   $T_d$   $T_e$  value specified and alpha a and alpha b and alpha d values are given then one can construct a design spectrum.


Generally if the  $T_c$  value may be given and from the  $T_c$  value one can obtained the other values that is  $T_a$   $T_b$  etcetera using the approximate values that was given by this relationship. Once you know this relationship approximate relationship one can obtained the values of  $T_b$  and  $T_a$ ,  $T_e$  and  $T_f$  generally lying this range.

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➤ **Design Earthquake**; many different descriptions of the level of severity of ground motions are available.

- MCE – Largest earthquake from a source
- SSE – Used for NP design
- Other terms denoting similar levels of earthquake are, credible, safety level maximum etc & are upper limits for two level concept.

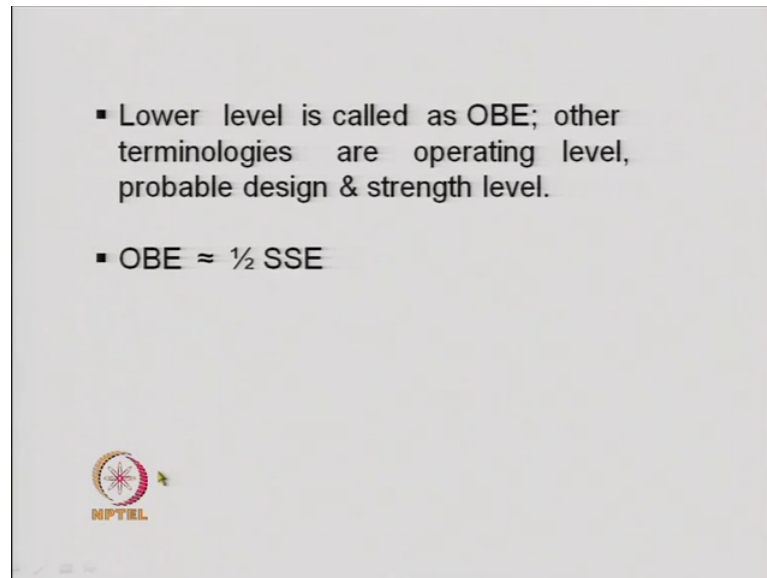


We have discussed how one can obtain a design response spectrum using that idealized segmented or idealized series of straight lines and plotted in a tripartite plot.


Now, we come to the different levels of earthquake that is described in the earthquake literature, design earthquake that generally specify certain value of peak ground acceleration and from that peak ground acceleration one can also estimate the expected peak ground acceleration for the extreme earthquake.

Now in the literature we get different terminologies for the design earthquake for example, one is MCE that is maximum credible earthquake there is a largest earthquake that we expect from a source, then SSE that is safely shutdown earthquake that is used in a nuclear power plant design. Other terms which denote similar levels are credible, safety level, maximum etcetera and they denote the upper limits for the two level concept; that means, they denote the earthquake PGA values for the extreme condition.

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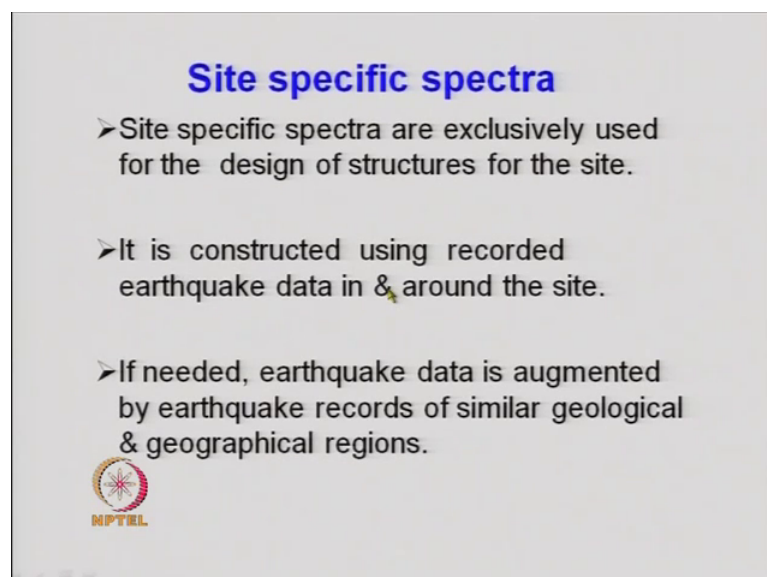


- Lower level is called as OBE; other terminologies are operating level, probable design & strength level.
- $OBE \approx \frac{1}{2} SSE$




The lower level that is the design PGA value is called OBE that is the ordinary base earthquake and other terminologies which are used or the similar terminology which are used for ordinary base earthquake is operating level, probable design and strength level earthquakes. And generally the OBE that is the design usual design earthquake that is the OBE that is the ordinary base earth earthquake is approximately equal to half of the shape shutdown earthquake used in the nuclear power plant design.

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### Site specific spectra

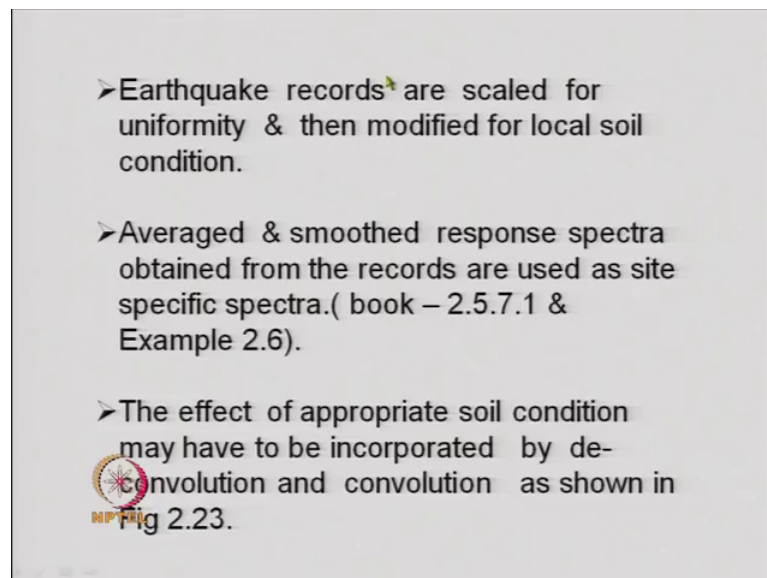
- Site specific spectra are exclusively used for the design of structures for the site.
- It is constructed using recorded earthquake data in & around the site.
- If needed, earthquake data is augmented by earthquake records of similar geological & geographical regions.



Next we come to the site specific spectra. The site specific spectra is called a site dependent response spectrum and they are used for designing speciality structures for a particular place and that particular place may have certain characteristics in terms of geology and geography. That is the design response spectrum that we talk of that design response spectrum may not be valid for that particular site because of the geological and geographical or conditions of the site.

Now, for that what we do is that we collect as many number of earthquake data possible for that particular site that is in and around that site sometimes if the collected data is not sufficient then we augment the earthquake data by collecting the data for similar geographical and geological regions.

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- Earthquake records are scaled for uniformity & then modified for local soil condition.
- Averaged & smoothed response spectra obtained from the records are used as site specific spectra. (book – 2.5.7.1 & Example 2.6).
- The effect of appropriate soil condition may have to be incorporated by deconvolution and convolution as shown in Fig 2.23.

Once we have collected enough number of earthquake records in and around that site after augmentation then we scale the earthquake records and also modify it for the soil condition. Now, the scaling is necessary because the different earthquake records that have been collected they may be valid for certain epicentral distances, certain peak ground acceleration or certain magnitude of earthquake. Whereas, the site specific spectrum that may have to be constructed for a given epicentral distance for a given magnitude of earthquake and for a given soil condition.

Therefore, all the earthquake records that have been collected they must be properly scaled so that it reflects the epicentral distance and the magnitude or the peak ground

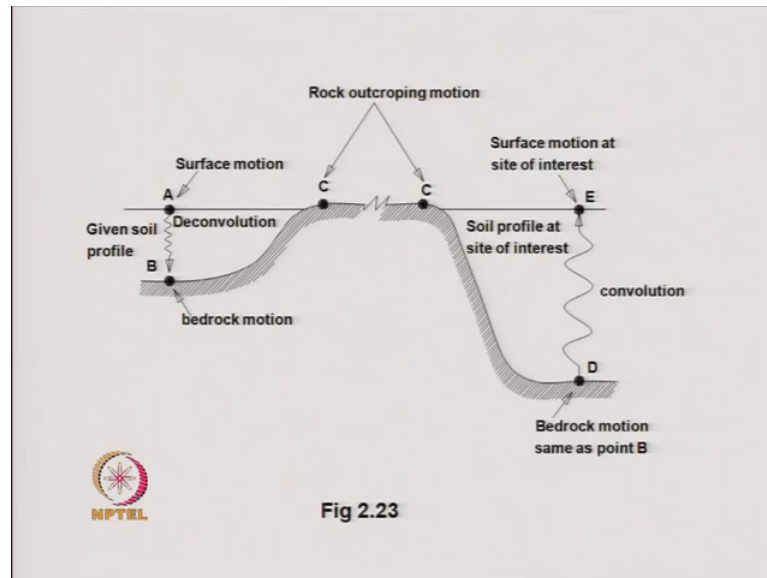
acceleration for which the site specific spectrum have to be constructed and also the site specific spectrum should be compatible with the soil condition. And in most of the cases the records that have been collected these records are collected on certain particular sites whose soil conditions may not be same as the soil condition that is existing for the site. Therefore, a certain techniques are used for scaling and after the scaling and we obtain the required response spectrum or site specific spectra.

The scaling depends upon the data that are available. In fact, if the data given in the form of the time history records then this time history records are Fourier synthesized and then the frequency contents of the ground motion of the each record they are normalized to provide a common base or common magnitude of earthquake for which the site specific spectrum is to be constructed and also it is normalized. So, that epicentral distance of the particular site for which we are constructing the site specific spectrum that epicentral distance is also reflected in the scaling process.

After the response or be earthquake records have been scaled properly then and modified for the local soil condition then a response spectrum corresponding to each one of those modified records of the ground motions are considered for obtaining the response spectrum from each one of them, and these response spectrums are then averaged and smoothen in order to obtain the site specific spectrum.

The effect of appropriate soil condition generally is incorporated by deconvolution and convolution.

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As shown in this figure in most of the cases the ground records that are available they may be on the surface of a particular ground and that background may be having a particular soil property. And the soil properties that is existing for the site is say different that is at point E say for example, we are interested in obtaining the site specific spectrum and say at point A we the some ground record that is available. Then in order get the appropriate soil condition in the site specific spectrum we do a deconvolution that is from the ground recorded ground motion at station A we obtain the ground motion at the back bedrock by a deconvolution process.

And once we get the bedrock motion and then we assume that the bedrock motion that is available at point D that will be same as that which is available at point B and assuming this two ground or bedrock motions to be same then we do a forward wave propagation problem or carry out a forward propagation problem at point D to point E and consider the appropriate soil condition while performing these convolution technique.

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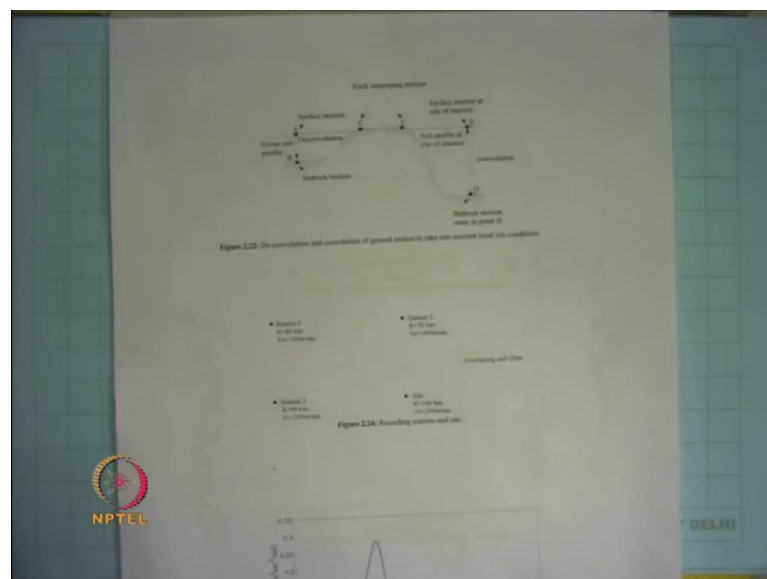
### Uniform hazard spectra

- Statistical analysis of available spectrum is performed to find distributions of PGA & spectral ordinate at each period.
- From these distributions, values of spectral ordinates with specified probability of exceedance are used to construct the uniform hazard spectra.
- Alternatively, seismic hazard analysis is carried out with spectral ordinate (at each period for a given  $\xi$ ) as parameter (not PGA).



Now this is explained with the help of a particular problem here.

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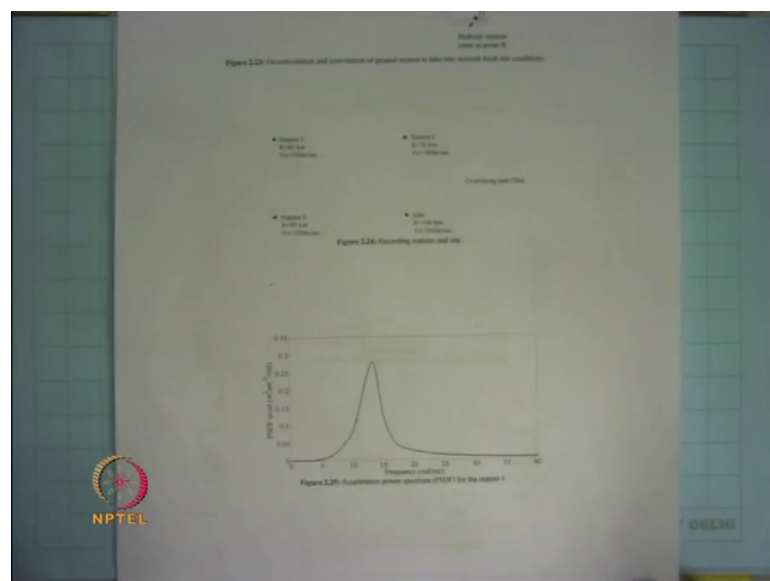


In place of the ground time history records we consider that there is a site over here and there are 3 stations where basically the data have been collected and at this 3 stations the epicentral distance is given and the shear wave velocities are given. These shear wave velocities and epicentral distances different for the 3 stations the shear wave velocity which is given basically indicates the soil condition for the particular station.



Now in place of the ground history or time history of ground motion for these stations it is given in the form of the power spectral density function of the ground motion or power spectral density function of the ground acceleration for these 3 stations. With the help of that information and the particular attenuation law that is valid for the region we perform the calculations in order to obtain the site specific spectrum at this particular site. And in the in that process we will see how we can do the scaling for the magnitude of peak ground acceleration and also do the scaling for the epicentral distance and finally, considered the appropriate soil condition that is existing at the site.

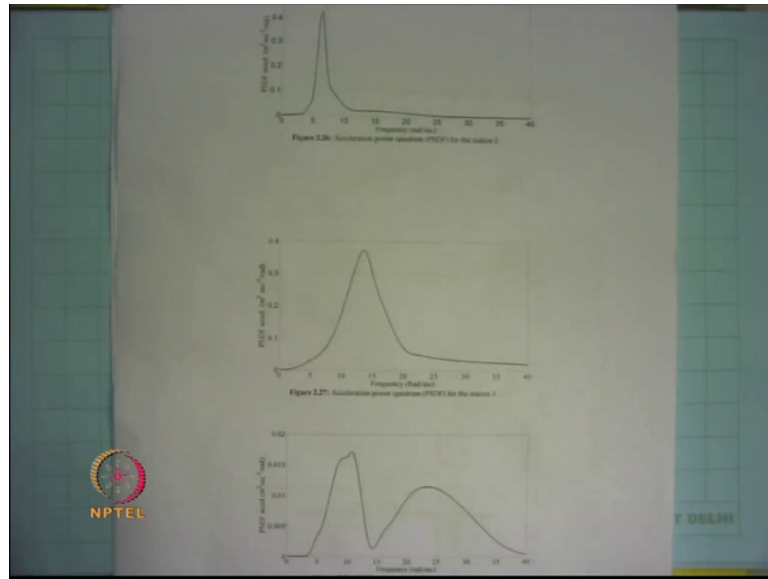
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The basis of the problem will be much more clear when we will cover the spectral analysis technique for solving a structures response to random ground motion. But for the time being the problem is explained with the help of the some excepted formula in order to show you how the scaling is done.

Now for the site 1 the power spectral density function of the ground acceleration is given by this.

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For site 2 this is the power spectral density function of the ground acceleration which is shown and this is for the site 3 the power spectral density function which is given. Now these power spectral density functions are available for the surface of the ground. And as I told you if we have the information available on the surface of the ground then many a time you have to do a deconvolution technique to obtain the corresponding quantity at the rock bed level and then only we take that particular rock bed level information and we assume that the same condition exist at the rock bed of the particular site.


So, what we do? We first convert this power spectral density function which is given at the surface at each site to the corresponding power spectral density function which will exist at the rock bed level just below site 1, site 2 and site 3.

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$$(PSDF)_{surface} = (PSDF)_{rockbed} (TF)^2$$

$$(PSDF)_{rockbed} = \frac{(PSDF)_{surface}}{(TF)^2}$$

$$(TF)^2 = \frac{1}{\left[ \cos^2(\omega H / V_s) + \left( \xi^2 H^2 \omega^2 / V_s^2 \right) \right]}$$


 $(\sigma_{v_z})_{rockbed} = 0.12g \quad p_1 = 3.2 \quad (PGA_1)_{rockbed} = p_1 \quad \sigma_{v_z} = 0.384g$   
 $(\sigma_{v_z})_{rockbed} = 0.08g \quad p_2 = 2.9 \quad (PGA_2)_{rockbed} = p_2 \quad \sigma_{v_z} = 0.24g$

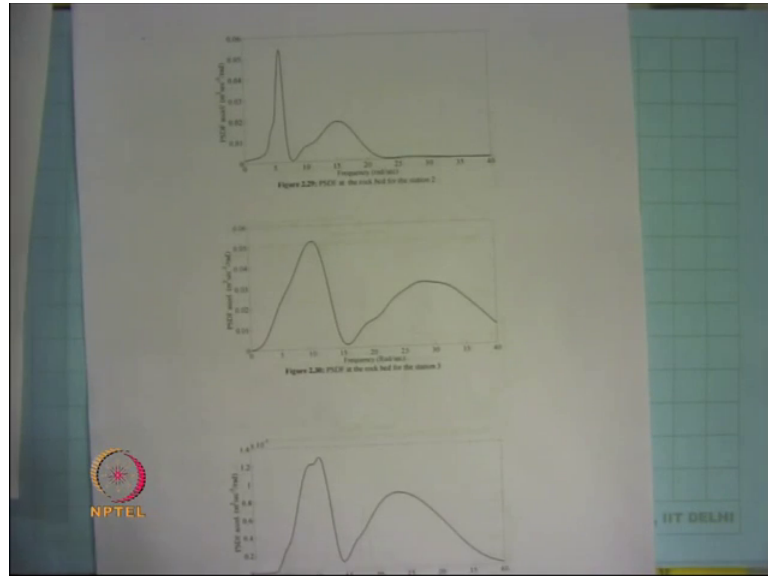
And for that we use this particular formula which will be discussed in chapter 4 where will be discussing the response analysis of structures for random ground motion and there we will show that if the power spectral density function is given for a particular system then and if it is an input to that particular system then output power spectral density function can be obtained by multiplying the power given power spectral density function with the transfer function square of the system.

So, here the power spectral density function at the surface that basically is obtained as a function of the product of power spectral density function at the rock bed multiplied by the transfer function square, that is a forward a problem or in other words forward wave propagation problem that is from the rock bed we are going to the surface. Now, if this equation holds good then from this equation we can work back what is the power spectral density function at the rock bed, that will be equal to power spectral density function at the surface divided by the transfer function square. And the transfer function square is given in terms of the depth of the soil and the shear wave velocity and damping of the system and this is obtained for a different frequencies.

Now, using these transfer function square for the specified values of the  $v_s$  for the 3 sites for the 3 sides we have seen that we have differential wave velocities and the soil layer dept is given as 20 meter,  $\xi$  is taken as 5 percent and then for each  $\omega$  value we calculated the values of transfer function square and we divided the surface power

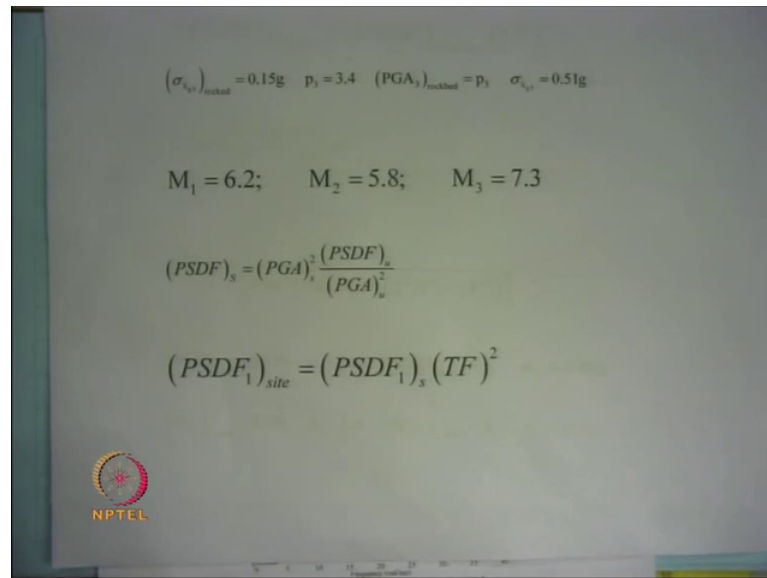
spectral density function of acceleration by the transfer function square, that give us the power spectral density function of the ground acceleration at the rock bed level just below site 1.

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Similarly the value of the power spectral density function at the rock bed level for site two is given over here and the power spectral density function of the ground acceleration at the rock bed level at site 3 is shown over here. So, once we got this power spectral density function then there are certain method to calculate the peak ground acceleration for the site at the rock bed level from the rock bed power spectral density function. Similarly, for the site two we can find out the peak ground acceleration at the rock bed level from the power spectral density function of the ground acceleration that you have obtained at the rock bed level and for site 3 also we did the same kind of calculation in order to obtain the peak ground acceleration at the rock bed level from the power spectral density function that is obtained for the site 3.

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$$(\sigma_w)_{\text{outlet}} = 0.15g \quad p_s = 3.4 \quad (PGA)_{\text{outlet}} = p_s \quad \sigma_w = 0.51g$$
$$M_1 = 6.2; \quad M_2 = 5.8; \quad M_3 = 7.3$$
$$(PSDF)_s = (PGA)_s^2 \frac{(PSDF)_w}{(PGA)_w^2}$$
$$(PSDF_1)_{\text{site}} = (PSDF_1)_s (TF)^2$$

Now, once we know the peak ground accelerations for the 3 sites at the rock bed level and we use their epicentral distance, then using the epicentral distance and the peak ground acceleration one can calculate the magnitudes of earthquake for which these power spectral density functions were obtained for the sites 1, 2 and 3 at the surface level. So, these magnitudes of the 3 sites for which the ground motion or the ground motion power spectral density functions were available are now calculated that is  $m_1$  is equal to 6.2,  $m_2$  was found to be 5.8 and  $m_3$  was found to be 7.3. So, these magnitudes were obtained by using an attenuation relationship given by Toro and if we recall most of that attenuation relationship they provide a peak ground acceleration as a function of the magnitude of earthquake and epicentral distance. Therefore the peak ground acceleration is given and epicentral distance is given then from that formula one can find out the magnitudes of the earthquake.

Now, it is specified that for the site the site specific spectrum should correspond to a magnitude of earthquake which is equal to 7 therefore, all the power spectral density functions which were obtained at the rock bed level for site 1, site 2 and site 3 and must be scaled for the magnitude of earthquake that is these magnitude levels should be brought to the level of 7. So, for that what is done is again we go back to the attenuation relationship and in that relationship we put the value of the magnitude at 7 and epicentral distance as the epicentral distance of the site which is specified. So, once we provide these two information from there we get the peak ground acceleration which should exist

at the rock bed level and for the site for which we are wanting to have the site specific spectrum.

So, once we know that peak ground acceleration of the site then one can obtain the scaled power spectral density function at the rock bed level is equal to the peak ground acceleration square of the site and the peak ground acceleration square of the different locations that is location 1, 2 and 3 for which we obtain the magnitudes of earth earthquakes are  $m_1$ ,  $m_2$ ,  $m_3$  for those locations the peak ground accelerations I have already been calculated before. So, we know those peak ground acceleration. So, we modify the PSDF of the ground acceleration at the rock bed level of the 3 locations in this particular way. This is the unmodified, that is the big the power spectral density function of the ground acceleration at any particular location say location 1 at the rock bed level and that is now multiplied by  $PGA^2$ , that is  $PGA$  that we obtain for the site for which we are wanting to draw there site specific spectrum and peak ground acceleration for that particular location.

Similarly, for location 2 and 3 we can get a power spectral density function modified power spectral density function which will be valid for that particular site in question at the rock bed level. Thus, we will get 3 scaled power spectral density function at the rock bed level of the site in question for which we are going to obtain the site specific spectrum. So, these scaling takes care of the scaling of magnitude or the peak ground acceleration also take care of the scaling of the epicentral distance.

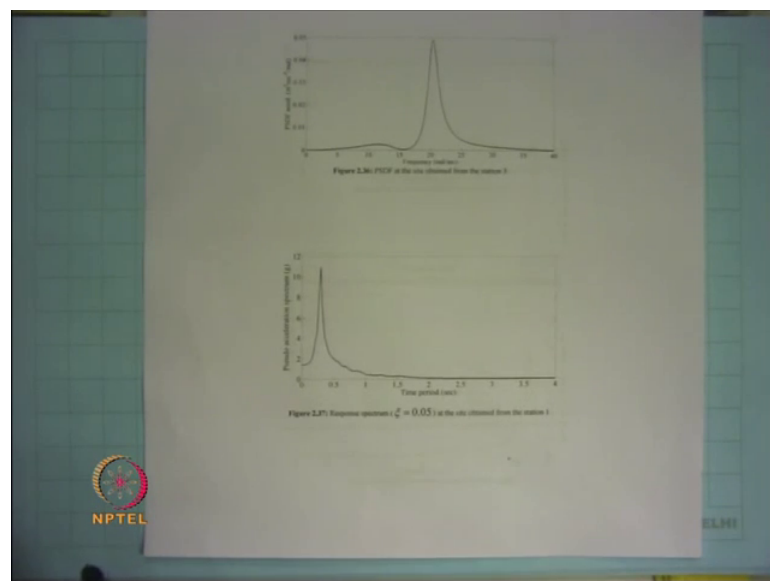
So, after we have done that then we considered the soil condition and for the particular site in question. So, the power spectral density function which you have obtained at the particular site coming from location 1 that you called as PSDF 1 and if we multiply that by the transfer function square then this will provide us a power spectral density function at the surface level.

So, this is the power spectral density function that we have obtained at the rock bed level coming from location 1 after scaling, and this will be the power spectral density function at the surface of the ground that is at the site itself. So, that way one can obtain the power spectral density function and for coming for location 2 to the site and similarly one can constructor PSDF 3 that will be coming from the location 3 which will be considering for obtaining the site specific spectrum here.

Now, once we get this 3 power spectral density functions at the site then we generate the time histories of the ground motion or synthetically generate the power time history of ground motion from the given power spectral density function and there are techniques for the for obtaining the synthetic ground motion where general synthetic ground motion for a given power spectral density function.

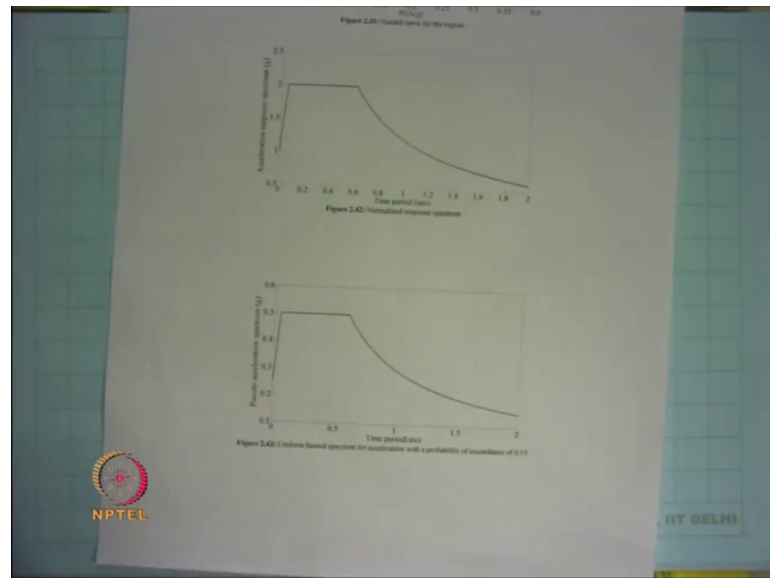
So, utilizing that one can obtained the power spectral or that obtained the time histories of the ground motion on for the 3 PSDFs or in other words we get 3 time histories of ground motions from these 3 time histories of ground motion, we obtained the response spectrum 3 response spectrums and these 3 response spectrums are average and smoothed in order to obtained the final response spectrum which is the site specific spectrum and which is shown here in the last figure.

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So, this last figure shows the site specific a spectrum which is an averaged and smooth spectrum coming from the 3 earthquake records that have been synthetically generated.

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And, so the procedure that you adopted depends upon the given quantity that we have and if the given quantity is a quantity which is only the time histories. Then this time histories are to be scaled for the peak ground acceleration on magnitude of earthquake and epicentral distance through Fourier series analysis if it is given in terms of the power spectral density function, then the procedure that are mentioned that can be obtained in order to get the site specific spectrum

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**Uniform hazard spectra**

- Statistical analysis of available spectrum is performed to find distributions of PGA & spectral ordinate at each period.
- From these distributions, values of spectral ordinates with specified probability of exceedance are used to construct the uniform hazard spectra.
- Alternatively, seismic hazard analysis is carried out with spectral ordinate (at each period for a given  $\xi$ ) as parameter (not PGA).

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Next we come to uniform hazard spectrum. The uniform hazard spectrum is also used in many cases when we are calculating the hazard analysis that is seismic risk analysis of structures at a particular site. And that may require the construction of a uniform hazard spectrum. Statistical analysis are available spectrum is performed to find distribution of peak ground acceleration and spectral ordinate at each period in order to obtain the uniform hazard spectrum.

So, what is done is that if we have a number of the response spectrums which are available at a particular region then what we do is that the peak ground accelerations which we get from each one of these response spectrums these peak ground accelerations are taken and they p d f and c d f that is cumulative distribution function and the probability density function of the peak ground acceleration is obtained from the given data. Similarly for a particular ordinate of the response spectrum at a given time period that is taken as a random variable and if we have got say 50 response spectrums available for the site then we get for a particular time period 50 values of the ordinate of the response spectrum.


So, we construct again a power spectral probably density function and the c d f for that particular ordinate of the spectrum at a particular time period. The same thing can be done for all the time period and we can have a distribution curve for a number of time periods and point peak ground acceleration. And once we get these probability density functions or the c d f cumulative distribution function then from these distributions the values of the spectral ordinates with specified probability of accidents are used to construct the uniform hazard spectrum.

Alternatively, one can obtain a seismic hazard analysis which can be carried out for spectral ordinates. We have explained the calculation of the seismic hazard curve for the peak ground acceleration. So, the same concept can be extended for the spectral ordinates at each time period for a given value of  $\psi$ . And once we have these hazard curve for the spectral ordinate at a particular time period then from that hazard curve again one can get the spectral ordinate corresponding to a certain probability of accidents and with the help of that data one can construct the uniform hazard spectrum.

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**Synthetic accelerograms**

- For many cases, response spectrum or PSDF compatible time history records are required as inputs for analysis.
- One such case is nonlinear analysis of structures for future earthquakes.
- Response spectrum compatible ground motion is generated by iteration to match a specified spectrum; iteration starts by generating a set of Gaussian random numbers.




Next we come to the synthetic accelerograms. For many cases one may have to obtain response spectrum or power spectral density function which are compatible with the time history records or for a given response spectrum or a given power spectral density function one may have to calculate synthetically a compatible time history record. So, this time history records which are generated from the response spectrum or given response spectrum or given power spectral density function these time history records if they are used for constructing back the response spectrum or power spectral density function they will be same as the ones from which they were generated. And this compatible time history records are many a time required for performing non-linear analysis of structures for a specified response spectrum or a specified power spectral density function and as all of you know that the response spectrum method of analysis and power spectral density function method of analysis. They are valid for the linear case therefore, if one has to perform a non-linear analysis then one has to obtain the compatible time history records.

Now response spectrum compatible ground motion is generated by iteration to match a specified spectrum say first it starts with the generation of a state of Gaussian random numbers and a set of iterations are performed.

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- Many standard programs are now available to obtain response spectrum compatible time histories; brief steps are given in the book (2.6.1).
- Generation of time history for a given PSDF essentially follows Monte Carlo simulation.
- By considering the time history as a summation of sinusoids having random phase differences, the time history is generated.




And these iterations are performed in such a way that at every iteration the calculated response spectrum is matched with the target response spectrum.

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$$a(t) = \sum A_i \sin(\omega_i t + \phi_i) \quad (2.39)$$

- Relationship between  $c_n$  &  $S d \omega$  discussed before is used to find amplitudes of the sinusoids (book – 2.6.2).
- Random phase angle  $\phi_i$  uniformly distributed between  $0 - 2\pi$  is used to find:



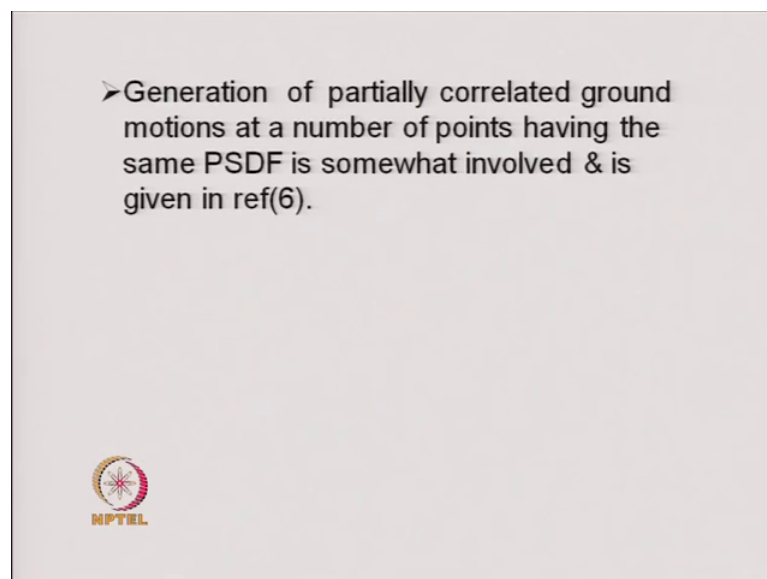
Now, in the case of the power spectral density function we obtained the compatible time history of ground motion by using equation 2.39 which is nothing, but the concept that we have explain for the case of the Fourier series and you can see here there any arbitrary function T can be written in the form of a Fourier series only difference over here is the values of  $\phi_i$  that is added now and  $A_i$  that is to be obtained. So,  $A_i$

basically can be obtained from the relationship that exist between  $c_n$  and the ordinate of the power spectral density function multiplied by  $d\omega$  if we recall we obtained the power spectral density function from the Fourier spectrum in one particular problem and they are we have just seen that how  $c_n$  is related to  $s$  into  $d\omega$ .

So, once we know the ordinate of the power spectral density function then one can find out the value of  $c_n$  and that  $c_n$  is nothing, but the  $A_i$  values. The  $\phi_i$  values are some random phase angles which is uniformly distributed between 0 to  $2\pi$  and we have got now many standard programs which can perform this that is they can generate the random angle which is uniformly distributed between 0 to  $2\pi$ .

So, utilizing that one can generate a time history of ground acceleration or ground motion for a given power spectral density function.

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Many standard programs are these available are this is available in order to obtain the time history of ground motion which are compatible with the power spectral density function or a given response spectrum. Generation of the partially correlated ground motion at a number of points having the same PSDF is somewhat involved and however, the methodology is given in the reference 6 of the book.