**Sustainable Material and Green Buildings**
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**Lecture – 21**
**Operational Energy: Thermal conductivity models**

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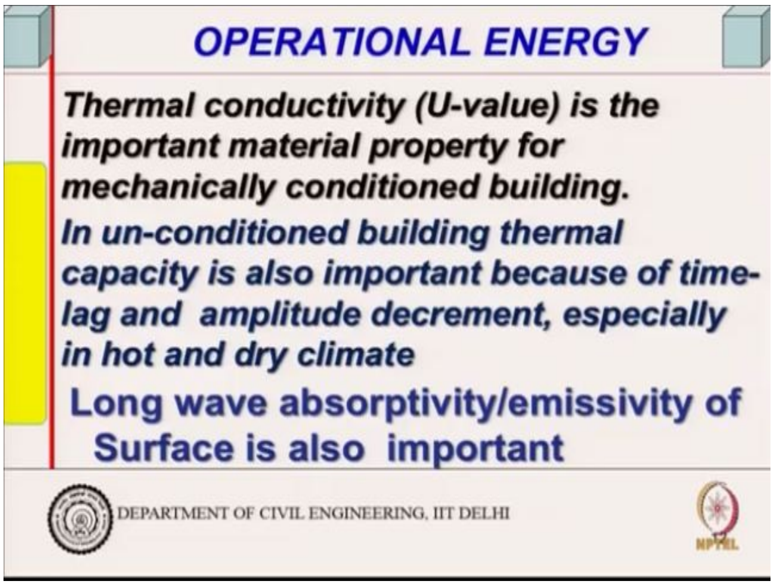


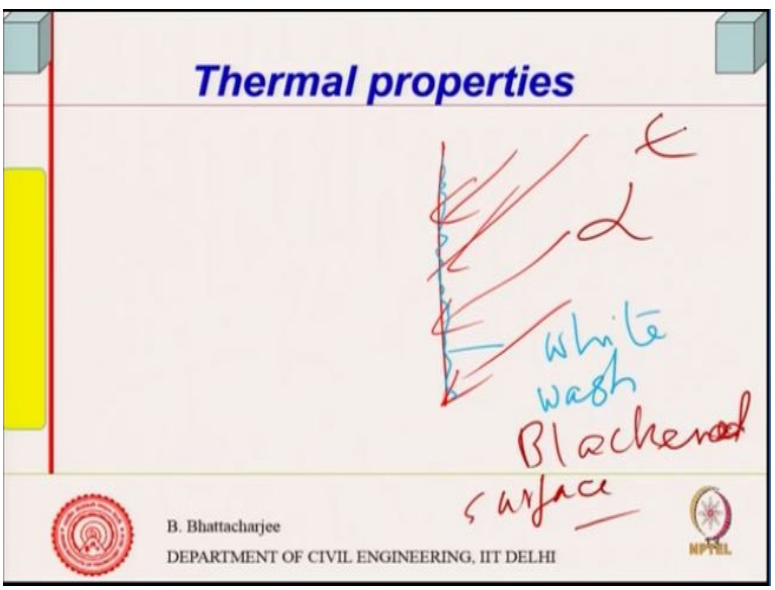


So we will continue from operational energy that is what we looked into. And we said that it is a U-value or thermal conductivity which is very important in terms of, you know, in terms of material properties for mechanically conditioned buildings, this is for conditioned buildings. Of course if it is a unconditioned building, then we would like to maintain comfort condition.

In that case, essentially thermal capacity is also important because there will be some storage in the wall system; wall or roof or ceiling or the envelope part, you know, there will be some storage in this. So it will be stored and released later on. So there is a time difference between maximum temperature outside and when its effect enters into the internal space.

So that time difference is what we call, we express it in mathematical form in dome manner called time lag and there will be decrement temperature inside, that would cause a heat flow. Heat flow would be, you know, its amplitude would be there would be some reduction in the amplitude if you look at the heat flux outside. So there is something called a decrement. So we define, we basically like to, in places where you have got high temperature variation outside, we like to keep decrement low.

That means ratio of heat coming in by temperature outside in terms of the fluctuating temperature that should be low. And also, we would like that there is a time difference because night temperature becomes cool in desert and such sense. So we are not interested right now in this because when you are looking at energy consumption, we are looking at energy consumption, right? But for unconditioned building, of course you have to look into these issues also. At the moment, we are not interested in that.

Similarly, the surface properties; long wave emissivity and short wave absorptivity. Sun's rays falls on the surface, right? Sun's rays fall onto the surface, let us say, okay? Sun's rays fall onto some surface, now how much of it will enter is a function of surface characteristics. How much of it will be absorbed is a function of surface characteristics. For example, a white washed surface will not absorb much. Black will absorb, black absorbs more.

Blackened surface absorbs more. So what we call alpha value absorptivity is also an important thing. But once it gets heated up, it also emits that energy out. And that is what we call long wave emissivity. So these properties are also important. There properties are also important. I think that is where we stopped. You know, long wave emissivity and short lag absorptivity these are important.

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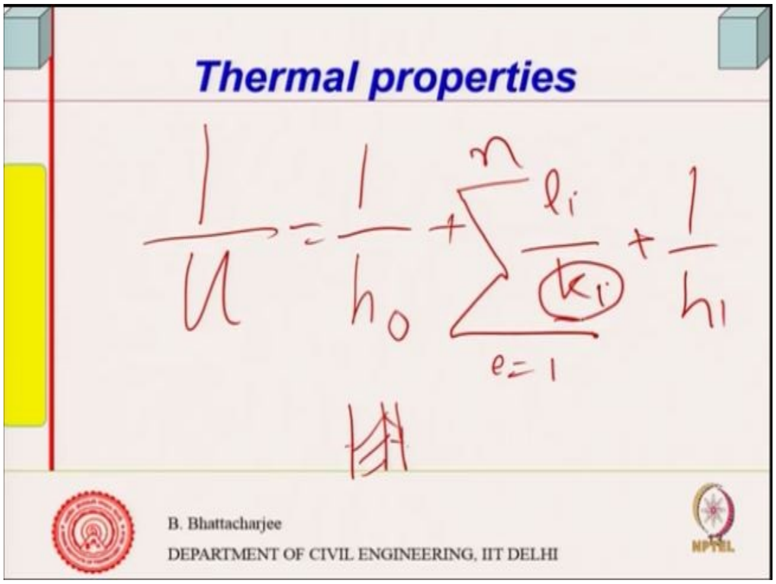


Now, therefore what is important is what is U and I think in the last class I was saying 1 over U is equal to, something I defined called surface conductance. I think last class I did that for one side and then we said that it would be sigma li over ki or rather; this should be written in this manner, not this. Sigma of the whole thing, sigma for different, suppose I have got a number of layers, right? One layer of the big plaster, other layer, etc. So I am going from 1 to n plus 1 by hi.

So see the conductivity of the material plays an important role in this property which is called transmittance or U-value. Higher the U-value, more heat will be transferred because we saw in the last class, Q is equal to UA into delta T temperature difference between two sides, two surfaces; this side and that side of the wall, you know it may have, including the air temperature here, air temperature here. So this is the heat flow. Therefore higher U means more heat flow for the temperature difference. And we would like to keep it lower. So insulation is 1 by U and for that K should be lower. So K is the important property that is why we are looking at it at the moment.

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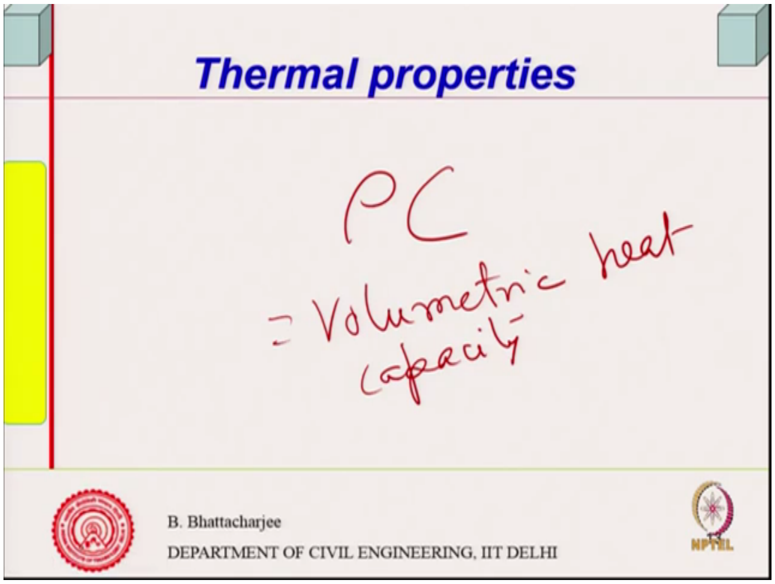


We will look a little bit into another property that is density into specific heat which is volumetric heat capacity. We will look a little bit into this also. So some understanding of thermal properties are important, right? We are looking at this actually because U value will be controlled by this, right?

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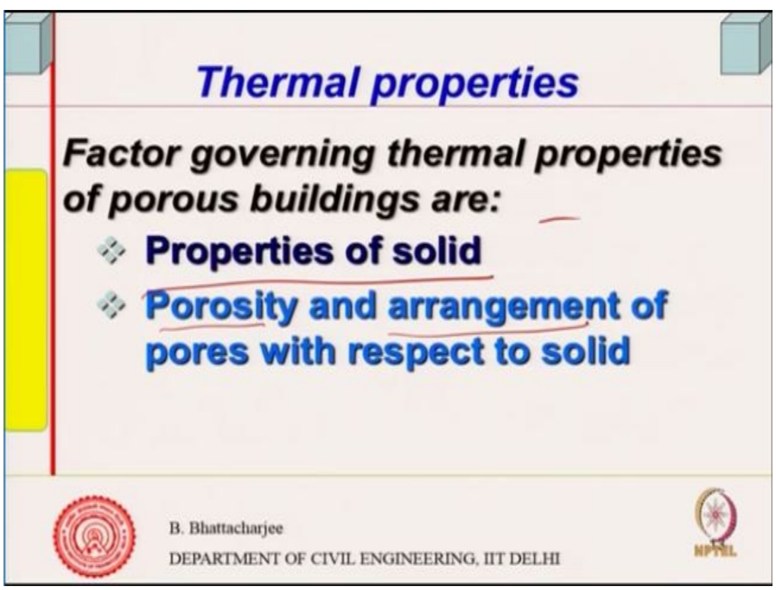


Alright, the factors which governs the thermal properties so thermal conductivity because most of our materials are porous. Most of our materials are porous. So we have solid and we have some pores and porosity and arrangement of pores with respect to the solid is important. Right, okay? So the arrangement also; we shall see that how it works. And this is very important moisture content.

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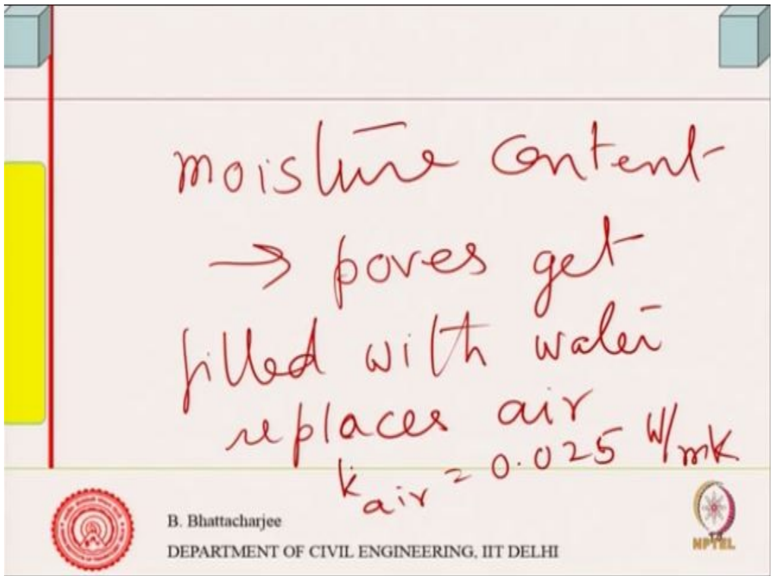


Right? Okay. Why moisture content? Moisture content, why? Because pores will get filled by, pores get filled by water, filled with water instead of air in place of air. So it replaces air. Conductivity of air at normal temperature is K air is something like 0.025 Watt/meter Kelvin. And conductivity of water, liquid water is 0.6 Watt/meter Kelvin or so. So you can see it is about 25 times. It is nearly about 25 times more. Water conducts nearly about 25 times more. 0.025 multiplied by 25 will give you around 0.625. So it conducts more. Therefore, when the pores are moist, saturated, it will conduct and also partially moist it will conduct. So let us see the mechanisms.

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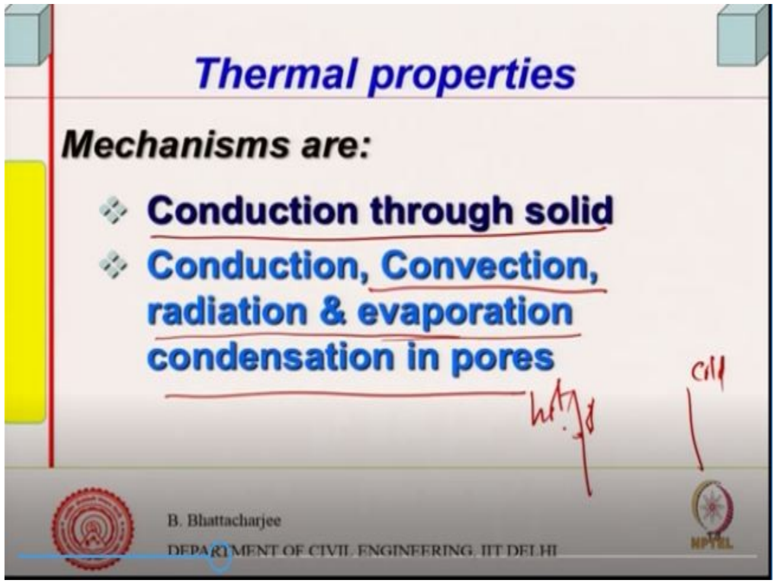


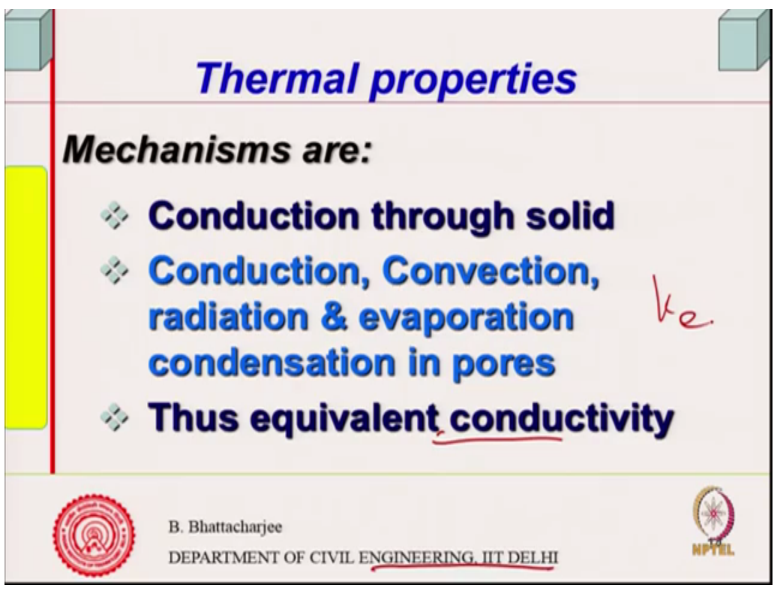


Mechanisms are conduction through the solid, conduction that is possible convection and radiation and evaporation condensation in pores. For example in the hot face, this is let us say the hot side and this is the cold side. Now water here might evaporate, the pores are filled with water, water might evaporate, travel to this place and condense. When it condenses, it will release that heat. So it will actually absorb heat while evaporating and while going to the other side where it condenses, it might, the transport, it might, you know this could be another parallel path. Right? Evaporation condensation in pores this can also occur. Right?

So what we talk about in case of porous material is the equivalent thermal conductivity of such material Ke. So most of our material like brick, concrete, light weight concrete and similar ones they are all porous materials. Even timber is porous and insulating materials are deliberately porous, some of them will make it deliberately porous, but if I understand the mechanism, then we know it becomes easier to look into that.

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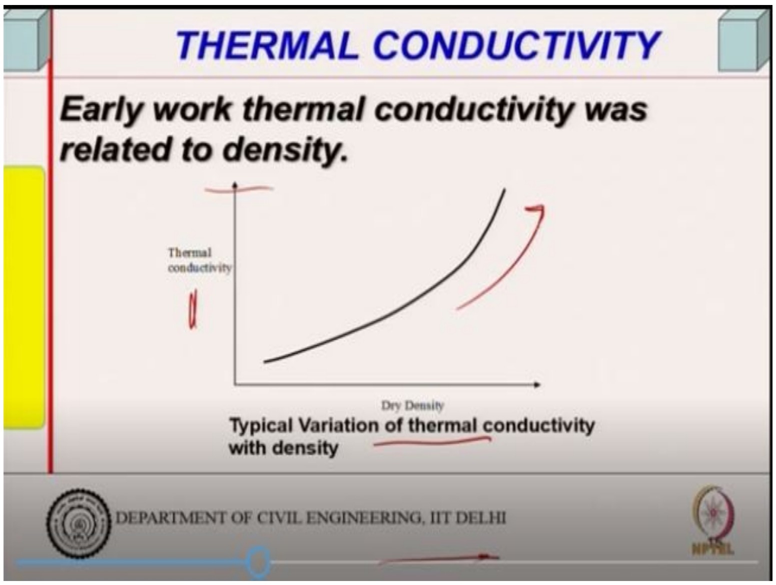


So (people) since it is a function of porosity, the porosity conductivity will be low. Higher the porosity, more air, less conduction through it. Let us take dry state, in dry state higher the conductivity, porosity will be low. So people try to relate it to density, simply. A dense material conducts more and a low density material will conduct less. So light weight aggregate system, light weight concrete and so on.

So dry density, if I look at it, versus thermal conductivity; as density increases, conductivity increases, you know this is the empirical observation and people try to relate some empirical relationship. So typical variation of thermal conductivity with density this is for the bulk, all the material people have been trying to deal with this, not looking in to details.

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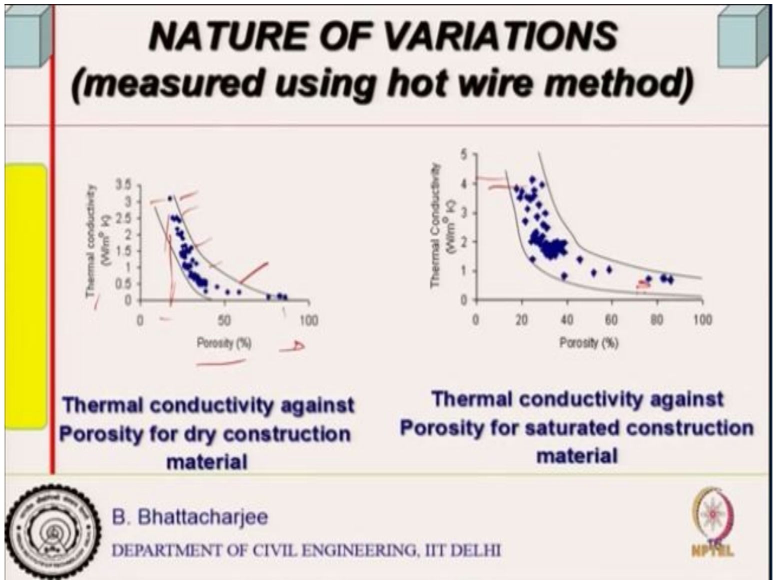


However, if you look at the porosity versus the thermal conductivity, for materials such as brick, concrete, concrete of different pores, or different porosity that is, so which means a weak concrete to a somewhat denser concrete. So if you look at it then, A rated concrete were deliberated from concrete. Where you create pores deliberately by adding foaming agent.

So this foams creates closed pores, fly ash bricks and similar sort of thing; their porosity will vary with thermal conductivity. If porosity increases, thermal conductivity reduces. Now here you can see that porosity can be as high as 80 percent or so in foam concrete or similar sort of thing. And in somewhat denser normal concrete, it could be somewhere around this. So conductivity varies between this sort of range.

But if it is saturated, this range increases. It goes to 4 here and this is surely the lower range increases. So saturated conductivity, saturated in the moist condition, the range shifts. So there is an increase in the conductivity. This goes to around 4.5 from 3 and this has also gone higher up from here. In fact the rate of increase will be more. So moisture plays a role, saturated state and dry state there is a difference.

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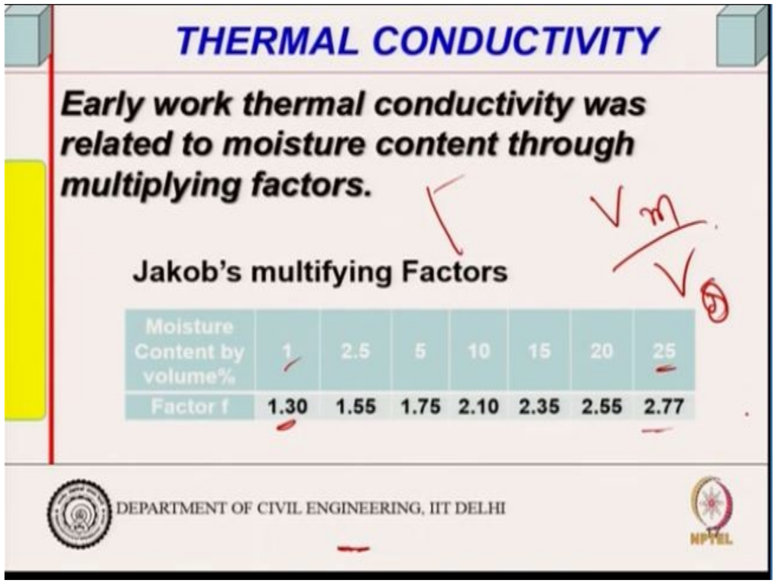


Now early work, as I said, thermal conductivity was related to moisture content through some multiplying factors. So these are called Jacob's multiplying factor. This you will find in books of 1950's or 40's around that time. It is simply said that if you have moisture content by volume 1 percent multiplied by a factor of 1.3, 2.5 volumetric moisture content. That means volume of moisture divided by the volume of dry volume of the solid.

Right? Which would be the same. Basically volume of the solid after all, this porous material so it is a bulk volume, it is bulk volume actually. So volumetric moisture content, not mass basis. And that is what it is. So that is why you find 25 percent. These are multiplying factors. If you know the dry conductivity, you can actually multiply this.

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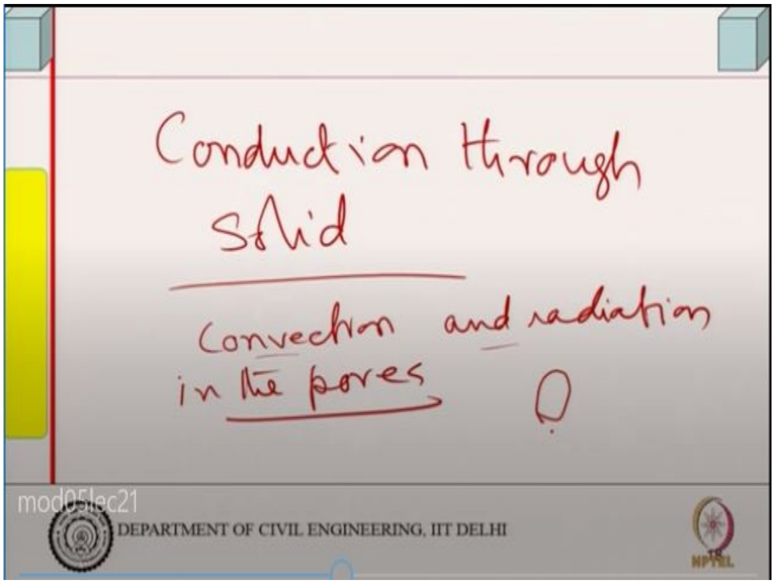


Now, what are the mechanisms? First of all, mechanism is conduction through the solid. And there can be convection and radiation in the pore. But this is very small because conduction is a phenomenon where the molecules move and then circulate and through circulatory motion they transfer the heat. Hot molecule will go and the cold molecule will you know so this kind of thing.

So in such a situation, if the pore sizes are small, then this does not occur. This is the function of the temperature difference between hot and cold surfaces. In a small sized pore, this temperature difference is very small. So pores less than 0.3 millimeter, 3 millimeter I do not get that. 0.3 millimeter is almost 300 microns. So pore size is less than 0.3, you are unlikely to get anything. And you will not get much of it in a closed pore system, convection and transportation.

So these are neglected in normal ambient conditions, but if you are dealing with high temperature then this becomes very important especially when porosity is very very high. So conduction through a solid is a main thing. And equivalent conduction through a pore which is a function of, mainly as a function of moisture content, not much of a function of radiation and all that.

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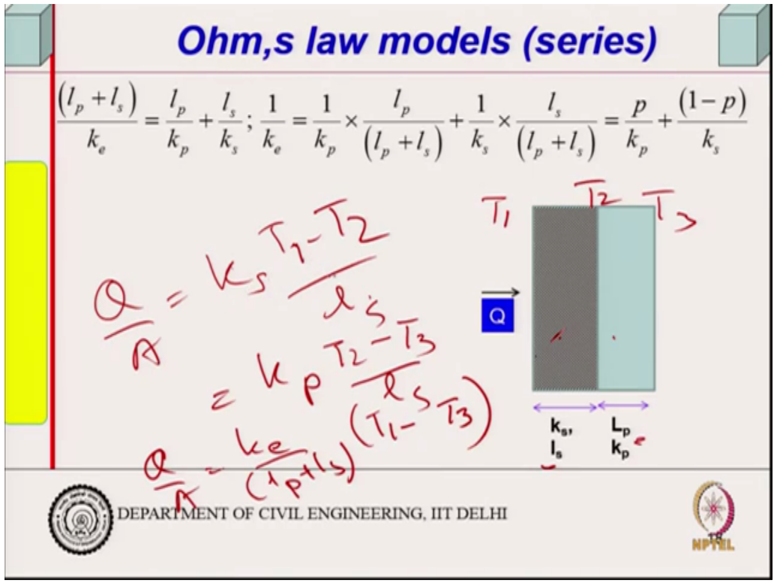


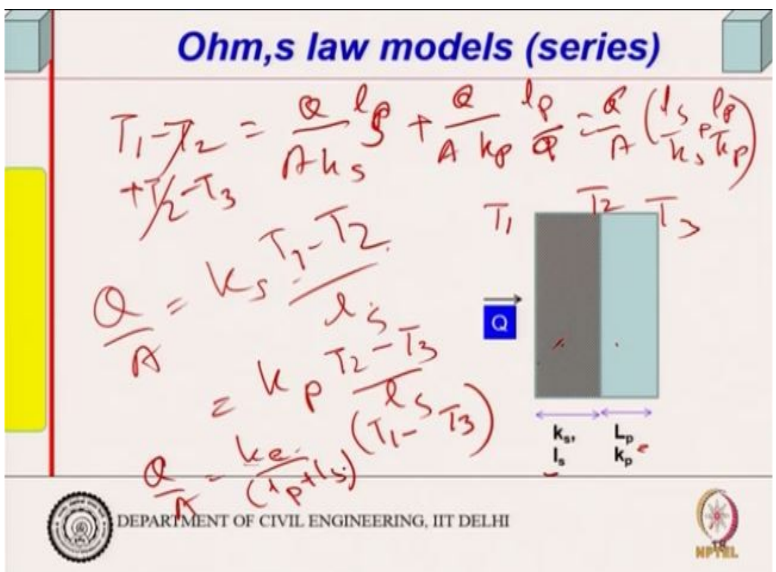


So people tried to do what some modeling, they assume the Ohm’s law model. That means this is the length of the pore, conductivity of the pore and heat is flowing in this direction. This is a solid series model. So keep flowing from this direction. So this is your solid, for example all pores are arranged in this form such that I have solid then pore, maybe then there is solid and pore. So these are in series.

So if this is in series, then simply it is something similar to you, what we did. Total length will be Ls plus Lp, right? And Ls plus Lp divided by K equivalent, how is K defined? K is heat flux Q divided by A. K T1 minus T2 divided by L. So that is what we said. So L by k, right? Same heat is flowing, same formula. So if this is T, this is T1 let us say this is T2, this is T3. So T1 minus T2 divided by L. So K solid divided by L solid and area is same; must be equal to K of the pore, T2 minus T3 divided by Ls and equivalent conductivity will be defined as Q into A into K equivalent divided by Lp plus Ls into T1 minus T3, right? T1 to T3.

So if I again do the same thing what I did in the last class, T1 minus T2 would be given as T1 minus T2 would be given as Q by AKs into L of the pore or L of the solid plus Q by AK pore. This is plus T2 minus T3, let us say, and this will cancel out T1 minus T3. Then this into L pore plus, you know, Q into L pore divided by A, I have already done that.

So this can be written as Q by A into Ls by Ks plus Lp by Kp. Q1 minus T3 if I do a little bit of algebra, that is what I will get. And doing this, therefore, what will be equivalent conductivity? Equivalent conductivity will be, you know, Q by A is equal to given by this formula. So this is the same flow that is taking place. So one can write it like this. So equivalent conductivity, one can derive like this one and that is what is done here.

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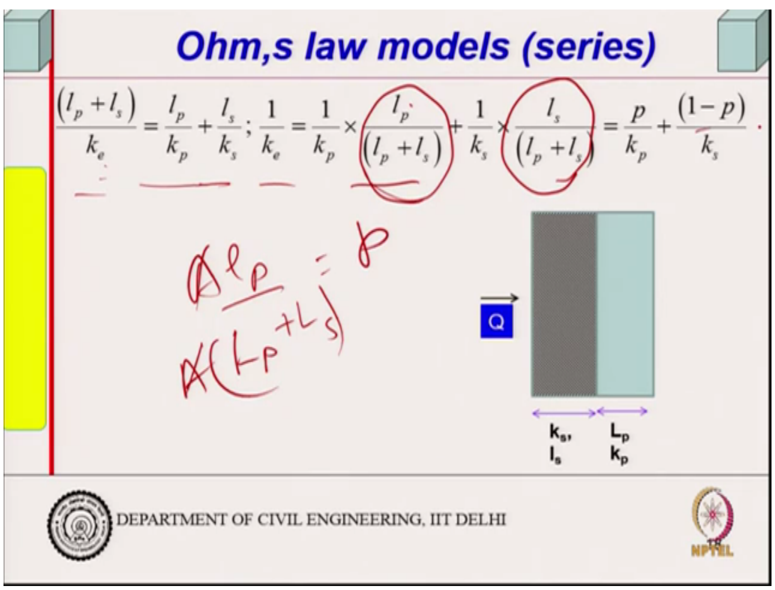

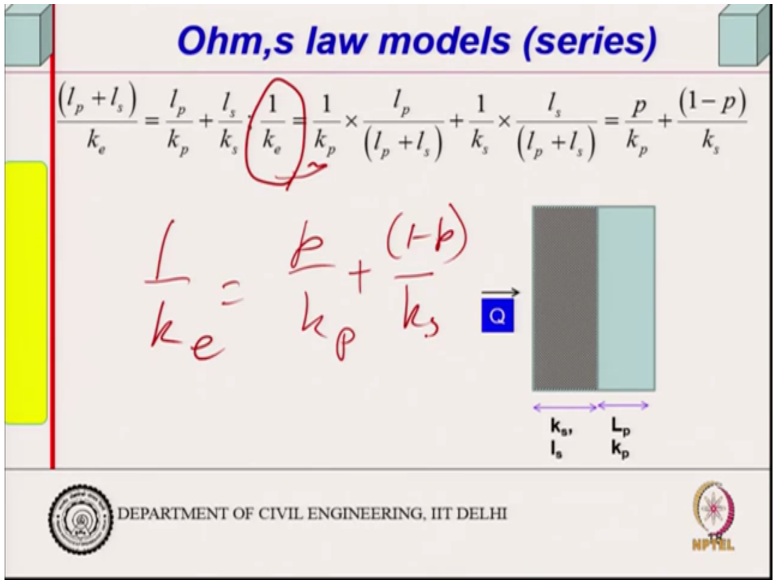


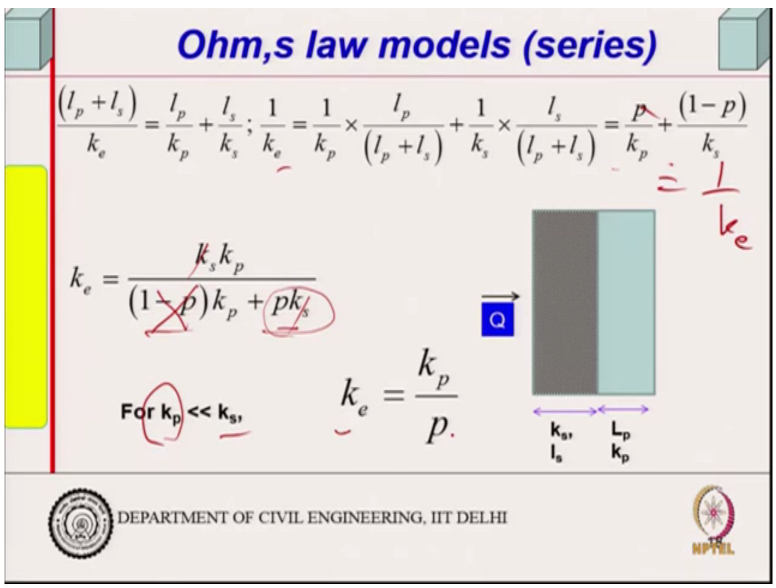


So this you can, you know, rest all the things, Q by A will get cancel from everywhere. So T1 minus T3, so this will be equivalent conductivity can be written in this manner, in terms of Lp plus Lp by Kp plus Ls by Ks, etc. So equivalent conductivity can be written in this manner. Now what is this Lp by Lp plus Ls? This is the proportion if the area is same then this can be written as, this can be simply written as, you know, this is basically Lp by Lp plus Ls is nothing but porosity if the area is same. Because this in A, this into A, right? So this is the volumetric, you know, volume of pores divided by total.

So this can therefore one can write this as 1 by Kp this is porosity. This is the volume of the porosity. And this is the volume of the solid which is 1 minus p. So when you have series model, you can simply write, you know, if you simply take 1 by Ke, this part, 1 by Ke is equal to P by Kp plus 1 minus P by Ks if it is series model.

If all pores are just solid then poured then again solid, they are in layered form. But in the real system, they are not in the layered form. People try to do that in the beginning because Ohm's law concepts are available. This is one way to look at it. So Ke can be written in this manner. If I write it from this one, Ke can be simply written because Ke, this is equal to 1 over Ke so Ke can be written as Kp, Ks. Ks multiplied by P and Kp multiplied by 1 minus p. So Ke can be written in this manner.

Now look at this, if the Kp is very small compared to Ks, which is the case for Ar then this time will be negligible compared to this term. So this term will be negligible compared to this term and simply Ke will be given as and Ks will cancel out so this is going away and Ks will cancel out with this I will be left with Kp by P.

So for series model, Ke is equal to Kp by P. So whatever is the conductivity of the pores divide by P. Right? Now this gives you a very relatively small value because you can see that Kp is 0.025 as I was saying, divided by let us say porosity of let us say 20 percent. So this is the value you will get. So this is one. People tried to look at the other model because this is one way of modeling. The other could be parallel model.

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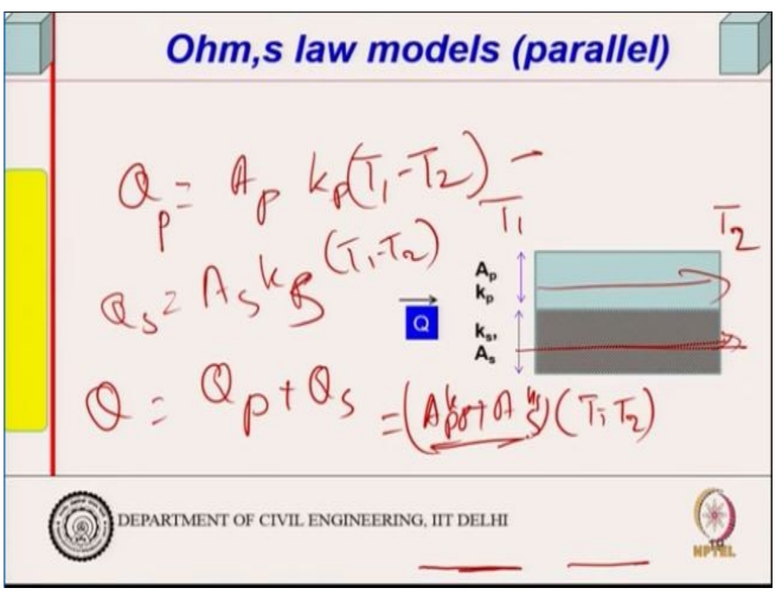

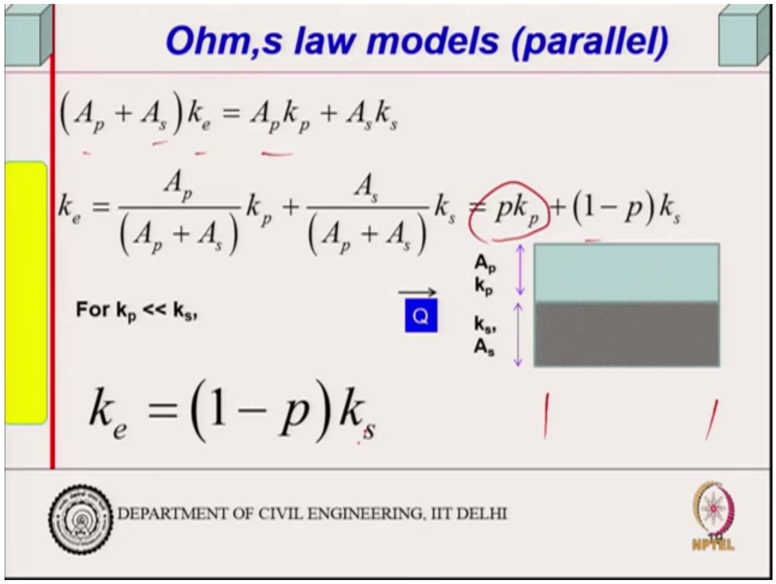


Parallel model is now I have got area. This is the area, this is the area of the solid and this is the area of the pores. So Q should be divided into two parts. Some heat will pass through this and some heat will pass through this. So the total Q will be given by, okay let me just write it. Q would be given by Ap into Kp, right?

Ap into Kp into temperature difference. Temperature difference is constant here; T1 minus T2. T1 minus T2, that is Q1 or Qp let me call it. And Qs will be As, Kp, Ks, T1 minus T2. Total heat Q is the sum total of all this. So Qp plus Qs which will be equal to Ap plus As. I can add this up, right? Or Ap, Kp, As Ks into this is not common, T1 minus T2 is common. So I can just do this algebra and therefore, the equivalent Ka will be given by this equivalent Ke into A. Equivalent Ke into A is this value.

So that is what I am doing; summing up the heat flow through each one of them and then I get Ap plus As into Ke must be equal to this. In other words then, Ke I can write divide this both by Ap by and since the length is same, this represents nothing but the porosity. This represents 1 minus porosity. So the law is Ke is equal to PKp plus 1 minus p into Ks. And if the Kp is very small, this will be neglected. So this would be (())(23:20). So you get two cases. This value will be definitely very high and the previous value was small.

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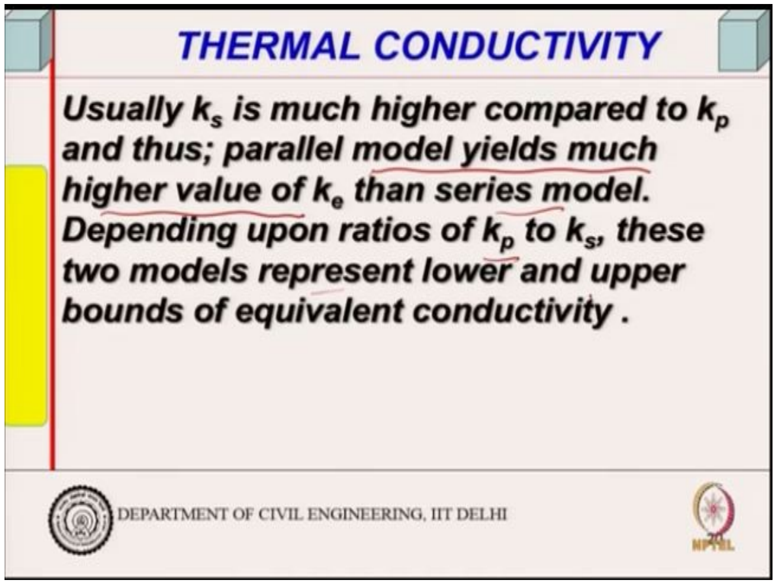


So in fact what it gives is range. So Kp is very very small therefore, parallel model yields much higher value of Ke and series model much smaller value depending upon ratio of Kp by Ks. These two models represent lower and upper bound of conductivity because in material, there will be neither porous, I mean neither parallel nor series but they are upper bound and lower bound. So one can use combinations. That is what people try to do sometimes; you will find it in some of the books.

You will find combinations for example, they would like to put some factor, so combinations for Ke would be something like some factor, let us say, lambda into let us say Kp into, like the model that was there, Kp Ks were two models. So we can just take it from here. This one we can take.

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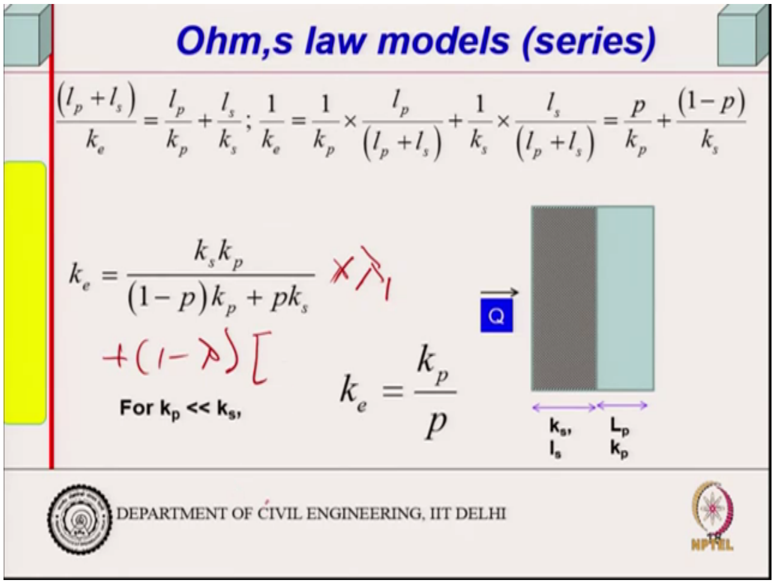


So K equivalent was given by this; this multiplied by lambda, this multipled by lambda and rest one is 1 minus lambda multiplied by this will be a K equivalent when you have a combination. Then it will be something like this, right? Or let me write it separately in the next one after this. So one can write it like this.

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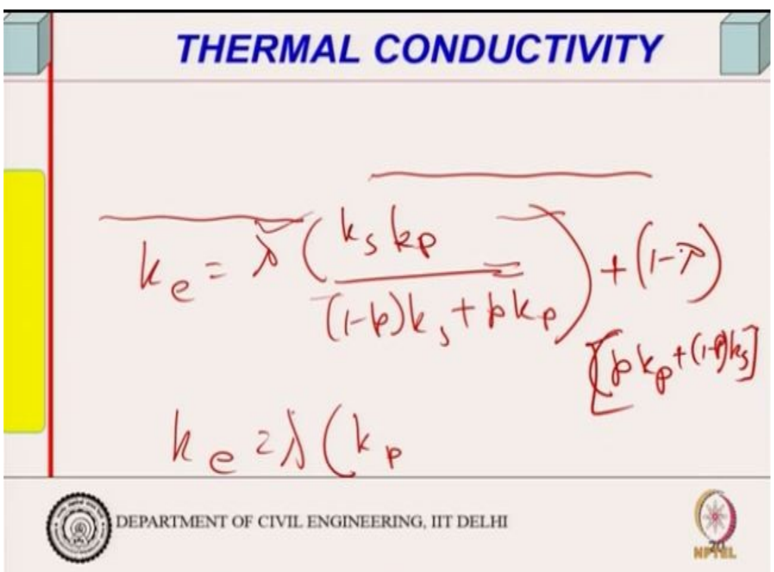


Ke will be equal to lambda, Ks, Kp divided by 1 minus P into Ks plus P into Kp that was the thing. Plus 1 minus lambda into P into Kp plus 1 minus P into Ks I think I should write like this.

So this kind has also been attempted. But then they do not work. You will again have to find out what the proportion of series and rest all is parallel and things like that. So it becomes difficult.

So there is another kind of thought process that went into thermal conductivity because if you can look into the details it becomes more and more important. Details of this behavior is becoming important. So that is why, approximate medium approximation.

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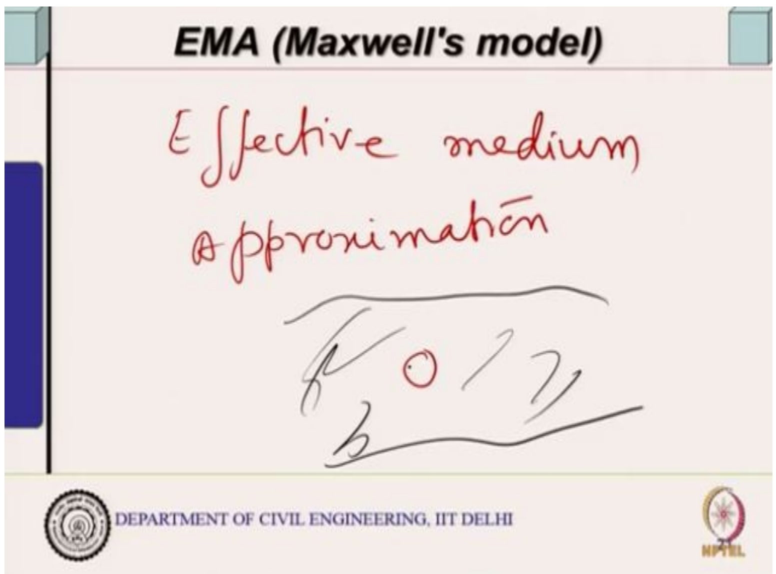


This is called EMA which stands for Effective Medium Approximation EMA Model. That actually came from if I have a spherical material, an inclusion in spherical inclusion in otherwise bulk material, the disturbance of the magnetic field electromagnetic field by this intrusion was derived by Maxwell. So based on this, effective mediumapproximation. Therefore, the net flow or its analogic to the electrical field.

There is a temperature field in the bulk. If I have another material, this temperature field will get disturbed. Therefore, the heat flow through this field will be disturbed due to presence of another intrusion, right? So this effective medium approximation concept takes care of that. So the disturbance of the temperature flow field, flow path because it has to flow the heat has to flow, it will be disturbed through this. And based on this EMA model are (())(27:35).

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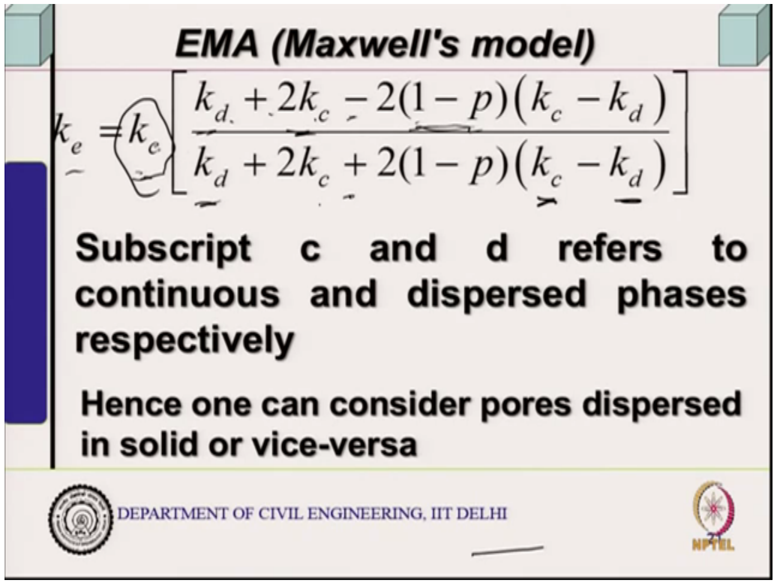


So this gives you K effective is equal to there are two components here, one is dispersed phase and a continuous phase. The bulk one is the continuous phase. And dispersed phase is the other phase material. So the equation, one of the equations this is what is used in thermal conductivity. Kc, if you assume, Kc is the continuous phase. This is the conductivity of the dispersed phase and this is how 1 minus P, P is the proportion of the dispersed phase which is similar to the porosity. So that is how it is given this is Kd plus 2Kc, 2 1 minus P and there is something wrong because this should become 1, something is wrong.

Subscript c refers to continuous and dispersed phases respectively because there is sign differences. So here is the minus and this is plus. So this is how it is actually. Subscript c and d refers to continuous and dispersed phase. Hence, one can consider pores as dispersed phase in solid and that is how the equation will be. This is plus and this is minus, you can see that this is some proportion of the Kc.

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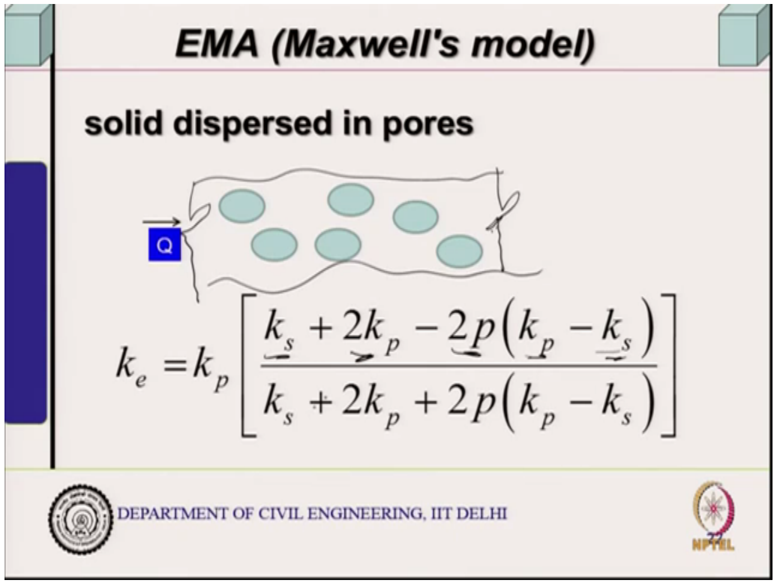


Solid dispersed in pores. Now another case could be solid dispersed in pores. That means higher porosity scenario the pores are interconnected and you have some portions. So if this solid is dispersed in pores, the equation will become K solid 2 Kp because continuous, you know, the equations were something like this. This continuous, 2K continuous, K dispersed so if the solid is dispersed in pores, then it will be something like this K solid and that's the pores. And 2porosity will be very much there plus Kp minus Ks. This is for the dispersed phase and this was the continuous phase. So if I neglect this, then I will get a slightly simple one and this is related to p. So I will get a simpler one.

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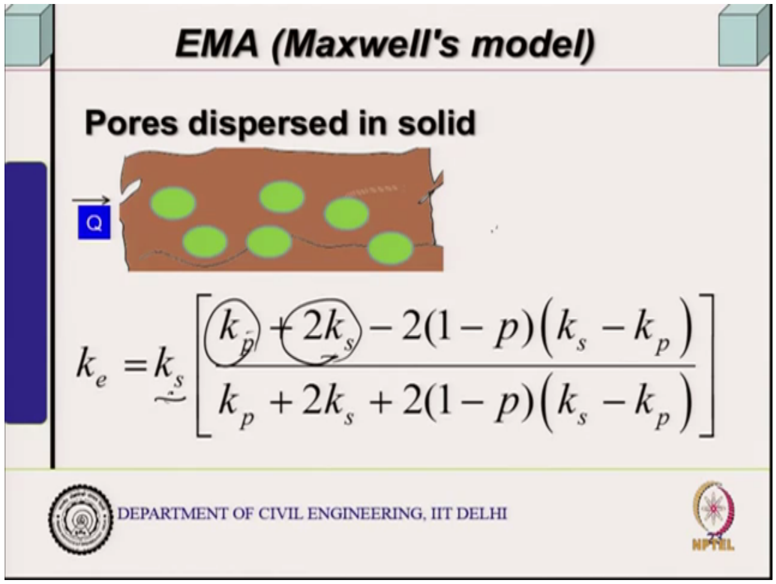


And if I look similarly, pores dispersed in solid. So this brown one is my solid and there are some pores. And then, Kp, 2Ks, so this time this is the dispersed phase so therefore Kp is here. This is the solid phase, this is the continuous phase and this is what it is. So you can see that this will again give you higher value because a fraction of Ks. And that will give you again you know solid dispersed in pores, highly porous material, it will be a fraction of the conductivity of the pores, right?

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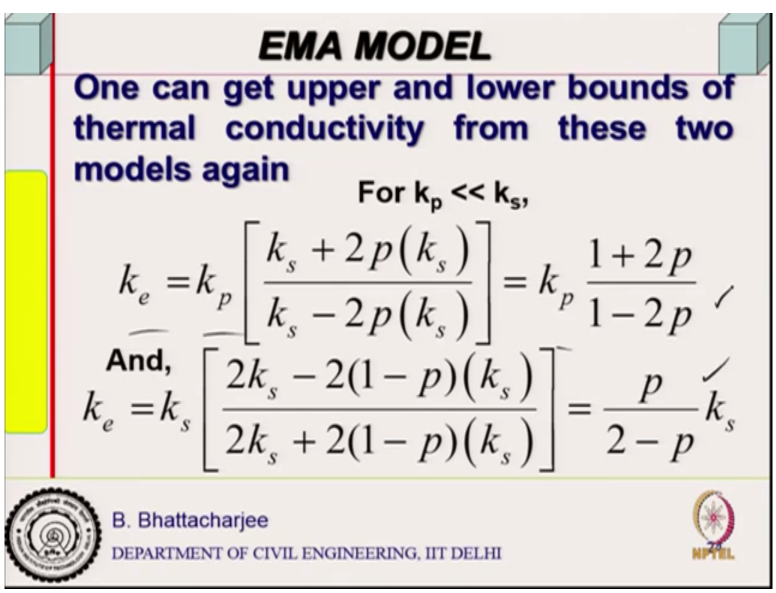

So one can get the upper and lower bounds of thermal conductivity with these two models again. So Ke is equal to Kp that is 1 minus2, rest all you neglect the, in this equation you can neglect these phases. Kp can be neglected with comparison to Ks. So this becomes plus, you know etc. So you can neglect this Kp, even neglect this Kp, you can get Ks plus 2P into Ks divided by Ks plus 2P, right?

So it can be simply Kp in extreme cases. So you can see that. Again I can get two ranges and this is what I will get P divided by 2P minus Ks. So by neglecting the Kp with respect to Ks I will get something of this kind. So these two gives again upper bound and lower bound. So none of them give, there are better models which try to, I mean this bounds are looked into more detail.

Especially similar bounds are available for elastic modulus because equations are similar. Suppose I have a material, series in parallel, you know two materials I have to find out the equivalent, modulus of elasticity. So models like Hashin and Strikespense model there are many models which can give upper and lower bounds of similar material. So I think we will look in to this. But then we can have 3D models as well. There are several types of concepts, I will just give you one.