Sustainable Materials and Green Buildings
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Lecture - 22
Operational Energy: Thermal conductivity models contd.

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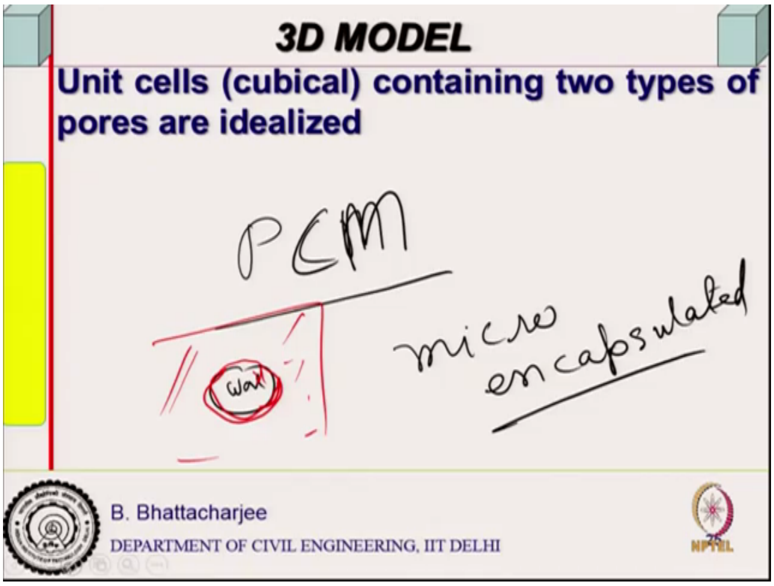

So, we now look into slightly different kind of modeling 3D models this is also attempted by some people right, now you can again consider a unit cell extension of this in 3 dimensional scenario, even there has been a extension of maximum models also and you know like 3 components the example is where it will be useful is say supposing the some materials which I talk about may be at the end sometime phase change material, phase change material phase change material which I talk of you know phase change material.

PCM or PCM whatever you call it phase change material what happens is they would melt at high temperature absorb heat so if you put them in the wall they will absorb heat, and from solid state they will go to molten liquid state and as the temperature outside or other side you know outside reduces, they will dissipate the heat and again solidify.

Supposing, I have some phase change material whose melting temperature is around 40 degree centigrade and it is the part of the wall then when heat comes in this will absorb heat and will not allowed it to come inside and when inside temperature falls down outside temperature falls down no heat will come inside it will try to dissipate it and solidify.

So, phase change material can store the heat for a longer period of time, now problem is such materials such as you can understand wax based materials wax melts right, similar sort of materials, so but you cannot carry if you put it is liquid it is run away through the pores.

So, they are micro encapsulated they are capsulated micro you know they are very small size (())(2:21), then what will you have? You have wax or something phase change material and you have to have a layer you have to layer thick layer outside, of the encapsulation which is polymeric, and then you have bulk concrete brick or whatever it is, you know concrete particular not concrete brick you can make it you know so brick concrete brick is anyway depending up on so block or whatever it is.

So, in such situation is now because 3 phase materials 3 phase materials, one is a bulk concrete or water then there is a polymeric layer thin however it is and then the this materials so conduction through this so, then there are 3 dimensional expansion 3 phase material multi-phase system.

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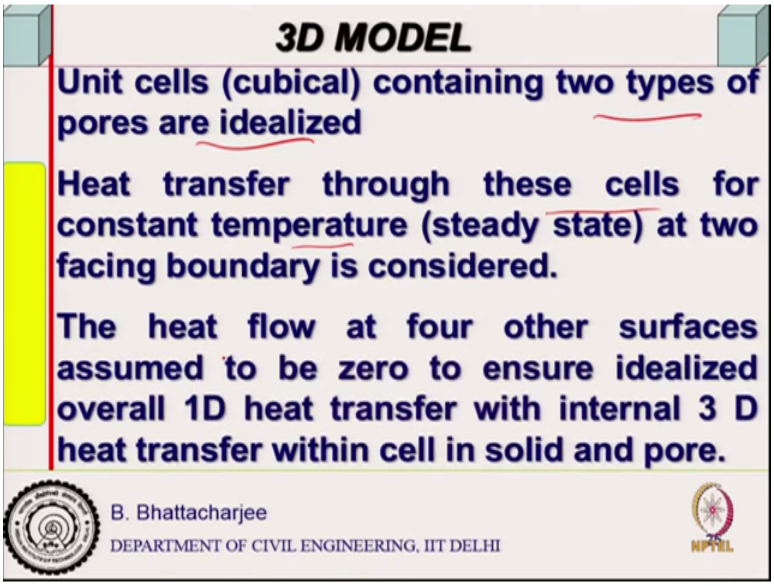

But this is again I am looking right now at the 2 phase system only 2 phase system it is possible to extend even this kind of concept to 3 phase system, so one can consider 2 types of pores so these cells for constant temperature of problem with the Maxwell's model was I did not discuss this did not mentioned this was, I initially it was taken only spherical pores, then it was extended to (())(3:48) actually.

So, you can not have any kind of regular shape so anything of that kind and problem with this model that I have been talking about I am dealing with cubes, so you need cell cubic you can consider and you can look at two types of spores and heat transfer through this cell for constant temperature at two facing boundary you can consider right.

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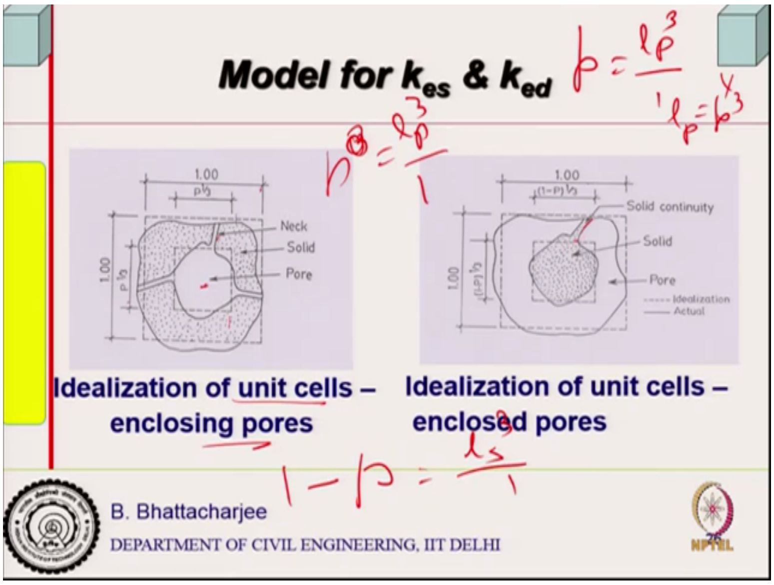

So, I will come to this possibly this is what it is, for example consider a unit cube and you have pore, and then there is a solid in that unit cube and this repeats itself in the material there are several cells all such cells, so every pore you can consider that is surrounded by sudden amount of solid and this proportional would depend up on porosity of course.

So, now it is 3D of the same kind and then you can consider another kind of pore where as solid inside and pores or here on all the three sides and this will be repeating in the this part of it will be repeating, so these two repeats itself these two repeats itself mean the material and they are randomly arrange and then you can look into their conductivity slide.

So, this are this is called solid enclosing the pores so enclosing pores, you can call this as enclosing pores and this is you know this is so this are should be unit cells of enclosed pores actually this enclosed pores actually the pores is enclosed here and this is actually enclosing pores right they are two types.

So, here the solid the material was been connected through so there are narrow necks, so that pores are if it is closed pores system you may not have necks, but in some cases materials are never there be some connected pores, there will be some unconnecting pores.

So, the connectivity depends of course and this is solid connectivity has to be there solid cannot be all the air cannot be isolated solid would not be there, so solids might also be connected where large quantity of pores, so one can idealize and this is unit cube.

So, 1 and therefore, this is this 1 so therefore, the porosity is known, this volume supposing I know the porosity and this actual pores should be of this size idealize into cubical shapes, because handling cubical shapes is easier, but currently one can handle anything but this is what is original done.

So, if you take the cube this one if you take this this particular one right, the volume of this one so how much will be this length if the porosity is p this should be p cube this should be you know the porosity is porosity is volume of pores this unity so p is nothing but this length cube l cube this is lp cube divided by 1, p will be lp cube divided by 1 so lp you know p is equals to l length of the pore cube divided by 1 because 1 is the overall volume right unit cube.

So, p lp would be equals to p to the power 1 third p to the power 1 third right, so one can estimate this from the porosity and p to the power 1 third on this side in this will be either around the solid will be 1 minus p to the power 1 third, because you know this one porosity is porosity is or rather 1 minus p is equal to length of the solid cube by 1, so length of the solid will be 1 minus p to the power 1 third, so that is how we can idealize this.

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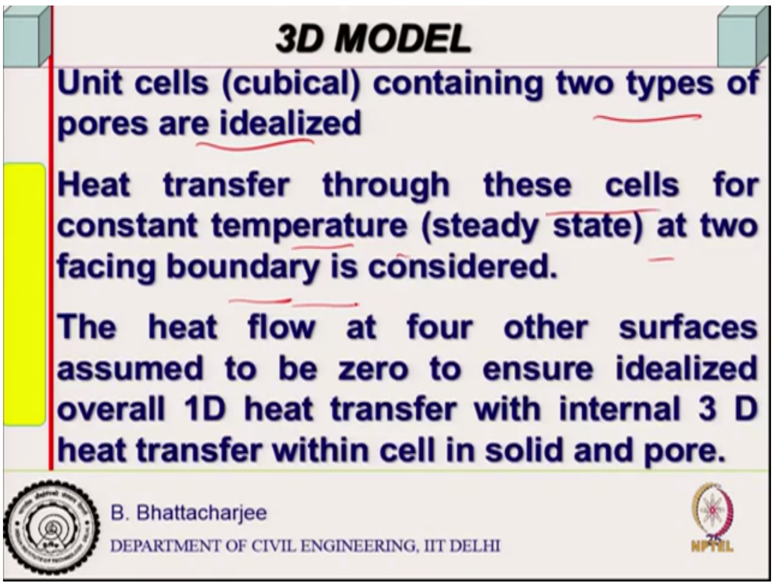

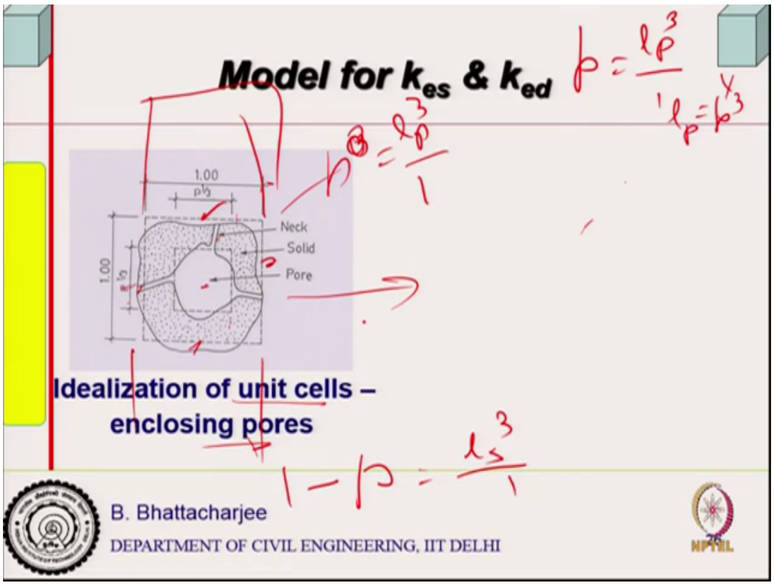

This is idealize in this manner, so now if you want to find the equivalent conductivity so what you assume, constant temperature at two phasing boundary, so you consider constant temperature here and here right at this boundary and this boundaries you assume since it is one dimensional heat transfer you assume that it is all insulated it is all insulated after all they will be in materials where this will be repeating this will be repeating and this will be repeating in the other direction also, so they are same and you assume that there is no heat transfer because they will have similar temperature so no heat transfer on the on those directions.

Only of those this direction, direction of the heat low in a wall or something like that one dimensional heat transfer is because conductivity is depend with respect to one dimensional heat transfer and this is same similar everywhere so you assume that are insulated, one dimensional steady state at two facing boundary you consider.

The heat flow at four other surfaces assumed to be zero to ensure idealized overall 1D heat transfer with internal 3D heat transfer within the cell and the pore, now inside this cell there will be 3D heat transfer by in from the inside he cell but nothing will go flow heat goes outside.

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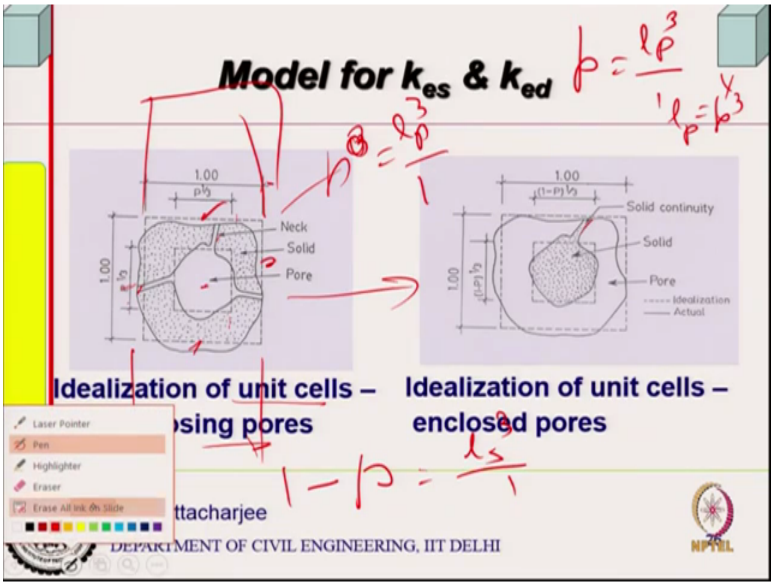

So, that is that is how it is idealized that is how it is idealized so this is an extension in a way three dimensional extension of the series parallel model right and one can solve this one can solve this heat transfer problem.

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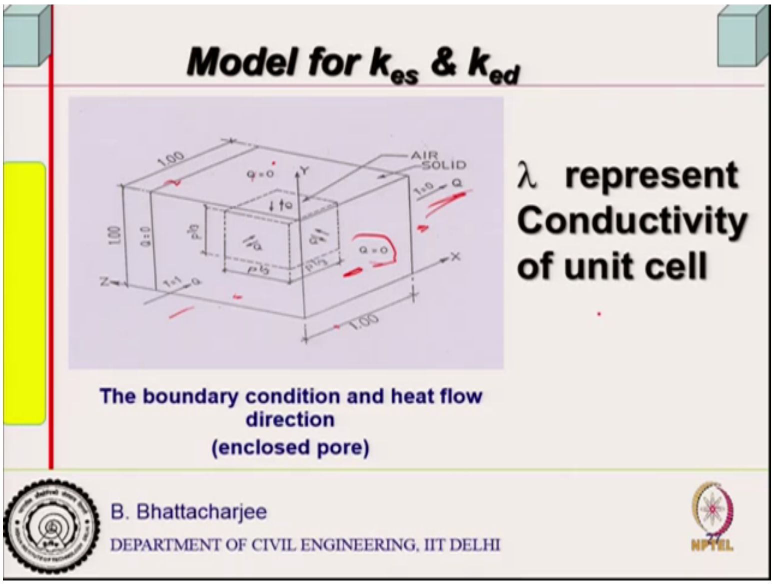

How does one solve? Simply like this right. So this is your unit cube heat is flowing you know heat is flowing in some of them will be 0, so this side is 0, this side is 1, this side no heat flow Q is equal to 0, this side no heat flow Q is equal to 0 except in this direction Q is going out and Q going out Q along this direction, this surface surface beyond this is 0 heat flow, bottom 0 heat flow, top 0 heat flow, four surface is heat flow is 0 and only in one direction Q is flowing coming in the temperature difference is all idealize is to 1 to 0 and if you find out this Q value that is the for this unit area that is a equivalent conductivity of the system.

So, this can be done these dimensions are given and this can be done you know this can be done by doing simple thing that is by using a non-dimensional form of the heat transfer equation and solve it, we are not solving it here.

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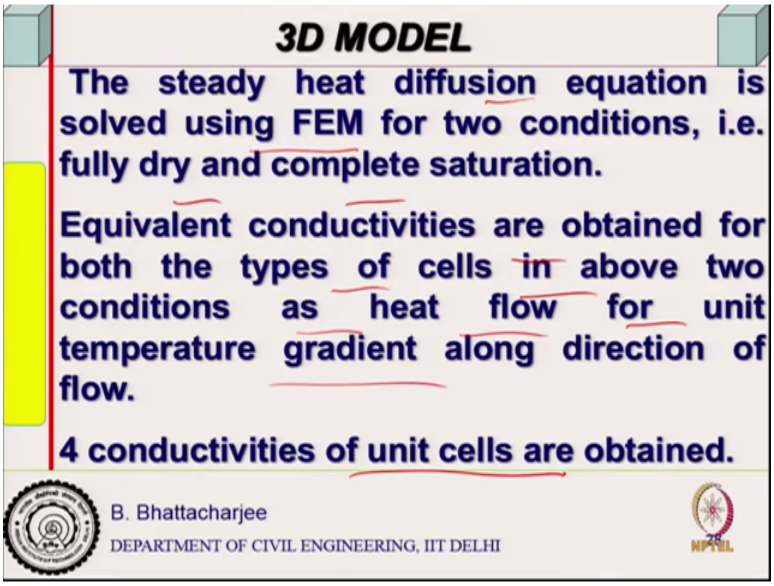

But, the steady heat diffusion equation is solved with finite element for two conditions fully dry and complete saturation, so there is water all inside. Equivalent conductivities then you can has been obtain for both type of cells in the above two conditions as heat flow for unit temperature gradient along direction of flow.

4 conductivities of unit cells then you will get, 4 cases of conductivities moisture, one is saturated, one for the condition when you know to 4 conditions one say one condition is pores inside solid outside, second condition is pores outside solid inside for dry, both this for dry, and for saturated condition you will get two similar situation, so you will get equivalent 4 equivalent conductivities of this unit cell.

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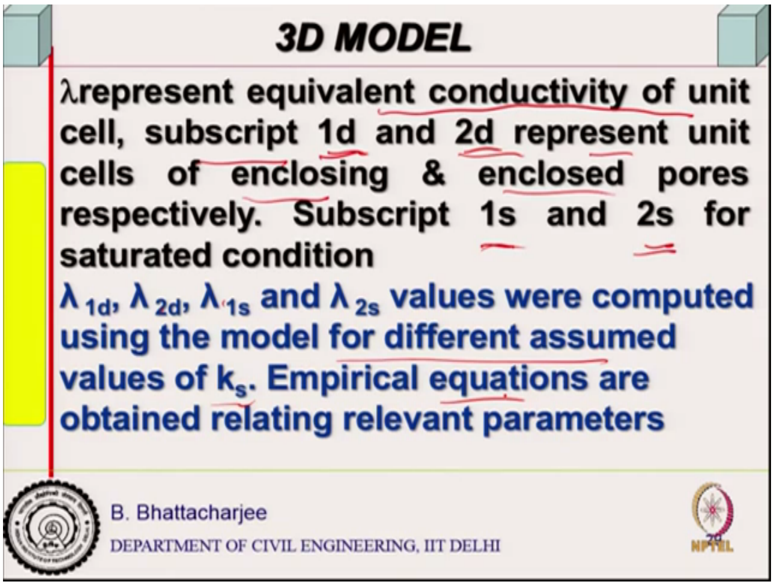

So, lambda represent equivalent conductivity of unit cell, because this is non-dimensional conductivity so that is why we knows this (())(11:30), subscript 1d, 2d represents unit cells of enclosing and enclosed pores and use 1s, 2s for saturated condition, so the 4 condition 1d, 2d, 1s, 2s and lambda

So, lambda 1d, lambda 2d, lambda 1s, lambda 2s values are computed using such finite elements solutions for different assume values of ks, because it will vary of the ks value ARS got constant conductivity but solids have got different conductivities, so therefore one can from this model solution one can generate what it is called empirical equation relating relevant parameters.

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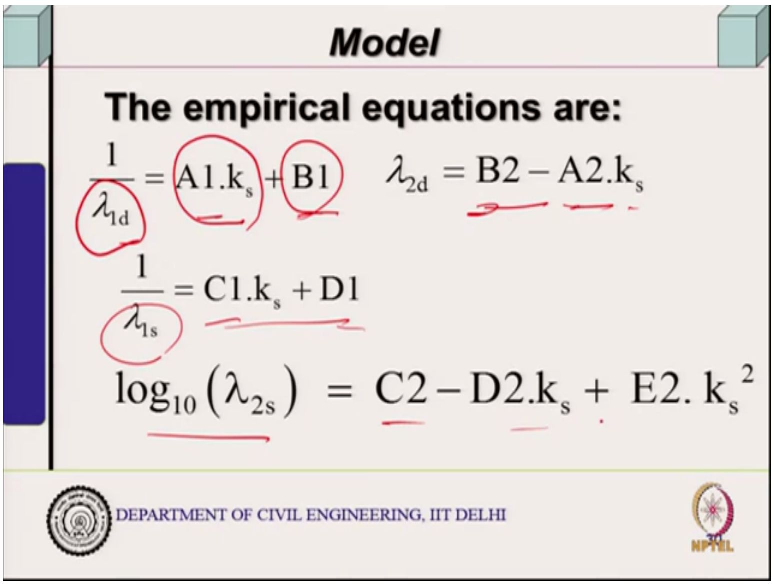

So, these empirical equations are something of this kind that is lambda 1d the dry condition is some constant into ks plus B1, lambda 2d is some constant A2 into ks and saturated condition you know something like this and 2s, now this is now used by lot of people first (())(12:39) where you need very huge time for computation this is not, this was not huge time for computation but time to get out get some simple equations because everybody cannot go and do final element solution all the time also this is a very simple finite element.

But, nowadays were there are very large calculations time required is very very high they develop what is called surrogate models the surrogate models are nothing, you do a numerical modeling of a some extent obviously choosing appropriate levels of values of etc etc.

And then fit an empirical equation which can be used by everybody, so complex problems are even handle today but when this was done such terminology (())(13:28) this is a kind of anyway one can say this something like a models surrogate models sort of things, which was derived from actually from computations results you obtain this.

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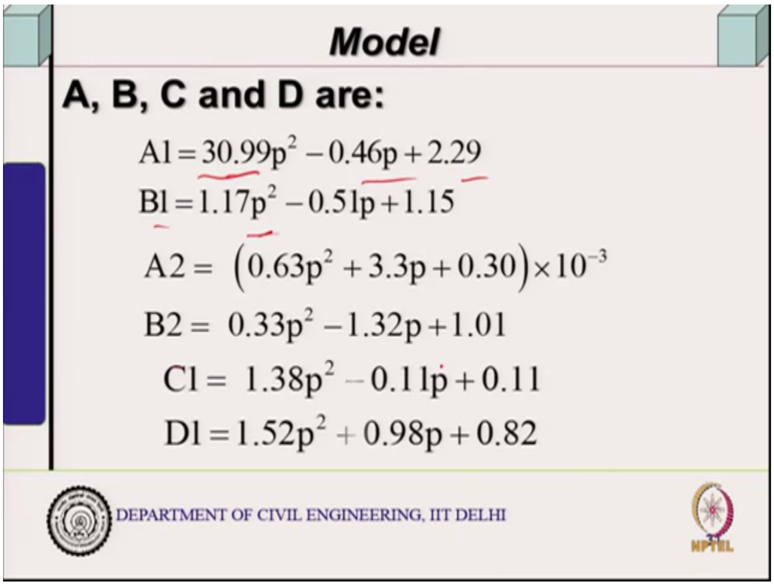

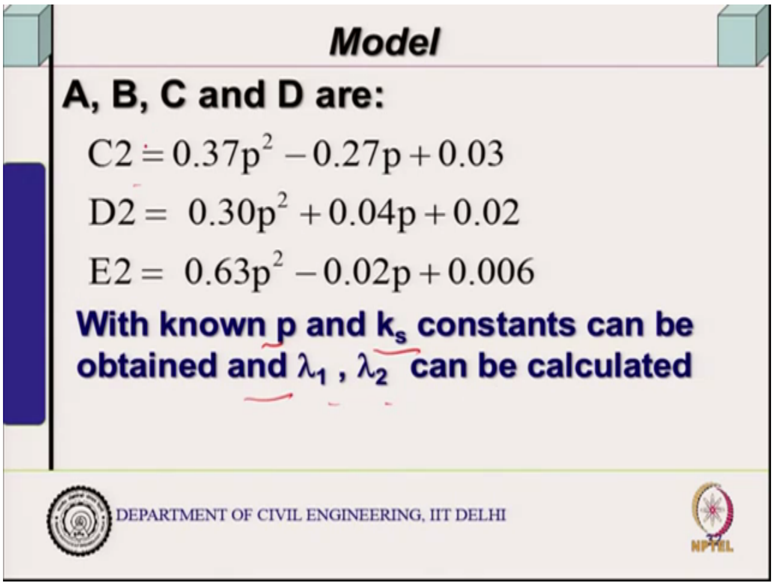

So, let us see how the A, B, C, D are functions of porosity is this A1, B1, B2, A2, C1, D1 these are functions of porosities and they are something like this so know some empirical results somebody does wider jobs this values might change but for the case of bricks and concrete and such materials not (())(14:01) this still may give a good results.

So, if you know porosity A1, B1 which is functional porosity A2, B2, C1, D1 and E1 these are all you know functions of porosity, so C2, D2, E2 these are functional porosity is given, with known p and ks solid conductivity constant can be obtained for lambda 1, lambda 2 etc, so these values can be obtain and you can find out lambda 1, lambda 2, you can find out lambda 1 and the etc then how do you come to the equivalent conductivity I will come to that gradually we will go to that and take an example calculations also.

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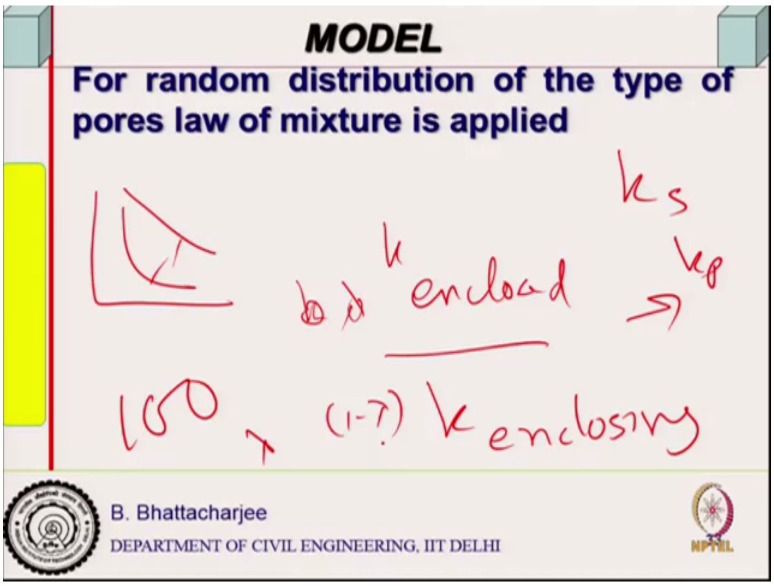

So, for the random distribution of these two types of pores, so these pores are the enclosed type and enclosing pores I do not know I can know the proportional from experiment here the advantage here, we will see that but these two types of pores there be random you know proportional of this and the conductivities are varying in order of not order of linear order.

If the conductivity of the, you know lambda 1 let us say or k enclosed let say enclosed to k enclosing the pores the solid inside here the ratio is very large order of 100 like upper bound and lower bound I was talking about this order of this one I was talking about these orders are vary 100 or even much higher depending upon depending upon the value of ks to kp.

If the ratios of ks by kp is very large this values are also very very large, so therefore, you need not combine them in linear manner linear proportional of lambda which we do it, we said lambda into 1 minus lambda rather you might take in geometric proportion you know for random distribution you can take geometric proportion, right.

So, it is the better way to take the geometric or geometric mean, so equation combining these two cells are combined to get overall effective thermal conductivity of the material, in both dry and saturated states and let us say how it is.

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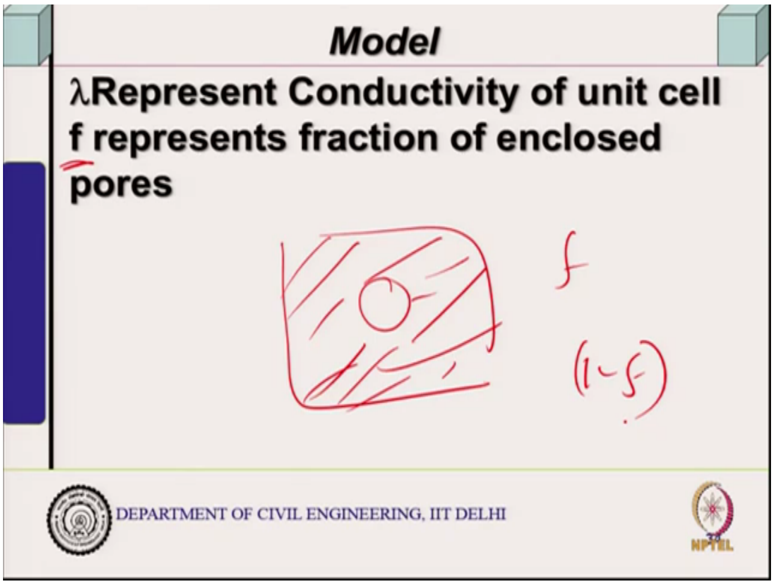

So, unit cell let say f is the fraction of enclosed pores where pore is inside, solid is outside, if this is the f then 1 minus f will be the proportion of enclosed you know other type of other type of enclosing pores right pores outside.

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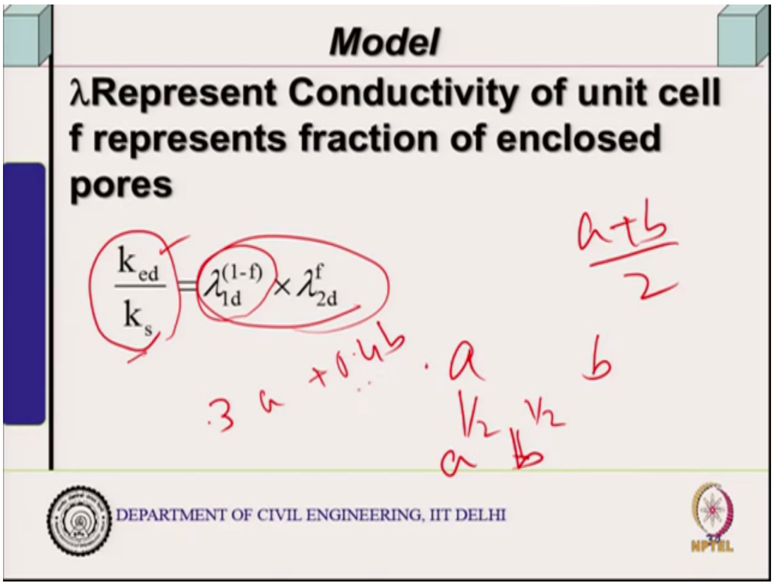

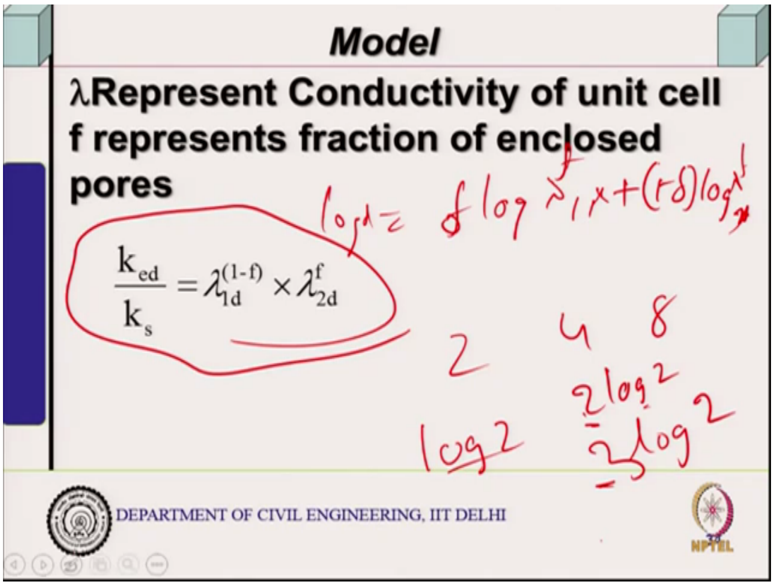

So, I can combines this and this is combine in this manner equivalent dry divided by ks solid, you know in to this is geometric mean, geometric mean is geometric means is you know to the power, geometric mean how do you take supposing I want to take geometric mean of a and b it is a to the power of half b to the power half, arithmetic means will be a plus b divided by 2, supposing it is you know weightage factor is not half it is 0.3 a, 0.3 a plus 0.4 b, so weighted average how do I find out you know like simply point weightage factor will be 0.3 and 0.4 but in case of geometric means this will go to the power because in geometric means when something is varying in geometric progression say 2, 4, 8, etc (())(17:57) rise.

You take log, so log 2, next one will be 2 log 2, then this will be 3 log 2, so the log varies in arithmetic progression, log varies in arithmetic progression, so if you take you know so therefore, the coefficient is coefficient is 2, 3 this varies in arithmetic progression, so f into log of you know if you combine this, this will log of lambda 1d plus 1 minus f into log of lambda 2d is equals to log you known d or whatever it is then if you if you can take this to the power and this also to the power and since you know log of these two were ended then the product will come so this is what would be derived the geometric average I am taking weightage average geometric mean I am taking.

Because the values very very high and I am assuming a random distribution so according to some proportion, so f is that fraction.

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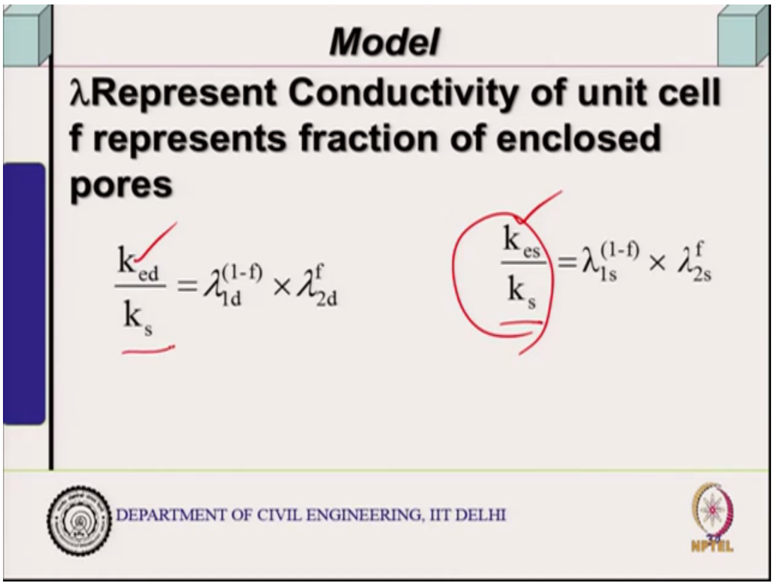

Similarly, for saturated condition I can do like this saturation condition I can do like this, now this can be experimentally determined fully saturated and dry (())(19:17) conductivities can be experimentally determine, ks may not be not known because you cannot make materials 0 pore, no material real material will not be of 0 pore size either you take concrete brick, etc etc.

And this can be possible determine with experiment, but generally you know) one can treat them as a unknown as a first place right treat them as a unknown as a first place, so there are two experimental results available and two unknown are there because same material have saturated it got the experiment done dry it and do the experiment you know, so there are two unknowns and they can be actually estimated from experimental results experimental results of conductivity of the solids dry and solid and once that is done for future peoples they do not have to do that they have to use the models only.

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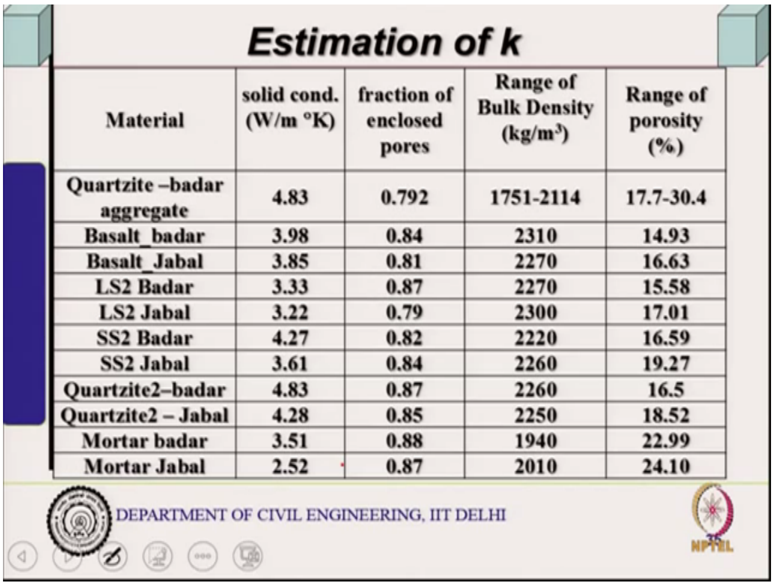

Estimation of k

Material	solid cond. (W/m °K)	fraction of enclosed pores	Range of Bulk Density (kg/m³)	Range of porosity (%)
Quartzite –badar aggregate	4.83	0.792	1751-2114	17.7-30.4
Basalt_badar	3.98	0.84	2310	14.93
Basalt_Jabal	3.85	0.81	2270	16.63
LS2 Badar	3.33	0.87	2270	15.58
LS2 Jabal	3.22	0.79	2300	17.01
SS2 Badar	4.27	0.82	2220	16.59
SS2 Jabal	3.61	0.84	2260	19.27
Quartzite2–badar	4.83	0.87	2260	16.5
Quartzite2 – Jabal	4.28	0.85	2250	18.52
Mortar badar	3.51	0.88	1940	22.99
Mortar Jabal	2.52	0.87	2010	24.10

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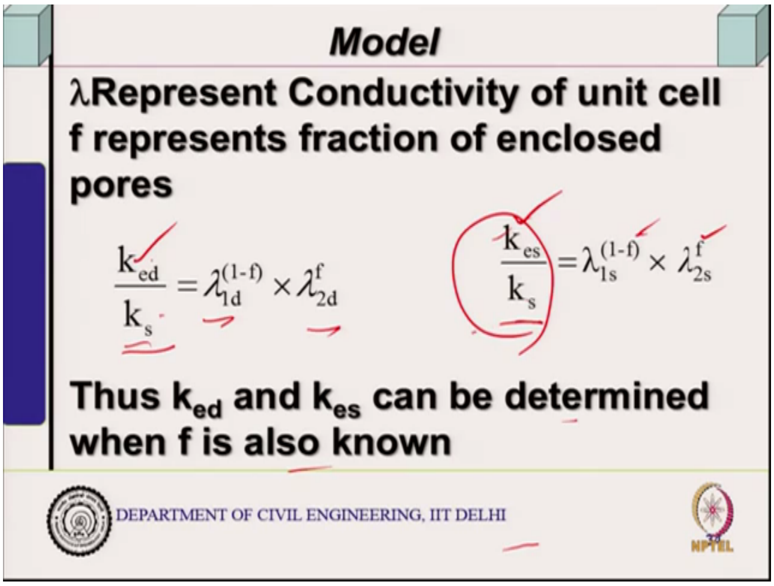

So, to arrive at the model can be determine and if f is known, so that is what it is so for example if look at this solid conductivities one could estimate them, one could estimate them from measurements, supposing I have these two equations available ked, ks, dry conditions, saturated conditions this available and this pore which I determine earlier than two unknown I can actually obtain ks and this I can obtain from this two equation together doing with the little bit of trial and error.

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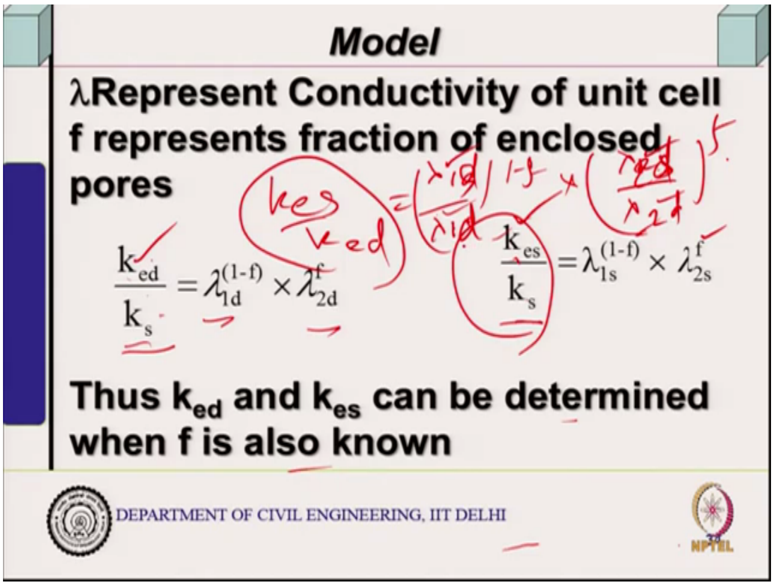

Because this is in can Trans candle you cannot straight way so you can obtain this you can find out the ratio of these two, so kes by ked I can find out if I want to solve for f this will be what? Lambda 1d to the power lambda 1d by lambda 1s to the power 1 minus f multiplied by lambda 2s divided by sorry lambda 2d divided by lambda you know I am just I am just finding out the ratio of ks so lambda, this will be s right right this would be 1k, this will be 1s divided by 1d to the power and this is 2s divided by 2d to the power f.

And this is known to be experimentally take log of both sides this has been obtained from the model, this few have been obtained from the model so I can find out the f, because it is now an equation of mean f, let me just redo this let me just redo this.

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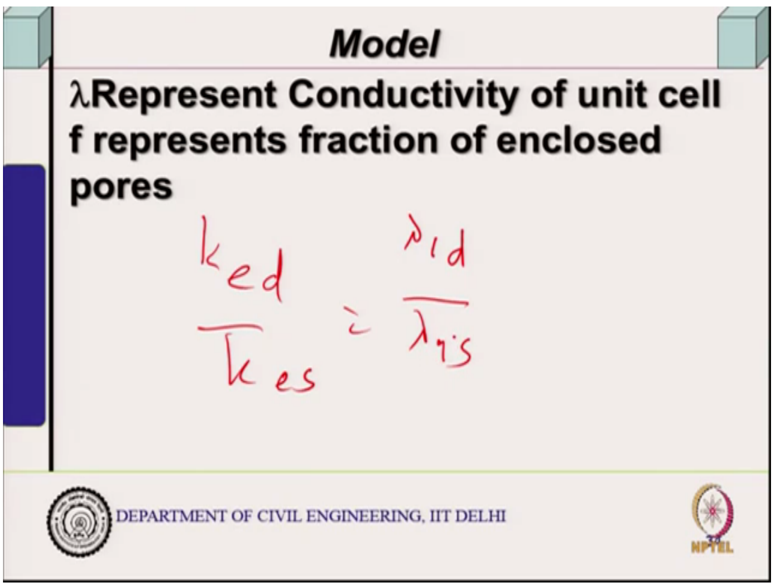

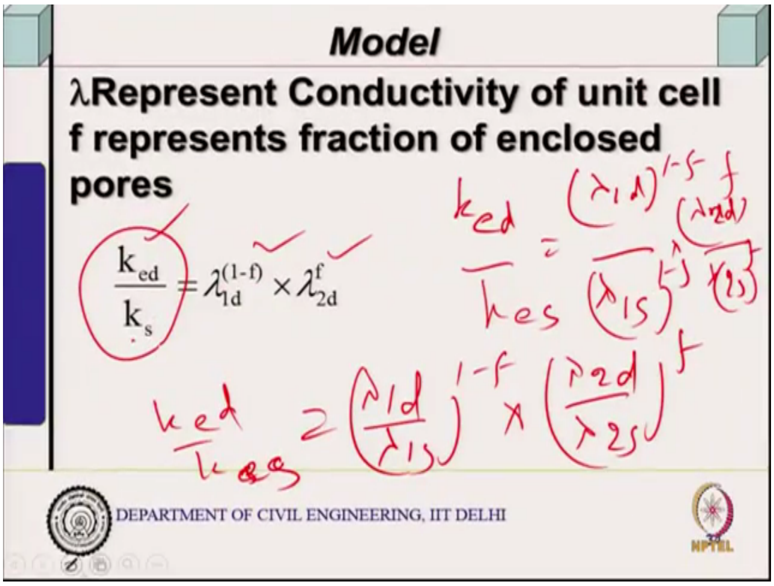

What I am saying is ked by kes experimentally or kes by ked experimentally, experimentally I can determine and this will be lambda 1d by lambda 1s to the power or 1d by 2d you know the formula was if I look at this formula if the look at the formula itself, 1d to the power 1 by f ked, ked is just let me write the full thing, ked by kes I am trying to write so this will be lambda 1 d to the power 1 by f 1 minus f, lambda 2d to the power f and this will be lambda 1s to the power 1 minus f multiplied by lambda 2s to the power f.

So, this is experimentally known this is experimentally known and this right hand side I can write is lambda 1d by lambda 1s to the power 1 minus f multiplied by lambda 2d by lambda 2s to the power f, so if I take log of ked by kes is known to be experimentally take log of both sides take log of both sides this is known to be from models is known to be from models, you know for various values ks I have found it out, so ks is unknown to be a moment but does not matter for same case I will find out the and I can really determine f and once I have determine the f calculate this value back for given value of experimentally this known to me.

So, assume ks so I would have assumed ks find out the value of f use that value of f, check for the value of ks if it matches fine otherwise this kind of trial and error I can find out both ks and kf for various cases and various cases and this has been done for various materials.

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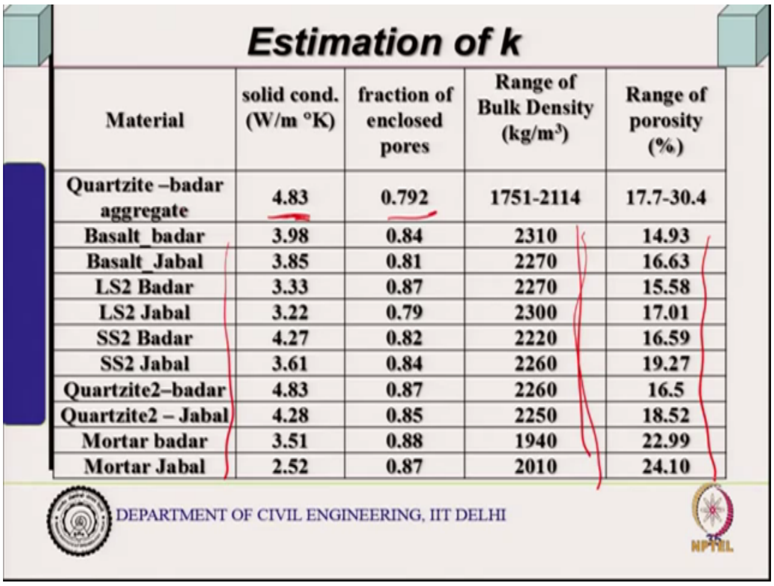

Estimation of k

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SS2 Jabal	3.61	0.84	2260	19.27
Quartzite2–badar	4.83	0.87	2260	16.5
Quartzite2 – Jabal	4.28	0.85	2250	18.52
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And if you do this say for quartz sent, quartzite aggregate both you find this fraction of enclosed pores where it idealizes and solid conductivity is 4.83, some sort of k you know these values can be obtain density ranges something of this kind difference sense different sense and from difference sources and ranges of porosity one can measure also by water permeable porosity simply.

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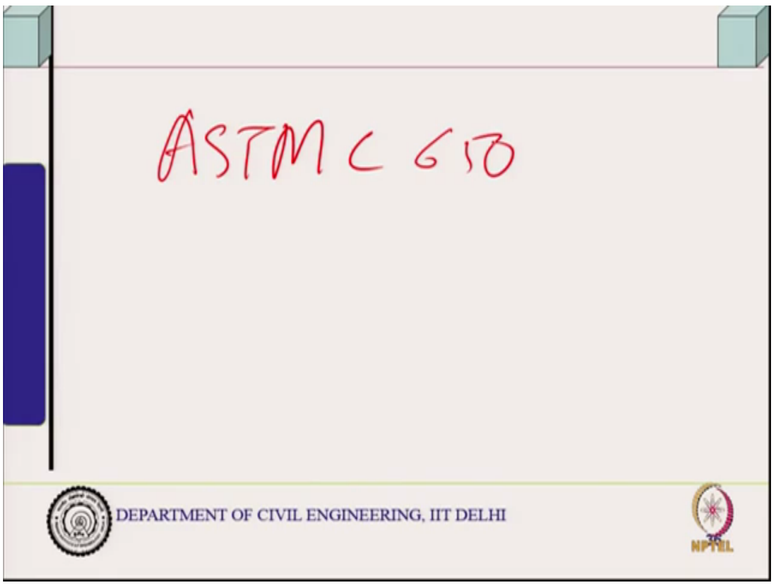

Porosity can be measure by I just tell you how do you measure I think I might have told somewhere ASTM procedure C 650 I think, water permeable porosity you can find out take your make your block put it in water for 48 hours right, see the mass change and put it for another 24 hours and see the mass change if the mass changes in two cases are not very different, then you assume that is your saturation state.

Then, boil it for 6 hours all force will be predict so completely saturation will occur because even the fine pores where water was not able to go so you can find out, so then measures its volume by suspended you know apparent to 18 emerge in water, so water displace you can find out so you can actually find out is volume since the volume is known, water volume is known specific gravity of water we assume knowing the temperature is not varying too much and then you can calculate the volume of pores.

So, permeable porosity can easily be find out, impermeable porosity would be difficult to find out for that you have to grind the grind the solid into powder, find out the specific gravity. The volume is known, if we find out the grind the solid into powder find specific gravity, total volume is known like mass is known mass divided by the specific gravity will give you the volume of the solid alone, overall volume is the volume of the pores can be found. So that is how you go about it, find out the pores and you can then using this porosity value, because p was needed in the model ks has been estimated from the p values because in the model A1, A2 etc are function of p.

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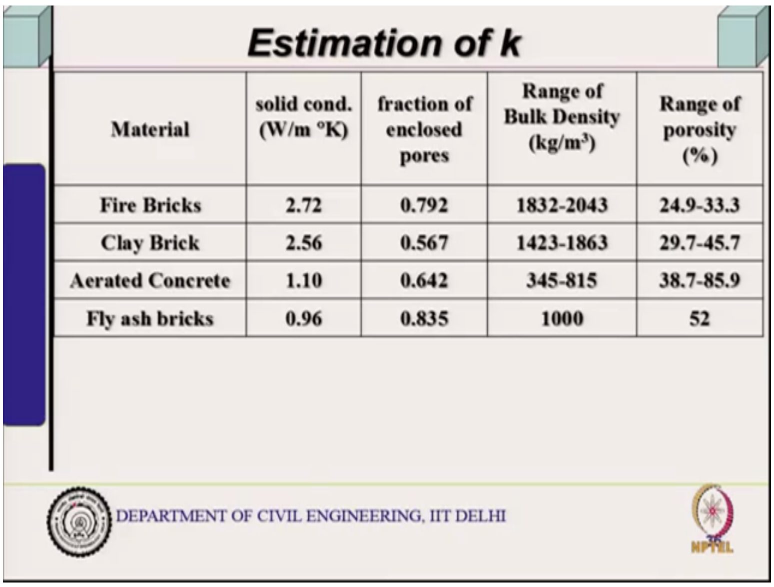

Material	solid cond. (W/m °K)	fraction of enclosed pores	Range of Bulk Density (kg/m³)	Range of porosity (%)
Fire Bricks	2.72	0.792	1832-2043	24.9-33.3
Clay Brick	2.56	0.567	1423-1863	29.7-45.7
Aerated Concrete	1.10	0.642	345-815	38.7-85.9
Fly ash bricks	0.96	0.835	1000	52

I will just take one example and show you, so solid conductivity of fire bricks, clay brick, aerated concrete, fly ash bricks and the fraction of enclosed pores, range of density, range of porosity this can be determined or you know they can be determined or determine, so you can use this model to find out let us say conductivity of clay bricks with porosity 30 percent.

So, if you know the permeable porosity for different types of for example clay bricks we know the solid conductivity is 2.56, fire bricks is materially different because they have larger element of amount of alumina, they are taken from deep below the ground so they have larger amount of alumina and therefore, their conductivity of solid is different.

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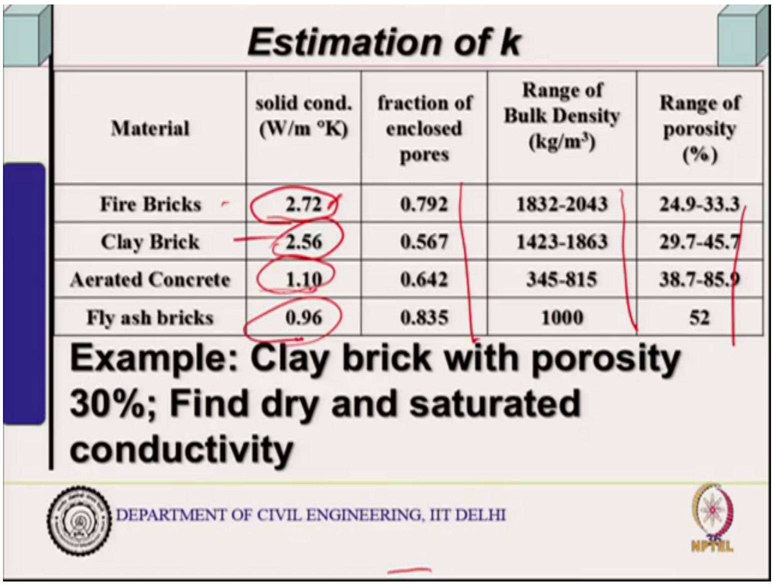

Aerated concrete has got generally they will have lot of other material added common sand and such material and affective conductive of solid turns out to be this, because the pores must be also include it in this. Fly ash bricks conductivity seems to be different because fly ash themselves is about low specific gravity, so this is but you can determine this let me see if I will come to this example later on but I think yes.

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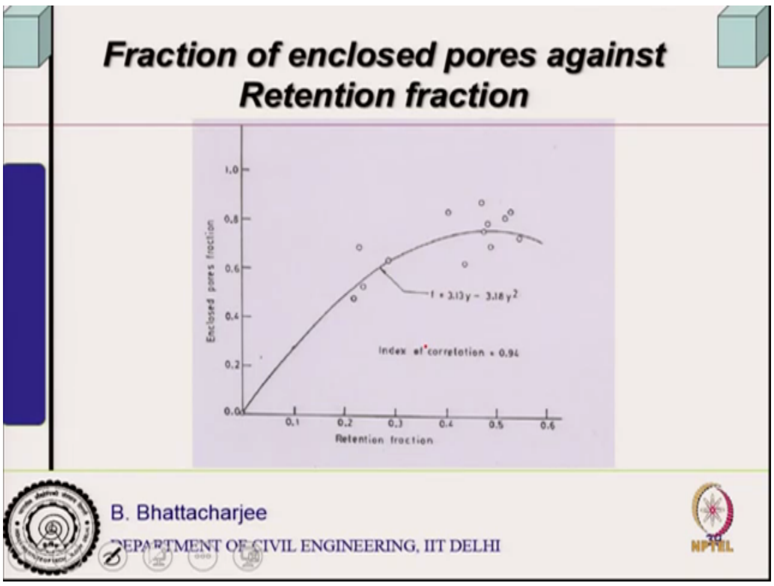

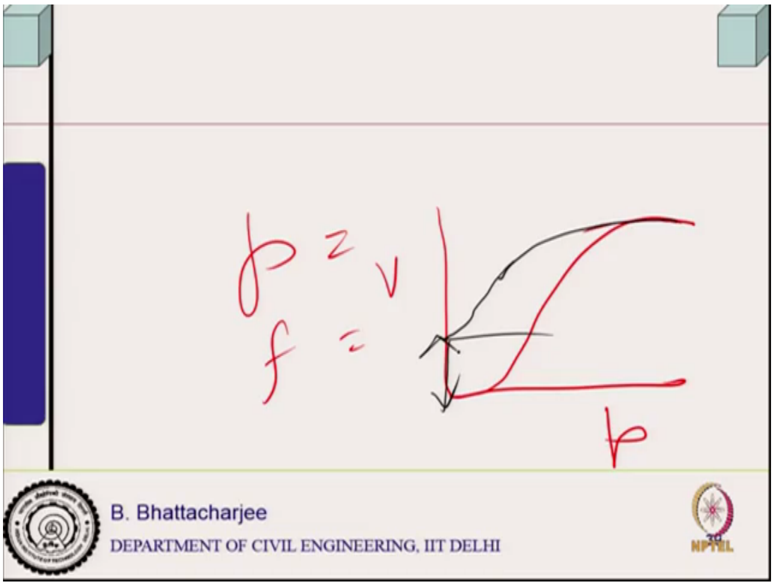

This, enclosed pores you can also determine from mercury into porosimetry, so this you know like f is determinable, p can be determine, porosity can be determine, ks you cannot determine, but f can also be experimentally determine by mercury intrusion porosimetry where what happens is you inject mercury into specimen under pressure.

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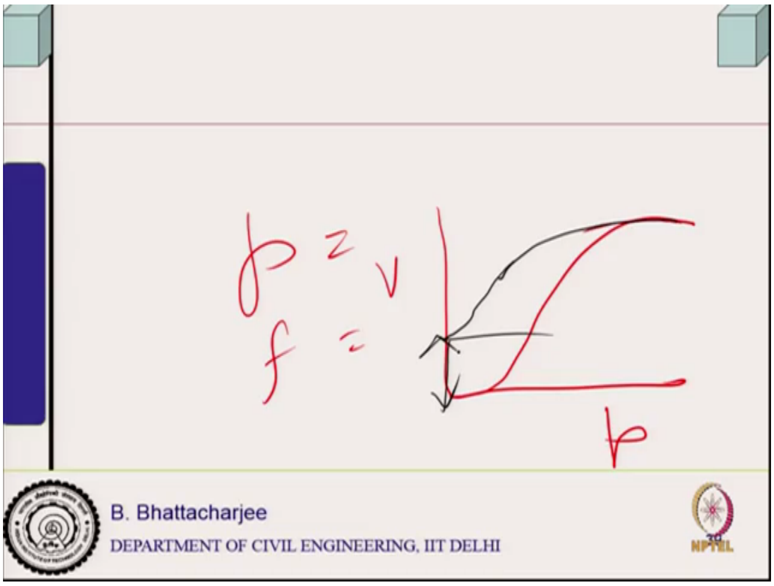

So, it follows a path like this it might follow a path like, this is pressure as your pressure increases volume of intrusion increases and when you release it when you release it, I will discuss in a next class again, when you release it follows, it does not follow the same path it follows a path like this, it follows a path like this, it follows a path like this.

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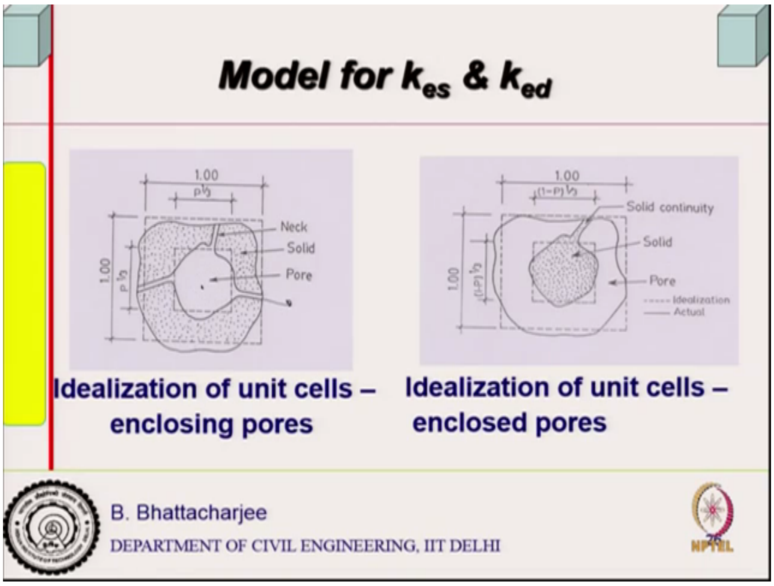

Some of the mercury is entrapped, this entrapped mercury is a measure of the pores which are enclose by solid that is why they could not come out pores of this kind, this is the measure of you know this kind of pores, pores which could not were pores of this kind, pores of this kind where mercury (())(29:15), but the total mercury could not come out it came out only when the at certain point through this but lot of lot of mercury, etc.

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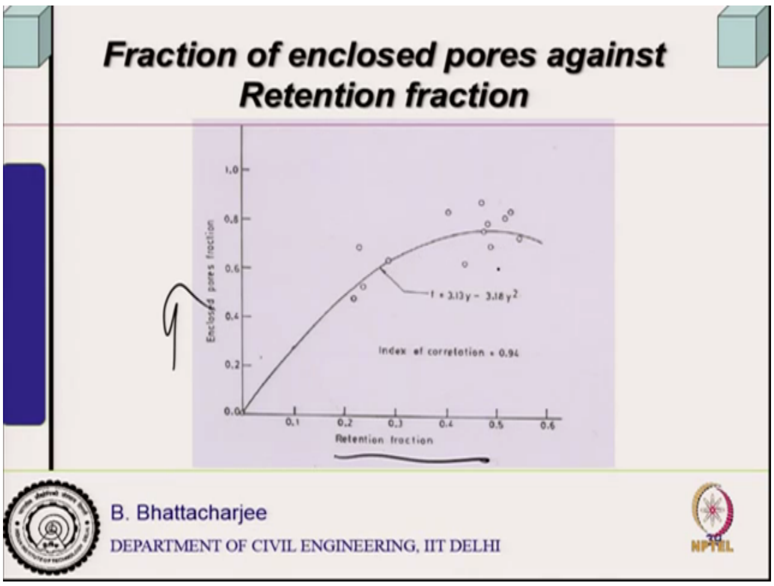

So, therefore that is fractional (())(29:25) so f you can so fraction of this type of pores you can find out from mercury intrusion porosimetry and this shows the result of experimental results of mercury intrusion porosimetry results you know this this this shows the kind of relationship of this is the this is called retention fraction in mercury intrusion porosimetry and this are enclosed pores, so they vary some or some sort of relationship exist you know one can get a rough estimate of this. Ok, we will stop here, next class we look into this from solving examples and continue with this work.