Seismic Analysis of Structures
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Lecture - 09
Response Analysis for Specified Ground Motion

In the last late eight lectures, we covered two major topics that is the seismology and seismic inputs. In the seismic inputs, we have saying that we have different forms of the seismic inputs and the one that is an used depends upon the analysis at hand. Out of the different forms of the seismic inputs, three different seismic inputs are quite often used. First one is the time history record of the ground motion, second one is the response spectrum or rather the design response spectrum and the third one is the power spectral density functions of the ground motion for designing the structures for earthquake forces.

Generally, we analyze the structure using the response spectral method of analysis; in which the response spectrum becomes the input. When we model the ground motion as a random process that is; the future earthquake considered as a probabilistic process, then the power spectral density function becomes the input and we analyze the structure using the principles of random vibration.

The time history of ground motion is used for the deterministic analysis of structures for earthquake and in that we are interested in the response of the structure at every instant of time t. The time history ground motion can be Fourier synthesized and the Fourier contents of the ground motion can be obtained, as you have seen before. And therefore, the two different types of analysis that can be performed for a specified time history of ground motion.

The first one is a time domain analysis in which we obtain the response of the structure with the time history of ground motion as a input, in time domain using certain time integration scheme. The second one in which we use the frequency contents of ground motion and perform the response analysis in frequency domain and then we perform an inverse Fourier transform to get the time history of response of the structure. Now both this topics that is the time history analysis of ground motion in time domain and frequency domain, will be discussed in the subsequent lectures.

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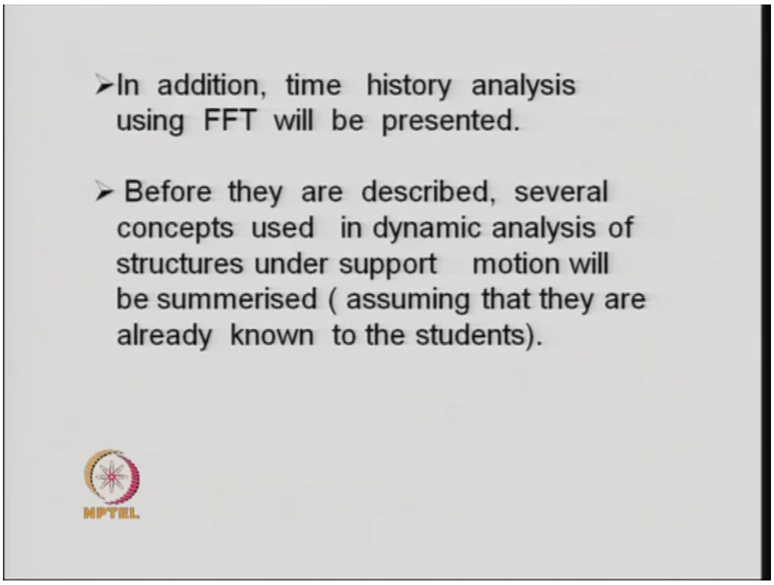

But before I do that let us try to look at some of the fundamentals of dynamics. I am sure you are quite conversant with them, but still for recapitulation. Let me take up those. First the equation of motion for a single degree freedom system is known to all of you.

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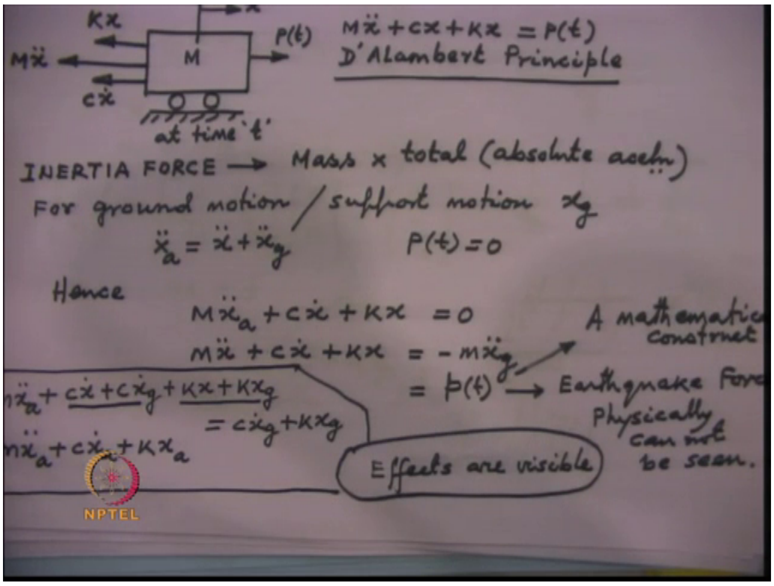

This is a mass of a single degree of freedom system, it is connected by a spring to a support and a dash pot also is connected to the support; dashpot indicates the damping and the spring denotes the restoring action of the elastic system and the M denotes the mass of the elastic system. Now if the mass moves in this direction x then an inertia force is developed due to the oscillatory motion of the mass.

All of us know more that for a oscillatory motion the acceleration, always is in the opposite direction to the displacement reduction. And therefore, if the displacement takes place in this direction then the inertia force which will be developed will be equal to mass times acceleration acting opposite to the direction of the motion.

Similarly, if the mass is displacing in this direction then the restoring force developed in the spring will be k x and this will be acting opposite to the direction of the motion. The c x dot which is the damping force, these damping force is called the viscous damping force; this is written as is equal to a coefficient of viscosity multiplied by the velocity, this damping force by definition x; opposite to the motion because it dampens the motion.

So, all the three quantities that is the restoring force, the inertia force and the damping force are acting opposite to the direction of the motion. And in the direction of motion the dynamic force is acting therefore, one can write down the simple equation that is M x double dot plus c x dot plus k x is equal to pt, these follows from the Beer-Lamberts principle, which is well known to all of you.

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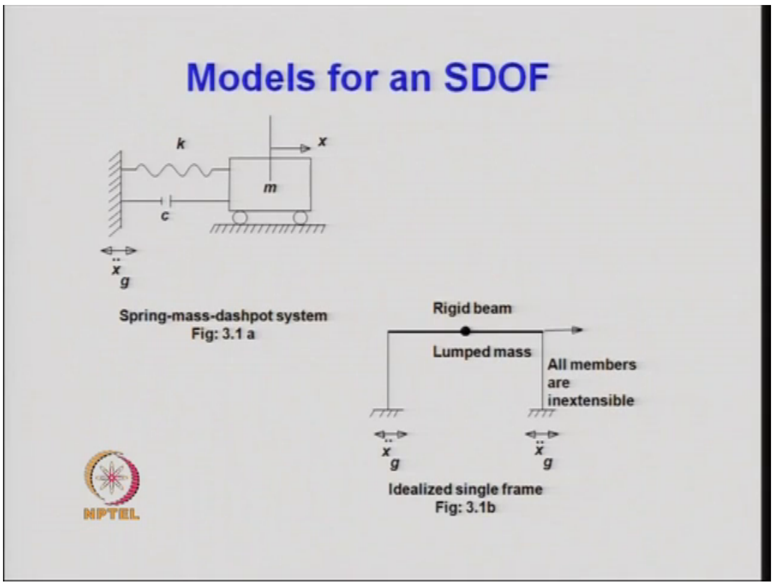

Now, when there is a support movement then the equation is slightly modified; let us see how it does get modified. Let us consider these portal frames, the beam here is a rigid beam and at the centre of the beam the mass of the entire system is lumped. There is a sway movement of the frame for a unit sway, the force which is required is equal to 13 v i by l cube for this column 12 v I by l cube for this column.

Or in other words 24 e i by l cube will be required to produce a unit displacement over here. So, we can say that the stiffness constant for this particular frame is equal to 24 e i by l cube. Now, this frame can now be modeled as a single degree of freedom system in this particular fashion. The m mass is this rigid mass, it is attached to a spring the spring has a stiffness coefficient of 24 e by l cube; it is attached to a dashpot in order to represent the damping.

Since the base is accelerated in this particular direction, the support over here is also accelerate in this direction and it is in the same direction as that of the motion. That is what is shown over here also, the acceleration and the displacement direction are in the same direction. So, this model is taken for that we write down the equation of motion.

So, for writing down the equation of motion we make a slight change; the change is that the mass now undergoes a total acceleration, a total displacement and total velocity. The definition of the total displacement, total velocity and total acceleration; they obvious from this equation, we can write down the total acceleration or the absolute acceleration to be is equal to x double dot plus x double dot g, where x double dot is the relative acceleration between the support and the mass the x double dot g, represents the acceleration of the support. So, if we add them together we get the total acceleration.

Similarly, one can get a total displacement; total velocity of the system. Now the most important thing is here that the inertia force is equal to mass times, the total or absolute acceleration if it is present. In the previous case, we did not have any absolute acceleration or the total acceleration because x dot itself was the total acceleration because there has no movement of the support.

But if there is a movement of the support then the inertia force which is generated is equal to mass times the total acceleration of the system. So, that is what we write over here m x double dot a becomes the inertia force, c x dot plus k x; they remain as before because the restoring force in the spring will depend upon the relative motion, that is the relative motion between the support. And the mass and it is assumed that the damping force is also equal to the coefficient of viscosity multiplied by the relative velocity.

Since there is no force acting over here, it is set to 0. Now if x double dot a the expression of this is substituted over here then keeping a make x double dot term on this side, we have a equation in on the left hand side of the equation all in terms of the relative quantities; that it m x double dot plus c x dot plus k x. And on the right hand side we can take in my mass times acceleration; that is it because minus m x double g. Now that we can write down as a effective force p t. So, this effective force p t that is minus m x double dot g that is called the earthquake force and these earthquake forces is a mathematical construct, it cannot be physically seen.

But the effect of this force can be filled or are visible. Therefore, the earthquake force that we talk of is in fact, a mathematical construct.

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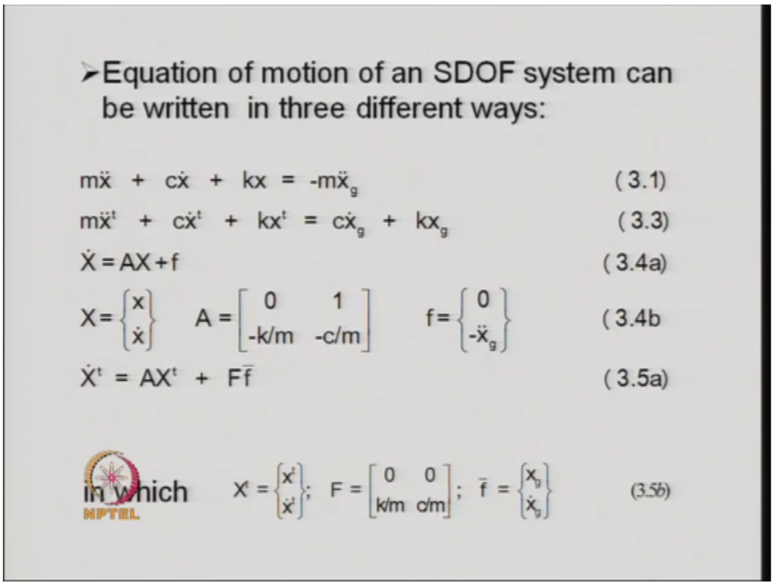

Now with this what you call the definitions or derivations let us look at this equation that is a m x double dot plus c x dot plus k x equal to minus m x double dot g that is what we have proved or derived. Now this can be written in three different forms; first these entire equations can be written in terms of not absolute displacement velocity and acceleration, but the total displacement velocity and acceleration.

For that what we do is that in this equation if we add on c x dot g; that is c x dot g if we add on over here and if we add on over here k x g, then they turn out to be k x t because x plus x g is equal to the total displacement. Similarly, here it turns out to be x dot t and since we have added k x g and c x dot g on the left hand side, we put them also on the right hand side.

Thus the equation of motion now becomes in the form of total displacement. If we solve this equation, we will get the total displacement of the mass rather than the relative displacement. One can note that on the right hand side now the requirement is the ground displacement; ground velocity not the ground acceleration. Now these two equations again can be written in the form of a state space. Now the concept of state space is that a dynamic system at an instant of time t can be represented by displacement and velocity.

If we know the displacement and velocity of a dynamic system at any instant of time t, then we know its state in the time space. Because knowing the displacement velocity at instant of time t one can also find out easily the acceleration. Therefore, we take x and x dot as the quantity of interest together represented by a vector called a capital X vector; these denotes the state of the system. If we try to write down the equation of motion in the form of these new vector or the new state variable X, then the equation X this form; X dot is equal to A X plus f.

If we look at this equation, we see that this is the first order differential equation rather than a second order differential equation. In fact, from the theory of differential equations one knows that if there is a n th order differential equation, then we can write down that n th order differential equation into a set of n; first order coupled differential equation. So, since it is a second order differential equation it can be written as a set of two coupled first order differential equation and that is what is seen over here. Let us know try to derived this a matrix and the f matrix.

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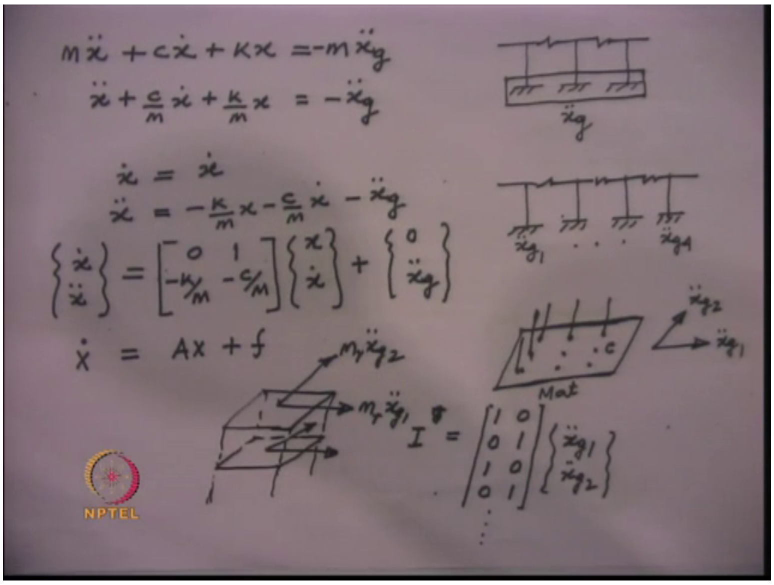

For that what we do is that we divide this original equation by m; we have c by m, k by m instead of c and k m on this side vanishes. Then we write down a equation which is a redundant equation; x dot is equal to x dot. Then write down this equation in the form that x double dot is equal to minus k by m x; that is we take these two terms on to the right hand side and we get a equation like this.

Now, these two equations we write together by defining x dot; x and x dot as a vector. Now these two equations can be written in this form; x dot and x double dot has a vector is equal to this matrix, multiplied by x into x dot y; where x and x dot is the state of the system or the value of capital X vector.

Now, if we expand this matrix then x get multiplied by 0 then x dot multiplied by x dot; on the left hand side you have got x dot. So, it is the first equation if we expand if multiplied this with this, then we get minus k m into x; that is this term and then you get minus c m into x dot this term. We add on to this 0; this will not be x double dot g, this will be minus x double dot g.

So, 0 minus x double dot g because the force there is no term over here, forcing term over here that is why it is 0. So, we get the matrix A; this vector f which is shown here in this equation. In the same fashion, this second order differential equation expressed in terms of the total displacement can also we written the state space, where the A matrix remains the same as before; there is no change. Whereas, the f bar that becomes x g, x dot g because on the right hand side of this equation, we can see that the force depends upon x g and x dot g, so this f bar becomes this and this is multiplied by this matrix in order to get this total force quantity.

So, the equation of motions can be written in the four different forms; one is in the relative displacement second order differential equation then in terms of the total displacement again a second order differential equation. Then two state space forms; one is in the form of relative displacement, other is in the form of the total displacement. So, the solution of these four types of equations provide us the response of the system, we can get the relative displacement as the response we can get the total displacement also as the response.

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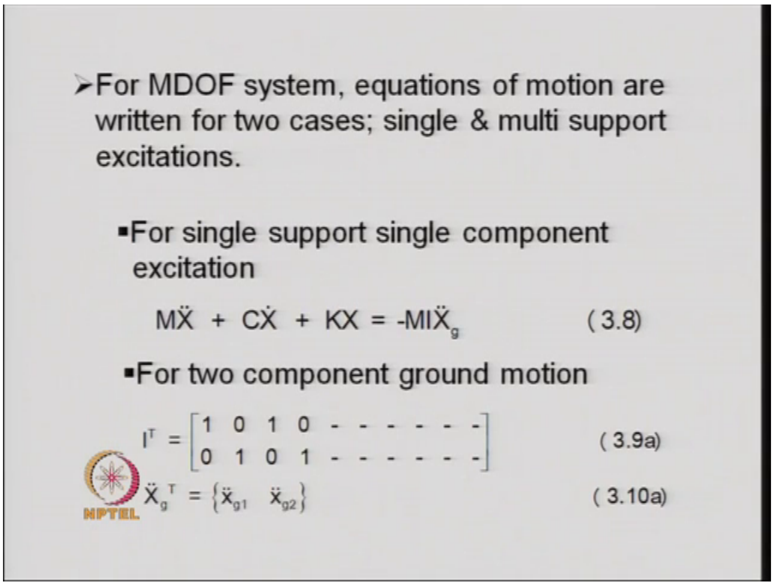

Now, when we this is for the second single degree freedom system; when we extend this to a multi degree of freedom system then what we can do is that in of the single degree of freedom system is replaced by a mass matrix; instead of a single m, it becomes a matrix x double dot becomes a vector of a n number of accelerations.

Similarly, instead of small c; we have a C matrix; x dot again is a vector K is not a stiffness constant, but a stiffness matrix of the system; X is the displacement vector. Here instead of writing down M; X double dot g, we write down matrix M; multiplied by influence coefficient vector I; into X double dot g. So, these are the modifications that are made when we try to write down the equation of motion for a multi degree of freedom system.

The proof of this is well known to all of you those who have done the dynamics therefore, I am not going to you know derive this equations, but a simple way to write down the equation of motion for a multi degree of freedom system is to consider on an extension of the single degree of freedom system and in that extension, we simply replace m c k by some matrices called mass matrix, damping matrix and stiffness matrix.

Mind you this stiffness matrix corresponds to the dynamic degrees of freedom. And all of you know that the dynamic degrees of freedom differ from the kinematic degrees of freedom. There may exist a number of kinematic degrees of freedom in a particular system, but all kinematic degrees of freedom are not the dynamic degrees of freedom. By definition the dynamic degrees of freedom should have a mass attached to it.

So, this K in fact, is the condensed stiffness matrix of the structure, corresponding to the dynamic degrees of freedom. So, what is generally done that we write down the full stiffness matrix of a particular system; considering all the possible independent kinematic degrees of freedom. And then condense it to the dynamic degrees of freedom for which we are able to attach some masses.

So, this condensation is a must for most of the dynamic problem. Now this I influence coefficient vector can take up different form, for that basically we can look at different kinds of problems. But before that let me also tell you that when you are solving a multi degree of freedom system; subjected to ground motion, then we categorize them into two distinct categories. One is the single support excitation and the second is a multi support excitation.

What we mean by single support excitation and multiple support excitation that we will see shortly. But, if you have a two component ground motion then I; this influence coefficient vector can be written like this. For example, this is a system in which you have got two ground motion one in this direction, other is in this direction.

Then one can write down; represent the acceleration by this formulation where I is a matrix of 1, 0, 0, 1, 1, 0, 0, 1. So, on this is a vector of x double dot g 1 and x double dot g 2. If we multiply this vector by the first row over here, then it become simply x double dot g 1, this becomes x double dot g 2. Again this becomes x double dot g 1, then it becomes x double dot g 2 and so, on.

In a multi degree freedom system like this; say this is a floor of a multi degree freedom system then it has a acceleration in this direction and then acceleration in this direction; so mass times the acceleration in this direction is one inertia force and mass times the acceleration in this direction is another inertia force. Therefore, for these two degrees of freedom; the forces which are inertia forces, which are acting is m x double dot g 1 and m x double dot g 2; that is what we get.

So, this is for one floor, this is for the next floor and so on. So, the I that is the influence coefficient vector that is what is shown over here, influence coefficient vector that is the I t is given by 1, 0, 1, 0 and then 0, 1, 0, 1 so on. And where x double dot g is represented by x double dot g 1 and in x double dot g 2; that is what we have shown you. So, for a two component ground motion; we get the I, the influence coefficient vector in this particular form.

Now in a concept of single excitation; single support excitation and multi support excitation is clear from these two diagrams. If all the three supports over here of a particular frame move by the same ground acceleration, then we call this as a single support excitation. And if the supports move differently; that means, the ground acceleration at this support is different from the ground acceleration at this support, then we have got a multi support excitation.

In fact, a multi storey frame; if we look at it then the multi storey frame is subjected to a multiple support excitation rather than the single support excitation. Because as the earthquake wave travels then the excitation that it will cause to the first column and the excitation that it will cost to the last column, they would be different. In fact, they will be different because there will a time lag between the two. Whenever there is some excitation at the first column, at that time the ground motion has not reached the last column therefore, at that time the excitation there is 0.

Therefore we see that there is a time lag between these two excitations; however, the earthquake motion or the ground motion travels with such a velocity; that in order to cross say a distance of about 30 meter or 40 meter or so, the time taken is very small. And therefore, the difference between the last column and the first column so, far as excitations are concerned. Generally is ignored and we say that all the bottoms of the columns are excited by the same time history of excitation.

So, this is an idealized condition and foremost of the practical problems we consider these idealizations and I will say that the entire building frame is a single support excitation system. However, if there is a long bridge supported on the peers and the peers are at a distance of about 100 meter or more than that. Then in that case, the excitations at the different peer supports are different and we consider the problem then as a multi support excitation problem.

Similarly, for a very long structure like dams we have a multi support excitation condition therefore, we treat the problem as a multi support excitation problem.

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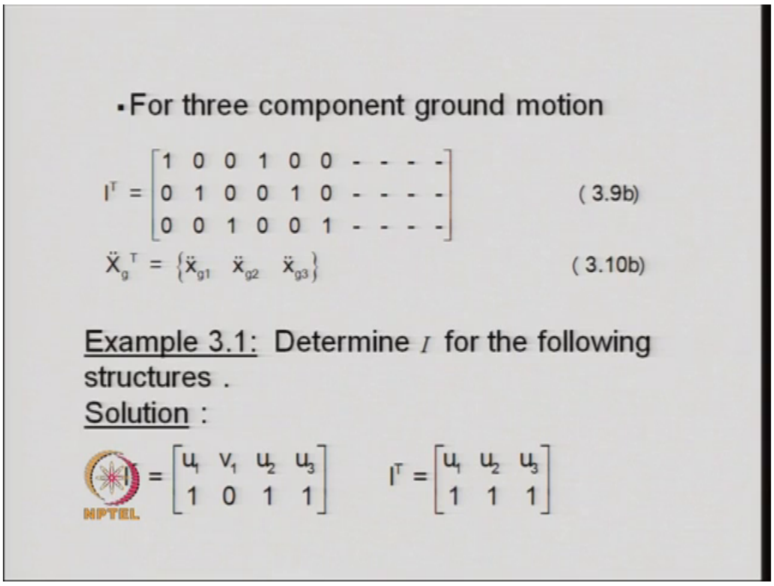

For three component ground motion in the similar fashion, we can show that the influence coefficient vector will be of this type 1, 0, 0 then 1, 0, 0 then 0, 1, 0, 0, 1, 0 and so on. So, in this way one can write down; for the three component ground motion the influence coefficient vector.

But we see an interesting thing is that the influence coefficient vector I; whether it is for a single component earthquake or a two component earthquake or a three component earthquake, they consist of generally 1 and 0’s. Now, let us try to look at some of the example problems to illustrate how the influence coefficient vector is obtained for a single point excitation system.

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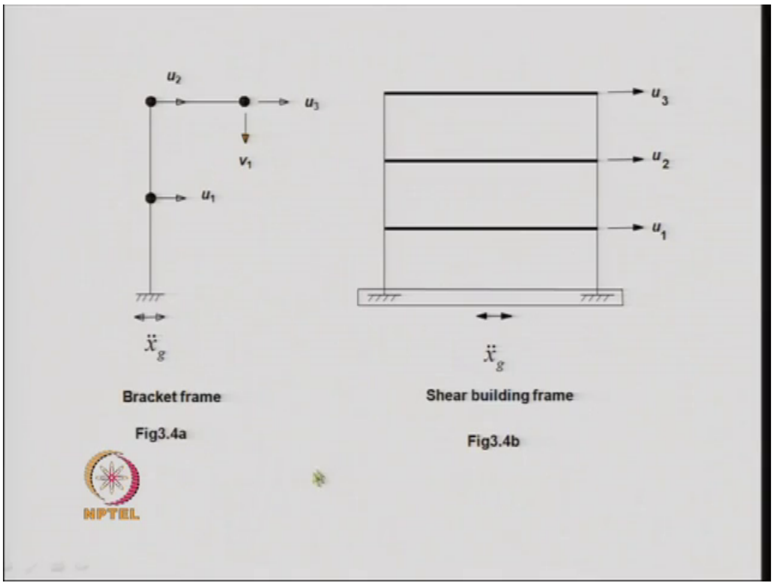

So, here is show two problems in this problem we have a ground acceleration in this direction, we have a mass here and accordingly we have a degree of freedom in this direction. The second mass is over here and the degree of freedom is this and in the third mass we have got two degrees of freedom; one is in this direction other in this direction.

So, in this case the I; the influence coefficient vector takes for u 1, it becomes 1 for v 1; it becomes 0 and for u 2 and u 3, they becomes 1. So, since there is no ground motion acting in this direction therefore, we have this form of the influence coefficient vector and the 0 coming into picture. Whereas, for this particular problem the influence coefficient vector corresponding to these three degrees of freedom are all once because the entire base is moving by the same excitation. And therefore, all the degrees of freedom moves by the same amount and therefore, we have the influence coefficient vector as 1, 1, 1.

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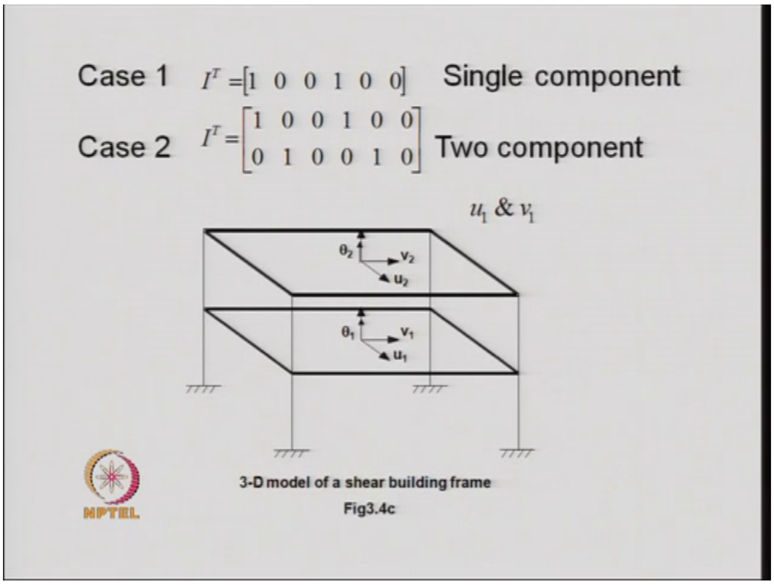

Similarly, if we wish to write down the influence coefficient vector for a these three dimensional frame, then for single component earthquake; obviously, we have the if the earthquake is having in this direction then we have got 1, 0, 0; that means, at the centre of the floor we have got three degrees of freedom, two translation and one rotation about a vertical axis. So, corresponding to this degree of freedom that is u 1; we have a ground motion. For the second direction here, there is no excitation that for the third; that is no torsional ground acceleration. Therefore, this is also set to 0. Again for the next floor; it becomes 1 for this degree of freedom then 0, 0.

So, that way for single component earthquake we have this definition of the influence coefficient vector. Similarly, if there is a two component earthquake; then the influence coefficient vector becomes 1, 0, 0; 1, 0, 0 in the first column and in the second column that is I T that. So, inside this is actually a column then for the second column we have 1, 0, 1, 0 these one indicates the ground motion in the v direction; zero here represent that there is no torsional ground motion. So, for the three dimensional frame we see that we have a I like this.

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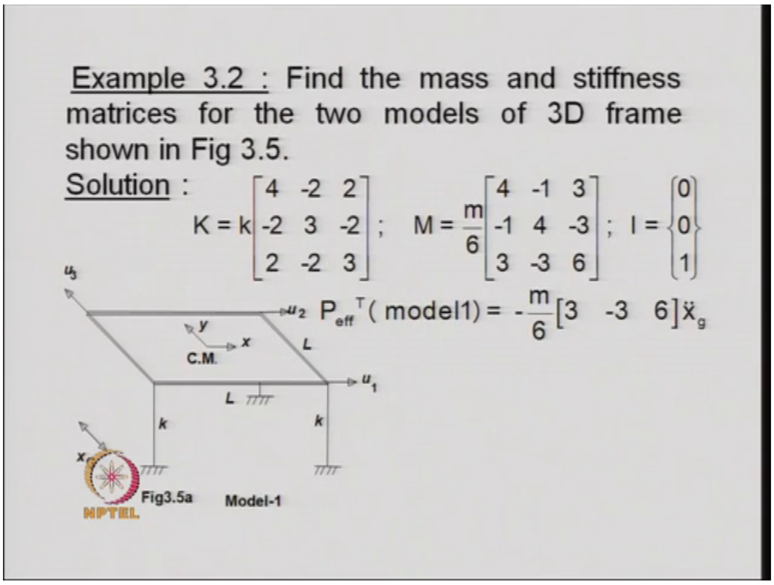

Next we take a problem of these type in which we have three columns; rigid floor this is asemmetric structure therefore, this is having some eccentricity that is the centre of mass, the centre of rigidity of the floor; they would be having some eccentricity.

So, this is an a symmetric system since it is a one storied three dimensional frame; it has three independent degrees of freedom. These three independent degrees of freedom can be chosen in different ways. Here we have considered a two different ways of choosing the independent degrees of freedom. The model one shows the first way of choosing the degrees of freedom.

Here since, we have got three independent degrees of freedom, we choose one as this degree of freedom other has this degree of freedom and the third one is these degrees of freedom. These three degrees of freedom are the three independent degrees of freedom.

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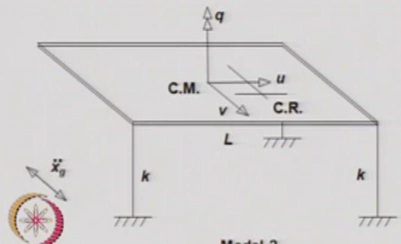
For Model -2

$$K = k\begin{bmatrix} 3 & 0 & 0.5L \\ 0 & 3 & 0.5L \\ 0.5L & 0.5L & 1.5L^2 \end{bmatrix}; \quad M = m\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{L}{6} \end{bmatrix}; \quad I = \begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix}$$

$$P_{eff}^T(\text{model2}) = -m \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \ddot{x}_g$$



Model-2 Fig3.5b

In the second model, what we have done is that we have taken at the centre of mass the degrees of freedom that is one degree of freedom in this direction, one degree of freedom in this direction and the third degree of freedom is a torsional degree of freedom; about a vertical axis. Again these are the three a different degrees of freedom for this particular model.

Now, for these two models we write down the stiffness matrix, the mass matrix and the influence coefficient vector. So, for the model one we see an interesting thing the K matrix since it is an a symmetric system, since one column is absent over here then the eccentricity of the system it will be having a two way eccentricity; that means, centre of mass and centre of rigidity will be placed such that there is e x and e y; two eccentricities will come into picture.

Therefore, we see that the stiffness matrix of the system is a coupled stiffness matrix. There is a coupling between x, y and theta not x y theta u 1, u 2 and u 3; all the three degrees of freedom, there is a coupling. Interestingly the mass matrix is also a coupled one, generally we think that if it is lumped mass system; it will have a diagonal mass matrix that is only at the diagonals, you will have the masses and of all off diagonal term should be 0; that is the kind of notion that we have for the all lumped mass systems.

However, we will see that in such a situation where the masses are lumped, but there is a coupling between the inertia forces is corresponding to all the three degrees of freedom.

Or in other words mass matrix over here will not be a diagonal mass matrix, the influence coefficient vector of course, is the simple here the influence coefficient vector is vector which is x double dot g.

So, therefore, there is no excitation in the u 1 direction, there is no excitation in the u 2 direction; only in the u 3 direction; there is a the ground acceleration is acting therefore, this is 0, 0, 1 that becomes the influence coefficient vector.

Now, let us see how we can obtain this coupled mass matrix; for that what we will do? I will try to explain to you how the first column of this mass matrix is derived. The other columns are derived in the similar fashion, so therefore I will not explain them. So, let us look at how we derive the first column of this matrix.

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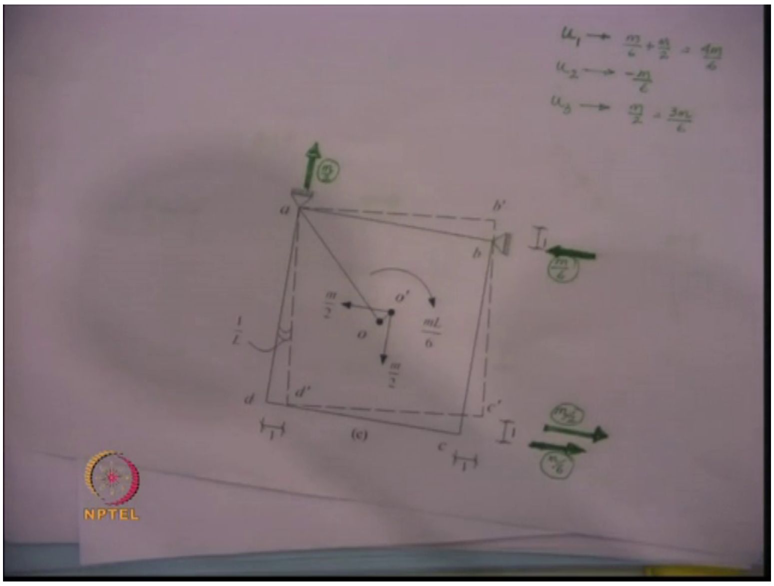

Now, if you look at this figure the degree of freedom over here is the first degree of freedom, here we have the second degree of freedom, here we have the third degree of freedom. So by definition if I wish to find out the stiffness matrix or the inertia force corresponding to the given acceleration; then we release one of the degrees of freedom, keeping the other two locked.

So, here this degree of freedom is locked and this degree of freedom is locked there we give a release to this degree of freedom. Now if we give an unit displacement for this degree of freedom in this direction; then point d; most to d dash perpendicular to line a d for small displacement. Since it moves by unity, here the movement is also unity so, that these length remains unchanged.

The rotation over here becomes equal to 1; divided by these arm length 1 by l. Now this point if I give a displacement of unity, this will also move in this direction by unity because there is no restraint provided over here in this direction. So, we do not have any restraint in this direction and therefore, it can move. Now, if it moves by unity in this direction then this point moves to this point that is b dashed and this movement is again perpendicular to this line.

So, that the length these lengths remain unchanged, these lengths remain unchanged because it has rotated by the same amount; this in this direction there is no movement. So, this point satisfies the condition that there is no movement on this direction, but a movement can take place perpendicular to this direction because there is no restraint in this direction. So, this line now becomes like this; so, the configuration of these rectangles becomes the dotted rectangle after we have given the unit translation over here.

Now in the process; the centre the original centre of this square; now moves to this point this movement is again perpendicular to this line. If it moves, it has to move perpendicular to this line so, that there is no change in the length. So, the centre of these moves by 2 o dashed, since there is a movement in this direction; there is a mass moment of inertia which is developed over here, which is equal to m l by 6.

Now note that the mass moment of inertia that is developed is in the opposite direction of the rotation. The rotation is taking place in this direction and the mass moment of inertia is obtained in the opposite direction. The reason is that the acceleration as I told before always acts opposite to the direction of motion, so therefore the inertia force is acting opposite to the direction.

Similarly since this point has moved in this direction and this centre has moved to this direction. Therefore, the mass total mass lumped at the centre that moves in this direction this other mass moves in this direction or of the other mass acts in this direction and this mass acts in this direction. And the value of them is m by 2 and m by 2 because the movement of this point in the horizontal and the vertical direction is equal to half and half if the movement of this point is 1 and 1.

So, the inertia force generated here is equal to m by 2 in this direction and the inertia force generated in this direction is m by 2. These inertia forces are again acting opposing opposite to the direction of motion, now the since the entire system has to be in moment equilibrium; there is a force generated here and here. These forces are m by 6 and m by 6 multiplied by l; that gives m l by 6, a moment which is opposing this m l by 6.

So, the reaction over here is m by 6; the force here which is generate is m by 6. Then this force and this force; these two forces are now generated, now these two forces equilibrate the inertia forces; that means, this and this makes a couple this two force makes a couple and this couple is opposed by this and this force as a couple.

So, therefore, m by 2 force is generated over here; so, that the entire system is in moment equilibrium. So, we see that if I gives a unit displacement over here this point not only moves in this direction, but also has to move in this direction by unity. The centre moves in this direction that is perpendicular to the diagonal; moves in this direction. The rotational inertia force is developed for moment equilibrium, we have these forces generated. Now, if we sum up these forces; then for a unit displacement giving in this direction, the force is generated in the same direction is equal to m by 6 plus m by 2; that becomes equal to 4 m by 6.

The force generated in this degree of freedom that is degree of freedom u 2 becomes equal to minus m by 6 because it is moving, the force is directed opposite to the direction of the degree of freedom. And the force generated in the third direction is equal to m by 2 or we can write 3; m by 6. So, we have the first diagonal m by 6 has taken as a common and you have 4 m by 6 for u 1, that is 4 m by 6. And then minus m by 6 that is this one and then three m by 6; that is this one. So, these are the three inertia forces which are generated given and unit acceleration to the degree of freedom u 1.

Similarly, when we give a unit accelerate or unit displacement over here then for unit acceleration the inertia forces which will be developed here, here and here; they form the second column of the matrix, when we will give the same unit movement over here keeping these two has locked then the inertia forces that will be developed here here and here they form the third column of the mass matrix.

So, the generation of the mass matrix may require some careful consideration depending upon the problem. So, for this kind of problem you can see that one can generate the mass matrix and this mass matrix is a coupled mass matrix. The P effective would become m multiplied by I. So, if I multiply m with I then we get these as that is the third column becomes the effective mass for the model 1.

For model two it is fairly simple and we can see that since we have the declared the degrees of freedom at the centre of the mass, then for this we have a mass of m, for the rotational degree of freedom; we have got m l by 6 that has the torsional mass or mass moment of inertia about the vertical axis other terms are 0. So, this is a classical lumped mass matrix that you generally see in most of the problems. The I influence coefficient vector becomes 0, 1, 0 because we have declared this as the first degree of freedom, this as the second degree of freedom, this as the third degree of freedom.

So, corresponding to the second degree of freedom; we have got x double dot g. So, we have got 0, 1, 0 as the influence coefficient vector and P effective becomes this; that means, this multiplied by this results in this particular expression for the P effective. The K matrix again you can see that there is no coupling between u and v; that is what we expect, but there is a coupling between this translation and the rotation and this translation and the rotation over here.

So, we have this form of the stiffness matrix. So, depending upon the problem we can have different types of modeling. Different types of modeling means how we choose the degrees of freedom that is important and depending upon the way we choose the degree of freedom, we have a K matrix derived and a mass matrix and not necessarily that a mass matrix should be a diagonal mass matrix.

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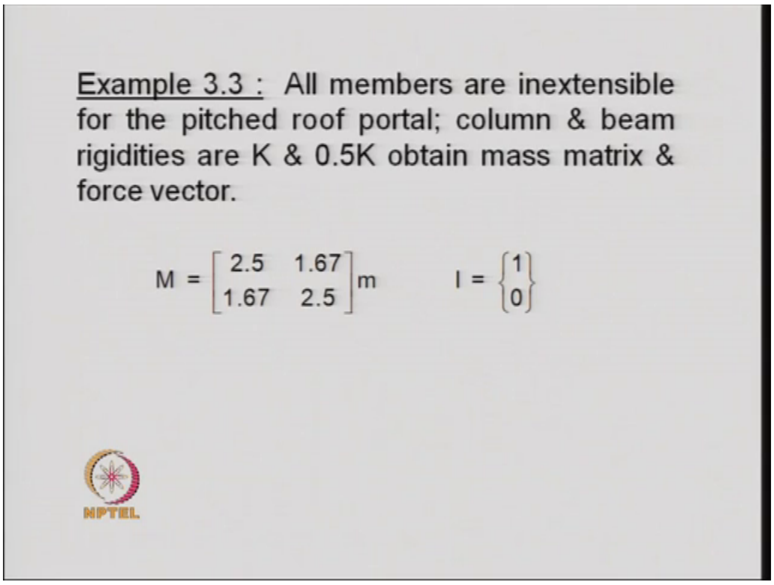

Again we emphasize this point by showing another example where the mass matrix is not a diagonal matrix, but a coupled mass matrix like this K matrix; obviously, also will be a coupled stiffness matrix; I will be equal to 1 and 0 and the problem is this one.

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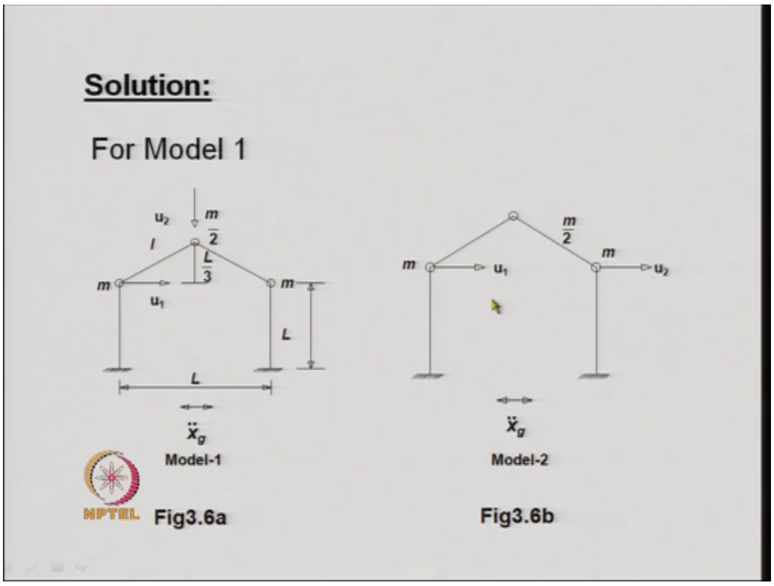

This is a pitch roof portal frame, this pitch roof portal frame we have a mass here, mass here and mass here. These two masses are m and m and this mass over here is m by 2 and one can choose the degrees of freedom in two different ways.

For example, this is one way of choosing that is in model 1; we choose the degree of freedom this as one independent degree of freedom and this is the another degree of freedom. Here in model 2; we choose the degrees of freedom as u 1 and u 2. So, they have the two degrees of freedom because we assumed that all the members are inextensible.

And in order to maintain the in extensibility, we have over here only two degrees of freedom because we can see here that for any one of them. So, for example, if we wish to give a movement here then automatically this one should move in the horizontal direction, otherwise this member will undergo a change in length. So, we can have two degrees of freedom for this model and there are the two different models that we can consider and for these two models the stiffness matrix and the mass matrix would be different.

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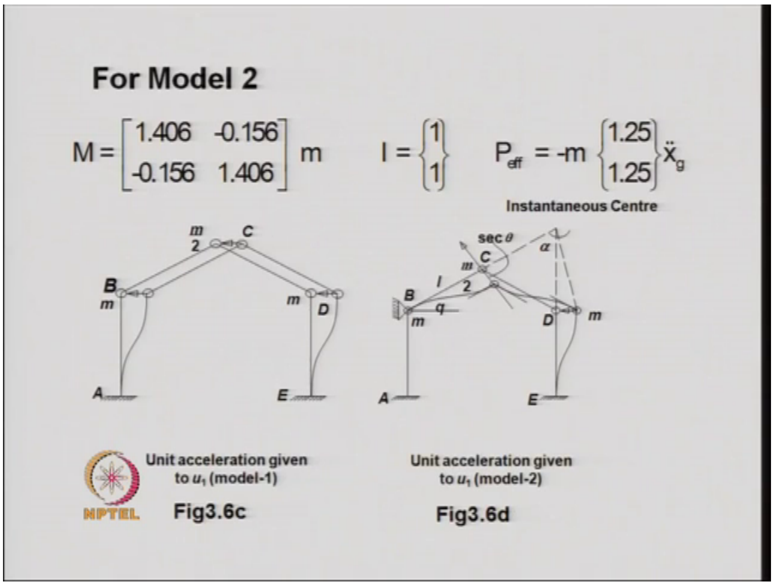

Let us see how we can consider or derive them. We will continue with the same problem in the next lecture.