**Seismic Analysis Of Structures**
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**Lecture – 08**
**Seismic Inputs (Contd.)**

In the previous lecture we discussed about how a design response spectrum is constructed. Then we discussed about the design earthquakes. Then we discussed on the size specific spectrum, then uniform hazard spectrum. And then we discussed about the how one can obtained the artificial ground motion from the response spectrum and the power spectral density function of the ground motion. The generation of this artificial ground motions are extremely important. Because many a time when we have to perform in non-linear analysis we cannot use the response spectrum or power spectral density function as the input.

In that case we have to provide a inputs as the time histories of ground motion to obtain the power spectral to obtain the time histories corresponding to a specified power spectral density function or to obtain a time history of ground motion corresponding to a target response spectrum, there are many standard programs which are available this days and one can use those standard programs the methodology how they these programs work that what we have discussed in the previous lecture.

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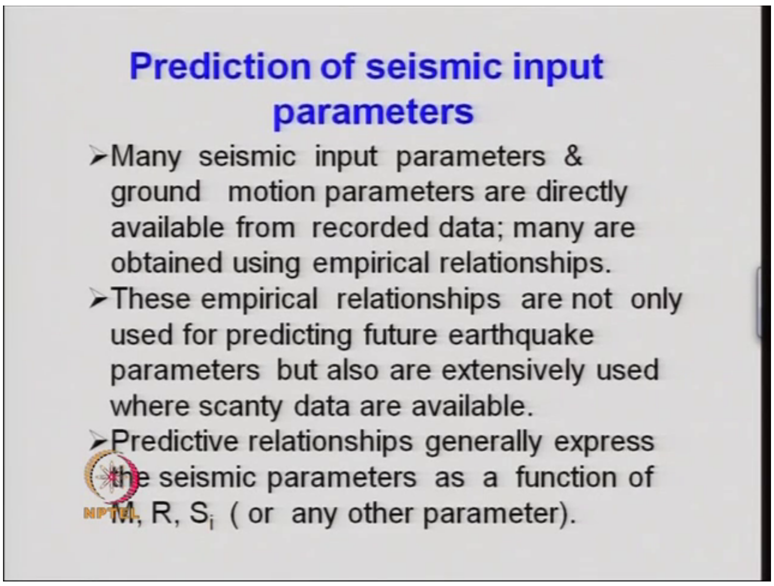

now in a region where we have a number of ground motion record available and other earthquake data measured data.

Then one can have a description of the seismic input as power spectral density function or a Fourier spectrum or a response spectrum or one can provide the possible peak ground acceleration. And many other seismic parameters that are used as input for analyzing the structures or performing any kind of seismic analysis of structures; however, there are many places where the recorded data are not sufficient and therefore, this kind of exercise cannot be done. Also wherever we have many data collected over a long period of time, then from those data the researchers wanted to find out some empirical equations for describing the power spectral density function of or for describing the response spectrum or for describe in the Fourier spectrum or the empirical relationship for duration of earthquake or different kinds of attenuation loss.

And these exercise have been done in the past and people came out with the different kinds of empirical formulas which can be used for predicting those seismic parameters. So, todays discussion is based on that that how we can obtain a different kinds of seismic parameter which will be used for future earthquake. Or in other words how to predict those seismic input parameters. Now the seismic input parameters are obtained from the past earthquake data. In different regions and there are several are predictive loss or predictive equations and one has to choose the most appropriate to one for the region in question and this selection of the selection of the predictive relationships.

they require careful consideration of the geology of the region geographical condition and the geotechnical condition that is the soil condition of a particular region. And one has to see what are the similar conditions existing for other regions for which the predictive equations are available. Then one can use those a predictive equations for the for predicting seismic input parameters. Many in many cases the some kind of adjustments are made based on the geological geographical and geotechnical parameters for the region and with those modifications those predictive relationships can be used.

Now, the seismic input parameters and ground motion parameters are generally directly available from the recorded data as I told you. And one uses those empirical equations for predictive purposes. The predictive relationships are generally expressed in the form of a function of magnitude the epicentral distance and any other important parameter. For example, it could be peak ground acceleration it could be intensity or it could be any other or quantity of interest.

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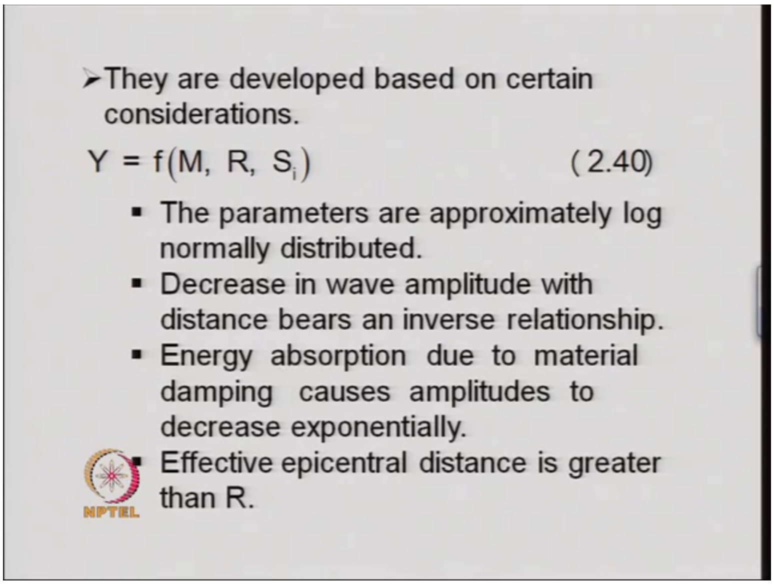


they are developed based on certain consideration. The equation 2.40 shows the function or the predictive equation and it is form it is generally function of magnitude epicentral distance and the any other important parameter and that is what I told before.

The important thing is that in obtaining this equation, it is generally assumed that the all the parameters or log normally distributed. Now this assumption has come because of the reason that the these parameters, which are recorded in different regions where statistically analyzed. And it was found that most of these parameters attain to follow a log normal distribution. Secondly, in obtaining the attenuation law it was observed that decrease in wave amplitude with distance bears an inverse relationship. Then the energy absorption due to material damping causes amplitudes decrease exponentially and this was also seen from the previous earthquake data. And the epicentral distance are that what is generally we considered is always a greater than on the actual epicentral distance because the earthquake source moves in a from one point to the other in a line source or in a area source.

Therefore, it is very difficult to precisely talk about a an epicentral distance therefore, the epicentral distance that is considered in this equations are always greater than the epi actual epicentral distance.

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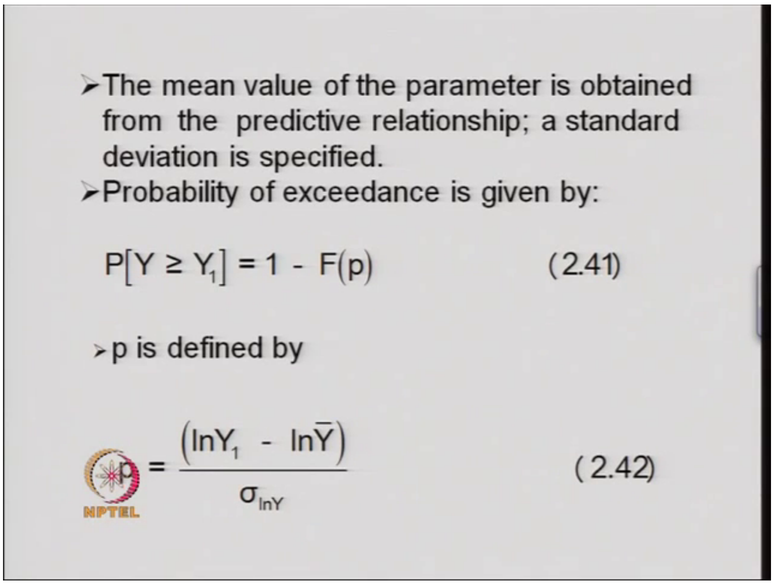


The mean value of the parameter is obtained from the equation or the predictive relationships and there is a standard deviation which is specified that is the mean value is the provided in terms of log value and standard deviation also is in terms of sigma l n of the parameter probability of accidents is given by this simple relationship that is probability of any seismic input parameter being greater than equal to a specified value is equal to 1 minus f p, where the p is defined by this that is l n y minus l n y bar divided by sigma l n y l n y bar is the log of the value specified value n l n y bar is the mean value of the parameter and sigma l n y is the standard deviation. So, this normalize quantity is taken as the value of p and f p; obviously, follows a normal distribution and there are standard charts which are available for are standard normal variate and from that one can easily find out the value of f p and hence probability of accidents of certain parameter are can be obtained easily.

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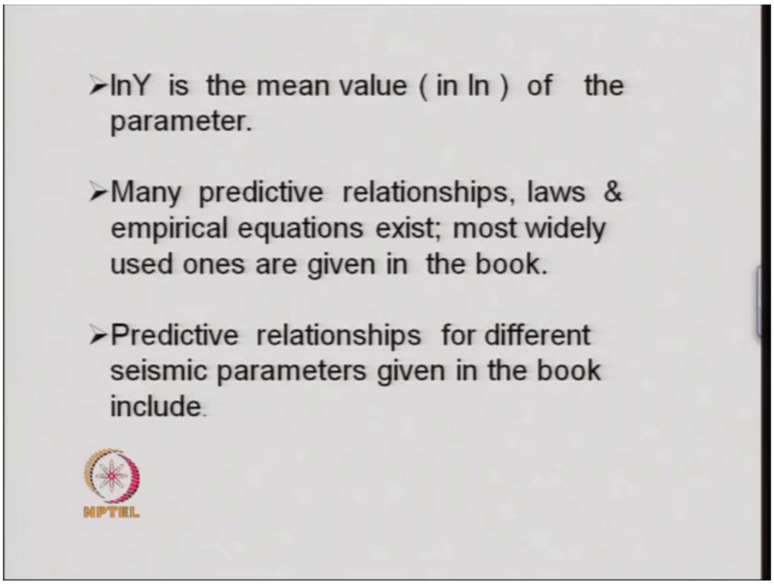


many predictive relationships loss and empirical equations exist. Most widely used ones are given in the textbook. And the ones which are widely used are described here.

Now, and let us see one by one those predictive relationship, that are widely used and given in the book. For example, the peak horizontal acceleration and peak horizontal velocity.

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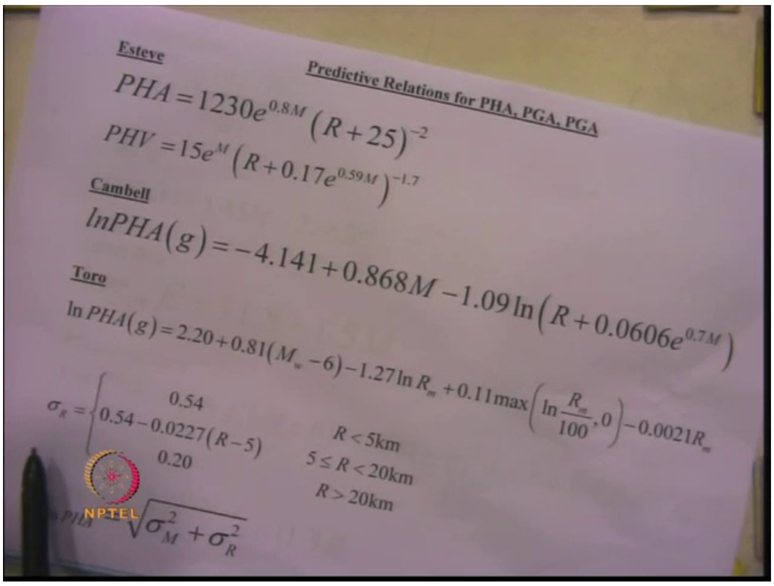


They were proposed by Esteve to be an exponential function like this, exponential function of magnitude m. And one can see that the r the epicentral distance is not is taken as r plus 25 that is what we discussed before; that means, the epicentral distance is increased by some value. Then the peak horizontal velocity that is again and exponential function of magnitude m, and one can see that here the epicentral distance also is increased by some value.

So, from these 2 equations given a particular value of magnitude m and an epicentral distance one can find out what is the peak horizontal acceleration and what is the peak horizontal velocity. Similarly, Campbell give another predictive law for the peak horizontal acceleration and that was from the data set that he analyzed. And they are the PHA is given in terms of the g unit here I forgot to mention that this PHA is given in terms of centimeter per second square and; obviously, this is in centimeter per second. So, log of PHA in g unit is again a linear function of magnitude m and the l n of the epicentral distance r. And we see that the epicentral distance r is again increase by certain quantity.

Toro obtained another expression for peak horizontal acceleration in g unit. And he is equation relates the moment magnitude m w with the PHA value and the epicentral distance. The sigma r value is a given by this; that means, the r m here the it is the mean value of the r and there is a standard deviation which is specified for epicentral distance r and they are given for this particular bounce over here. And the standard deviation for the PHA is given by this that is sigma square m plus sigma square r. So, we assume here in this particular equation not tode toror assumed that the sigma value of pij will be related to the sigma value of m and sigma value of r that is r is considered here again as a random variable like magnitude m.

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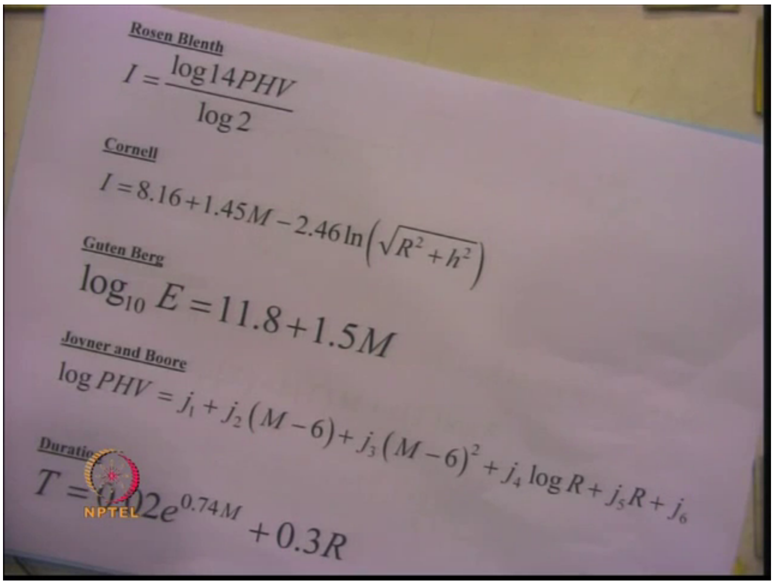


Next there was an attempt to relate the peak horizontal velocity of the ground with the intensity of earthquake. And if you remember I said that the intensity of earthquake is a subjective measure, whereas, PHV is something which we expect that we should be able to measure, but these subjective quantity has been used to find out a quantity which can measure. So, this is the equation that Rosen blenth observed that the PHV maintains with the intensity of earthquake. Similarly, the Cornell obtained in relationship between the intensity of earthquake and the magnitude of earthquake, where the magnitude of earthquake again is a quantity which can be measured whereas, intensity of earthquake is a subjective one, but from the recorded data of the earthquake and the kinds of damages and destructions that we are observed the people have tried to equate i with m using this particular equation and this Cornell's equation is widely used in relating i with m.

Where r is the epicentral distance and h is the focal depth. Then guten berg give a very important relationship between the energy released and the magnitude of earthquake. So, for a given magnitude of earthquake one can assess water is the energy released and this equation is widely used in the literature in relating the magnitude of earthquake with the energy release. Joyner and boore provided another empirical equation for the peak horizontal velocity and that is a function of magnitude of earthquake this magnitude of earthquake is the local magnitude and epicentral distance r and we can see that the it is not simply one r, but r is a you know is increased by some value.

Here the j 1 j 2 j 3 j 4 j 5 and j 6 they are the constants which are variable in the sense that it may vary from region to region and one can get a particular set of value for this con constants, while using this equation for a particular region duration of earthquake also has been attempted to for prediction. So, the duration of earthquake T which people observed could be related to the magnitude of earthquake and the epicentral distance and this is a exponential function of magnitude.

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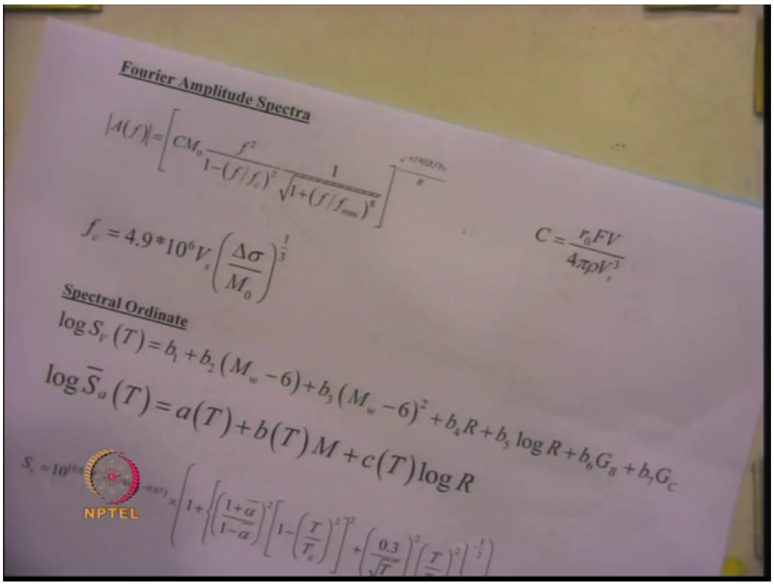


So, with the help of this the duration of earthquake can be predicted and for a particular region if enough data is not available about the duration of earthquake.

Next is the Fourier amplitude spectra that we discussed in the previously. And Fourier amplitude spectra provides the frequency contents of the ground motion and as an input as an important predictive parameter for performing a frequency domain analysis of structures. And this is given with the help of this equation. And one can see here the constants which are involved over here is f c, and that is the a frequency which is called the on the critical frequency or cut of frequency. And then f is the usual frequency. So, the amplitude Fourier amplitude spectrum is expressed in terms of the frequency; obviously, it can be converted to time period because frequency b s and inverse relationship with time period. And these f c is related to the moment magnitude m 0 and the shear wave velocity. And the constant c over here is dependent on the shear a velocity with the help of this relationship.

Now, these constants; that means, the value of the shear wave velocity f and delta sigma. So, these values may differ from the side to side and using the specific combination of values for a particular site one can obtained a Fourier amplitude spectrum by using this equation. Next people also tried to obtain the empirical a relationship for the response spectrum or velocity response spectrum Sv. So, this is time period dependent. So, log Sv T is given by this equation. And one can see that it is related to the mw magnitude and epicentral distance r and a number of constant. So, this constants again vary from region to region and by substituting appropriate values of this constant for a particular region one can get an estimate of the Sv, that is the pseudo velocity spectrum ordinates at different time period t.

Similarly, this equation is used for obtaining the acceleration response spectrum ordinate for different period T. And this is related to the magnitude again magnitude of earthquake and the epicentral distance r in this a T b T and cT there again site dependent and one has to find out this constants a T b T c T. And in in some cases these at bT cT they are plotted the plots are available and this plots are with respect to the period T. Another expression for the pseudo velocity spectrum ordinate Sv, that was obtained by this and here one can see that for different period given period T, one can calculate the value of the pseudo velocity spectrum. So, therefore, this in fact, will be a function of T and this is also dependent upon the surface magnitude ms and the epicentral distance R.

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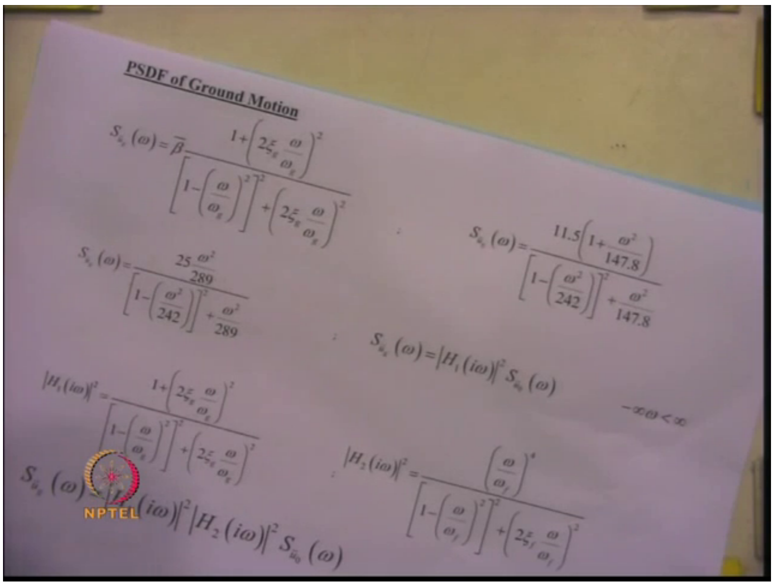

Next with relationship predictive relationships were obtained for power spectral density function of ground motion. And there are several such expressions which are available. Now one can see that this is expressed in terms of the ratio of omega by omega g, where omega is the frequency against which we plot the power spectral density of ground motion. Omega g is the predominant frequency of the ground through which the seismic waves passes from the rock bed to the surface. So, for that soil medium on can find out the predominant frequency of that ground, n z g is the again damping associated with the soil. The concept here is that the ground motion while travelling from the rock bed to the surface get modified due to the soil condition and depending upon the soil predominant frequency and damping it takes a shape of the power spectral density function.

That is the shape there is a change in shape between the power spectral density function which is recorded or which is obtained at the rock bed and which is obtained at the surface. Now this is again another what we call expression empirical equation for the power spectral density function. And these values are instead of omega g etcetera they are specified values present for a particular class of these soil condition this is valid. There is another equation which is of these type where these constants are specified. So, they are valid for a particular region.

With general type of the power spectral density function equation was provided by clough and penziene. Here the concept is that the power spectral density function that exist at the rock bed level. That gets filtered through to filtering medium and the frequency response functions square or absolute value of the frequency response function square are given by this equation and by this equation. So, this is for filter one and this is for the filter 2. And the predominant frequency for the filter one is omega g and for filter 2 it is omega f.

Similarly, the damping constant for the 2 filters they vary. Now this was a modification of the power spectral density function expression given by this equation. Now here the form of the equation is of this type; that means, there is only one particular filter exiting and the rock bed power spectral density function gets modified through one filter; that means, the entire soil is considered as one filter and that obtain that that gave a relationship between the surface power spectral density function and the rock bed power spectral density function. Now this was a this was modified to this 2 filter concept of power spectral density function because of the reason that this is not able to provide a an inadequate or correct value of the power spectral density function of displacement at the 0 frequency.

Whereas these expression when we use double filter they can and provide the correct value or some finite value to the power spectral density function of displacement at 0 frequency.

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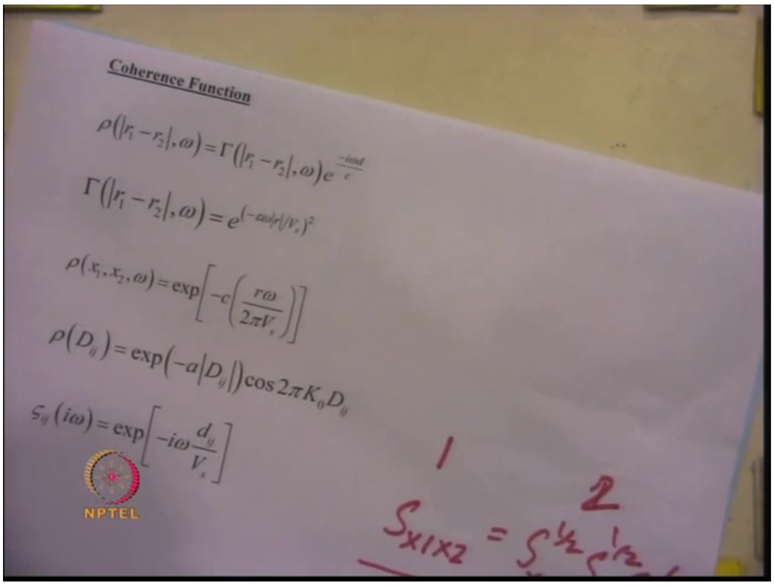


Next we come to the coherence function that we discussed before. The covariance functions they are used for obtaining the cross power spectral density function of the ground motion between 2 points. So, we work out say the power cross power spectral density function between 2 points 1 and 2 as s x1 s x 2 to be is equal to s x1 to the power half into s x 2 to the power half multiplied by a coherence function that was a function of omega and the distance between the 2 points r.

So, this is how a cross power spectral density function between 2 points that is computed, s x1 and s x 2 are the power spectral density function of the ground motion omega power spectral density function of acceleration at this 2 points. And for a homogenous field we generally assume that the power spectral density function of ground acceleration at these 2 points of the same as a result of that one can write down the cross power spectral density function as s x of this coherence function. So, if one knows or if the in power spectral density function of the ground motion in terms of acceleration or displacement. They are given then one can find out the cross power spectral density function provided one knows the coherence function.

So, different forms of the coherence function that is given in the literature. Here this is a coherence function and one can see that this is a exponentially again decaying function with a gamma function that is the distance between the 2 points and is a function of omega and this function is more precisely written over here this is also a exponentially decaying function with the shear velocity coming into picture. So, using this into this over here one can get a coherence function which would be a multiplication of 2 exponentially decaying function a very popular coherence function, which is used in many cases is given by this expression.

So, this is again exponentially decaying function with a constancies specified r is the distance between the 2 points. And omega is the frequency and v is the shear wave velocity of the earthquake. So, this particular equation is in is a real quantity does not have any imaginary component. And as a result of that provided we know the value of r that is the distance between the point x1 and x 2. Then one can easily obtain a value of the coherence function only in terms of omega because v s will be specified c would be specified. Then one can I get the coherence function and only as a function of omega and s x is also given as a function of omega therefore, s x1 s x 2 can be easily express for each frequency that you considered in our analysis.

This is another coherence function that is reported in the literature this is in the form of a cos harmonic function that is a cosine function. And an exponential function that is with the distance this exponentially decays and this form of the coherence function was obtained and from the recorded data in tau i 1. And an exercise was done there with the available data to find out different forms of the coherence function. And one set of data happened to coincide with this kind of empirical equation. The general form of the coherence function represented by this equation is given over here, where we can see that this is a exponential function of a imaginary quantity i. So, therefore, to one can write it in the form of a real part cosine and imaginary part which will be a sign.

So, this particular form of the coherence function is a complex quantity becomes a complex quantity epi in the form of a plus id. And then s x1 s x 2 no more remains a real quantity, but a complex quantity, but in the in the problems of a probabilistic analysis of structures using spectral analysis there is absolutely no problem in tackling s x1 s x 2 as a complex function. In fact, it is logical that the cross power spectral density function terms would be a complex function or a complex quantity rather than the real quantity; however, if we use this particular form of the equation which is a special form of this one. Then we get the cross power spectral density function as the real quantity.

Then we have a number of expressions for the modulating function. And the modulating function that we talked of in the previous lecture that modulates a stationary process to a uniformly modulated non stationary process are you called uniformly modulated power spectral density function which is also known as the evolutionary power spectral density function.

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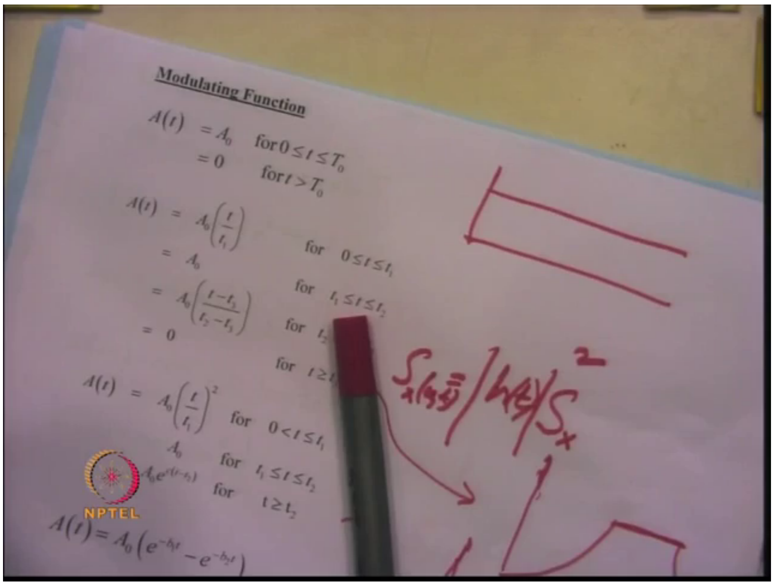


And they are we had seen that the power spectral density function xx of the earthquake is multiplied by a modulating function h T square. And this gives the value of the power spectral density function as a function of omega n T both.

So, various forms of the moderating function that were observed from the recorded data. This is a very simple one which is a moderating function of this type that is a rectangular model modulating function. Then one can have a modulating function which is of a type like this that is not exactly trapezoidal type because these 2 are a non-linear curves, but it has a form of a trapezoidal function. Then one can have an exponential function as the these are for the exponential function rather the rather this is an exponential function.

And with the help of this modulating function one can obtain a power spectral density function which is a as a uniformly model modulated at power spectral density function used for obtaining the response of structures for evolutionary power evolutionary power spectral density function of earthquake.

So, we see that a number of predictive relationship that exist in the literature. And one can use any one of these predictive relationships for the performing the seismic analysis of structures. Mostly in regions where we do not have enough earthquake data we look for these predictive relationship to predict the future form of the future earthquake that will be given as an input for the analysis. Now in using this predictive relationship one has to take care of the local zoological geotechnical and geographical conditions and many a time the constants of the empirical equations are adjusted for this conditions.

So, the protective relationship as such are very useful in obtaining the seismic response of structures for a variety of cases. And we will just see later that for the random vibration analysis of structures using the using the spectral analysis technique, we can I use this equations of the power spectral density functions or various forms of p d power spectral density functions, that I have shown you and out of that the double filter power spectral density function is widely used for obtaining the response of the structure to a the specified power spectral density function. And this double filter power spectral density function is obtained with the concept that the power spectral density function of ground motion that exist at the rock bed gets filtered through 2 filters. And the concept of 2 filters where used for finding out a finite or a reasonable value of the power spectral density function of displacement at 0 frequency.

If we use the single or filter for obtaining the power spectral density function or at the ground surface given the power spectral density function of ground motion at the rock bed. Then we get a situation get into a situation where the power spectral density of displacement at 0 frequency remains undefined. Similarly, the power the response spectrum ordinates for future earthquakes can be used and the analysis of structures can be carried out with the help of these predictive relationships given for the response spectrum of acceleration or velocity. There are cases where the response spectrum ordinates using the empirical relationship has been also used for obtaining the seismic hazard analysis of structures or seismic hazard analysis of regions in order to find out the hazard of the probability of accidents of certain value of the response spectrum ordinates.

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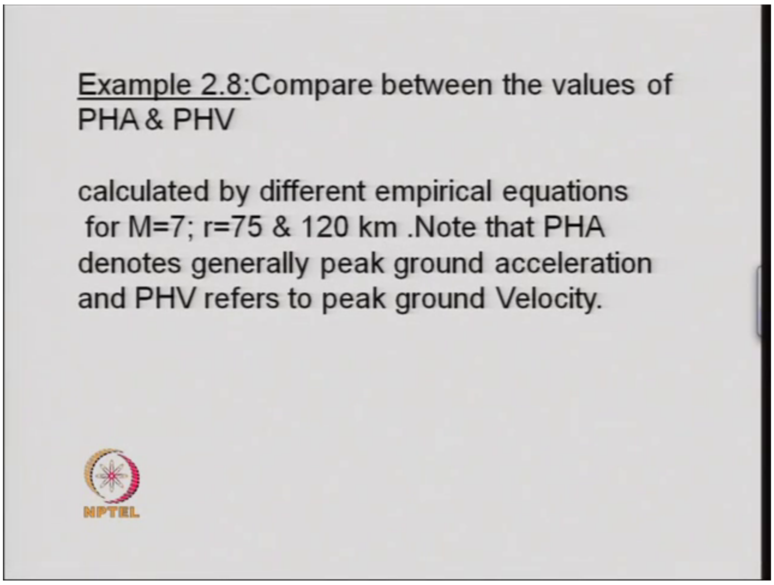


Now, here 3 examples are solved over here. PHA and PHV they were calculated using different empirical equations. And compare they were obtained for the magnitude of earthquake 7 and epicentral distance of 75 and one a 120.

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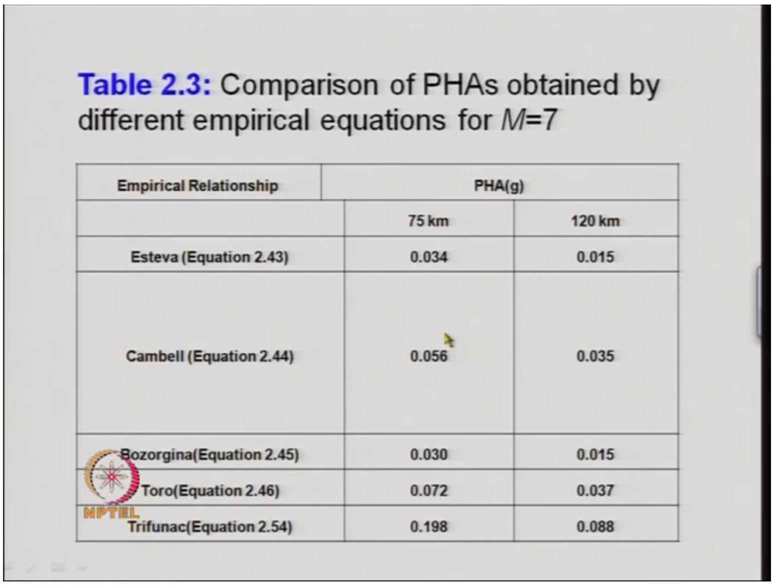


| Empirical Relationship | PHA(g) | |
|---|---|---|
| | 75 km | 120 km |
| Esteva (Equation 2.43) | 0.034 | 0.015 |
| Cambell (Equation 2.44) | 0.056 | 0.035 |
| Bozorgina(Equation 2.45) | 0.030 | 0.015 |
| Toro(Equation 2.46) | 0.072 | 0.037 |
| Trifunac(Equation 2.54) | 0.198 | 0.088 |

The comparison shows that the PHA calculated by different equations give different values for the same epicentral distance. For example, Esteva he is equation provided PHA of the order of 0.034 Campbell. He is equation provided a PHA of 0.056. Then bozorgina he is equation provided 0.03 which is quite comfortable with 0.034. Then toro is 0.072 and trifunac he is equation gave a value which is a wide departure from all other values. At 120 kilometer distance we can see that there is a similarity between esteva and bozorgina. And this and these they have a good similarity toros and the cambell and again there is a large departure or of the PHA calculated by the equation of the trifunac.

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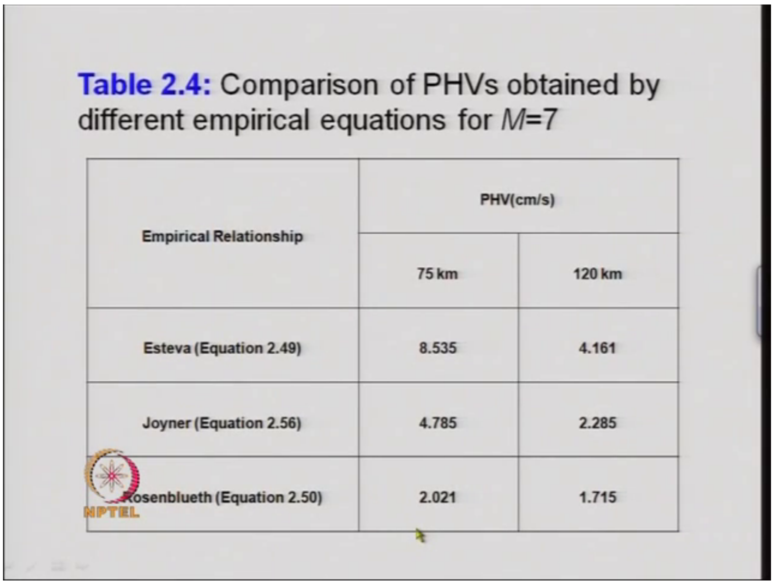
**Table 2.4:** Comparison of PHVs obtained by different empirical equations for $M$=7

| Empirical Relationship | PHV(cm/s) | |
|---|---|---|
| | 75 km | 120 km |
| Esteva (Equation 2.49) | 8.535 | 4.161 |
| Joyner (Equation 2.56) | 4.785 | 2.285 |
| Rosenblueth (Equation 2.50) | 2.021 | 1.715 |

similarly, the PHV was a compared estevas equation provided 8.535 which is very high compared to the other 2 equations given by Joyner and rosenblueth.

At 120 kilometer distance they were more or less they are they are very much near to each other whereas, this was a very much different.

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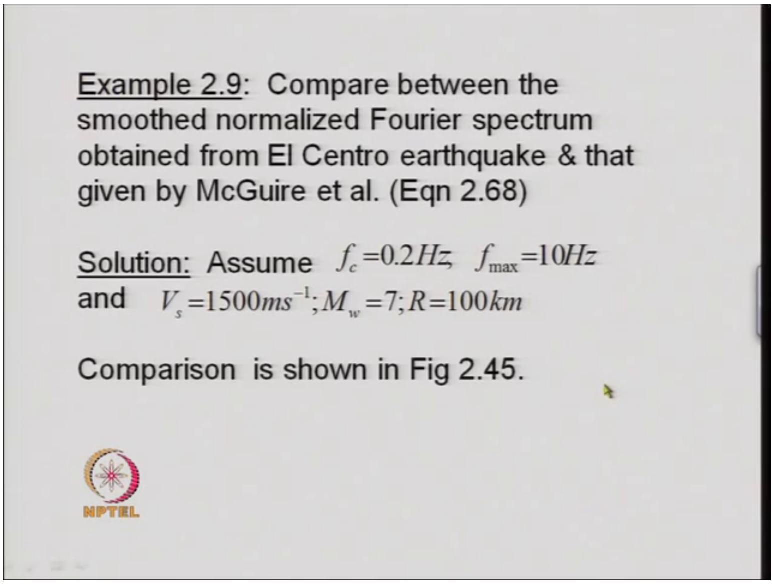


then we can have compared the smooth normalize Fourier spectrum obtained from el Centro earthquake. And that given by McGuire's equation. In the this was for the specified values f max was taken as 10 hertz f c was taken as 0.2 hertz m w was taken as 7 r was taken as 100 kilometer v as 1500 meter per second.

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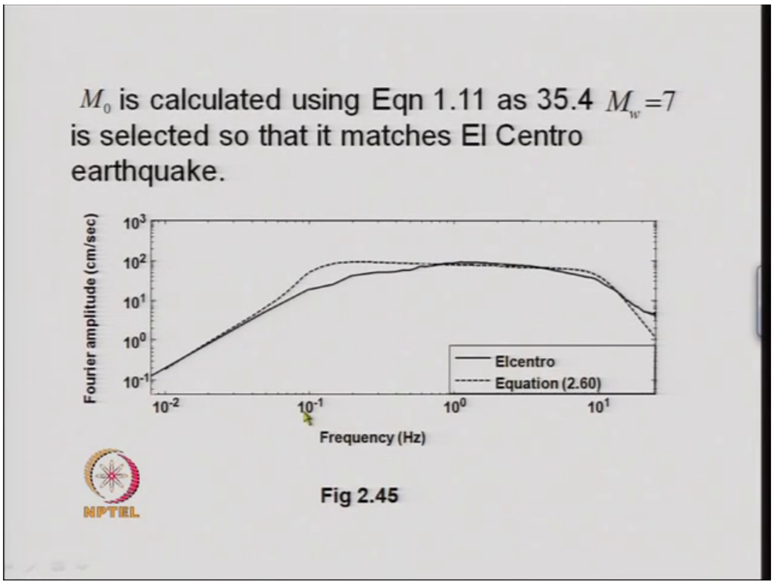


And we can see that the Fourier amplitude spectrum obtained by this equation and the elcentro earthquake they seem to compare quite well. So, on the expression that is given by McGuire's equation for the Fourier amplitude spectrum seems to provide a good approximation to the Fourier amplitude spectrum, when the earthquake is broad banded earthquake because we know that all el Centro earthquake is a broadband earthquake.

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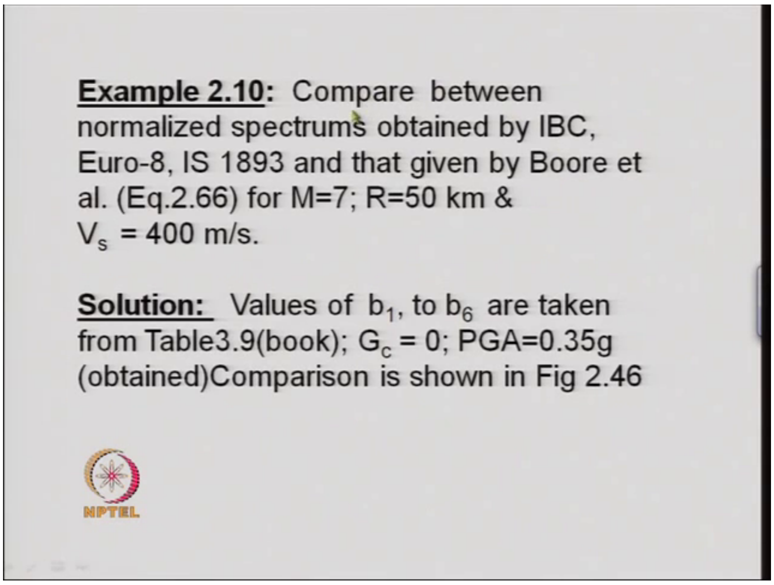


Then compare the between the normalized spectrums of 10 by I b c euro 8 is 1 8 9 3 and that given by boore et all. So, here the response spectrums which was normalized of course, with PGA value. So, the shape of this response spectrums were obtained for the various constants which are given in the boores equation like b 1 to b 6 this constants who were taken for table 3.9 given in the book and g c is equal to 0.

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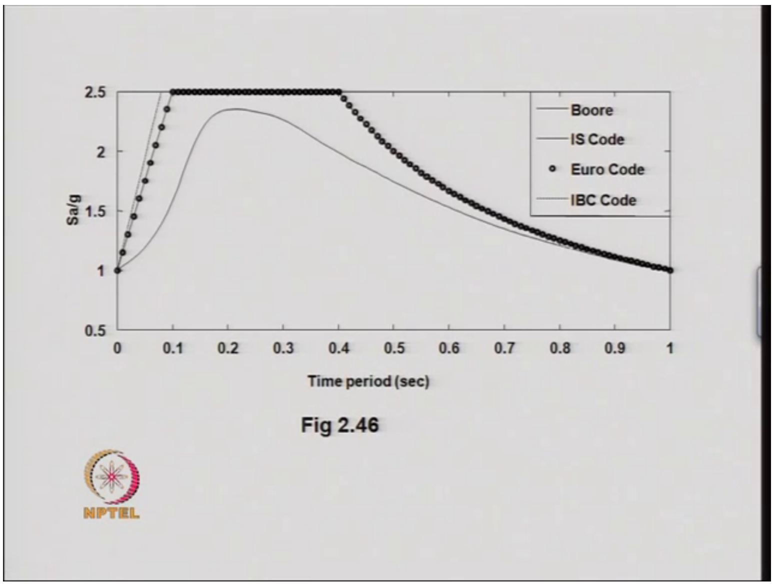

And PGA was taken as 0.35 g that is the normalized spectrums where normalized with respect to 0.35 g. And this is the comparison which we see we can see that the euro code n I s code they are more or less the same ibc code and the boores they were ibc code was this and the boores response spectrum is this.

So, at this region of course, is fairly matching in this region. And in fact, in this following region is more or less matching, but there is a departure here white departure in the beginning of the time period.

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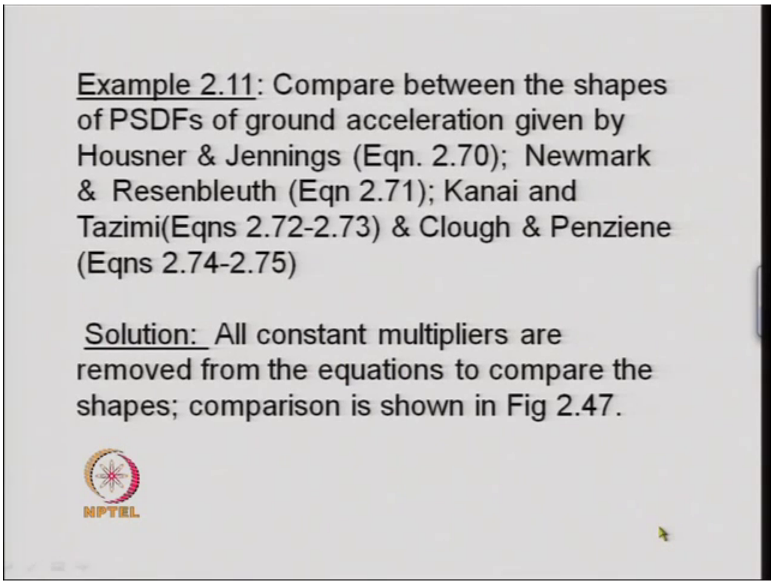


then we compare the power spectral density functions of ground acceleration given by different expressions. For example, Hous Housner and Jennings Newmark and Resenbleuth Kanai and Tazimi and Clough and Penzien Kanai Tazimi and clough and Penzien, they are widely used for most of the analysis.

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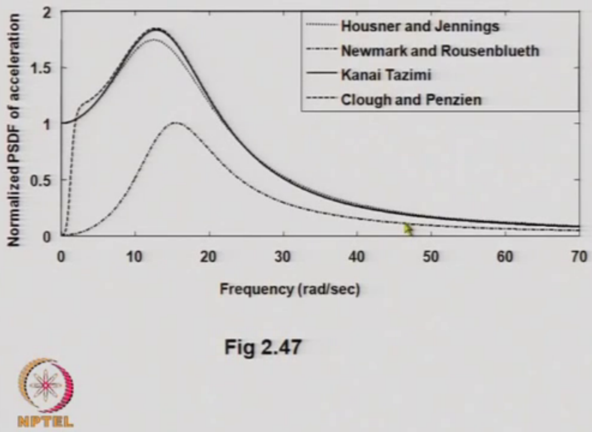


Fig 2.47

And we see that the there is a very good what to call a matching between the clough and Penzien Kanai Tazimi and Housner and Jennings. whereas, the Newmark and Resenbleuth that tends to differ quite a bit from them.

So, this shows that we have different kinds of spectrums given by different equations and which one is to be used depends upon the local condition, but the most widely once which are used in the literature they have a they are found to match with most of the power spectral density function that are obtained in the database. And therefore, the once we generally accept for predicting the future earthquake can be taken for those or from those equations. In general, the predictive relationships all the predictive relationship that have shown that is for the peak ground acceleration, peak ground velocity, peak horizontal acceleration, peak horizontal velocity, then response spectrum Fourier spectrum and the power spectral density function including the modulating functions they are available in the literature some of them I have shown over here.

There are more and in the many websites this predictive relationships are given. For analyzing a structure, we use various forms of seismic inputs the size peaking inputs. If they are not directly a given as a time history, then one has to rely on this predictive relationships, in order to describe the Fourier amplitude spectrum or the response spectrum or the power spectral density function of earthquake. And attenuation relationships are quite extensively used in determining the peak ground acceleration and peak velocity of earthquake, if give for the given magnitude and epicentral distance.

So, these quantities are used in both probabilistic analysis of structures as well as the deterministic earthquake analysis of structures, when you use that deterministic analysis of structures generally either we use response spectrum provided. And these response spectrums are usually the design response spectrum specified in the code with the help of these design response spectrum one can perform a seismic response spectrum method of analysis which is an elegant method and very easy to follow. And therefore, widely used in earthquake engineering. The main reason for this is that the entire earthquake analysis can be carried out statically.