Engineering Hydrology
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Lecture 56
Numerical examples on Unit Hydrograph

Hello, all. Welcome back. In the previous two lectures, we were discussing about the derivation of unit hydrograph and also direct runoff hydrograph based on linear system theory. We were making use of the principle of proportionality and principle of superposition for deriving the response of a catchment based on certain input rainfall data. Today, let us understand those concepts in detail by solving some of the numerical examples.

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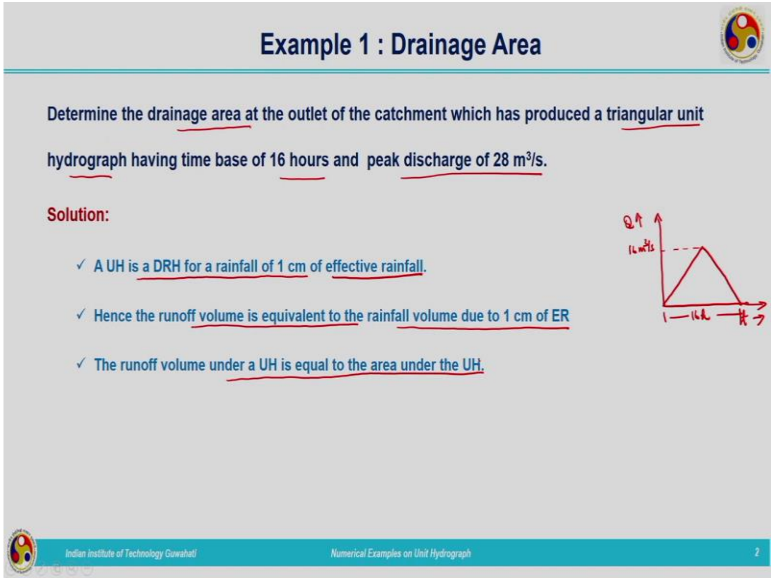

First example is to determine the drainage area. So, first let me read out the question. Determine the drainage area at the outlet of the catchment which has produced a triangular unit hydrograph having time base of 16 hours and peak discharge of 28-meter cube per second. We are having a catchment which has produced a runoff at the outlet. The details related to runoff is given to us, that is represented by means of a triangular unit hydrograph and the time base of the unit hydrograph is also given to us that is 16 hours. We need to find out the drainage area, the peak discharge is given to us. So, it is like this we are having a unit hydrograph represented by a triangle which is having a peak discharge of 16-meter cube per second and the time base is 16 hours. From this given data we need to calculate the drainage area. How can we calculate it?

Unit hydrograph is the response of the catchment for a unit rainfall that is 1 centimeter of rainfall which is acted on the catchment uniformly. if you are calculating the area under the curve, it will give you the volume corresponding to that particular rainfall. How much is the volume of water which is drained at the outlet of the catchment that can be obtained by taking the area under the curve. So, if you are having the volume of water which is drained at the outlet of the catchment that is representative of the area, that volume divided by the rainfall depth will be giving you the area of the catchment that is what we are going to do here.

A unit hydrograph is a direct runoff hydrograph for a rainfall of 1-centimeter effective rainfall. The runoff volume is equivalent to the rainfall volume due to 1 centimeter of effective rainfall that is after the deduction of initial abstractions from the total rainfall we are having an effective rainfall that is equivalent to 1 centimeter. So, the complete volume of water will be representing the volume of runoff. So, the runoff volume under a UH is equal to the area under the unit hydrograph.

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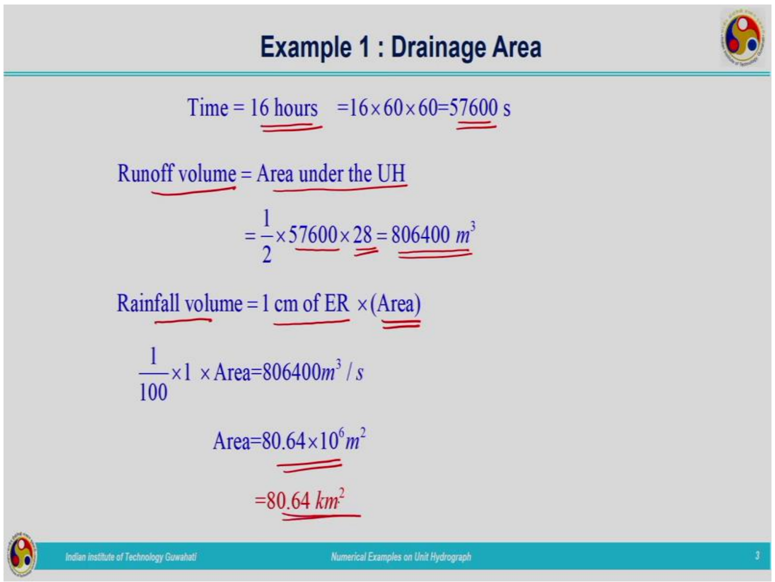

Here, the time base of the unit hydrograph is 16 hours that can be converted into seconds because the peak flow is given in meter cube per second. So, that is why we are converting it into seconds,

Time = 16 hours=$16\times 60\times 60$=57600 s

It will be coming out to be 57,600 seconds and we know runoff volume is equal to area under the unit hydrograph. Triangular unit hydrograph is there, area under that triangular unit hydrograph will be representing the runoff volume. So, we know the formula for calculating area of triangle. The time base is 57,600 and the peak is 28-meter cube per second. So, half into base into height it will be coming out to be

$$\text{Runoff volume} = \text{Area under the UH } = \frac{1}{2}\times 57600 \times 28 = 806400 \; m^3$$

So, rainfall volume will be 1 centimeter of effective rainfall multiplied by area. So, this rainfall volume and the runoff volume both are equal. This much of rainfall is producing runoff at the outlet. So, both should be equal. We can create these volumes and we can find out the drainage area as

$$\text{Rainfall volume} = 1 \text{ cm of ER } \times (\text{Area})$$

$$\frac{1}{100}\times 1 \;\times \text{Area}=806400 m^3 / s$$

$$\text{Area}=80.64\times 10^6 m^2 = 80.64 \; km^2$$

This is the case with the unit hydrograph. The same principle can be applied to the case of direct runoff hydrograph also.

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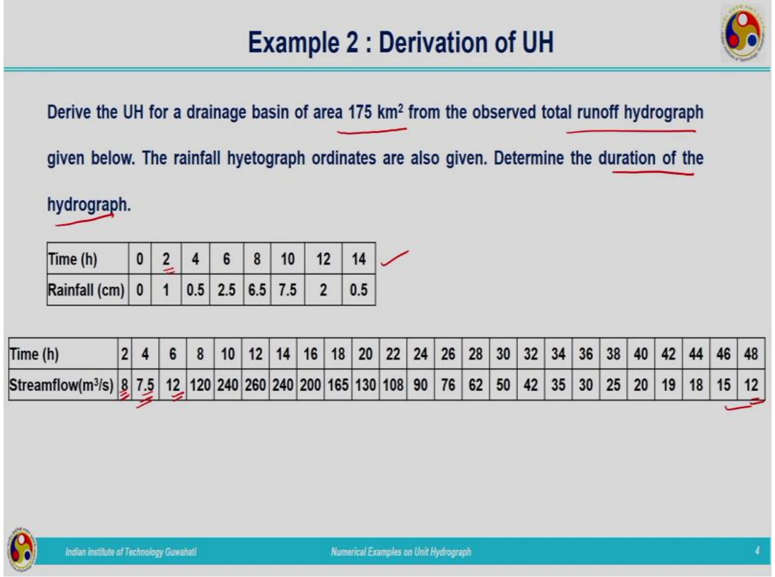

Now, we will move on to the second example, derivation of unit hydrograph. How to derive a unit hydrograph if we are having the streamflow data? Initially itself when explaining the theory related to unit hydrograph we have discussed about the data requirements for the derivation of unit hydrograph. We need to have the details related to streamflow and also the corresponding effective rainfall. Effective rainfall should be chosen in such a way that it is uniformly distributed for that particular duration over the entire catchment. So, it is very difficult to identify such an effective rainfall.

So, let us see how it can be done by with the help of this numerical example. Derive the unit hydrograph for a drainage basin of area 175 kilometers square from the observed total runoff hydrograph given below. The rainfall hyetograph ordinates are also given. Determine the duration of the hydrograph. So, you have been given the rainfall hyetograph and also the streamflow data. So, the rainfall hyetograph is given to you and the runoff hydrograph, total runoff hydrograph is also given to you. Look at the data carefully that is I am talking about the runoff data, you can see that it is starting from 2 hours to 48 hours and the streamflow data is varying from 8-meter cube per second to 12-meter cube per second. So, you can understand that here it is not starting with 0. We are having the rainfall hyetograph given to us and from that we need to determine the effective rainfall and also while looking into the runoff data it is having contribution from baseflow because before the rainfall itself there is some streamflow data present in the runoff hydrograph that is mainly because of the baseflow. So, first what we need to do we need to detect the baseflow from the runoff hydrograph for getting the direct runoff hydrograph. How to get the baseflow? Baseflow data will be either given to you or by looking into the runoff data you can understand what can be the possible baseflow. So, here you look at the rainfall data, it is starting it 2 hours. Here at 2 hours we are having streamflow. So, definitely at t is equal to 2-hour rainfall is starting, response we cannot observe at 2 hours, the response due to a rainfall at t is equal to 2 hours will be observed after that time only. So, this 8-meter cube per second is not the contribution from the rainfall. So, we can understand that this is the contribution coming from baseflow. Then here you can see at the 4^{th} hour there is a reduction in streamflow taking place that is baseflow recession is taking place. Now, look at the 6^{th} hour, you are having a streamflow of 12-meter cube per second. So, definitely you can understand that streamflow is increasing. So, from the data you can understand that streamflow is increasing from 6^{th} hour up to that whatever flow was there it was from the baseflow.

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Time (h)	Streamflow (m^3/s)	Direct Runoff (m^3/s)	UH Ordinates (m^3/s)
2	8		
4	7.5		
6	12	0	0.00
8	120	108	15.40
10	240	228	32.50
12	260	248	35.35
14	240	228	32.50
16	200	188	26.80
18	165	153	21.81
20	130	118	16.82
22	108	96	13.69
24	90	78	11.12
26	76	64	9.12
28	62	50	7.13
30	50	38	5.42
32	42	30	4.28
34	35	23	3.28
36	30	18	2.57
38	25	13	1.85
40	20	8	1.14
42	19	7	1.00
44	18	6	0.86
46	15	3	0.43
48	12	0	0.00

Sum= 1705 m^3/s

Volume of Direct Runoff= $1705 \times 2 \times 60 \times 60$

$$= 1.228 \times 10^7 m^3$$

Drainage Area = 175 km^2

$$\text{Depth of Direct Runoff} = \frac{1.228 \times 10^7}{175 \times 10^6} m$$

$$= 0.0701 m$$

$$= 7.01 cm$$

So, here we are finalizing a base flow of 12-meter cube per second. So, streamflow 8-meter cube per second, 7.5-meter cube per second at the time t is equal to 4 hours, these we can conclude that coming from baseflow. And when time t is equal to 6^{th} hours, 12-meter cube per second. So, that is there is a slight increase from 7.5 to 12-meter cube per second. So, what we are doing that, we are assuming that this 12-meter cube per second is the contribution from baseflow that we need to deduct from the total streamflow in order to get the direct runoff. So, if, in the question if the baseflow is given to you that has to be deducted. So, we will get direct runoff after detecting the baseflow of 12-meter cube per second from the streamflow.

So, you can understand that the response due to that particular effective rainfall is starting from 6^{th} hour, 6 to 8 hours it is increasing, 6 it is 0 and t is equal to 8 hours it is 108, when it comes to 48 hours again runoff is 0. So, this is representing the direct runoff at the outlet of the catchment.

Now, we can calculate the volume of the direct runoff. What is our aim? We are going to find out the unit hydrograph corresponding to this direct runoff hydrograph. So, we need to calculate the volume of runoff first. Why we are calculating? We need to have the understanding about the effective rainfall or the depth of the runoff which is produced at the outlet value. So, we are calculating sum of these direct runoff that is coming out to be 1705-meter cube per second. Why do we want to calculate the sum? Because our aim is to find out the volume of direct runoff. How can we get the volume of direct runoff? Volume of direct runoff can be obtained by taking the area under the curve. Here, we are having the direct runoff hydrograph, area under this direct runoff hydrograph will be giving you the volume of water corresponding to this direct runoff, that is for example, if I am drawing the direct runoff hydrograph like this. So, for example, let this be the direction of hydrograph and along the y axis it is the runoff in meters per second along the x axis it is the time in hours. So, you just multiply this, area under the curve if you are taking this is divided into small, small strips like this. So, each strip can be considered as a rectangle. Time varying from, here it is starting from 2, 4, 6 that way but all those ordinates corresponding to 2 hours, 4 hours and all as 0. It is starting from 6. So, 6 hours can be considered as 0. So, these areas small, small strips can be considered as rectangle and you can calculate the total area under the curve by summing up the areas of each strip. How can we get the area of this strip? This multiplied by this depth.

So, this is representing the ordinate of the DRH and this is representing the time. So, meter cube per second multiplied by time unit. We will be converting it into seconds, here the x axis is in hours that will be converted into seconds. So, meter cube per second multiplied by seconds it will be in meter cube. So, this area is representing the volume, that is why we are calculating the area under this runoff hydrograph, that area under this runoff hydrograph will be giving you the volume of water corresponding to this runoff at the outlet of the catchment.

For carrying out the area calculation, we have taken the sum of these ordinates, direct runoff ordinates at different, different times are there, sum of that we have taken and now we will multiply the sum of the ordinates with the time interval. Here, the time interval is 2 hours, 2 hours need to be converted it into seconds. So, volume of direct runoff is given by

$$\text{Volume of Direct Runoff} = 1705 \times 2 \times 60 \times 60 = 1.228 \times 10^{7}\, m^{3}$$

This much volume of water is measured at the outlet of the watershed. The drainage area is given to you that is the catchment area is 175 kilometers square. We are having the volume of water drained at the outlet of the catchment and we are having the area contributing that flow. So, definitely corresponding runoff depth can be calculated by dividing the volume by the area of the catchment. So, that is depth of direct runoff can be obtained by dividing this volume of direct runoff with drainage area. So, that can be calculated as

$$\text{Depth of Direct Runoff} = \frac{1.228 \times 10^{7}}{175 \times 10^{6}}\, m = 0.0701 m = 7.01 cm$$

So, the depth of direct runoff is 7.01 centimeters. What we have done? We have considered the given runoff data, by observing the data we could understand that it is giving the storm hydrograph we need to develop the direct runoff hydrograph for that we have identified the baseflow contribution by comparing with the rainfall data and that baseflow is deducted from the ordinates of the storm hydrograph in order to develop the direct runoff hydrograph. The area under the direct runoff hydrograph will be giving you the volume of water drained at the outlet of the catchment. Once volume of water which is drained at the outlet of the catchment is calculated that divided by the catchment area will be giving you the depth of direct runoff. That is what we have calculated here as 7.01 centimeters.

Now, what we are going to do we will be dividing each ordinate of direct runoff hydrograph with this direct runoff depth that we will be providing you the ordinates of unit hydrograph. So, here what principle we have applied, we have made use of the principle of proportionality that is we are having the solution that multiplied with a certain constant value will also be the solution of the particular problem. So, here we will be dividing the direct runoff ordinates with this direct runoff depth. So, that is given here in this column that is the unit hydrograph ordinates. So, each and every ordinate of direct runoff hydrograph is divided by the direction of depth to get the unit hydrograph ordinates. Here, we have determined the unit hydrograph from the direct runoff hydrograph. Initially, storm hydrograph is given to you, we have developed the DRH from there by finding out the direct runoff depth and after dividing the ordinates, dividing each coordinate with the direct runoff depth, we have found out the unit hydrograph.

Now, what will be the duration of this unit hydrograph because the duration of the direct runoff hydrograph is not given to us. The observed streamflow data is collected, based on that we have derived the unit hydrograph. We have been given the rainfall data from that we need to identify what will be the duration of the unit hydrograph which is developed now.

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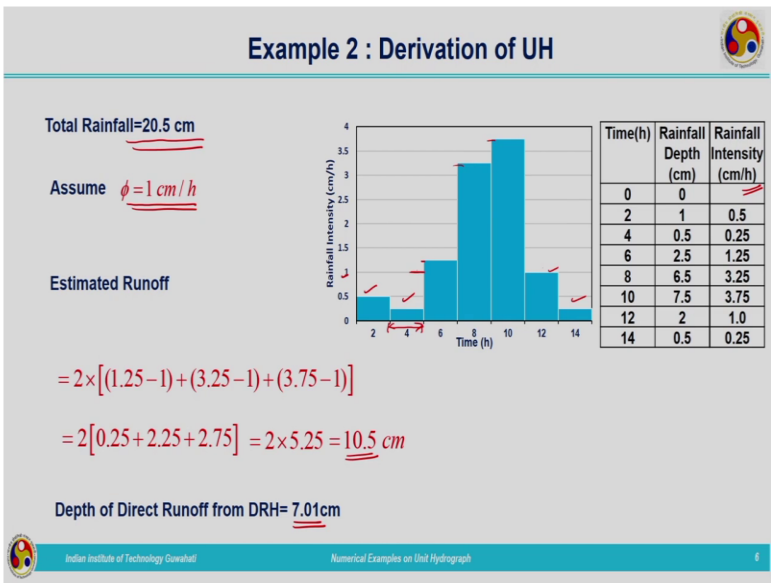

The rainfall data is given to us, from this we need to calculate the intensity of rainfall. This you know already. So, rainfall intensity is calculated, the unit of this is centimeters per hour. Now, if we are plotting the rainfall hyetograph it will be looking like this, rainfall intensity along the y axis and time along the x axis. This is representing the total rainfall, which is given to us. We need to first identify what is the effective rainfall, that effective rainfall can be obtained by detecting the abstractions, initial abstractions from the total rainfall. So, we need to first find out the technique for finding out the initial abstraction and after finding out the initial abstraction we have to deduct that from the rainfall data, total rainfall depth to get the effective rainfall. So, for that here we are going to make use of φ-index method.

We are having the total rainfall 20.5 centimeters. We will assume a φ-index of 1 centimeter per hour. So, we can compute the runoff, estimated runoff or the direct runoff from this rainfall will be, if we are assuming φ is equal to 1 centimeter per hour all these rainfall pulses will be going as abstractions. Above this 1 centimeter per hour whatever coming that will be contributing towards direct runoff.

$$\text{Estimated Runoff } = 2\times\left[(1.25-1)+(3.25-1)+(3.75-1)\right]$$

$$=2\left[0.25+2.25+2.75\right]=2\times5.25=10.5\ cm/h$$

So, the runoff is coming out to be 10.5 centimeters. So, if you are assuming a φ -index value 1 centimeters per hour we can compute the runoff at the outlet as 10.5 centimeters, but what is the value of direct runoff depth, which we have computed in the previous slide based on the runoff volume that is equal to 7.01 centimeters. So, that is less than that of the estimated runoff depth based on φ -index value. So, what we have to do, we have to change the assumed phi-index value. 1 centimeter per hour is producing a runoff of 10.5 centimeters which is more than that of the estimated value from the hydrograph. So, what we have to do we have to reduce the estimated runoff depth. We need to get a less value of runoff. For that we need to increase the φ -index value.

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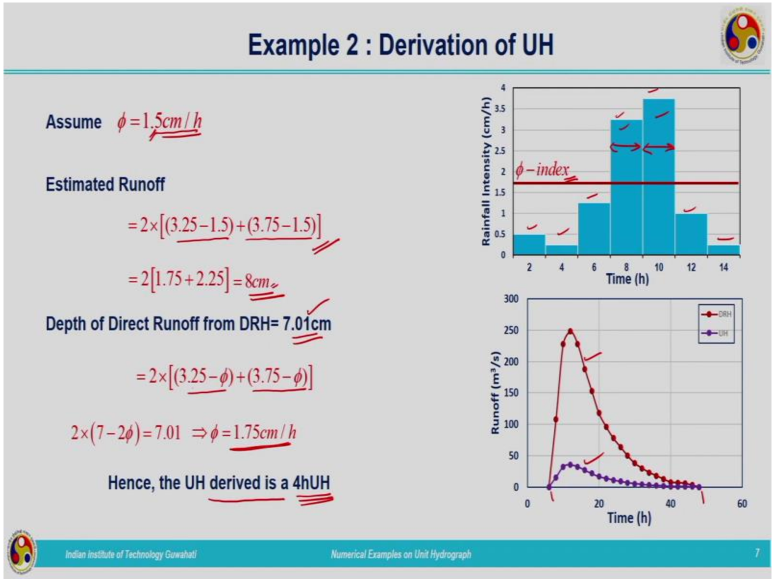

We will increase the φ-index value to 1.5 centimeters per hour. If φ-index value is 1.5 centimeters per hour, all these pulses will be going as abstractions, only these two pulses will be contributing runoff. So, we can compute the runoff depth as

$$\text{Estimated Runoff} = 2\times\left[(3.25-1.5)+(3.75-1.5)\right]$$
$$= 2\left[1.75+2.25\right] = 8cm$$

So, it can be calculated as 8 centimeters. Again, it is high. Depth of direct runoff from DRH is 7.01 centimeter. So, again we need to iterate on the value corresponding to φ-index. But we know the depth of direct runoff is 7.01. Here it is 8 centimeters. So, estimated runoff is 8 centimeter and the value corresponding to runoff depth from the DRH is 7.01 centimeters. So, these are approximately coming nearer to each other. So, we can fix the interval for φ-index because there is not much difference between these runoff depth values or you have to go for identifying the interval first, then we need to find out the φ-index. So, here approximately I can tell that the interval is correct, now, we need to get the φ-index value. So, what we can assume this excess rainfall is equivalent to direct runoff depth computed based on DRH. So,

$$\text{Estimated Runoff} = 2\times\left[(3.25-\phi)+(3.75-\phi)\right]$$
$$2\times\left(7-2\phi\right) = 7.01$$
$$\phi = 1.75cm/h$$

So, it is coming out to be 1.75 centimeter because here when it was 1.5 centimeters, it was coming out to be 8 centimeters. So, φ-index value should be increased then only the estimated runoff depth will be reduced, that way we have found out the interval with which the φ-index will be lying and we have equated it with 7.01 that is the direct run of depth which is computed based on DRH and we found out the exact value corresponding to φ-index. So, that is coming out to be 1.75 centimeters per hour. This is the value representing the initial abstractions. Now, above this what are the pulses which is contributing to effective rainfall we are having these two, that is corresponding to 3.25 and 3.75 centimeters per hour of rainfall. So, there is not much difference, even though, actually based on the assumption under unit hydrograph the rainfall should be uniformly distributed for the particular duration. Here initial 2 hours it is uniformly distributed and again increased to 3.75 centimeters per hour, there is a difference of 0.5 centimeters per hour in the rainfall intensity. That much approximation is considerable, so that is why we can assume that the effective rainfall which is producing a runoff depth of 7.01 is due to these two pulses of effective rainfall. So, the derived unit hydrograph can be considered as a 4-hour unit hydrograph because, this is 2-hour pulse and this is also off 2-hour pulse. So, total rainfall which is produced a unit hydrograph is based on 4-hour rainfall. Even though the pulses are of slightly different values, we can assume that it is uniformly distributed over that interval. Actually, both should be of same interval then we can tell this unit hydrograph is having a duration of 4 hours. But in actual practice it is very difficult to get the uniform interval to get the duration having rainfall uniformly distributed within that time interval. So, we are assuming that this is approximately fine and the duration of unit hydrograph can be considered as 4 hours. So, this is the way in which we will be deriving unit hydrograph based on the direct runoff hydrograph. So, whenever we are going to derive a unit hydrograph for a particular catchment, we need to have the streamflow records and the corresponding rainfall data which is of uniformly distributed between certain duration, then only we can determine the unit hydrograph of certain duration.

So, these are the steps which we will be utilizing for deriving the unit hydrograph from the direct runoff hydrograph. Otherwise, if you are having already idea about the duration of the direct runoff hydrograph, the duration of the unit hydrograph can be considered as the same if it is produced due to single pulse. Otherwise, we have to go for deconvolution principle to develop the unit hydrograph if the direct runoff hydrograph is produced due to effective rainfall having different pulses.

Now, we can plot the unit hydrograph and DRH. This is the DRH which is given in the question and this is the DRH develop based on the given streamflow data and the corresponding UH unit hydrograph is given by this plot. So, if you look at the plot you can understand that both are having the same time base and the duration of the unit hydrograph is found out to be 4 hours based on the effective rainfall.

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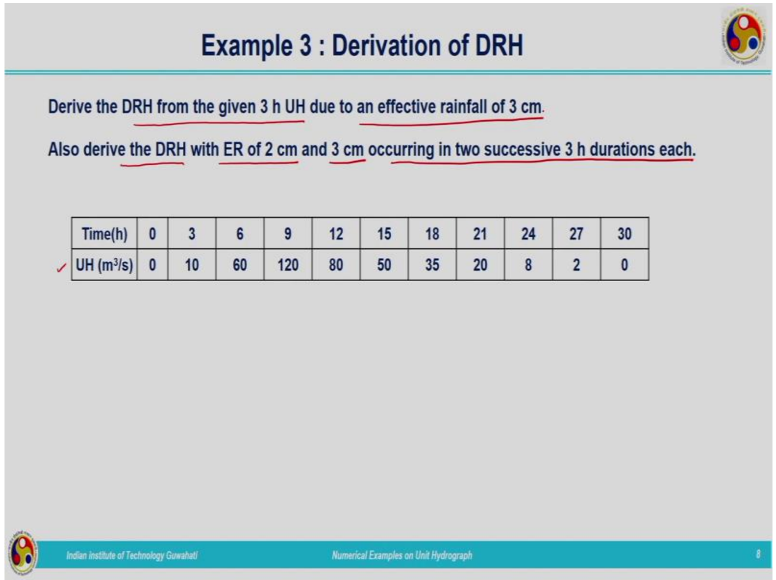

Now, we will move on to the next example, derivation of direct runoff hydrograph. If unit hydrograph is given to you how to obtain the direct runoff hydrograph, that is based on the discrete convolution equation. In the previous example, we have made use of the principle of proportionality. Here, what we will be doing we will be making use of principle of proportionality and also superpositions.

Let me read out the question. Derive the DRH from the given 3-hour unit hydrograph due to an effective rainfall of 3 centimeters. Also, derive the DRH with ER of 2 centimeter and 3 centimeter occurring in two successive 3-hour durations each. The data given are 3-hour unit hydrograph is given to you, time is varying from 0 to 30 hours and unit hydrograph ordinates in meter cube per second is given to you.

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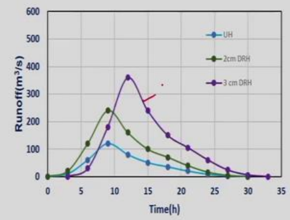

Time (h)	UH (m^3/s)	DRH due to ER=2cm (m^3/s)	DRH due to ER=3cm (m^3/s)
0	0	0	-
3	10	20	0
6	60	120	30
9	120	240	180
12	80	160	360
15	50	100	240
18	35	70	150
21	20	40	105
24	8	16	60
27	2	4	24
30	0	0	6
33	-	-	0

So, for the derivation of DRH, the data is again represented over here in these columns. These are given data only, time in hours and unit hydrograph ordinates in meter cube per second. First, we need to find out the direct runoff hydrograph corresponding to 2 centimeters of rainfall. So, we will be applying the principle of proportionality here and we will find out the response of the catchment for an effective rainfall of 2 centimeters. So, we are just multiplying the ordinates of unit hydrograph with 2 because the unit hydrograph ordinates are the response of 1-centimeter rainfall. Here we need to find out the DRH due to 2 centimeters of rainfall, just multiplying the ordinates of the UH by 2 centimeters. So, these are the data ordinates which are computed, simply multiplying by making use of the principle of proportionality. So, if we plot the 2-centimeter DRH, it will be like this, ordinates are increased, we have multiplied the ordinates of unit hydrograph with 2 but the time base remains the same, because single pulse input is considered 2 centimeters of input is considered and corresponding to that we have developed the DRH. So, the time base is not going to change. Another pulse is contributing runoff and if you are computing the total response of the catchment, then the time base of the direct runoff hydrograph will be different than that of the unit hydrograph because it is a combination of two different unit hydrographs. First part is over.

First part of the question was to determine the direct runoff hydrograph from 2 centimeters of effective rainfall. Second part is that 2 centimeters of effective rainfall is occurred and after that 3 centimeters of rainfall occurred with a 3-hour gap. So, we need to find out the DRH due to effective rainfall of 3 centimeters which has occurred after 3 hours. So, it will be starting after 3 hours only. So, what we have done, we have multiplied the ordinates of unit hydrograph with 3 and lagged it by that much time. So, here the lagging is done by 3 hours. So, you can see, it has not started from 0 over here, it has started at 3-hour lag. So, this is the direct runoff hydrograph corresponding to 3 centimeters.

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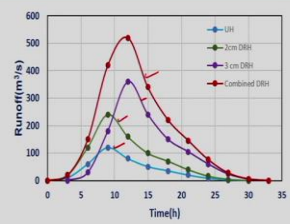
Example 3 : Derivation of DRH

Time (h)	UH (m^3/s)	DRH due to ER=2cm (m^3/s)	DRH due to ER=3cm (m^3/s)	Combined DRH (m^3/s)
0	0	0	-	0
3	10	20	0	20
6	60	120	30	150
9	120	240	180	420
12	80	160	360	520
15	50	100	240	340
18	35	70	150	220
21	20	40	105	145
24	8	16	60	76
27	2	4	24	28
30	0	0	6	6
33	-	-	0	0



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Now, we can compute the combined direct runoff hydrograph by making use of the principle of superposition. What we have done we have added the ordinates of direct runoff hydrograph due to 2 centimeters and DRH due to 3 centimeters. So, if you are plotting the combined DRH, you can understand that the combined DRH is represented by this red graph. So, you can see the time base of the combined DRH from 0 to 33, it is 33 hours, or when we find out the combined direct runoff hydrograph the time base will be changing. It is the combined response of the 2-centimeter effective rainfall and 3-centimeter effective rainfall. So, that is what is plotted over here. So, combine DRH can be derived by making use of the principle of proportionality and also principle of superposition.

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So, various numerical examples can be obtained from these textbooks. So, here I am winding up this problem-solving session. Thank you.