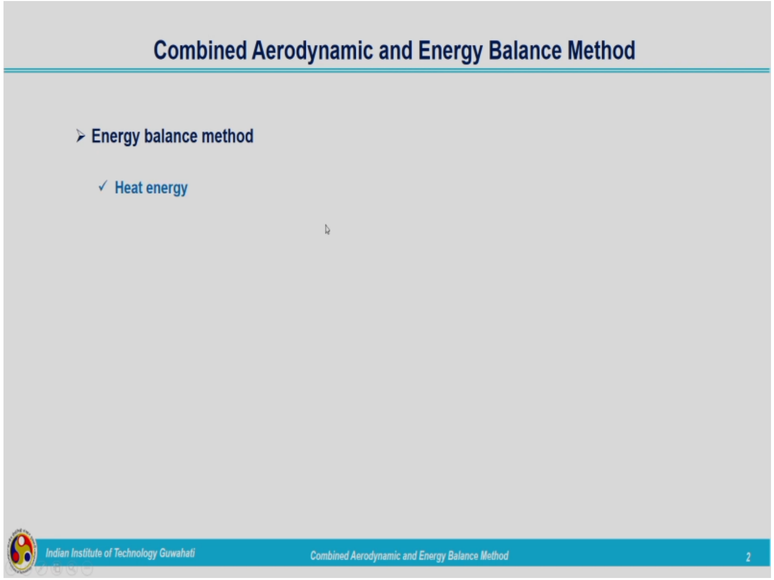
Engineering Hydrology
Professor Dr. Sreeja Pekkat
Department of Civil Engineering
Indian Institute of Technology, Guwahati
Lecture 26
Evaporation - Combined Aerodynamic and Energy Balance Method

Hello All. Welcome back. In the previous lectures, we were discussing about the estimation of evaporation. Two methods we have already seen, energy balance method and aerodynamic method. In the energy balance method, we have considered the important factor, which is influencing evaporation as heat energy from the sun.

And in the second method, that is the aerodynamic method, we have considered the wind velocity and the specific humidity. But we know when we are talking about the process of evaporation, all the three, that is the wind velocity, specific humidity and also heat energy, all the three factors are very important.

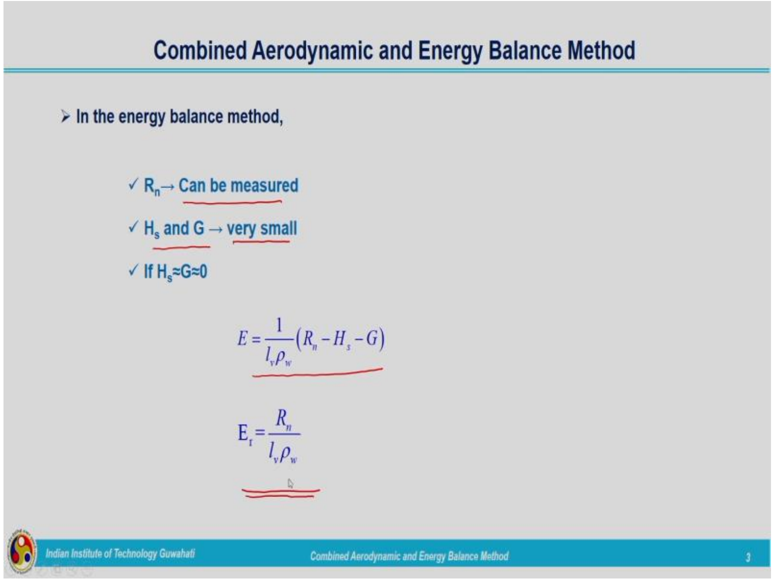
So, in this lecture, that is today's lecture, we will be discussing about a combined method, which incorporates all these three factors, which are influencing evaporation. Let us move on to the lecture.

(Refer Slide Time: 01:35)



As I already told, energy balance method incorporates heat energy, and in the aerodynamic method, it incorporates humidity and wind velocity. So, we need to have a method, which combines all the three factors together for finding out the evaporation. So, we will be deriving a method, which is the combination of the two methods, which are already seen. That is the combination of energy balance method and aerodynamic method.

(Refer Slide Time: 2:10)



In the energy balance method, what we have considered? We have considered the heat energy from the sun, that is R_n heat radiation that can be measured. But the two other terms, H_s and G, sensible heat and the heat lost to the ground surface were very small, those values we have already neglected. The equation which is used in the energy balance method, or the equation which is derived by the energy balance method is given by this expression,

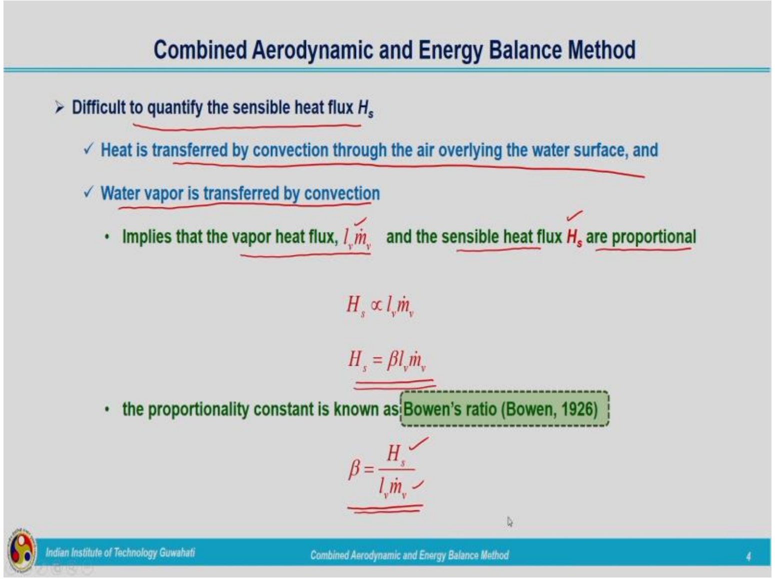
$$E = \frac{1}{l_v \rho_w}(R_n - H_s - G)$$

In this, we have neglected the terms H_s and G, and finally, we got the expression for evaporation due to net radiation as

$$E = \frac{R_n}{l_v \rho_w}$$

The complete detailed derivation of this particular equation, we have seen under the topic of energy balance method.

(Refer Slide Time: 3:16)



Now, we should understand that the two quantities have been neglected, that is the sensible heat, that is heat loss to the atmosphere and also the heat loss to the ground. But when we are talking about the heat lost to the atmosphere, that cannot be neglected. But at the same time, it is difficult to quantify also, that is the sensible heat flux or H_s is very difficult to quantify. So, how can we incorporate this particular term, even though it is difficult to quantify? That is the next question.

Heat is transferred by convection through the air overlying the water surface. We have seen two transport processes, such as conduction and convection. And in the previous lecture, that is while deriving the expression for aerodynamic method, we have seen the vapor flux and also the momentum flux, or the vapor transport, or the momentum transport is taking place through the transport process termed as convection. In the similar way, heat is also getting transferred to the atmosphere, that is through air by convection only.

And at the same time, we have seen, water vapor is also transferred by convection. So, because of these two processes, water vapor is transferred by convection and also heat flux is also taking place due to convection. This implies that, the vapor heat flux and the sensible heat flux are proportional to each other. What is vapor heat flux? Given by the expression $l_v\dot{m}_v$. and the notation which we are using for heat flux is H_s, that is nothing but our sensible heat.

So, since these two processes are taking place, because of convection, we can assume that the same mechanism that is heat transfer and also the vapor transfer is taking place due to convection. Both of these processes are due to convection. So, what we are going to conclude that the vapor heat flux, and the heat flux can be considered to be proportional. That is $l_v\dot{m}_v$ and H_s can be considered to be proportional to each other.

$$H_s \propto l_v\dot{m}_v$$

You should clearly understand why we have considered like this? Because both of these transport processes are due to convection.

We can write

$$H_s = \beta l_v\dot{m}_v$$

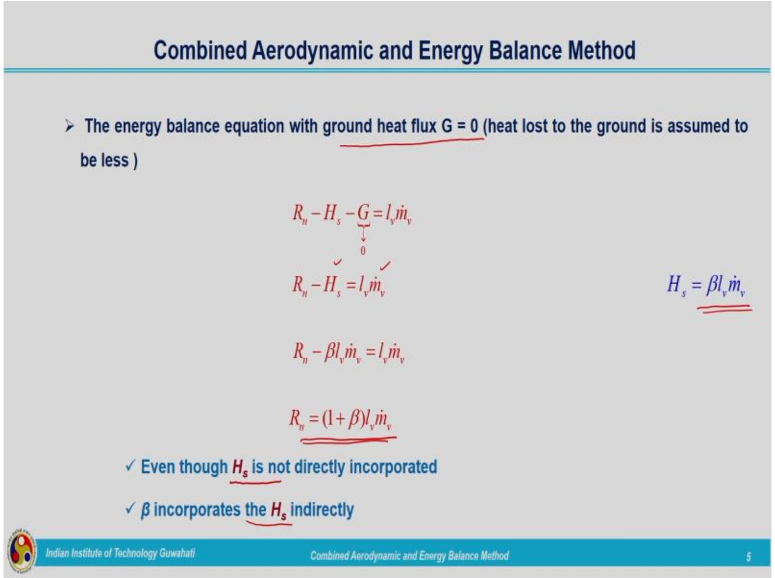
This β is known as Bowen's ratio. So,

$$\beta = \frac{H_s}{l_v\dot{m}_v}$$

So, you look at the equation. We are having sensible heat incorporated here, and we are having the vapor heat flux, that is the expression is $l_v\dot{m}_v$. What we have assumed? We have considered these two processes. We know these two processes are taking place due to convection.

So, accordingly since both are taking place under same transport phenomena, we can consider these two are proportional to each other. That way, we have considered a coefficient of proportionality as Bowen's ratio and β is written as $\frac{H_s}{l_v\dot{m}_v}$.

(Refer Slide Time: 06:57)



Now in the energy balance equation, we can put only the ground heat flux G is equal to zero. When energy balance equation was finalized, we have considered H_S and G equal to zero. Here, what we are going to do, we are going to incorporate sensible heat H_s by means of Bowen's ratio. So there only remains another term related to heat flux that is the heat lost into the ground surface that is G. That can be, since it is of very small amount, that can be neglected. It is not going to affect the final expression for evaporation.

So how our equation will be looking

$$R_n - H_s - G = l_v\dot{m}_v$$

In this, what we are going to do, we are going to substitute G is equal to zero. So, it will be

$$R_n - H_s = l_v\dot{m}_v$$

Now what we are going to do, we are having the relationship between H_s and $l_v\dot{m}_v$ right in terms of Bowen's ratio.

$$H_s = \beta l_v\dot{m}_v$$

So, for H_s, we can substitute $\beta l_v \dot{m}_v$ and we get

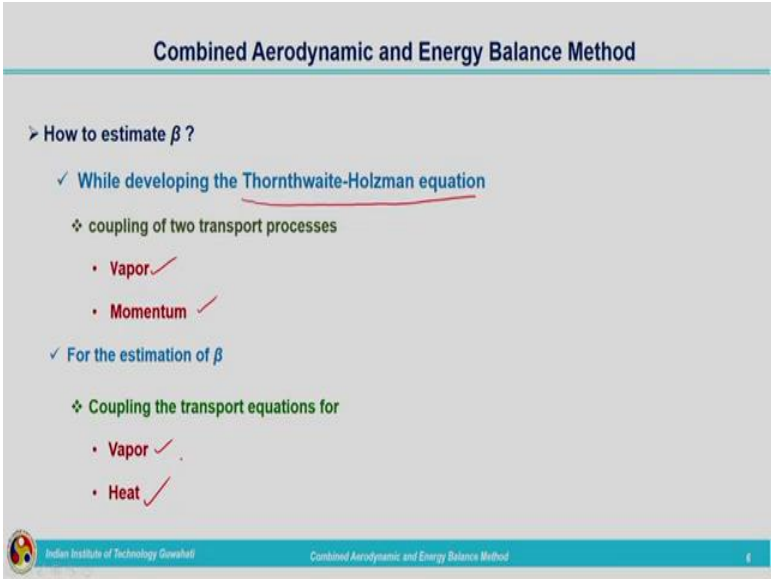
$$R_n - \beta l_v \dot{m}_v = l_v \dot{m}_v$$

The terms can be readjusted and we will get the expression for R_n as

$$R_n = (1+\beta) l_v \dot{m}_v$$

So here in this you can see, we have not neglected the sensible heat, we have incorporated the sensible heat in terms of Bowen's ratio. Even though directly we have not considered H_s, indirectly β is dictating the effect of H_s.

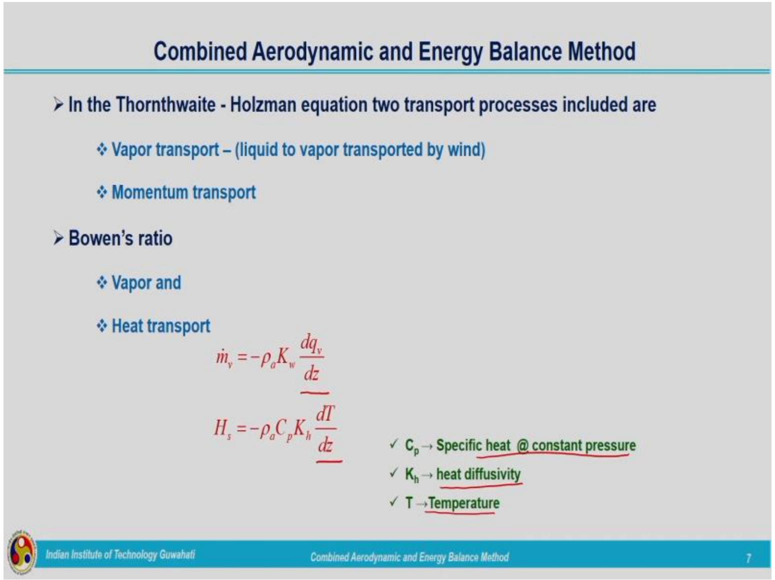
(Refer Slide Time: 8:49)



Now, next question is how to estimate the value corresponding to β. We are just going back to Thornthwaite-Holzman equation, that is the equation which is derived based on aerodynamic method. While developing the Thornthwaite-Holzman equation, we have considered two transport processes and combined together. Those two transport processes are related to vapor transport and momentum flux.

So, there we were considering vapor flux and momentum flux are due to same transport processes that is due to convection. In the similar way, here also we are going to couple two transport processes for the estimation of β, that is we are going to couple the transport equations for vapor and heat, that is vapor flux and heat flux are mainly due to the process of convection. So, these two processes will be coupled together. These two equations will be coupled together for the determination of β.

(Refer Slide Time: 09:58)



So, we can write the vapor transport $\dot{m}_v$ as

$$\dot{m}_v = -\rho_a K_w \frac{dq_v}{dz}$$

And also, the sensible heat H_s is due to the process convection, and we can write it as

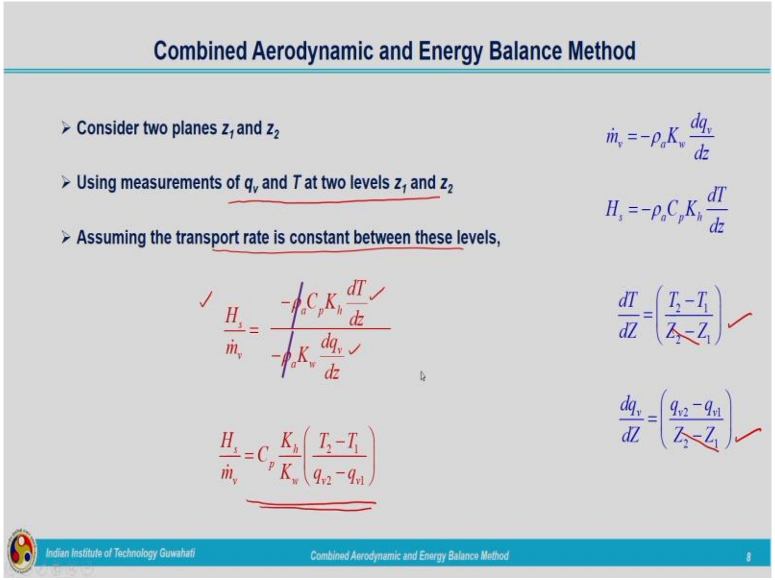
$$H_s = -\rho_a C_p K_h \frac{dT}{dz}$$

So here in this case, C_p is the specific heat at constant pressure, K_h is heat diffusivity, T is the temperature. You can compare the equation. $\dot{m}_v$ is directly proportional to specific humidity gradient, and also heat flux is directly proportional to temperature gradient. So that humidity gradient is $\frac{dq_v}{dz}$ and temperature gradient is $\frac{dT}{dz}$.

So other terms are coming due to convection, that is in the case of vapor flux, it is $\dot{m}_v$ is proportional to particular gradient, that is the humidity gradient. And in the case of heat flux, it is something related to temperature. So, it is related to temperature gradient.

And after that, you can look at the equation, proportionality coefficient as taken by these two terms ($\rho_a K_w$) in $\dot{m}_v$ and these three terms ($\rho_a C_p K_h$) in H_s, that is in the case of the sensible heat. Now what we are going to do, these two processes are proportional to each other, since both of these are taking place due to the same transport process that is due to convection. So, this sensible heat flux and also the vapor flux, both can be taken as proportional to each other. That is what we are going to do in the next step. And for that also, since there are gradient, we are taking between two planes, we are considering two planes, one is near to the water surface and the other one is just above that, two planes we are considering z_1 and z_2.

(Refer Slide Time: 11:58)



And what we will be doing using the measurements of q_v and T at two different levels, z_1 and z_2, we can calculate the corresponding values. And also, the distance between z_1 and z_2 is taken in such a way that the transport rate is constant between these two levels.

And also, we can assume that the temperature variation and also the specific humidity variation, all these are linear process. This vapor flux is given by this equation

$$\dot{m}_v = -\rho_a K_w \frac{dq_v}{dz}$$

and also, sensible heat is given by this equation

$$H_s = -\rho_a C_p K_h \frac{dT}{dz}$$

And what we are going to do? We have already mentioned that these two processes are proportional to each other, so we can consider in that way. $\frac{H_s}{\dot{m}_v}$ ratio we are taking. So, just substituting in that,

$$\frac{H_s}{\dot{m}_v} = \frac{-\rho_a C_p K_h \frac{dT}{dz}}{-\rho_a K_w \frac{dq_v}{dz}}$$

Certain terms will get cancelled, - ρ_a a divided by - ρ_a will get cancelled. Now we can look at the denominator and numerator. We are having two gradients, temperature gradient and specific humidity gradient. Temperature gradient, we can write $\frac{dT}{dz}$ and specific humidity gradient $\frac{dq_v}{dz}$.

So,

$$\frac{dT}{dz} = \left(\frac{T_2 - T_1}{z_2 - z_1} \right)$$

That is why I told you the distance between two layers, which we have considered, z_1 and z_2 should be very small, then only we can assume the linear variation.

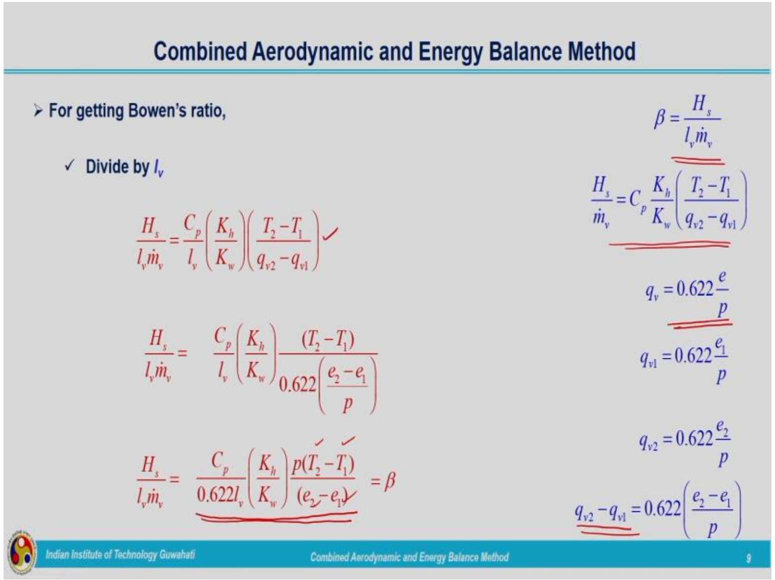
Then regarding the $\frac{dq_v}{dz}$, we can write it as

$$\frac{dq_v}{dz} = \frac{q_{v2} - q_{v1}}{z_2 - z_1}$$

After that, what we will do, we will be substituting this in this particular equation of $\frac{H_s}{\dot{m}_v}$. So here, when we are substituting these $z_2 - z_1$ from these two terms will get cancelled and we will get the expression as

$$\frac{H_s}{\dot{m}_v} = C_p \frac{K_h}{K_w}\left(\frac{T_2 - T_1}{q_{v2} - q_{v1}}\right)$$

(Refer Slide Time: 14:15)



Now for getting the Bowen's ratio, we need to that is we know already, β Bowen's ratio is given by

$$\beta = \frac{H_s}{l_v \dot{m}_v}$$

We already have $\frac{H_s}{l_v \dot{m}_v}$ from the previous slide given by this equation:

$$\frac{H_s}{\dot{m}_v} = C_p \frac{K_h}{K_w}\left(\frac{T_2 - T_1}{q_{v2} - q_{v1}}\right)$$

So, what we have to do? We need to have some l_v, latent heat of vaporization in the denominator here in the left-hand side, $\frac{H_s}{l_v \dot{m}_v}$. For that we will be dividing the entire expression that is on the left-hand side of the equation and right-hand side of the equation by latent heat of vaporization. So, we are going to divide this equation by l_v. So, we will get the equation to be like this,

$$\frac{H_s}{l_v \dot{m}_v} = \frac{C_p}{l_v}\left(\frac{K_h}{K_w}\right)\left(\frac{T_2 - T_1}{q_{v2} - q_{v1}}\right)$$

Here you can see left-hand side is representing the Bowen's ratio, β. Now, what we have to do, we know the expression for q_v from the water vapor dynamics we are taking this equation.

$$q_v = 0.622\frac{e}{p}$$

And we are having two terms, q_{v1} and q_{v2}.

$$q_{v1} = 0.622\frac{e_1}{p}$$

and

$$q_{v2} = 0.622\frac{e_2}{p}$$

We can take the difference between q_{v2} and q_{v1}. It will be taking this form

$$q_{v2} - q_{v1} = 0.622\left(\frac{e_2 - e_1}{p}\right)$$

The same question will arise here also. Why not p_1 and p_2? So, we are considering the distance between these two layers, which we have considered in such a way that there is not much variation or the pressure, at the level z_1 and z_2 are more or less equal. That is why we are not considering p_1 and p_2 separately we are considering only p.

So, this we can substitute here in the equation. So, when we substitute in this particular equation, this equation will be taking the form,

$$\frac{H_s}{l_v \dot{m}_v} = \frac{C_p}{l_v}\left(\frac{K_h}{K_w}\right)\frac{(T_2 - T_1)}{0.622\left(\frac{e_2 - e_1}{p}\right)}$$

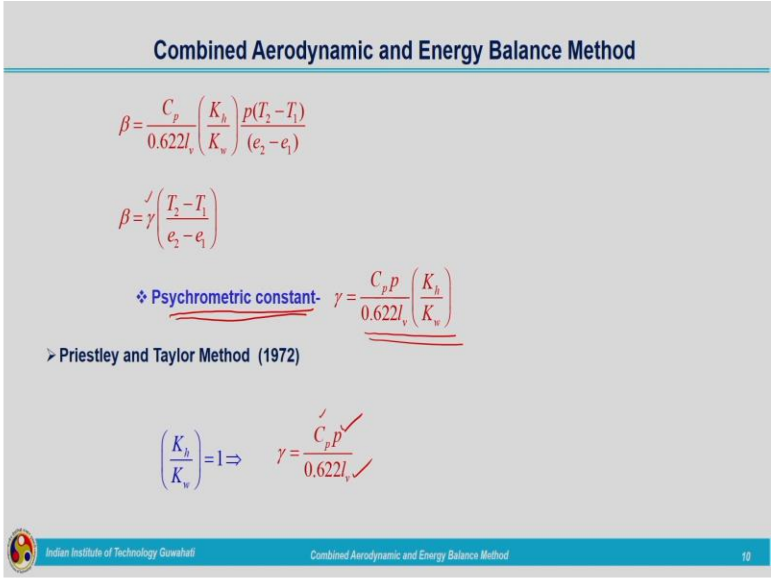
We can make some rearrangements of the terms. So, then $\frac{H_s}{l_v \dot{m}_v}$ will be looking like this,

$$\frac{H_s}{l_v \dot{m}_v} = \frac{C_p}{0.622 l_v}\left(\frac{K_h}{K_w}\right)\frac{p(T_2 - T_1)}{(e_2 - e_1)}$$

So, you look at the left-hand side, left-hand side is $\frac{H_s}{l_v \dot{m}_v}$, that is nothing but our β, Bowen's ratio. So, this is the expression for β.

So, you look at the terms which are contained in the equation for β. C_p, $0.622 l_v$, l_v can be computed by means of the empirical equation which we have discussed earlier. K_h and K_w are known values, coefficients and the atmospheric pressure or small p can be calculated T_2, T_1, e_2, e_1, either temperature can be measured and the temperature at one level will be measured and T_2 can be calculated from the lapse rate equation. e_1, e_2 are vapor pressure and can be measured or calculated by using the corresponding formula. So, we are getting the value corresponding to β if we are making use of this equation.

(Refer Slide Time: 18:05)



So, beta is given by this equation

$$\beta = \frac{C_p}{0.622l_v}\left(\frac{K_h}{K_w}\right)\frac{p\left(T_2-T_1\right)}{\left(e_2-e_1\right)}$$

And what we are going to do? We are going to club certain terms together.

$$\beta = \gamma\frac{\left(T_2-T_1\right)}{\left(e_2-e_1\right)}$$

What is γ then? γ is termed as psychometric constant. So, whenever you are dealing with evaporation, you may come across the term psychometric constant, it is nothing but γ, that is given by

$$\gamma = \frac{C_p p}{0.622l_v}\left(\frac{K_h}{K_w}\right)$$

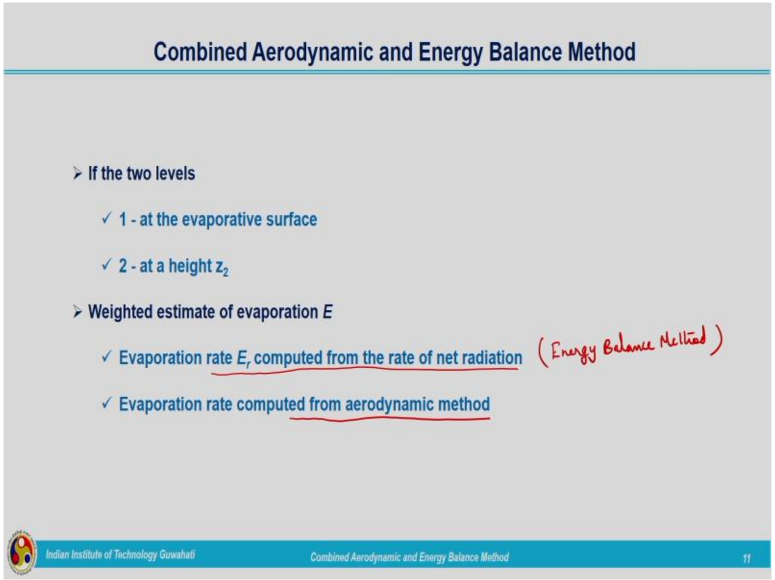
So, this is the equation for Bowen's ratio.

Now, according to Priestly and Taylor Method, this is a common method, combined method used for finding out evaporation. What they have assumed? They have conducted so many evaporation experiments and they have found out a combined expression for evaporation. So Priestly and Taylor assumed that $\frac{K_h}{K_w}$ can be taken as 1. Then γ will be taking the form of

$$\gamma = \frac{C_p p}{0.622 l_v}$$

So, pressure we can measure, latent heat of vaporization we can calculate, C_p is known to us. So, γ can be obtained. Once γ is obtained, γ multiplied by $\frac{T_2 - T_1}{e_2 - e_1}$ will be giving you β.

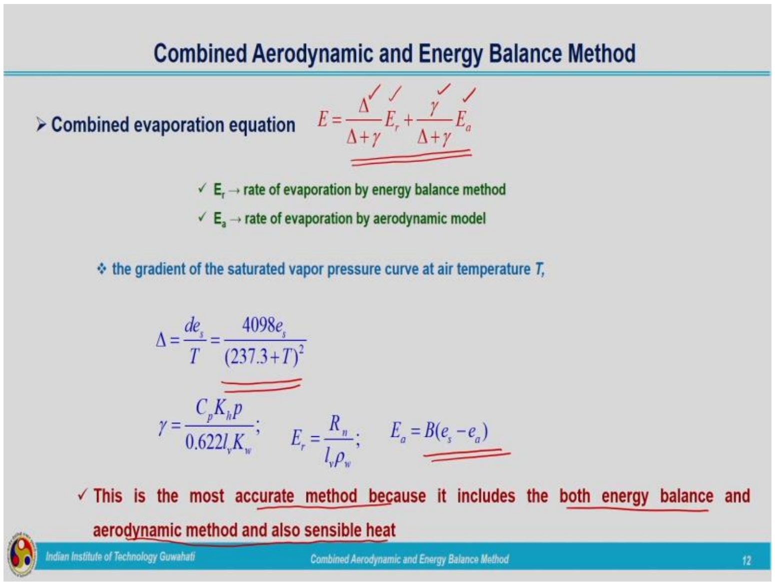
(Refer Slide Time: 19:30)



Now we are considering the two levels in such a way that what we have done in the case of aerodynamic methods, same consideration we are going to do here also. Level 1 is the evaporative surface that is on the water surface. From the water body, evaporation is taking place. The surface of that water body is considered as the level 1 and the level 2 is considered at a distance away from the water surface at a distance of z_2 as we have denoted, this gap should not be taken in such a way that our assumptions of linear variation should be valid.

Now weighted estimate of evaporation. Why we are telling weighted estimate? We are considering, we are going to consider both energy balance method and also the aerodynamic method. We are going to combine together evaporation rate, E_r, computed from the rate of net radiation, that is nothing but our energy balance method. Energy balance method, we have derived the equation and also the second one evaporation rate computed using aerodynamic method.

(Refer Slide Time: 20:56)



Now combined evaporation equation is given by E is equal to combination of evaporation due to net radiation and also evaporation due to aerodynamic method. Evaporation due to net radiation is incorporating the heat energy and the aerodynamic method incorporates the case of wind velocity and also the specific humidity.

So that is given by the equation

$$E=\frac{\Delta}{\Delta+\gamma}E_r+\frac{\gamma}{\Delta+\gamma}E_a$$

E_r is representing the evaporation due to net radiation and E_a is the evaporation due to aerodynamic method.

Now we have discussed about some term gradient Δ when we were discussing about the saturation vapor pressure curve. The gradient of the saturated vapor pressure curve that is given by Δ, Δ is given by

$$\Delta = \frac{de_s}{T} = \frac{4098 e_s}{(237.3+T)^2}$$

So, if we are having the temperature and the corresponding saturation vapor pressure, we can calculate the gradient of that particular saturation vapor pressure curve. That gradient is required while calculating the evaporation due to combined method.

And here in this equation, γ is psychometric constant, and Δ is the gradient of saturation vapor pressure. We are having the expression for calculating E_r, evaporation due to net radiation and also E_a is the evaporation calculated by using aerodynamic method that is due to wind velocity and also specific humidity gradient.

That way, E_a also can be calculated. So, the total evaporation capital E is given by the combination of E_r and E_a. γ is given by the equation

$$\gamma = \frac{C_p K_h p}{0.622 l_v K_w}$$

and E_r is given by

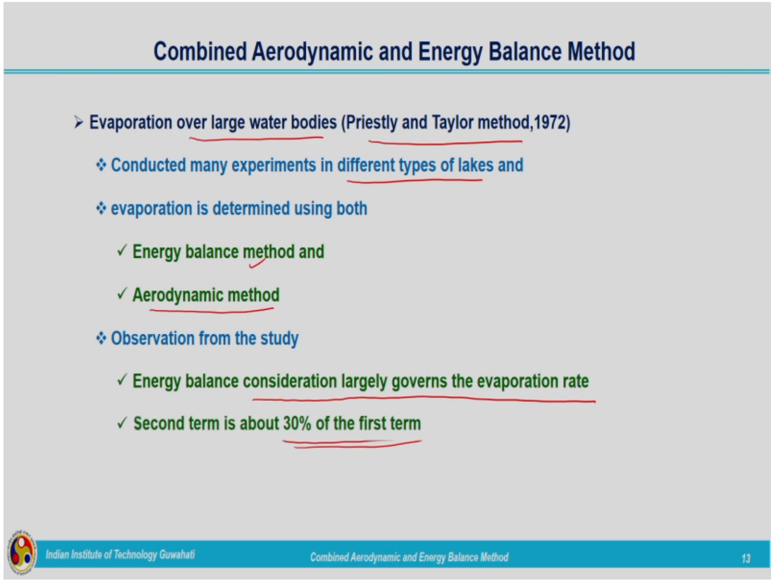
$$E_r = \frac{R_n}{l_v \rho_w}$$

and E_a evaporation due to aerodynamic method is given by

$$E_a = B(e_s - e_a)$$

Now this is the most accurate method for calculation of evaporation, because it incorporates both the aerodynamic method and also the energy balance equation. Both the energy balance and the aerodynamic method and at the same time, energy balance method was ignoring sensible heat. Here, we have incorporated the sensible heat also. So altogether, this equation will be giving you an accurate estimate of evaporation.

(Refer Slide Time: 23:53)

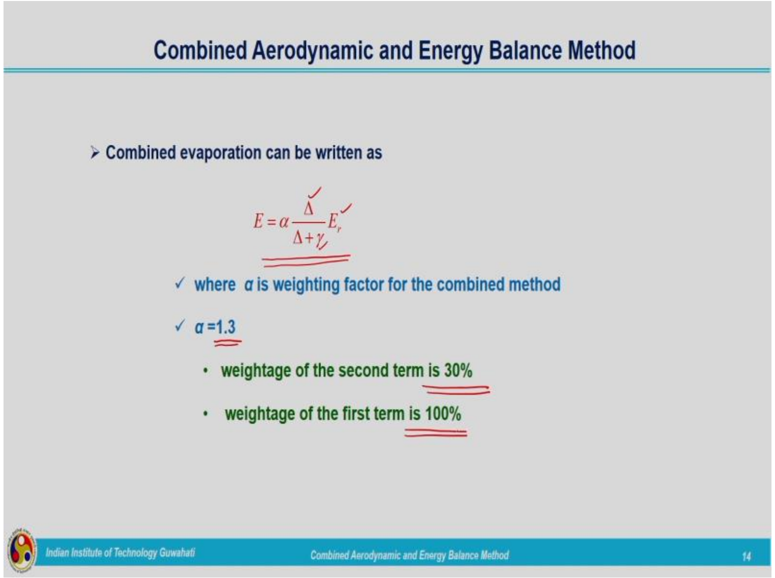


Now, when we are talking about evaporation over large water bodies, Priestly and Taylor, this method is termed as Priestly and Taylor Method, a combination method for determination of evaporation. They have conducted experiments in different types of lakes and evaporation is determined using energy balance method and also aerodynamic method.

They have conducted experiments, experimental measurements they have done. They have calculated the evaporation due to energy balance method and also aerodynamic method. Then what they have found out that the observation from the study states that energy balance consideration largely governs the evaporation rate, that we know, if sunlight is not there, what is the energy source for this process.

So, energy balance method incorporates the net radiation coming from the sun and that is the predominant factor which is influencing. The second part that is due to the vapor transport, due to wind and also specific humidity gradient, it was found that only 30% of the first term is coming from the aerodynamic equation.

(Refer Slide Time: 25:11)



So combined evaporation can be written as E is equal to

$$E = \alpha \frac{\Delta}{\Delta + \gamma} E_r$$

So, what is α then? Δ gradient of the saturation vapor pressure curve, γ psychometric coefficient, and E_r is the evaporation due to net radiation. α is nothing but the value is 1.3. How this value is coming out to be 1.3? That is the weightage for the second term due to aerodynamic case is coming out to be 30% and weightage of the first term is a 100%.

The weightage given to evaporation due to net radiation is 100% and we are considering 30% of that is the only contribution from evaporation due to aerodynamic method. So, combined together, they have taken a factor α and put the combined equation as

$$E = \alpha \frac{\Delta}{\Delta + \gamma} E_r$$

So here ends this topic of Combined Method of Evaporation.