Engineering Hydrology

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Lecture 38

Estimation of Infiltration – Theoretical Equation

Hello, all. Welcome back. We were discussing about different infiltration equations, that is, estimation of infiltration. We have seen empirical equations and the corresponding numerical examples. So today we will move on to the theoretical equations.

So many empirical equations are there. Only three of the equations, we have seen, and some of the examples also solved related to Kostiakov equation and Horton's equation. Now let us move on to the theoretical equations.

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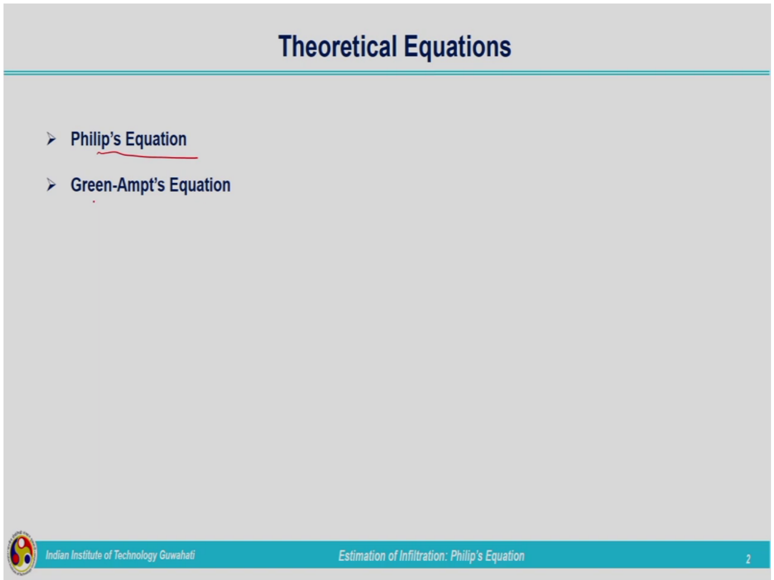

So mainly, we are going to look at two equations, Philip's Equation and Green-Ampt's Equation. So these are the analytical solutions for the flow equation. We have already seen the unsteady flow equation corresponding to the flow taking place in the unsaturated porous media, that is, Richard's equation, detailed derivation, we have seen.

So, the theoretical equations or the analytical solution is giving you the solution corresponding to the flow equation, that is, based on the fundamental principles we are finding out the solution for the flow equation. So first we will see Philip's equation, then we will move on to Green-Ampt equation.

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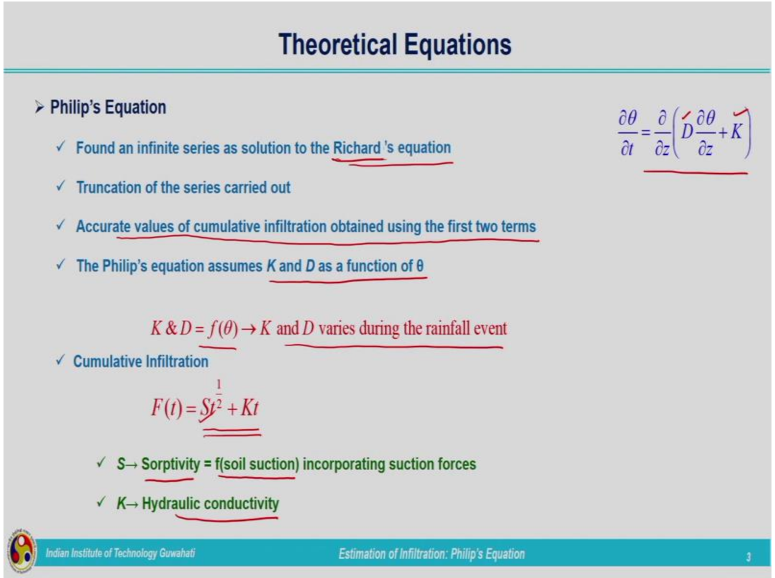

Richard's equation is representing the unsteady flow taking place through the unsaturated porous media. And Philip's equation is giving the solution to Richard's equation by making use of infinite series. So this is our Richard's equation,

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z}\left(D\frac{\partial \theta}{\partial z} + K\right)$$

This is a combination of continuity and momentum equations. Mass conservation equation and Darcy's Law is taken for the representation of momentum equation and combination of those two equations are done to get the final form of Richard's equation. And since infinite series is utilized we need to go for truncation of the series.

So truncation is carried out and it is found that accurate values of cumulative infiltration can be obtained from the first two terms. In the Philip's equation, the assumption is *K* (hydraulic conductivity) and *D* (diffusivity) are functions of θ. [In Horton's equation, it is assumed to be constant. We have not discussed about that because it is an empirical equation. We have just seen the importance of the equation and the parameters of the equation while discussing about Horton's equation].

θ is the moisture content. Whenever infiltration is taking place, θ is changing, that is, the soil moisture content present in the soil is changing because more and more water is infiltrating into the ground and the pores which are present in the soil will be filled with water.

So, the soil moisture present at the end of the infiltration or at the end of the rainfall process will not be the same which was present in the beginning. So, *K* and *D* vary during the rainfall event and are functions of moisture content theta.

$$K \,\&\, D = f(\theta)$$

Now the expression for cumulative infiltration is consisting of two terms (the truncation of the series is done for giving the accurate solution) and is given by

$$F(t) = St^{\frac{1}{2}} + Kt$$

So, two terms are present in Philip's equation representing the cumulative infiltration and also infiltration rate. *S* is the sorptivity, a function of soil suction. And *K* is the hydraulic conductivity.

So, separately it is addressing two driving forces. The main forces which are causing the flow through unsaturated porous media is suction force and also gravity force. So suction force will be predominant when it is in a drier state, or it is in the unsaturated state. During unsaturated condition relatively dry soil, suction force will be very high, that is, the first term $St^{1/2}$ will be predominant.

As more and more water is infiltrating, pores will get filled up with water and the effect of the suction force reduces and the main driving force will be gravity. So that is represented by the second term *Kt*.

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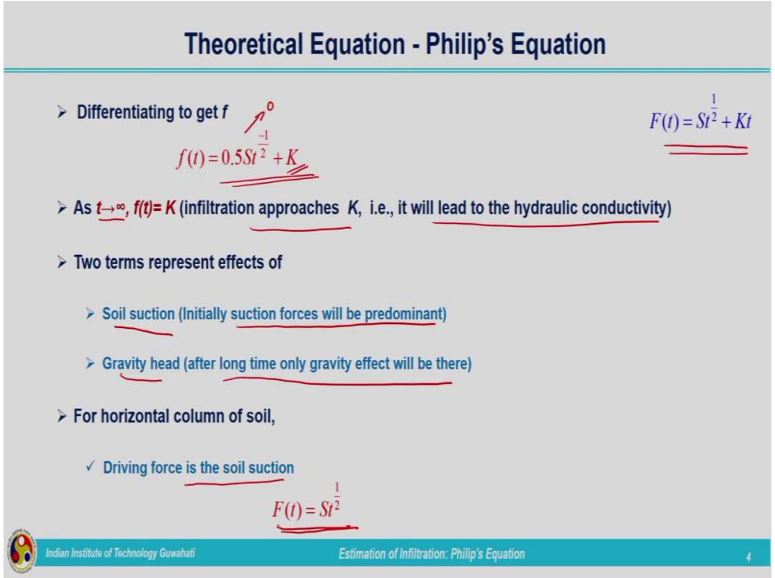

So this is our cumulative infiltration equation. For determining the infiltration rate we will be differentiating the cumulative infiltration curve.

$$f(t)=0.5St^{\frac{-1}{2}}+K$$

Now you look at the equation, as $t \rightarrow \infty$, that is t is increasing to a large value, then $f(t)=K$ or the infiltration rate approaches to hydraulic conductivity. When the soil is in the unsaturated condition or in the relatively dry state, just after the rainfall starts, the pores will start getting filled up with water, soil moisture will be increasing.

So initially, the suction forces will be predominant and as time passes, time increases to a certain value, the influence of first term will be very, very slow. The first term will be approaching to 0, and the total sum corresponding to infiltration rate will be hydraulic conductivity only.

So it will be at that time all the pores will be completely filled with water and the water will be flowing based on the action of gravity. Suction effects will be nullified at that time. So the two terms represent the effects of soil suction and gravity head; initially, suction forces will be predominant and after long time, only gravity effect will be there, suction effect will not be the because all the pores are completely filled with water.

The advantage of this equation is that it can be utilized in the case of horizontal soil also. In the case of horizontal soil, the flow will be taking place due to suction effect only. There will not be any action of gravity taking place for the flow to take place. So driving force is mainly soil suction in the case of horizontal soil column. We can write the cumulative infiltration as

$$F(t) = St^{\frac{1}{2}}$$

That is, we are having only the suction component, sorptivity term will be present, hydraulic conductivity term will not be present.

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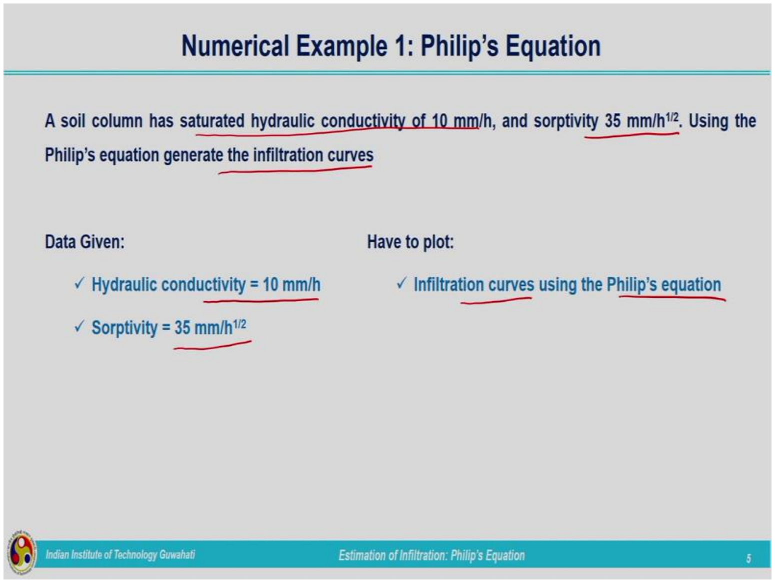

Now let us solve one numerical example related to Philip's equation.

Q- A soil column has saturated hydraulic conductivity of 10 mm/h, and sorptivity of 35 mm/h$^{1/2}$. Using Philip's equation, generate infiltration curves.

In this question, we are not given any value corresponding to cumulative infiltration or infiltration rate. In previous problems, you have been given the cumulative infiltration data. When we were solving Horton's equation or Kostiakov equation we have been given the infiltration data. But here in this case, infiltration data is not given to us. We have been given the soprtivity value and also hydraulic conductivity value.

We know the equation corresponding to Philip's equation, corresponding to cumulative infiltration, and also infiltration rate. By making use of these two values, that is, sorptivity and hydraulic conductivity and by making use of the equations corresponding to cumulative infiltration and infiltration rate, we can find out the infiltration rate curve and cumulative infiltration curve.

So the data given are, hydraulic conductivity K =10 mm/h, and S = 35 mm/h$^{1/2}$.

We have to plot infiltration curves using Philip's equation. Infiltration curves means infiltration rate curve and also cumulative infiltration curve.

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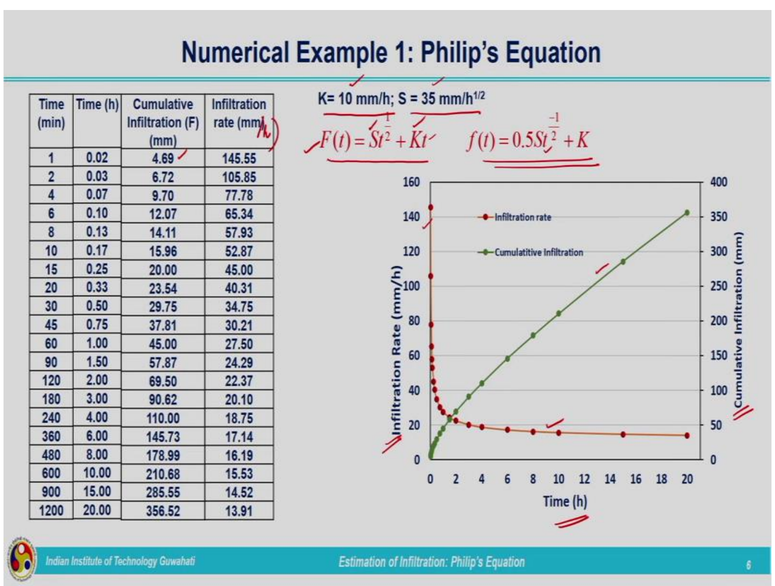

Time (min)	Time (h)	Cumulative Infiltration (F) (mm)	Infiltration rate (mm/h)
1	0.02	4.69	145.55
2	0.03	6.72	105.85
4	0.07	9.70	77.78
6	0.10	12.07	65.34
8	0.13	14.11	57.93
10	0.17	15.96	52.87
15	0.25	20.00	45.00
20	0.33	23.54	40.31
30	0.50	29.75	34.75
45	0.75	37.81	30.21
60	1.00	45.00	27.50
90	1.50	57.87	24.29
120	2.00	69.50	22.37
180	3.00	90.62	20.10
240	4.00	110.00	18.75
360	6.00	145.73	17.14
480	8.00	178.99	16.19
600	10.00	210.68	15.53
900	15.00	285.55	14.52
1200	20.00	356.52	13.91

So, we are going to assume the time starting from just greater than 0, that is, time in minutes we are taking, we are starting from 1 minute. Time is converted to hours, and we have been given the data corresponding to hydraulic conductivity and sorptivity.

Philip's equation corresponding to cumulative infiltration is given by,

$$F(t) = St^{\frac{1}{2}} + Kt$$

S is known to us, *K* is known to us, *t* we are assuming from 1 to 1,200 minutes. After substituting for *t* for different time intervals, we can calculate cumulative infiltration (mm) as listed in this table.

Now corresponding to infiltration rate, either we can calculate from the cumulative infiltration obtained, that is, incremental depth we can calculate, and incremental time also can be calculated. Incremental depth divided by incremental time will be giving you the infiltration rate. And another way is that directly making use of the Philip's equation for infiltration rate. So Philip's equation for infiltration rate is given by this,

$$f(t) = 0.5St^{\frac{-1}{2}} + K$$

So here also, *S* and *K* are known to us. And as we have done in the calculation of cumulative infiltration, here in this formula, we will substitute values corresponding to various *t*, and infiltration rate (mm/h) calculated as listed in this column,

Now, we can plot cumulative infiltration versus time and infiltration rate versus time. So this is the curve. Along the primary y-axis, infiltration rate is taken, and along the x-axis time is taken and along the secondary y-axis, cumulative infiltration is taken. The red curve describes our infiltration rate calculated using the Philip's equation and the green curve is representing the cumulative infiltration corresponding to the hydraulic conductivity K = 10 mm/h and S = 35 mm/h$^{1/2}$.

So this way, by making use of the formula, you can calculate the cumulative infiltration and also infiltration rate. And sometimes infiltration rate and cumulative infiltration, any one of these will be given to you. From that, you may have to calculate the Philip's parameters. So that can also be done as we have done in the case of Horton and Kostiakov equations. So I hope it is clear to you how the Philip's equation can be utilized for calculating cumulative infiltration and infiltration rate.

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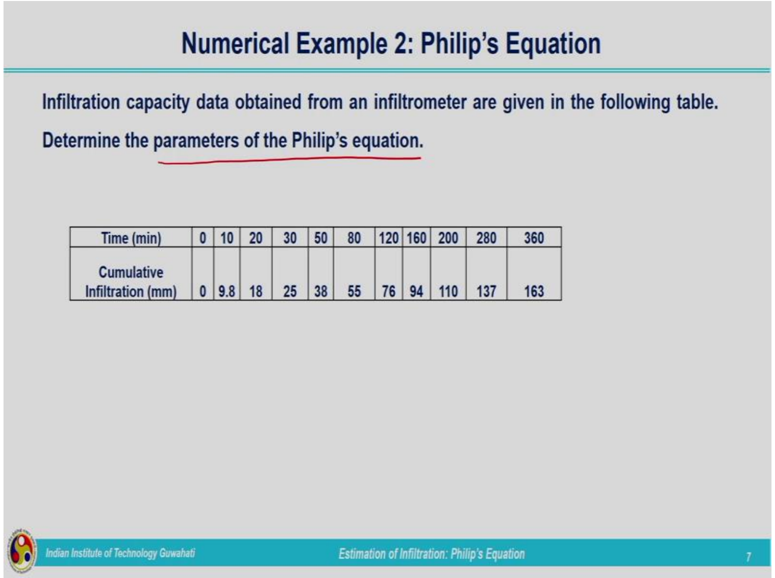
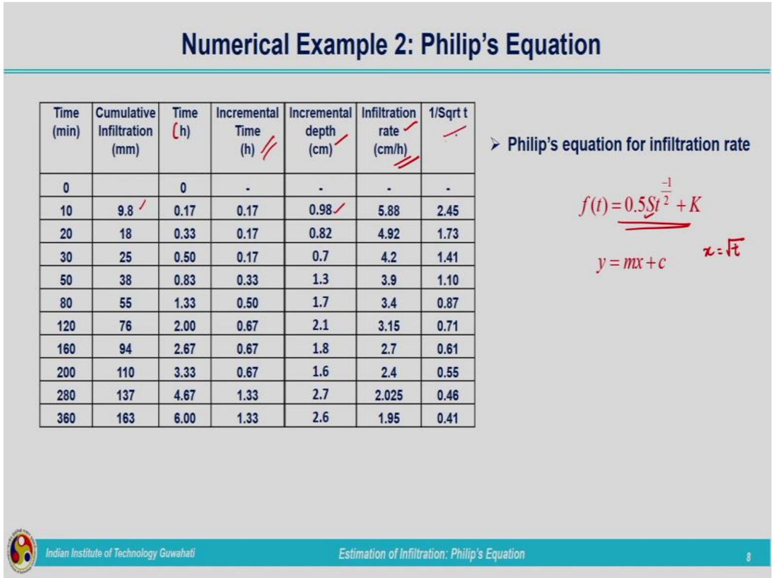


Now we can solve another example which can be utilized for finding out the Philip's parameters (K and S).

Q- Infiltration capacity data obtained from an infiltrometer are given in the following table. Determine the parameters of Philip's equation.

So the data given are shown in this table, time (min) and cumulative infiltration (mm). (Refer Slide Time: 16:33)



And time in hours is calculated, and incremental time in hours is calculated, as we have done in the case of previous couple of problems. And incremental depth (cm) corresponding to each time is calculated.

Infiltration rate (cm/h) (i.e., incremental depth divided by incremental time) is calculated and given over here in this column. So, we are having the infiltration rate and cumulative infiltration is already given to us.

Now, Philip's equation for infiltration rate is given by $f(t) = 0.5St^{\frac{-1}{2}} + K$

So, if we are plotting, $f(t)$ versus $1/\sqrt{t}$, it will be in the form of $y = mx + c$. So, once straight line is fitted, we can get the values corresponding to slope and intercept. From the slope you will get the value corresponding to sorptivity, and from the intercept you will be getting the value corresponding to hydraulic conductivity.

That is the reason why we have calculated infiltration rate over here. Cumulative infiltration is given to you. We have been asked to find out the Philip's parameters. So for getting the Philip's parameters, we have calculated the infiltration rate. And what we are going to plot? We are going to plot between infiltration rate versus $1/\sqrt{t}$.

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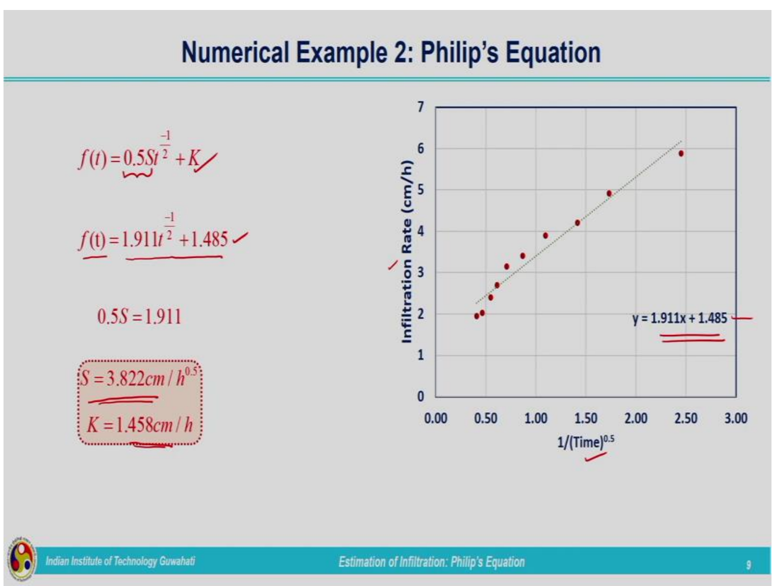

So this is the curve, data points are marked by the red dots, and with those data points, a straight line is fitted given by the equation $y = 1.911\, x + 1.485$.

So we can compare these two equation, this particular equation is written in the form of Philip's equation,

$$f(\mathrm{t}) = 1.911 t^{\frac{-1}{2}} + 1.485$$

So, $K = 1.485$ cm/h, and $0.5S = 1.911$ or $S = 3.822$ cm/h$^{1/2}$

So the units will be depending on our problem. In the question, whatever units you have taken, because this is an analytical solution, you do not have to worry about the units. So the same units, in which the data is there, based on that you will be getting the parameters also. So, if you are using the data in mm and time in minutes, the unit of hydraulic conductivity will be mm/min. So this way we can find out the parameters related to Philip's equation.

So this S and K are the important parameters corresponding to Philip's equation. Once these S and K are known to us, we can make use of those values for calculating the infiltration taking place in that particular soil. So these are the variety of problems related to it.

And next theoretical equation which we are going to discuss is Green-Ampt equation. That, we will see in the next lecture. We will derive it from the starting from the fundamental principles. It is the combination of continuity and momentum equation. Philip's made use of the Richard's equation, and Richard's equation solution is found out, and finally the equations have been given by making use of first two terms of the solution, infinite series. And in the case of Green-Ampt equation, we will start from continuity and momentum equation. That, we will see in the next lecture.

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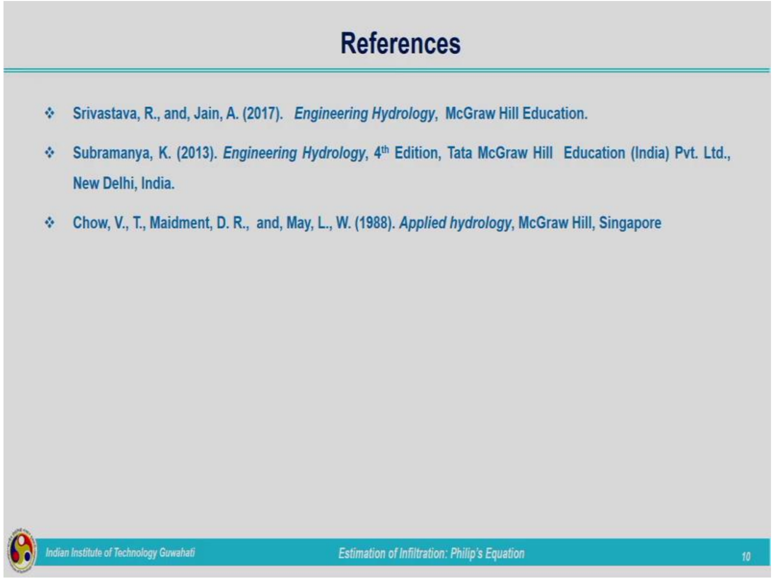

So related to this Philip's equation also, you can get so many problems from these textbooks. So here, I am winding up this lecture. Thank you.