

Sustainable Material and Green Building
Professor B. Bhattacharjee
Department of Civil Engineering
Indian Institute of Technology Delhi
Lecture – 20

Crushing and Grinding: Bond's Law + Operational Energy: U-Value
 (Refer Slide Time: 00:17)

Bond's law

–Bond's law: Work required is proportional to crack tip length at fracture,

–At fracture surface energy T required is proportional to c ,

$$T = \frac{1}{2} \frac{\pi \sigma^2}{E} c$$

– c is proportional to square root of surface area, surface area / volume is thus inversely proportional to D

B. Bhattacharjee
 DEPARTMENT OF CIVIL ENGINEERING, IIT DELHI

So that was I saying what required is proportional to crack tip length at the fracture. At fracture energy T required is proportional to C that is what I was explaining you. So he takes it T half sigma square this is what already we have looked into this you have understood. C is proportional to square root of surface area. C is proportional to the square root of the surface area per unit volume is thus inversely proportional to D .

So this C supposing C means you know large aggregate I am breaking it down to a smaller aggregate. So the C would be related to this now. The new one I am creating and surface area per unit you know it is related to square root of surface area, this is related to square root of surface area because if the diameter changes surface area is proportional to pi D square. In a spherical one pi D square is a surface area.

So surface area is C is you know D is this any way this C , because I have broken it down to C . So it is proportional to square root of surface areas, square root of surface areas. So surface area per unit volume. So surface area per unit volume we have seen was 6 by D , and the D if I reduce it down so it will be you know square root of C is proportional to square root of surface area per unit volume is thus inversely proportional what it takes is inversely proportional to D .

Because this should be this comes to this place again 6 to D. So this C is inversely proportional to D you have a large aggregate breaking into smaller aggregate. So you have to create supposing it was originally D originally it was D, now you got to make it another sphere. So D by 2 let us say or sphere becomes difficult, rectangular one you can easily understand.

So this becomes D by 2, this becomes D by 2 or rectangular you know parallel or cubic one, if you want to divided into 2 you will get 8 cubes out of this actually if you break down into D by 2 and so on. So basically that means fracture will be creating in 8 surfaces D by 2, 8 surfaces you will be creating. So it is inversely proportional to D same or 6 by D larger the D, smaller the D more number of surface area. So this is what surface is inversely proportional to D, D smaller more surface area per minute volume.

(Refer Slide Time: 3:19)

Bond's law

**Assumption is for irregular particles
crack length is proportional to $\frac{1}{\sqrt{D}}$**

$$\frac{P}{\dot{m}} = K_r \left(\frac{1}{\sqrt{D_{sb}}} - \frac{1}{\sqrt{D_{sa}}} \right)$$

K_r is expressed in terms of work index W_i

B. Bhattacharjee
DEPARTMENT OF CIVIL ENGINEERING, IIT DELHI

So his law is come from this kind of concept assuming is for irregular particles crack length is proportional to he take 1 by D to the power sb before n. So instead of taking 1 by D that was the first law I talked about (())(3:41) then Kicks and then Bond's law is something like under root D. So he use under root D because surface area pi D square, so if you reduce down the you know size surface area will be increasing and it will be proportional to surface area is proportional to root over D, because pi D square is the surface area.

So S is pi D square. So he takes therefore root over instead of taking log and all that and he write in this manner. So finally these are empirical as I said this is also empirical instead of taking D linearly he takes this neither logarithmically it takes this. So this is what, and that

apparently has been most popular in this grinding and crushing. K_r is expressed in terms of work index this K_r has been expressed in terms of work index. So it is define it in a manner.

(Refer Slide Time: 04:43)

Bond's law



–Work index is defined as the energy required to crush infinite size particles to, Particles having size 100 μ corresponding to 80% passing in sieve analysis

D values in equations corresponds to 80% passing rather than average,

$$\frac{P}{\dot{m}} = W_i = K_r \left(\frac{1}{\sqrt{100}} - \frac{1}{\sqrt{\infty}} \right) = k \left(\frac{1}{\sqrt{D}} \right)$$

$$K_r = 10 W_i$$

B. Bhattacharjee
DEPARTMENT OF CIVIL ENGINEERING, IIT DELHI

So bond's coefficient whatever it is, so bond's coefficient so it is work index is energy required to crush infinite size particle, to particles having size 100 micron corresponding to 80 percent passing in sieve analysis. So if you want to crush it to 100 microns, then what is again? The total energy required, K is bond energy bond's or work index is defined and the energy required at crush very large system to a very fine system.

System having 80 percent passing through 100 microns 80 percent passes. So that means total energy required work index is a total energy required to crush it from infinite size to a very finite size. Energy required to crush infinite size particle to particles having 100 microns 80 percent should pass through 100 microns. So D value equation corresponds to 80 percent passing, rather than average.

So D if you are talking of D in this equation is 80 percent passing not average size. So therefore, P this is the power per unit mass flow rate feet this is W_i is equal to $K_r \left(\frac{1}{\sqrt{100}} - \frac{1}{\sqrt{\infty}} \right)$ to infinite size larger size. So before it was large size finally it is 100 micron, so the value this is what is called work index. So K_r is equals you know this if you take this simply nothing but K_r into 1 by 10, K_r into 1 by 10.

So 10, K_r into 1 by 10 so 10 W_i is equals to K_r . So this is K_r is related to 10 W_i . So work index can be found out in a given system this can be found out because I take large mass sufficiently large mass compared to 100 micron. I mean it could a very large rock or

something that kind or depending on the crusher that I am using specially for cement if I am using I do not have to go even to rock size clinker largest clinker size and so on from the 100 micron.

So micron to millimetre you know centimetre size is that should be in the middle that middle that will give you 0 sufficiently large so that is K_r is equal to 10 work index and work index is defined in this manner which gives measurable and this value are given for certain things.

(Refer Slide Time: 07:18)

Bond's law

Writing P and F for D (80%) product and D(80%) feed,

$$W = 10W_i \left(\frac{1}{\sqrt{P}} - \frac{1}{\sqrt{F}} \right); \quad P = K_r \quad m = 10i$$

Wi for many materials are known from experiments, thus power/mass can be calculated

B. Bhattacharjee
DEPARTMENT OF CIVIL ENGINEERING, IIT DELHI

So bond law then therefore, writing P and F D 80 percent then takes size as 80 percent you know they said D_{80} that would be in that size 80 percent passing through you know the size through which 80 percent passing through 100 micron I am saying so they do not take average size rather 80 percent passing size that I take. So D 80 percent is taken.

So it will something like this 10 W_i is the constant now work index 10 and this you can find as I said this is measurable this measurable because you can take large size crush them to 100 micron 80 percent passing through 100 micron, whatever you get you know it will be 10 times I mean you get basically a 10 W_i is case so you find out is the power P by m dot for that case and that is equals to nothing but K_r which is equals to 10 K_i you know which is equals to 10 W_i .

So you can actually this measurable W_i is measurable. So, W_i is the materials are known from experiments thus power per unit mass can be calculated right. So writing P and F for D diameter of the product 80 percent product and this is for feed. So this is the feed and this is

the product. So feed size 80 percent passing through, supposing I had all 80 percent 10 mm size of the clinker hypothetically 10 mm is too large actually it will be much smaller.

10 mm size of the clinker 80 percent passing through 10 mm only 20 percent is above 10 mm then that will be my feed and then finally I grind it to let us say 50 micron, now 50 micron then if I know the W_i work index which is measured already measured experiment work, I can find out what will be the power per unit mass, power per unit mass. So that is the idea that is Bond's law.

So that is what used people use at this concept they use quite often now this concept can also be use in aggregate if you mechanize aggregate processing system this use in cement or similar kind of material in chemical technology bond's this long bond's work index. So that is the energy required or power required P by m dot required for any crusher you know the feed rate, power required can be measured from very large size to crush it down to 100 micron down 80 percent down and then you know the power you measured.

So that is your work index, 10 of the work index is the K value constant value and we will use this one to 10 W_i and if you know the feed size let us say now you know it is a 3 millimeter and you want to make it to 50 millimeter or 5 millimeter to 50 millimeter what is the power per unit mass that you can find or some grinding energy can be found out with this.

So this will depend on what? This will depend upon the type of material you have cement clinker have very high value, fly ash will have much less value this will be much higher for cement because the it is clinkerised material very hard material fly ash is which is produced it will require much less. Similarly, their sizes are also would be different. So it will depend upon the sizes and also this value.

Similarly, if you take limestone powder its W_y might be different and these values are actually listed down people have listed down doing experiments they have actually listed down. So different types of grinders, different type of grinder crushers they have actually listed down. So one can use them to find out what is the power required.

Now so that is what I am saying when you use a high belite cement, grinding energy is also less, remember some of those cements is said grinding energy is also less, just because the mass that is to be grinded is less harder. So they require less energy of production. So if you are replacing fly ash not only that you are actually reducing down you know the carbon

dioxide production from there but the grinding energy which is not directly it is only for the grinding part later on.

Because you might mixing the fly ash together with the cement and into grind them together. So grinding energy also will reduce depending upon the percentage should reduce. Similarly, GGBFS they will have different W_i (12:19). So I think that is related to some idea about grinding.

(Refer Slide Time: 12:26)

OPERATIONAL ENERGY

Thermal conductivity (U-value) is the important material property for mechanically conditioned building.

$$Q = kA \frac{T_1 - T_2}{l}$$

$$= h_o A (T_{1a} - T_1)$$

$$= h_i A (T_2 - T_{2a})$$

$$T_1 - T_2 = \frac{Q l}{kA}$$

DEPARTMENT OF CIVIL ENGINEERING, IIT DELHI

OPERATIONAL ENERGY

$$T_{1a} - T_1 = \frac{Q}{h_o A}$$

$$\frac{T_2 - T_{2a}}{h_i A}$$

$$Q = kA \frac{T_1 - T_2}{l}$$

$$= h_o A (T_{1a} - T_1)$$

$$= h_i A (T_2 - T_{2a})$$

$$T_1 - T_2 = \frac{Q l}{kA}$$

DEPARTMENT OF CIVIL ENGINEERING, IIT DELHI

And then we will look into some issues of operational energy, some issue of operational energy right some issue of operational energy. Now, if you see operational energy which you look into later on, we will look into later on. If you see the code for example any code energy code you will find that in condition building where energy is used it largely depends on

insulation, they would suggest that insulation value so when we look into let us say ECBC in energy conservation right. So there define there are talking mostly the insulation. So when you have a condition space main thing is a insulation part of it.

And the property that we deal is called U-value those who are not familiar for them quickly you see if you have. If I have heat flowing at constant temperature from this side to this side let me say T_1 and T_2 , then heat is flowing long this direction. So this temperatures are at constant that means there is no increase in temperature here, whatever Q heat is coming out here is going out at the out of periphery.

Wall or slab whatever it is and I that case you will have a temperature gradient existing from T_1 to T_2 . Temperature higher here, temperature lower here and it is governed by basically furious law, right. So this Q is given as $kA(T_1 - T_2)$ divided by this thickness is there it is given by $T_1 - T_2$ divided by l . So this is called thermal conductivity of the building material or material that I am dealing with if it is a homogeneous material.

And what you have seen further also that there will be some, there will be some you know there will be some sudden if the air temperature is 2 you know here and let me say $2T$, no this colour is not happy with this colour let me change T same showing, T_{2a} let me call it T_{1a} to denote the air temperature. Air temperature will be higher here and there will be a sudden drop here and there will be a sudden drop there to T_{2a} .

So there is a fine air layer through which equivalent conduction or you know heat transfer take place through radiation and conduction because there is air layer solid can never be there alone it is surrounded usually by air. So the air there is a heat transfer here to here and from air to here, and from air to this place and that also same Q must be coming there same Q must be going out to the air as well because the temperatures are not changing everything is remaining constant.

So nobody is absorbing the heat no material is absorbing the heat, the heat coming in must be equal to heat going out. In such cases this Q can also be written as if I call this as equivalent conductivity of this one is h_i and this is h_o let me call it equivalent conductivity or equivalent conductance as we call it because their thickness is not known to me. So l over K I take l over you know l , K over l . So we write it as this in this manner h into same area into $T_{1a} - T_1$ must be equals to h into h I was calling it h_o , $h_i A(T_2 - T_{2a})$.

So this, this is the expression one can write and from this if one combines, one can actually a simple exercise because one can write like this you know this q are all same. So I can write $T_1 - T_2$ divided by $T_1 - T_2$, $T_1 - T_2$ is equals to Q into l divided by kA similarly, $T_{1a} - T_1$ I can write let me just go a little bit for those one who have not done this course or similar thing before T_1 can be written as Q by h naught into A and $T_2 - T_{2a}$ can be written as Q by h_{ia} .

And if I sum this 3 up you can see that this will first cancel out this T_1 because there is a minus sign and you know this will cancel out with this one and I am summing up everything what I will be left is $T_{1a} - T_{2a}$ these 3 I sum up. And I can show that if I sum this up I will just clear this of little bit.

(Refer Slide Time: 17:34)

OPERATIONAL ENERGY

$$\frac{1}{U} = \frac{l}{k} + \frac{1}{h_o} + \frac{1}{h_i}$$

$$\frac{Q}{A} = \frac{Q}{A} + \frac{Q}{A} + \frac{Q}{A}$$

$$\frac{Q}{A} = \frac{Q}{h_o A} + \frac{Q}{k A} + \frac{Q}{h_i A}$$

$$T_{1a} - T_{2a} = \frac{Q}{A} \left(\frac{l}{h_o} + \frac{l}{k} + \frac{1}{h_i} \right)$$

$\rightarrow \frac{1}{U}$

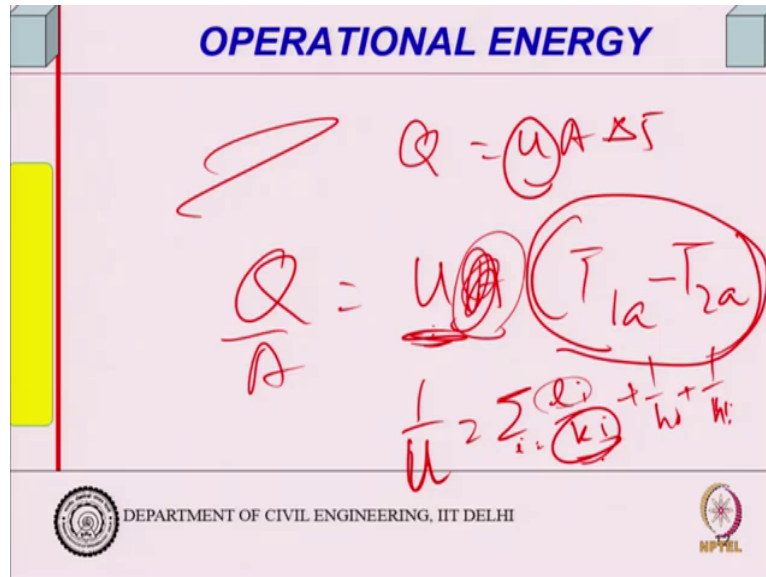
DEPARTMENT OF CIVIL ENGINEERING, IIT DELHI

If I sum this up $T_{1a} - T_1 + T_1 - T_2 + T_2 - T_{2a}$ let me write this out I wrote this as a Q divided by $h_o A$ plus Q divided by you know Q divided by $k l A$ again plus Q divided by $h_i A$ if you remember this and this all will cancel out living $T_{1a} - T_{2a}$ is equals to $T_{1a} - T_{2a}$ is equals to Q into 1 by h_o plus 1 by l over k plus 1 by h_i .

So you see there is a similarity between all the expressions there. So combining this we call this 1 over u you know this one we call as 1 over u , combining the conduction through 3 layers the solid one layer outside, another layer outside. So I call them and we call them as u . So u is nothing but transmittance between air to air, transmittance between air to air and you can see that the k that is thermal conductivity of the material plays a very strong role there.

So if you want to improve insulation u should be, u should be low, higher k means higher u, 1 over u is equals to 1 by h_i plus 1 over k plus 1 by h_o . So 1 by u is this so higher k means higher u insulation means u should be less, insulation means u should be less, k should be less right insulation means u should be less, k should be less.

(Refer Slide Time: 19:40)



In other words, the Q can be written as, Q can be written as from this it follows, just I will write at the top, from this it follows erase this out and write this from this it follows that Q is equals to Q you know u Q is it is not cleaned properly. Q is equal to uA T1a minus T2a that is how we do it. So is the property of the wall or slab or a room or roof's ceiling or roof or any of that kind 2 dimensional surfaces which is called transmittance or we call it u value.

And it is a, this multiplied by the area so if you take Q by A, flow per unit area this will be u multiplied by temperature difference. So Q is equals to Q is equals to uA delta T as we write it so generally you write it like this Q is equal to uA delta T. So if I reduce u my heat flow will be less, so lower u means better the insulation, lower u means better insulation, lower u means better insulation.

And you have seen that 1 over u is equals to sigma 1 over k_i if there are many of them number of layers then I can write it in this manner plus 1 by h_i plus 1 by h_i . So in other words conductivity of the material is higher, I have higher u value. Higher conductivity means higher u value because there in a same 1 by u is 1 by k l and thickness higher the thickness lesser is the u value. So thicker the material more is the insulation, more is the conductivity less is the insulation.

So u should be low, better insulation means u should be low, transmittance should be low. So higher insulation means lower u which means lower k as well but thickness should be more. So thermal conductivity is an important parameter for all the walling and roofing material ceiling and roofing material because heat transfer take place through the envelope and their insulation you will find that the coats actually restricts this u values u values for you know if the energy coat suggest they would suggest that ok u should be between this or this or something of that kind.

If it is prescriptive as we shall see later on or you want to get lead rating etc. Energy has to be you can get a lot of points from energy efficiency and there the you value (U) (22:40) ignored. So we would be talking about the u value.

(Refer Slide Time: 22:47)

OPERATIONAL ENERGY

Thermal conductivity (U-value) is the important material property for mechanically conditioned building.

In un-conditioned building thermal capacity is also important because of time-lag and amplitude decrement, especially in hot and dry climate

The slide features a graph with a red sine wave and a black line, with handwritten red annotations. The text '24h' is written below the graph. The slide footer includes the Department of Civil Engineering, IIT Delhi logo and the NPTEL logo.

Therefore, it is important to look into thermal conductivity. So U -value is important material for mechanically condition building that is what I am trying to explain that is what I am trying to explain, and we will look into the next room un-conditional building of course thermal capacity is also important. In unconditional building thermal capacity is also important because of something called time lag and amplitude decrement I am not interested in this class related to this too much because you see there you can look into the temperature variation 24 hours in this manner.

So what we would like to have is inside temperature, inside temperature we would like to have, inside temperature obviously we would like to be something like this 24 hours same 24 hours this is called amplitude decrement this ratios and this is called phase lag or time lag you

know from here to here this is the time lag pick to pick this is time difference that is called time lag.

So this is also important in hot dry climate because day time heat if you utilise in the night cool time that is very good in desert we are not interested in this in this particular class we are interested in this part right, so I think we will stop here.