Engineering Hydrology
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Module: 2
Lecture – 25
Evaporation Aerodynamic Method

Hello all, welcome back. In the previous few lectures, we were discussing about the estimation of evaporation. Initially, we have discussed the experimental method of estimation of evaporation, that is measurement of evaporation by using experimental techniques, after that we have started with the analytical methods which are used for the estimation of evaporation. In that first method, we have already seen in the previous lecture, that is the energy balance method. In that what we have done? We have made use of the continuity or the mass balance equation along with the energy conservation equation for deriving the expression for rate of evaporation.

So, in that particular equation both the energy conservation and the mass balance together considered. The main factor which we have considered in the energy balance method was the heat energy, that is heat energy provided by the sun is absorbed by the water in the water body. That was the predominant factor, which we have considered while deriving the energy balance equation. Today we are going to see aerodynamic method. In aerodynamic method what are the extra factors which we are considering other than heat energy, let us see one by one in today's lecture.

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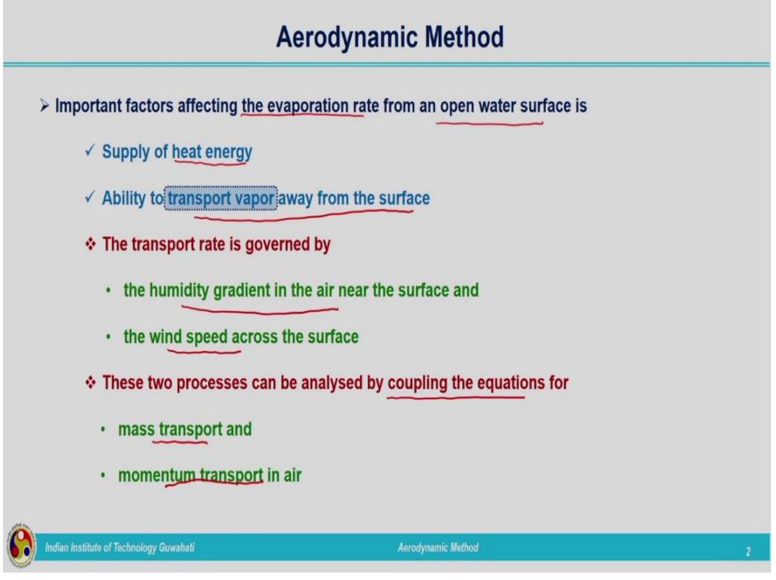

So, the important factors affecting the evaporation rate from an open water surface, which we have seen supply of heat energy from the sun and the ability to transport vapor away from the surface. Evaporation is the process by which the water is converted to water vapor. So, this water vapor will be moved from the water body to the atmosphere. So, this has to be removed continuously from that particular location, if it is there only again and again addition of water vapor will not be possible, it is possible only up to the saturation point, beyond that it is not possible.

So, continuous transport of these water vapor from the above the water body is required. So, the two important factors, which are influencing evaporation are the heat energy and the vapor transport away from the water body.

So, now here in this aerodynamic method, we will be incorporating the transport of vapor, vapor transport will be considered. In the previous method, we have been using heat energy, that is the emphasis was given to heat energy, heat radiation, but in this case, we will be considering the vapor transport.

So, the transport rate is governed by humidity gradient in the atmosphere and wind speed. That is humidity gradient means, we are having a certain vapor pressure corresponding to the given temperature and there is a maximum capacity for the air to hold water vapor, so at that point the vapor pressure will be saturation vapor pressure. So, from the existing vapor pressure to saturation vapor pressure that much can be added, that much pressure can be increased by the extra addition of the water vapor. So, there is a deficit in the humidity or the water vapor. So, the humidity gradient near the water surface is very, very important.

Second factor is that more and more water vapor is added to the atmosphere, this thing should be transported by means of wind action. So, how this transport is taking place? The wind will be carrying away this water vapor, which is added to the atmosphere. So, once the water vapor is getting transported away from the water body, more and more water vapor will be added to that particular location. So, in that case wind is playing the predominant role.

So, here in this method of aerodynamic approach, we are making use of the humidity gradient and also the wind velocity which is prevailing at that particular location. So, these two processes can be analysed by coupling the equations for mass transport and also momentum transport. That is these mass transport and momentum transport related to the water vapor, we are going to consider and these two processes such as the humidity gradient and wind velocity gradient will be considered by coupling the equations related to mass transport and momentum transport, mass and momentum transport related to the water vapor.

For this also what we are going to do? We are going to consider a control volume. So, control volume within the water body will be considering, so will be considering a control volume just above the water body.

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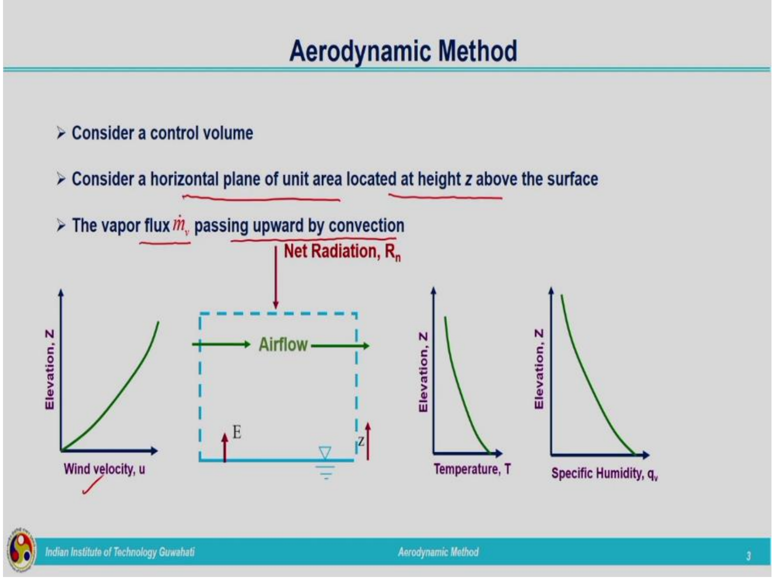

So, let this be the water surface and consider a horizontal unit area located at a height of z above the water surface. So, we will be considering the vapor movement in the upward direction, that is we are considering the z direction in the upward direction. So, this is our control volume and we are, heat energy is absorbed by the water. That net radiation is denoted by R_n and evaporation in the upward direction is taking place from the water body.

Now, our interest is to find out the vapor flux $\dot{m}_v$ passing upward by convection. The transport process, which is responsible for the vapor transport is the process termed as convection. Different transport mechanisms are there, so out of that we will be making use of the principle of convection here in the vapor transport. Once the water vapor is added to the atmosphere due to wind action, it will be removed from that location, or it will be transported from one location to another. So, the air flow direction is marked in this way.

Now, we know already as the elevation is increasing wind velocity is increasing, that relationship is shown in this particular figure, as the value of z is increasing, wind velocity is also increasing. Then coming to the temperature, as the elevation is increasing, we know the temperature is decreasing. That we have already seen.

And then coming to specific humidity, as the elevation is increasing specific humidity is also decreasing, because we know altitude is high means it is far away from the water body and the moisture, which is present in the atmosphere will be less compared to the location which is near to the water surface. So, as the altitude is increasing the specific humidity will be decreasing. So, these are the principles, which we are supposed to know while deriving this equation.

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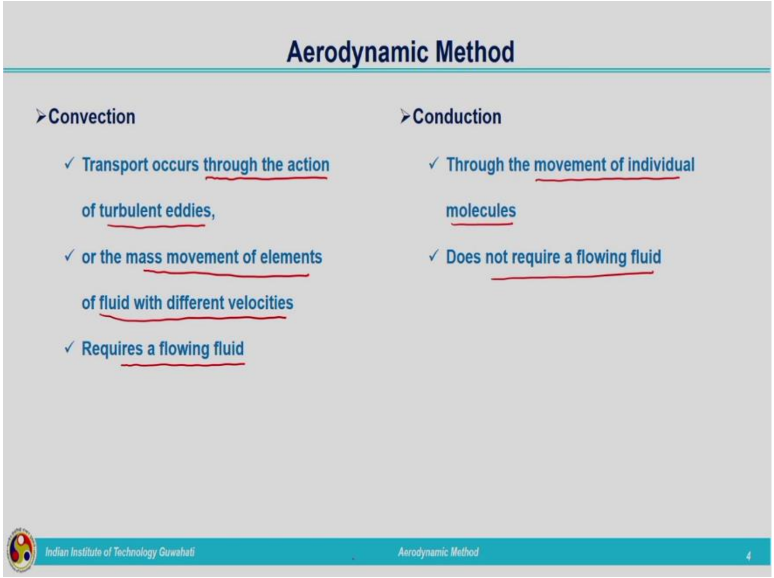

So, two different transport mechanisms I am explaining here and the mechanism which we will be utilizing here or the vapor transport is taking place is due to convection. So, two different transport processes are convection and conduction. So, in the case of convection, transport occurs through the action of turbulent eddies. So, turbulent eddies will be present the transport of water vapor will be taking place due to the action of turbulent eddies, or the mass movement of the elements of fluid with different velocities, the fluid elements mass movement will be taking place.

But in the case of conduction, it is due to the movement of individual molecules, that is conduction process is taking place through the movement of individual molecules. But in the case of convection, it is not due to movement of individual molecules, it is due to the mass movement of elements of fluid.

So, convection is possible, if we are having flowing fluid, it requires a flowing fluid. But in the case of conduction, it does not require a flowing fluid. So, the difference between convection and conduction you are supposed to know, and the transport process, which we are going to make use in this particular derivation is the convection process.

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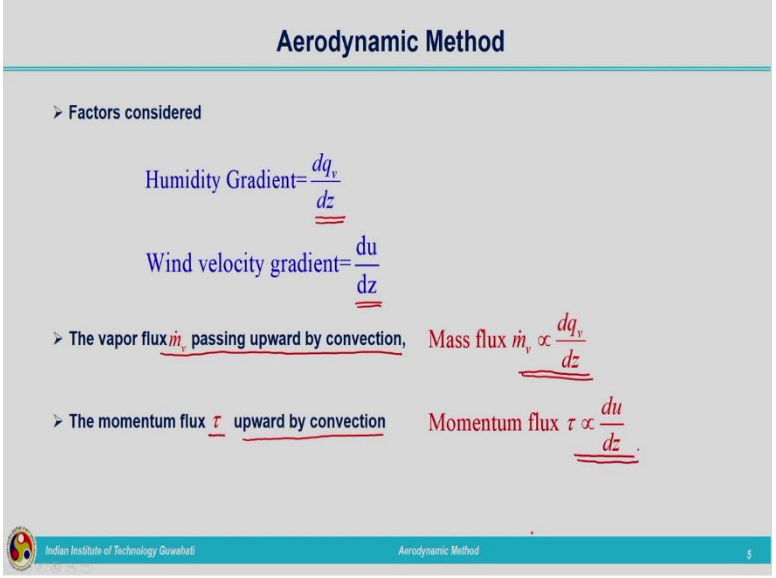

And in the aerodynamic method, we have already mentioned the factors considered are the humidity gradient. So, what is the notation used for humidity? That is q_v, so when it comes to humidity gradient, as the two different altitudes are there what is the difference in humidity with respect to difference in altitude. So, that can be represented by $\frac{dq_v}{dz}$

And second one is the wind velocity gradient, that is represented by $\frac{du}{dz}$, u is the notation used for wind velocity and the du value corresponding to $\frac{du}{dz}$ is considered as the wind velocity gradient. Why we are taking these two gradients? Because, these are the two predominant factors which are responsible for evaporation and these two are considered in the method known as aerodynamic method.

Now, coming to the vapor flux, vapor flux notation is $\dot{m}_v$, that we have already used. So, the vapor flux $\dot{m}_v$ passing upward by convection. What we are doing? We have seen the control volume above the water surface, we will be considering a plane parallel to the water body, through that plane how much vapor transport is taking place due to the process termed as convection.

So, that can be written as that is,

$$\dot{m}_v \propto \frac{dq_v}{dz},$$

$\frac{dq_v}{dz}$ is our humidity gradient and humidity gradient is representing the difference in the humidity at two different altitudes. So, because why we are considering that we can add more and more water vapor only if it has not reached the saturation vapor pressure.

So, we can add up to or the evaporation process can add an amount of vapor into the atmosphere until it reaches the saturation condition. So, that is why these humidity gradients is coming into picture. So, the vapor flux $\dot{m}_v$ is represented by this particular expression, that is it is proportional to how much gradient is there corresponding to specific humidity,

$$\dot{m}_v \propto \frac{dq_v}{dz}$$

Next is the momentum flux. Momentum flux is represented by the notation τ . τ , momentum flux which is taking place in the upward direction again due to convection. The transport process, which we are explaining here is by means of convection. So, vapor flux passing upward by convection, we have written and in the similar way momentum flux is also taking place due to convection, that is written by

$$\tau \propto \frac{du}{dz}$$

Mass flux is proportional to humidity gradient and the momentum flux depends upon the wind velocity, that is why it is proportional to $\frac{du}{dz}$, wind velocity gradient.

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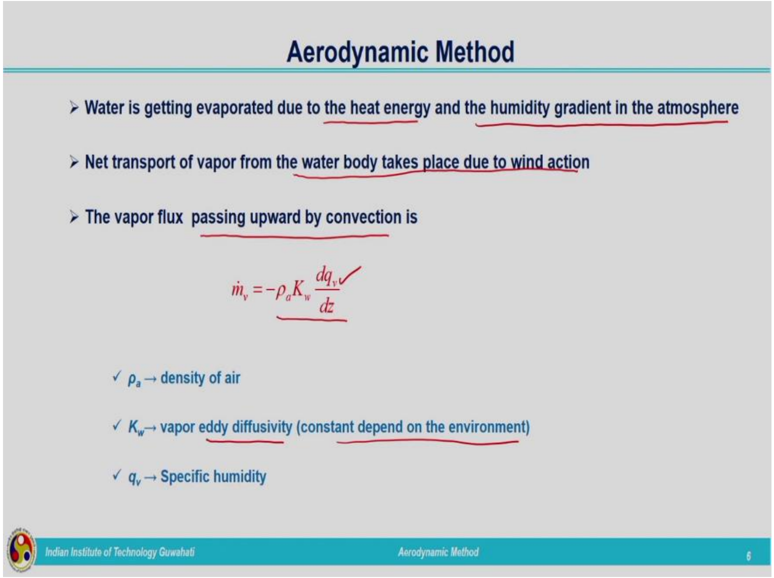

We know already water is getting evaporated due to the heat energy and the humidity gradient in the atmosphere and due to the action of wind, the transport of water vapor is taking place. So, net transport of water vapor from the water body is taking place due to wind action. So, three important factors heat energy, then comes the humidity gradient, and wind velocity gradient. Heat energy we have considered in the case of energy balance equation, in the case of aerodynamic equation, we are considering the wind velocity gradient and the humidity gradient.

So, the vapor flux passing upward by convection can be written as

$$\dot{m}_v = -\rho_a K_w \frac{dq_v}{dz}$$

What we have written in the previous slide?

$$\dot{m}_v \propto \frac{dq_v}{dz}$$

Vapor flux is directly proportional to or the mass transport is directly proportional to humidity gradient and that proportionality is removed by means of the constant. So, the expression has changed the form. Any process, any expression related to convection, different actions will be taking place, the equation will be having this form.

So, here we are having

$$\dot{m}_v = -\rho_a K_w \frac{dq_v}{dz}$$

So, here ρ_a is the density of air, K_w is the vapor eddy diffusivity, this is a constant depending on the environment. Here we will be considering K_w to be constant. Then we know $\frac{dq_v}{dz}$ is the specific humidity gradient, q_v is the specific humidity.

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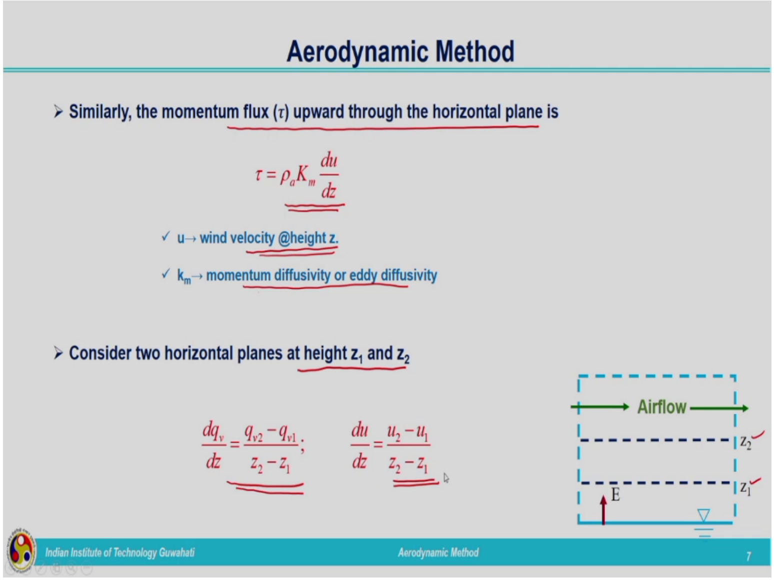

In the similar way, momentum flux τ upward through the horizontal plane can also be written. The process is same that is the convection process. So, you compare these two expressions, that is one is related to vapor flux mass transport and the other one is related to momentum flux. Here

$$\tau = \rho_a K_m \frac{du}{dz}$$

So, here we are having ρ_a is the density of air, u is the wind velocity at height z, $\frac{du}{dz}$ we are taking, in the previous case $\frac{dq_v}{dz}$ specific humidity q_v is considered. When it comes to momentum transport, it is related to wind velocity, that is the wind velocity u at a height of z is considered and K_m is the momentum diffusivity or eddy diffusivity. Momentum diffusivity or eddy diffusivity is the constant of proportionality we are considering and ρ_a is the density of air.

Now, what we are going to do? We are going to consider the two horizontal planes. So as the altitude increases, that was our gradient we were considering, that is two heights. Difference in the specific humidity at two different altitudes. In the similar way difference in wind velocity at two different altitudes. So, we need to consider two planes at heights z_1 and z_2. So same control volume we are considering, evaporation is taking place, we are having the air flow, airflow velocity is the wind velocity, which we are denoting as u.

And we are considering two planes z_1 and z_2. z_1 and z_2 both are considered to be very close to each other, because we need to make an assumption that the variation in specific humidity or in the wind velocity is linear, that is why the gap between these two planes, which we are considering should be minimal.

So,

$$\frac{dq_v}{dz} = \frac{q_{v2} - q_{v1}}{z_2 - z_1}$$

And then coming to velocity gradient, wind velocity gradient, it can be written as

$$\frac{du}{dz} = \frac{u_2 - u_1}{z_2 - z_1}$$

Why we are able to write this expression in the difference form? Because, we are assuming that the variability, or the variation of the specific humidity, or the wind velocity between these two planes z_1 and z_2 are varying linearly, because the gap between these two planes are very less.

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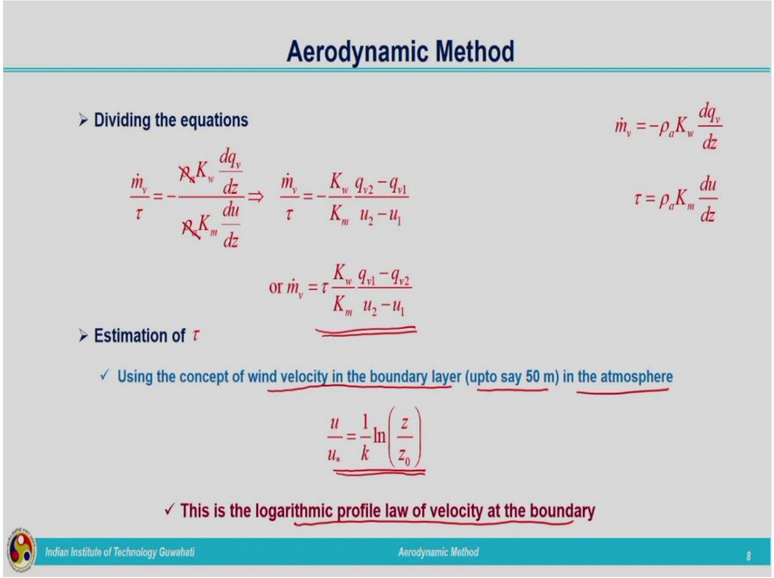

Now, what we are going to do? We are going to divide the two equation, which is representing the vapor transport, mass transport and mass flux and the momentum flux. $\dot{m}_v$ and τ, $\dot{m}_v$ is the mass flux and τ is representing the momentum flux. So, what we are going to do? We are going to divide these two equations, $\frac{\dot{m}_v}{\tau}$.

$$\frac{\dot{m}_v}{\tau} = -\frac{\rho_a K_w \frac{dq_v}{dz}}{\rho_a K_m \frac{du}{dz}}$$

So right hand side negative sign is there.

So, ρ_a gets cancelled and we will get this.

$$\frac{\dot{m}_v}{\tau} = -\frac{K_w \frac{dq_v}{dz}}{K_m \frac{du}{dz}}$$

$\frac{dq_v}{dz}$ and $\frac{du}{dz}$ can be written as the difference equation, as

$$\frac{\dot{m}_v}{\tau} = -\frac{K_w}{K_m}\frac{\frac{q_{v2}-q_{v1}}{z_2-z_1}}{\frac{u_2-u_1}{z_2-z_1}}$$

and from that $z_2 - z_1$ will be getting cancelled and our expression will be taking the form

$$\frac{\dot{m}_v}{\tau} = -\frac{K_w}{K_m}\frac{q_{v2} - q_{v1}}{u_2 - u_1}$$

That is mass transport divided by momentum transport.

$$\frac{\dot{m}_v}{\tau} = -\frac{K_w}{K_m}\frac{q_{v2} - q_{v1}}{u_2 - u_1}$$

Or

$$\dot{m}_v = \tau \frac{K_w}{K_m}\frac{q_{v1} - q_{v2}}{u_2 - u_1}$$

Now, next step is to estimate the value corresponding to momentum flux τ. Momentum flux is related to wind velocity gradient, that you have to keep in your mind, and using the concept of wind velocity in the boundary layer. In fluid mechanics, you have you might have studied the boundary layer concepts, here we are going to make use of that concept related to wind velocity. So, we are going to make use of the concept of wind velocity in the boundary layer almost up to 50 meters in the atmosphere.

So, what is the expression corresponding to velocity distribution, in the boundary layer theory. That is given by

$$\frac{u}{u_*} = \frac{1}{k}\ln\left(\frac{z}{z_0}\right)$$

This is the logarithmic distribution of velocity, logarithmic profile law of velocity at the boundary. So, if you go through the boundary layer theory, which you have studied in fluid mechanics, you will be reminded of this particular equation at the boundary. So, this particular equation we will be making use here in the case of estimation of momentum flux.

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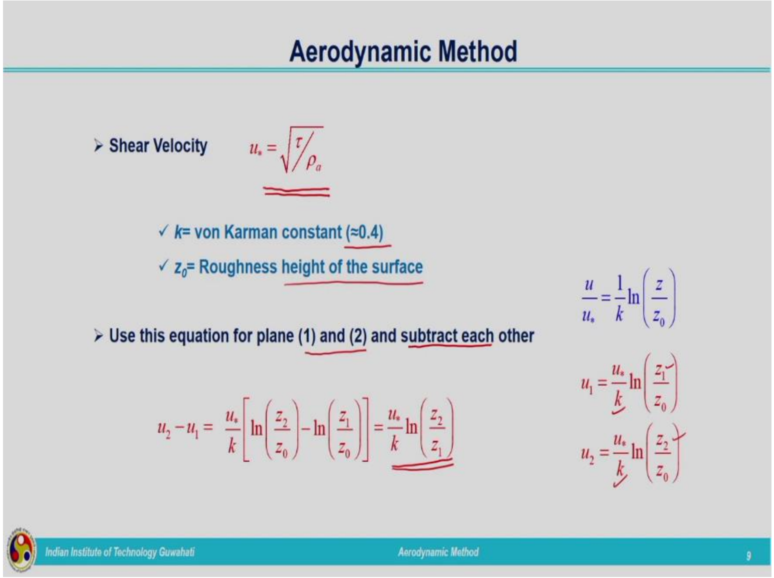

So, here we are having the term shear velocity u_*. Shear velocity, how can we get the value or the expression corresponding to shear velocity? It is given by

$$u_* = \sqrt{\frac{\tau}{\rho_a}}$$

So, τ is the momentum flux. So, $u_* = \sqrt{\frac{\tau}{\rho_a}}$, do not worry about these expressions, these are taken from the boundary layer theory. So, that we are applying here for the wind velocity.

So, k which we have used in the previous slide is the von karman constant, this value is taken to be approximately 0.4 and z_0 is the roughness height of the surface. Now, use these equations for two planes, plane 1 and plane 2, which we are considered, that is we have considered two planes at z_1 and z_2.

So, what we are going to do? We are going to write the velocity distribution expression for two different planes, that is our velocity distribution equation is

$$\frac{u}{u_*} = \frac{1}{k}\ln\left(\frac{z}{z_0}\right)$$

Here this k is the von karman constant, and z is the level which we are considering, z_0 is the roughness height of the particular surface.

We will write the equation for two different altitudes, that is

$$u_1 = \frac{u_*}{k} \ln\left(\frac{z_1}{z_0}\right)$$

and

$$u_2 = \frac{u_*}{k} \ln\left(\frac{z_2}{z_0}\right)$$

After that what we are going to do? We are going to subtract these two equations, subtract one from the other.

So, $u_2 - u_1$ we are going to write these as

$$u_2 - u_1 = \frac{u_*}{k}\left[\ln\left(\frac{z_2}{z_0}\right) - \ln\left(\frac{z_1}{z_0}\right)\right]$$

$\frac{u_*}{k}$ is coming in both equations, that can be taken as common. We got the equation like this, so we are having the logarithmic term within the bracket, that can be rearranged. So, it can be written as

$$u_2 - u_1 = \frac{u_*}{k} \ln\left(\frac{z_2}{z_1}\right)$$

z_0 is getting cancelled.

So, the expression for wind velocity, difference in wind velocity at section 2 and section 1, that is $u_2 - u_1$ can be written as

$$u_2 - u_1 = \frac{u_*}{k} \ln\left(\frac{z_2}{z_1}\right)$$

Why we are doing all this exercise? We need to get an expression corresponding to our momentum flux. So, how can we relate this thing? Our momentum flux term is here in the expression for shear velocity, u_* is given as

$$u_* = \sqrt{\frac{\tau}{\rho_a}}$$

Let us see, how can it be obtained? How we can get the expression for momentum flux? Here we are having the shear velocity.

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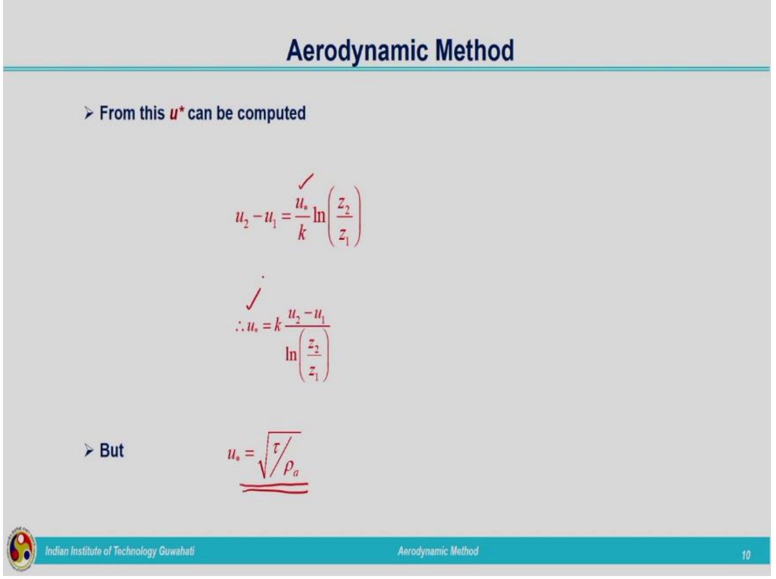

Now, if from the previous expression, that is from this expression,

$$u_2 - u_1 = \frac{u_*}{k}\ln\left(\frac{z_2}{z_1}\right)$$

we are going to get the expression for shear velocity u_*, i.e., we are rearranging the equation and u_* is equals to

$$u_* = k\frac{u_2 - u_1}{\ln\left(\frac{z_2}{z_1}\right)}$$

And then we will substitute the expression containing momentum flux here and from that we can find out the expression for momentum flux. So, everything converted in terms of wind velocity. Shear velocity is given by this particular expression

$$u_* = \sqrt{\frac{\tau}{\rho_a}}$$

So, that will be substituting here.

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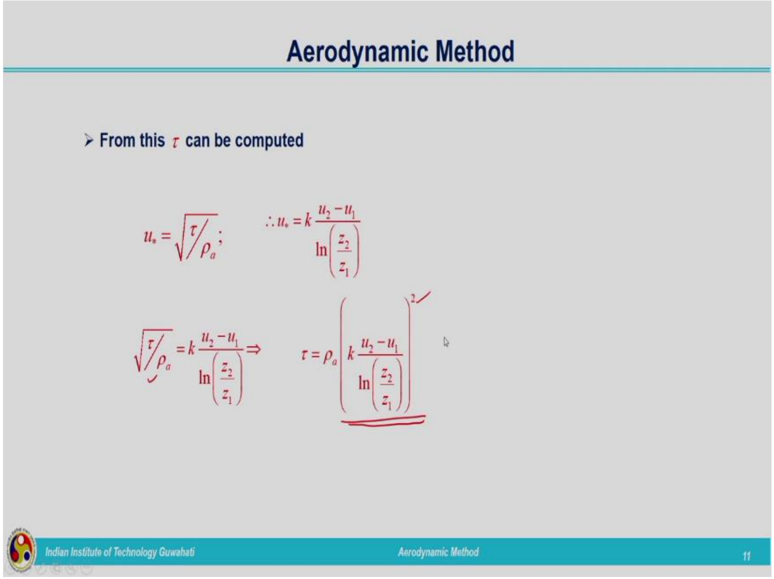

So,

$$u_* = \sqrt{\frac{\tau}{\rho_a}}$$

$$u_* = k\frac{u_2 - u_1}{\ln\left(\frac{z_2}{z_1}\right)}$$

$$\sqrt{\frac{\tau}{\rho_a}} = k\frac{u_2 - u_1}{\ln\left(\frac{z_2}{z_1}\right)}$$

Momentum flux τ is within the root. So, we need to square the expression and after that rearranging the terms, we can get the expression corresponding to momentum flux τ.

So, we have squared the expression and the entire right-hand side will be having the power 2 and the left-hand side will be $\frac{\tau}{\rho_a}$. So,

$$\frac{\tau}{\rho_a} = \left(k\frac{u_2 - u_1}{\ln\left(\frac{z_2}{z_1}\right)}\right)^2$$

So, from that we will be finding out the expression for τ, it will be coming out like this,

$$\tau = \rho_a \left(k \frac{u_2 - u_1}{\ln\left(\frac{z_2}{z_1}\right)} \right)^2$$

So, this is the equation corresponding to momentum flux tau.

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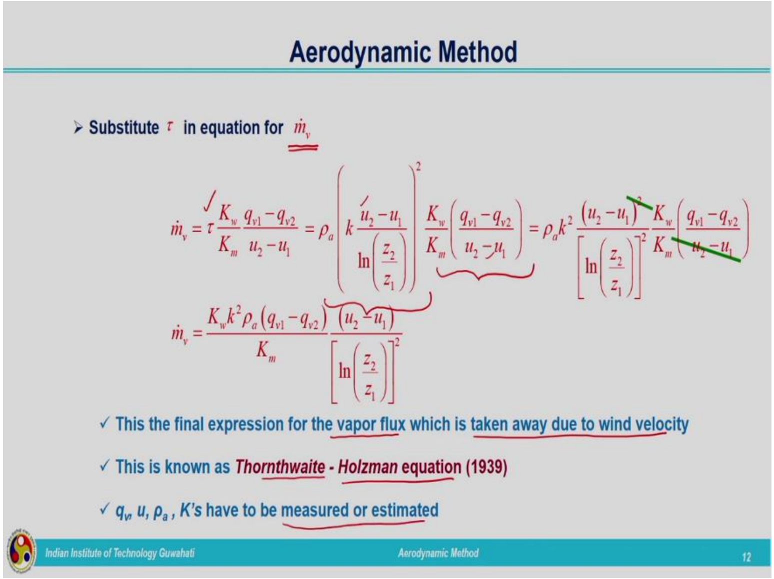

Now, what we are going to do? We are having the equation corresponding to $\dot{m}_v$, that is we have divided the mass flux and the momentum flux, $\dot{m}_v$ divided by τ we have done initially and $\dot{m}_v$ in terms of τ we have written there. Now, we are having the expression corresponding to momentum flux tau from the velocity distribution law near the boundary layer.

So, we will do the substitution for τ, that is

$$\dot{m}_v = \tau \frac{K_w}{K_m} \frac{q_{v1} - q_{v2}}{u_2 - u_1}$$

this is the equation for $\dot{m}_v$ that is our mass flux, so here for τ will be substituting the expression, which we got in the previous slide i.e.,

$$\dot{m}_v = \rho_a \left(k \frac{u_2 - u_1}{\ln\left(\frac{z_2}{z_1}\right)} \right)^2 \frac{K_w}{K_m} \frac{q_{v1} - q_{v2}}{u_2 - u_1}$$

So, we got a lengthy expression for mass transport, $\dot{m}_v$ is given by, ρ_a multiplied by, this entire term, this entire term is representing our momentum flux that is τ, that is substituted over here multiplied by $\frac{K_w}{K_m}$ multiplied by $\frac{q_{v1} - q_{v2}}{u_2 - u_1}$. This equation can be rearranged, somewhat we can make some rearrangements, because we are having $u_2 - u_1$ here and also $u_2 - u_1$ here. So, that can be cancelled.

So, it will be

$$\dot{m}_v = \rho_a k^2 \frac{(u_2 - u_1)^2}{\left[\ln\left(\frac{z_2}{z_1}\right)\right]^2} \frac{K_w}{K_m} \left(\frac{q_{v1} - q_{v2}}{u_2 - u_1} \right)$$

So, this $u_2 - u_1$ and in the numerator, we are having $(u_2 - u_1)^2$, so that gets cancelled, and the final expression takes this form

$$\dot{m}_v = \frac{K_w k^2 \rho_a (q_{v1} - q_{v2})}{K_m} \frac{(u_2 - u_1)}{\left[\ln\left(\frac{z_2}{z_1}\right)\right]^2}$$

When you derive step by step you will get, you will get this equation. It will be slightly difficult to remember, but step by step derivation will be leading you to this final expression.

So, this final expression is for the vapor flux taken away due to wind velocity. That is, we are having the process of evaporation taking place near the surface of water body, water vapor is getting added to the atmosphere, that has been transported by means of wind velocity. So, the process which is taking place, how much vapor transport or the mass flux is taking place due to the action of wind velocity can be obtained by using this equation corresponding to $\dot{m}_v$.

This is the well-known Thornthwaite– Holzman equation. This equation is very commonly used in the case of evaporation. So, this equation is based on the aerodynamic method. The equation which is derived based on the aerodynamic method is termed as Thornthwaite – Holzman equation.

Now, we are having different terms. q_{v1} and q_{v2} that is the specific humidity at level z_1 and z_2 and different k's are the, these things have to be measured or estimated. Wind velocity can be measured, specific humidity can be calculated, we are having the expression corresponding to specific humidity in the water vapor dynamics we have studied.

So, those equations will be utilizing here and we can calculate these values or the values which are measured, such as wind velocity at different levels or it can also be calculated by using different formula or relationships. So, those equations will be utilized and we can finally calculate how much is the vapor flux taking place at that particular location.

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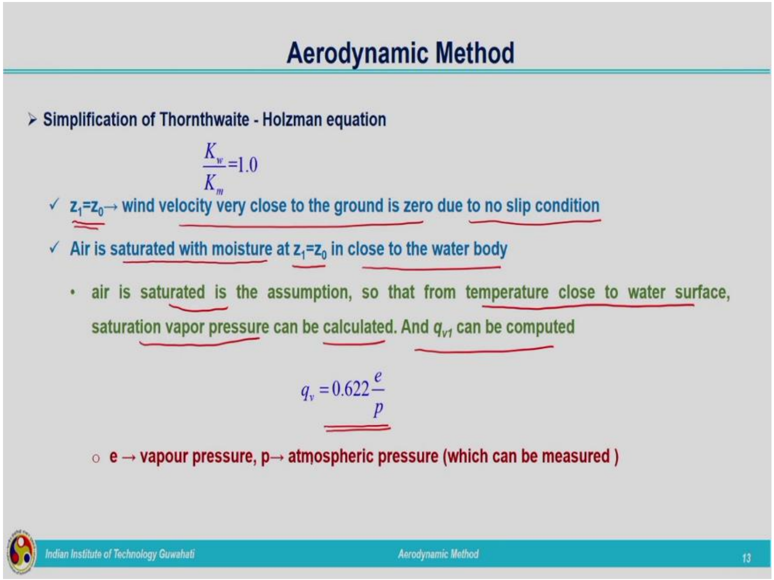

Now, that equation you have seen slightly lengthy, and it is difficult to remember. So, what we are going to do? We are going to simplify that particular equation. The simplification of Thornthwaite–Holzman equation can be done by assuming K_w and K_m are the constant of proportionality and $\frac{K_w}{K_m}$ can be taken to be unity. Then what we are doing? We are having the wind velocity at two levels z_1 and z_2. What we are going to assume? We can consider z_1 very close to the water body, on the surface of the water body.

So, z_1 is considered as z_0 and wind velocity close to the ground is zero due to no slip condition. That is water surface is not moving water body, in the water body, water surface is not having any velocity. So, based on the no slip condition, we can assume that the air which is moving near to that surface also will be having the velocity as that of the water surface.

What is the velocity of the water surface? Since it is not moving, since it is a water body like lake, or which is not moving, then we can assume the velocity of the water surface to be zero. So, that can be considered, the wind velocity at section one can be considered to be zero, because we are assuming that section 1 is at the water surface. So, section 1 we are considering at the water surface. So, there the air will be saturated, because the more and more water vapor is added continuously, so the air will be saturated near the water surface. So, the air is saturated with moisture at $z_1 = z_0$ very close to the water body. All these are simplifications, we need to make the simple form of Thornthwaite – Holzman equation.

So, air is saturated, this assumption is giving us that from the temperature close to the water surface, the saturation vapor pressure can be calculated and q_{v1} can be calculated, that is z_1 is equal to z_2. So, corresponding to that temperature, we can calculate the saturation vapor pressure and also specific humidity. What is the expression for specific humidity? It is given by q_v is equal to

$$q_v = 0.622\frac{e}{p}$$

Where, e is the vapor pressure, p is the atmospheric pressure, these can be measured.

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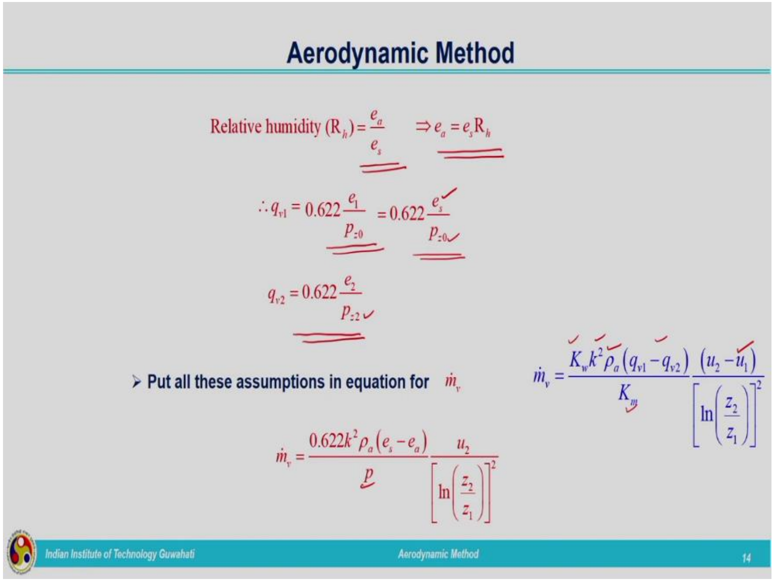

Now, coming to relative humidity we know, it is given by

$$R_h = \frac{e_a}{e_s}$$

Tthese equations we have already seen. So, e_a can be calculated by using the equation,

$$e_a = R_h e_s$$

Now,

$$q_{v1} = 0.622 \frac{e_1}{p_{z_0}}$$

p_{z_0} is the level pressure at the water surface. It is given by

$$q_{v1} = 0.622 \frac{e_s}{p_{z_0}}$$

With the assumption, near the water body, we are taking the value $z_1 = z_0$, so at that location just above the water body, the air will be fully saturated and we can take the vapor pressure to be saturation vapor pressure. So, q_{v1} expression is simplified, so q_{v2} is corresponding to

$$q_{v2} = 0.622 \frac{e_2}{p_{z_2}}$$

Put all these assumptions in equation for vapor transport. So, this is our vapor transport equation,

$$\dot{m}_v = \frac{K_w k^2 \rho_a \left(q_{v1} - q_{v2}\right)}{K_m} \frac{\left(u_2 - u_1\right)}{\left[\ln\left(\frac{z_2}{z_1}\right)\right]^2}$$

We have assumed $\frac{K_w}{K_m} = 1$ and then we are having the expressions for q_{v1}, q_{v2}. So, then we can substitute in the equation. So, the equation will be taking the form

$$\dot{m}_v = \frac{0.622 k^2 \rho_a \left(e_s - e_2\right)}{p} \frac{u_2}{\left[\ln\left(\frac{z_2}{z_1}\right)\right]^2}$$

$\dot{m}_v$ is equal to q_{v1} and q_{v2}, we are having the expression in terms of e_s and e_2. So, when it is $q_{v1} - q_{v2}$ is calculated, it will be coming out to be will be

$$q_{v1} - q_{v2} = 0.622 \frac{(e_s - e_2)}{p}$$

So, that we are considering $p_{z_0} - p_{z_2}$ as *p*.

So, it will be

$$\dot{m}_v = \frac{0.622 k^2 \rho_a (e_s - e_2)}{p} \frac{u_2}{\left[\ln\left(\frac{z_2}{z_1}\right)\right]^2}$$

So, you may get confused where this p_{z_2} and p_{z_0} have gone, it has assumed to be the atmospheric pressure. So, that is represented by *p* which is not varying, that variation we are not considering.

So, the expression takes the form,

$$\dot{m}_v = \frac{0.622 k^2 \rho_a (e_s - e_2)}{p} \frac{u_2}{\left[\ln\left(\frac{z_2}{z_1}\right)\right]^2}$$

Why u_2? $u_1 = 0$ based on the no slip condition, so $u_2 - u_1 = u_2$. The surface layer of air which is close to the water surface will be having the same velocity as that of the water surface, that is equal to zero.

$$\dot{m}_v = \frac{0.622 k^2 \rho_a (e_s - e_2)}{p} \frac{u_2}{\left[\ln\left(\frac{z_2}{z_1}\right)\right]^2}$$

So, this is the simplified form of the Thornthwaite–Holzman equation.

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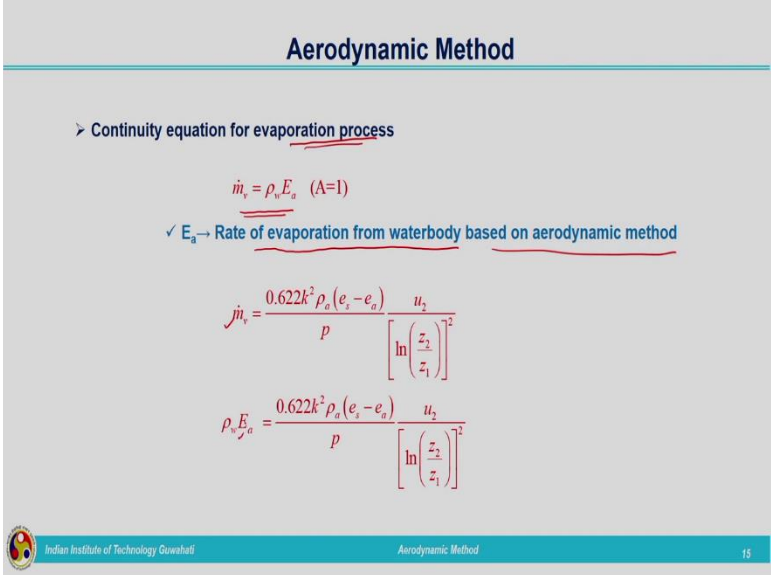

Now, we know continuity equation for evaporation process, this we have seen, mass balance equation we have seen in the previous class. It is given by

$$\dot{m}_v = \rho_w E_a A$$

A is area from which the evaporation is taking place, that area we are considering unity, A is equal to 1. So,

$$\dot{m}_v = \rho_w E_a$$

Why do we want to make use of this equation, continuity equation here? Because our aim is to find out the evaporation rate, that is contained in the mass conservation equation.

So, $\dot{m}_v = \rho_w E_a$, so that we will be substituting here, E_a is the rate of evaporation from water body based on aerodynamic method and that will be substituting here for this vapor transport, we will be substituting $\rho_w E_a$ and from this we will calculate the expression for E_a.

$$\rho_w E_a = \frac{0.622k^2 \rho_a (e_s - e_2)}{p} \frac{u_2}{\left[\ln\left(\frac{z_2}{z_1}\right)\right]^2}$$

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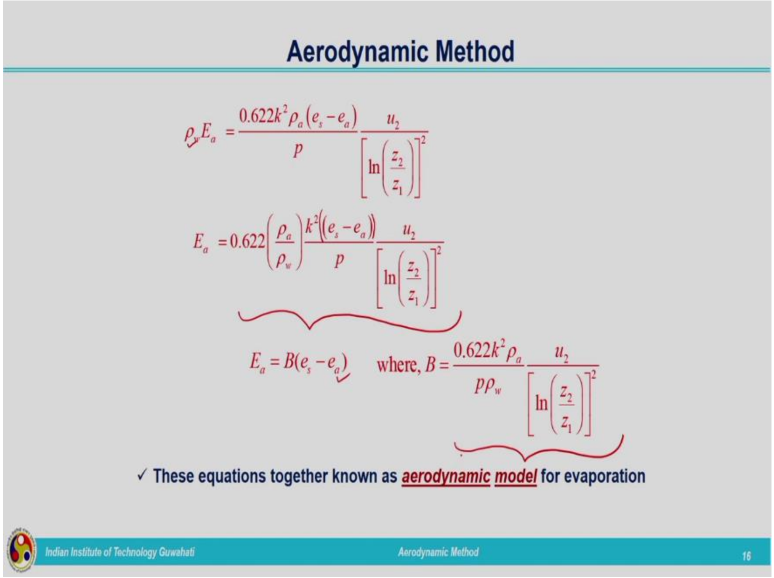

So,

$$\rho_w E_a = \frac{0.622k^2\rho_a(e_s - e_2)}{p}\frac{u_2}{\left[\ln\left(\frac{z_2}{z_1}\right)\right]^2}$$

$$E_a = 0.622\left(\frac{\rho_a}{\rho_w}\right)\frac{k^2(e_s - e_2)}{p}\frac{u_2}{\left[\ln\left(\frac{z_2}{z_1}\right)\right]^2}$$

So, E_a is given by, so we can write this expression as such entire expression we are considering and we are simplifying it in such a way that

$$E_a = B(e_s - e_2)$$

What is $e_s - e_2$? It is the saturation vapor pressure minus the vapor pressure corresponding to that particular temperature, deficit in the vapor pressure. So,

$$E_a = B(e_s - e_2)$$

So, we are keeping only this term, $e_s - e_2$ here and all other terms combined together under the term *B*. So, all terms combined together we are substituting as *B* for getting a simplified equation.

$$B = \frac{0.622k^2\rho_a}{p\rho_w} \frac{u_2}{\left[\ln\left(\frac{z_2}{z_1}\right)\right]^2}$$

So, $E_a = B(e_s - e_2)$.

These equations together known as aerodynamic model for evaporation. In aerodynamic equation we have considered the specific humidity and the wind velocity gradient. And we have written the mass flux and the momentum flux. Mass flux and momentum flux expressions for the transport process, which we have considered is convection. So, for a convection process, how the equation will be looking like and based on that particular equation, we have used the equation corresponding to mass transport and momentum transport and divided each other. And we have made use of the velocity distribution, expression near the boundary layer, for getting the expression for wind velocity and by making use of all these theories, we have finally calculated the evaporation. How did we get the evaporation? By incorporating the continuity equation.

So, here we have made use of the continuity or the mass balance equation, specific humidity gradient and also wind velocity gradient. So, by making use of all these principles, finally we got this simplified equation for calculating the evaporation rate based on aerodynamic method, that is why we are having E_a, based on heat energy, heat radiation it was represented E_r in the previous lecture and today we have, we are representing E_a. E_a is given as $E_a = B(e_s - e_2)$. But B is a complicated expression, but final expression we can tell E_a is equal to B multiplied by deficit in vapor pressure. This is the aerodynamic model for the calculation of evaporation.

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So, the reference textbooks which I have utilized are these books and the derivation is mainly I have taken from applied hydrology by V.T Chow and others. So, here I am winding up this lecture, thank you.