Engineering Hydrology
Dr. Sreeja Pekkat
Department of Civil Engineering
Indian Institute of Technology, Guwahati
Module 5 - Lecture 68
Numerical Example on Reservoir Routing

Hello all, welcome back. In the previous lecture we were discussing about reservoir routing. Today we will solve one numerical example related to reservoir routing.

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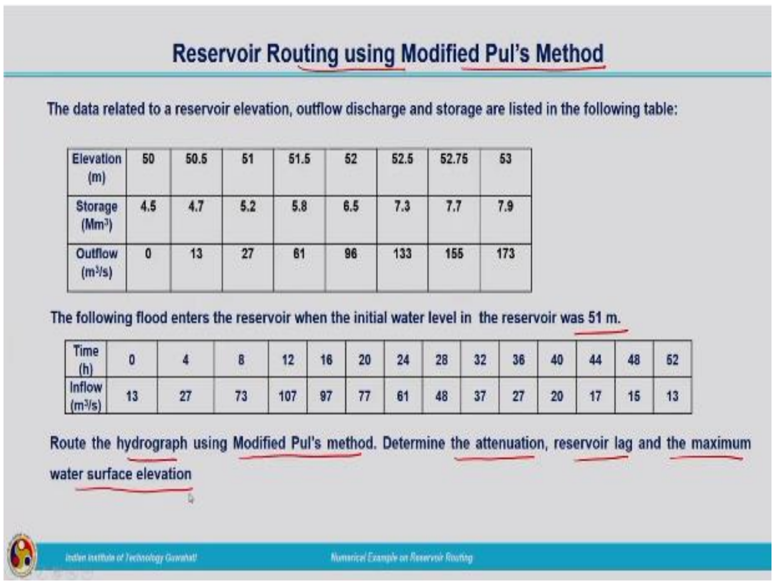

This example is related to reservoir routing using Modified Pul's method. First let me read out the question. The data related to a reservoir elevation, outflow discharge and storage are listed in the following table. Elevation, storage, outflow these details are given to you. The following flood enters the reservoir when the initial water level in the reservoir was 51 meters. The inflow hydrograph details are given to you, inflow versus time data is there. Route the hydrograph using Modified Pul's method. Determine the attenuation, reservoir lag and the maximum water surface elevation. We have been given the details related to storage, outflow with respect to elevation and also inflow hydrograph is given to you. Now, we need to route the flood and we need to find out the outflow hydrograph. After finding out the outflow hydrograph we need to find out how much is the reduction taken place in the peak values and also reservoir lag and also the water surface elevation.

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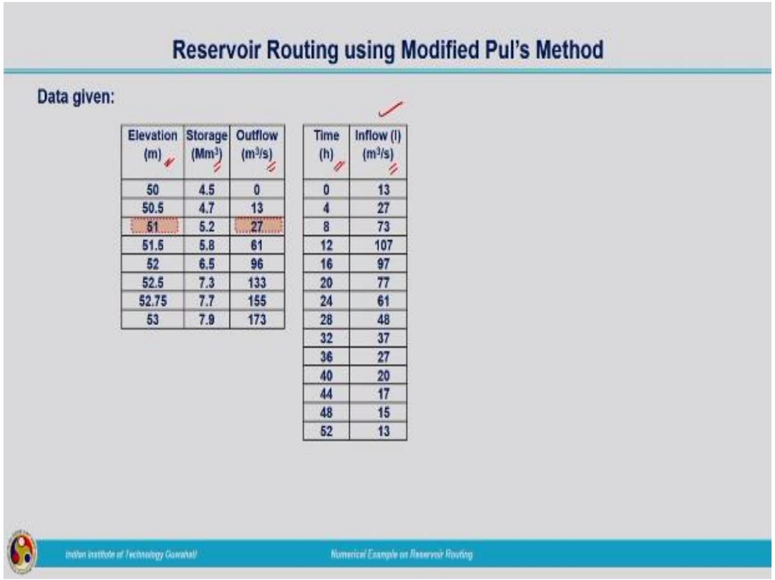

Elevation (m)	Storage (Mm^3)	Outflow (m^3/s)
50	4.5	0
50.5	4.7	13
51	5.2	27
51.5	5.8	61
52	6.5	96
52.5	7.3	133
52.75	7.7	155
53	7.9	173

Time (h)	Inflow (I) (m^3/s)
0	13
4	27
8	73
12	107
16	97
20	77
24	61
28	48
32	37
36	27
40	20
44	17
48	15
52	13

Given data are: same data tabulated there in the first slide is repeated here: elevation in meter, storage million-meter cube, outflow meter cube per second, then we are given with the inflow hydrograph time in hours, inflow *I* in meter cube per second. While discussing about reservoir routing we have seen we are making use of the continuity equation along with some storage function, that is storage is a function of water surface elevation and outflow is also a function of water surface elevation. From that we need to find out a relationship between *Q* and storage. So, that is the storage function, storage is represented as a function of outflow in the case of reservoir routing and then we will be making use of the continuity equation for finding out the outflow hydrograph. In the question it is also given that this inflow hydrograph or the inflow is entering the reservoir when the elevation is at 51 meters and the corresponding outflow is 27-meter cube per second. When the reservoir elevation is 51 meters this flood enters, till that it was in a steady condition.

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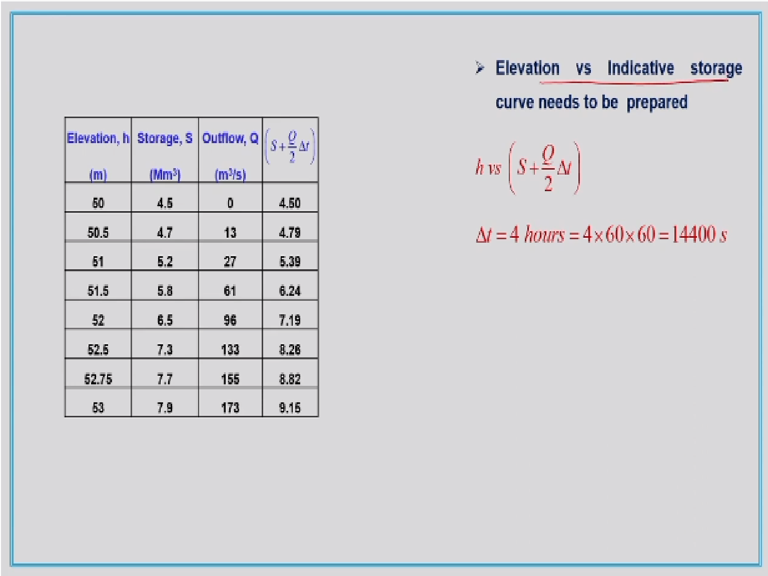

Elevation, h (m)	Storage, S (Mm³)	Outflow, Q (m³/s)	$\left(S+\frac{Q}{2}\Delta t\right)$
50	4.5	0	4.50
50.5	4.7	13	4.79
51	5.2	27	5.39
51.5	5.8	61	6.24
52	6.5	96	7.19
52.5	7.3	133	8.26
52.75	7.7	155	8.82
53	7.9	173	9.15

Now, we will proceed for solving the example. First, we have to plot the elevation versus indicative storage curve. What is indicative storage? Indicative storage is $\left(S+\frac{Q}{2}\Delta t\right)$. So that is h versus $\left(S+\frac{Q}{2}\Delta t\right)$ curve has to be derived, h is given to us, S is given to us, Q is given to us. So, these are the data given to us, for varying elevation corresponding storage and outflow Q are given storage is given in million-meter cube and outflow is given in meter cube per second. So, you have to be careful about the units while calculating the indicative storage. We can calculate indicative storage $\left(S+\frac{Q}{2}\Delta t\right)$, so S is in million-meter cube and outflow Q is in meter cube per second, this unit conversion should be properly done while carrying out the calculation. Inflow hydrograph which is given to us is with an interval of 4 hours, so we will consider Δt to be 4 hours. 4 hours that can be converted into seconds because our outflow is in meter cube per second that is

$$\Delta t = 4\ hours = 4\times 60\times 60 = 14400\ s$$

So, $\frac{Q}{2}\Delta t$ we need to calculate, that is why Δt we are converting it into seconds. So, we can compute the indicative storage and it is listed in this column. So, $\left(S+\frac{Q}{2}\Delta t\right)$ is with us. Now, what we are going to do, we are going to plot the curve elevation versus indicative storage, h versus $\left(S+\frac{Q}{2}\Delta t\right)$ curve will be plotted.

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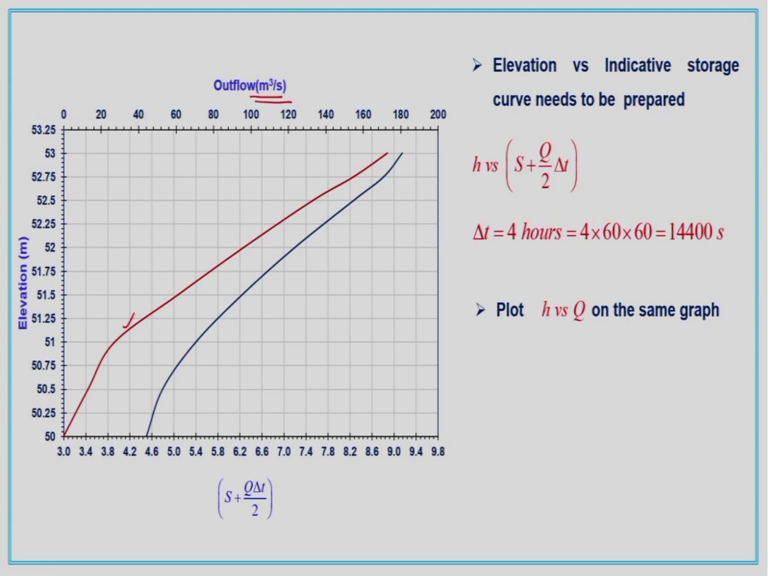

So, this is the curve representing elevation versus $\left(S+\frac{Q}{2}\Delta t\right)$, that is our indicative storage. Elevation is plotted along the y axis and indicative storage along the x axis, $\left(S+\frac{Q}{2}\Delta t\right)$ along the x axis. So, the curve is plotted here like this. Now we have to plot h versus Q on the same graph, outflow elevation relationship is there, that we are going to plot here in the same graph. So, that is plotted here, this red curve is representing the elevation versus outflow, but here one thing you need to be careful, we are plotting the outflow along the secondary x axis, indicative storage is plotted along the primary x axis and outflow is plotted along the secondary x axis, elevation axis is the same, not separately we are making because elevation is the same for both storage and the discharge. So, here we are having both the curves, that is the elevation versus indicative storage and also elevation versus outflow. Now, we can go for calculation related to reservoir routing.

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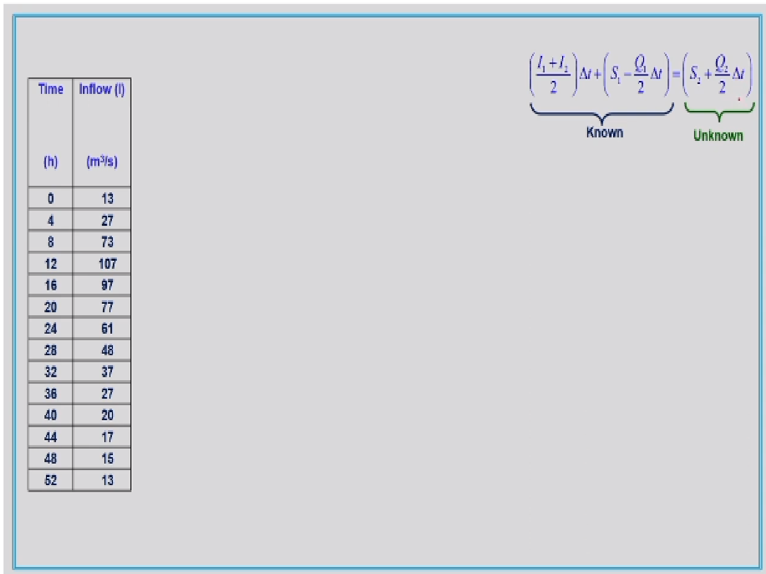

Time (h)	Inflow (I) (m³/s)
0	13
4	27
8	73
12	107
16	97
20	77
24	61
28	48
32	37
36	27
40	20
44	17
48	15
52	13

So, we will consider inflow hydrograph, time in hours and inflow in meter cube per second is given to you. We have made certain rearrangements with the terms of continuity equation and we got the equation in this form with the known values on one side and unknown values on the other side, i.e.,

$$\left(\frac{I_1+I_2}{2}\right)\Delta t+\left(S_1-\frac{Q_1}{2}\Delta t\right)=\left(S_2+\frac{Q_2}{2}\Delta t\right)$$

So, this $\left(S_2+\frac{Q_2}{2}\Delta t\right)$ is unknown. This is our indicative storage. So, with the known storage elevation data and the Q elevation data, discharge elevation data we have plotted a curve with the elevation versus indicative storage and elevation versus the outflow data. By making use of that curve and also with the known data at time t is equal to 0, initial values are known to us related to storage and outflow and also the inflow flood hydrograph we will further calculate the required values.

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$$\left(\frac{I_1+I_2}{2}\right)\Delta t+\left(S_1-\frac{Q_1}{2}\Delta t\right)=\left(S_2+\frac{Q_2}{2}\Delta t\right)$$

Time (h)	Inflow (I) (m^3/s)	$\left(\frac{I_1+I_2}{2}\right)$ (m^3/s)	$\left(\frac{I_1+I_2}{2}\right)\Delta t$ (Mm^3)	$\left(S_2-\frac{Q_2\Delta t}{2}\right)$ (Mm^3)	$\left(S_2+\frac{Q_2\Delta t}{2}\right)$ (Mm^3)	h (m)	Outflow, Q (m^3/s)
0	13					51.0	27
4	27	20.00	0.288	5.0056	5.29		
8	73	50.00	0.72				
12	107	90.00	1.29				
16	97	102.00	1.47				
20	77	87.33	1.26				
24	61	69.33	0.99				
28	48	54.67	0.79				
32	37	42.67	0.61				
36	27	32.00	0.46				
40	20	23.33	0.34				
44	17	18.67	0.27				
48	15	16.00	0.23				
52	13	14.00	0.20				

So, this is the table required for us. In the equation we are having $\left(\frac{I_1+I_2}{2}\right)\Delta t$ we have already assumed and $\left(S_1-\frac{Q_1}{2}\Delta t\right)$ and $\left(S_2+\frac{Q_2}{2}\Delta t\right)$. So, incorporating these terms we have made different columns $\left(\frac{I_1+I_2}{2}\right)$ will be calculating but our term is $\left(\frac{I_1+I_2}{2}\right)\Delta t$ and we are having the second known term is $\left(S_2-\frac{Q_2\Delta t}{2}\right)$ and the unknown is $\left(S_2+\frac{Q_2\Delta t}{2}\right)$. Elevation h and outflow Q corresponding to initial condition we will be taking from the given data. So, we can calculate the $\left(\frac{I_1+I_2}{2}\right)$, inflow hydrograph is known to us, so getting the values corresponding to this column is not difficult. So, we can calculate first entry related to $\left(\frac{I_1+I_2}{2}\right)$. So, the column 3 related to $\left(\frac{I_1+I_2}{2}\right)$ can be calculated. So, this is listed here in this column and after that we will multiply $\left(\frac{I_1+I_2}{2}\right)$ with Δt, that is listed here it will be in million-meter cube.

Now, next step is to calculate $\left(S_2 - \frac{Q_2 \Delta t}{2}\right)$. So, the first entry related to $\left(S_2 - \frac{Q_2 \Delta t}{2}\right)$ is regarding the initial condition, when elevation h is equal to 51 meters, inflow wave or flood wave is entering the reservoir. So, at that time initially what is the water surface elevation that is the initial condition we are considering, corresponding to that what storage is there and also outflow is there that is considered as the outflow Q and storage S for time t is equal to 0. So, the h is 51 meters and corresponding Q is 27-meter cube per second. So, based on these values and the storage corresponding to 51 meter we will calculate the value of $\left(S_2 - \frac{Q_2 \Delta t}{2}\right)$, that is coming out to be 5.01-million-meter cube. Now, from this equation look at this equation $\left(\frac{I_1 + I_2}{2}\right)\Delta t + \left(S_1 - \frac{Q_1}{2}\Delta t\right)$ is giving us the value corresponding to $\left(S_2 + \frac{Q_2}{2}\Delta t\right)$. So, we will add the values in this column $\left(\frac{I_1 + I_2}{2}\right)\Delta t$ and $\left(S_2 - \frac{Q_2 \Delta t}{2}\right)$ will be added together as 5.29, corresponding to our indicative storage $\left(S_2 + \frac{Q_2}{2}\Delta t\right)$.

Now, what is the next step, we will move on to our graph, $\left(S_2 + \frac{Q_2}{2}\Delta t\right)$ that is our indicative storage is plotted against the elevation, in the same graph we have plotted the outflow also. So, we will make use of this value $\left(S_2 + \frac{Q_2}{2}\Delta t\right)$ as 5.29.

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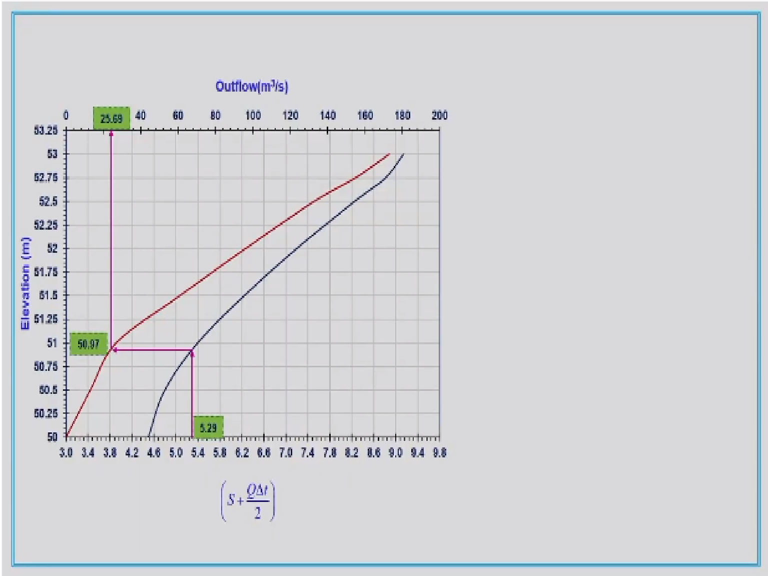

We will look into the graph, so $\left(S_2 + \frac{Q_2}{2}\Delta t\right)$ is 5.29 that will be coming to be here as marked and from there we will take the value or we will find out the value corresponding to *h* or the elevation. Elevation is found out to be 50.97 and this elevation that is corresponding to this elevation 50.97 what is the outflow value, that we will take from the second curve, outflow is coming out to be 25.69. So, what we have done for the initial value at *t* is equal to 0 at the moment when the flood wave is entering the reservoir, we have considered it as the initial condition corresponding to storage and *Q*, based on that storage and *Q* value, we have found out the value corresponding to $\left(S - \frac{Q}{2}\Delta t\right)$. By adding the terms on the left-hand side of the equation we got the indicative storage corresponding to that, that indicative storage is taken and we have made use of the graph plotted and we got the elevation and outflow corresponding to that indicative storage.

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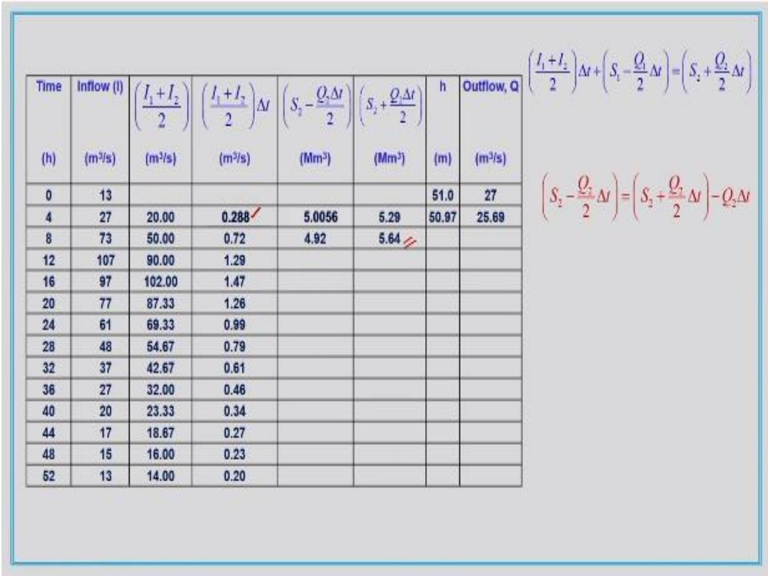

Time (h)	Inflow (I) (m^3/s)	$\left(\frac{I_1+I_2}{2}\right)$ (m^3/s)	$\left(\frac{I_1+I_2}{2}\right)\Delta t$ (m^3/s)	$\left(S_2-\frac{Q_2\Delta t}{2}\right)$ (Mm^3)	$\left(S_2+\frac{Q_2\Delta t}{2}\right)$ (Mm^3)	h (m)	Outflow, Q (m^3/s)
0	13					51.0	27
4	27	20.00	0.288	5.0056	5.29	50.97	25.69
8	73	50.00	0.72	4.92	5.64		
12	107	90.00	1.29				
16	97	102.00	1.47				
20	77	87.33	1.26				
24	61	69.33	0.99				
28	48	54.67	0.79				
32	37	42.67	0.61				
36	27	32.00	0.46				
40	20	23.33	0.34				
44	17	18.67	0.27				
48	15	16.00	0.23				
52	13	14.00	0.20				

$$\left(\frac{I_1+I_2}{2}\right)\Delta t+\left(S_1-\frac{Q_1}{2}\Delta t\right)=\left(S_2+\frac{Q_2}{2}\Delta t\right)$$

$$\left(S_2-\frac{Q_2}{2}\Delta t\right)=\left(S_2+\frac{Q_2}{2}\Delta t\right)-Q_2\Delta t$$

So, that is put over here, the indicative storage was 5.29, corresponding to that the elevation was found out to be 50.97 and the outflow is 25.69-meter cube per second. Now, this outflow is utilized for calculating the value of $\left(S_2-\frac{Q_2\Delta t}{2}\right)$ for the next time.

Now, we will move on to the calculation of $\left(S_2-\frac{Q_2\Delta t}{2}\right)$ from this $\left(S_2+\frac{Q_2}{2}\Delta t\right)$. How can we get $\left(S_2-\frac{Q_2\Delta t}{2}\right)$? $\left(S_2-\frac{Q_2\Delta t}{2}\right)$ is nothing but $\left(S_2+\frac{Q_2}{2}\Delta t\right)-Q_2 t$. So, with this value with this 5.29 value we will subtract $Q_2\Delta t$, in this Q_2 is the new Q_2 which we have found out from the graph that is 25.69, Δt is corresponding to 4 hours. So, that can be calculated as 4.92. 4.92 is the value corresponding to $\left(S_2-\frac{Q_2\Delta t}{2}\right)$. Now, we will repeat the step that is we will add 0.72 and 4.92 to get the value corresponding to $\left(S_2+\frac{Q_2}{2}\Delta t\right)$ that is 5.64.

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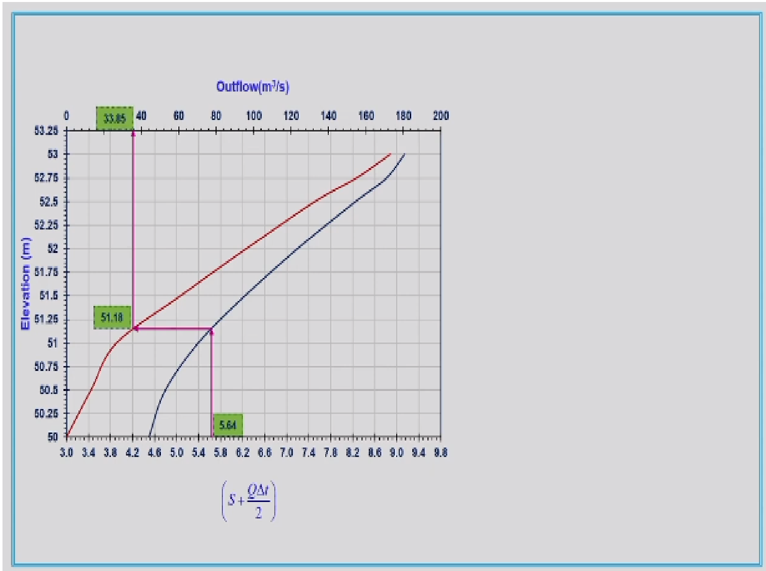

Now, we will again look into the graph. The value corresponding to our indicative storage is 5.64, $\left(S+\frac{Q\Delta t}{2}\right)$ is 5.64, corresponding to that what will be the value corresponding to elevation that is coming out to be 51.18 and that will be extended to the secondary x axis to get the corresponding outflow value in meter cube per second, that is 33.85. So, 33.85 is the outflow corresponding to time t is equal to 8 hours.

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Time (h)	Inflow (I) (m^3/s)	$\left(\frac{I_1+I_2}{2}\right)$ (m^3/s)	$\left(\frac{I_1+I_2}{2}\right)\Delta t$ (m^3/s)	$\left(S_2-\frac{Q_2\Delta t}{2}\right)$ (Mm^3)	$\left(S_2+\frac{Q_2\Delta t}{2}\right)$ (Mm^3)	h (m)	Outflow, Q (m^3/s)
0	13					51.0	27
4	27	20.00	0.288	5.0056	5.29	50.97	25.69
8	73	50.00	0.72	4.92	5.64	51.18	33.85
12	107	90.00	1.29	5.16	6.45	51.69	76.11
16	97	102.00	1.47	5.36	6.83	51.89	89.55
20	77	87.33	1.26	5.54	6.79	51.88	88.19
24	61	69.33	0.99	5.52	6.52	51.73	78.71
28	48	54.67	0.79	5.39	6.18	51.53	64.03
32	37	42.67	0.61	5.25	5.87	51.33	43.32
36	27	32.00	0.46	5.24	5.70	51.22	35.89
40	20	23.33	0.34	5.19	5.52	51.11	30.53
44	17	18.67	0.27	5.08	5.35	51.01	26.78
48	15	16.00	0.23	4.97	5.20	50.92	24.09
52	13	14.00	0.20	4.85	5.05	50.81	21.03

$$\left(\frac{I_1+I_2}{2}\right)\Delta t+\left(S_1-\frac{Q_1}{2}\Delta t\right)=\left(S_2+\frac{Q_2}{2}\Delta t\right)$$

$$\left(S_2-\frac{Q_2}{2}\Delta t\right)=\left(S_2+\frac{Q_2}{2}\Delta t\right)-Q_2\Delta t$$

Now, this step is repeated. So, this Q value is utilized along with this $\left(S_2 + \frac{Q_2}{2}\Delta t\right)$, to get $\left(S_2 - \frac{Q_2 \Delta t}{2}\right)$ in this cell. So that is calculated to be 5.16. Now, summing up these two values 1.29 and 5.16 we will get the value corresponding to $\left(S_2 + \frac{Q_2}{2}\Delta t\right)$ for t is equal to 12 hours. This value is utilized and corresponding values of elevation and the outflow will be taken from the graph. So, this procedure is repeated until we reach the end of the inflow hydrograph. So, the values that we computed are listed here in this table. So, we got the outflow value corresponding to this flood which is entered into the reservoir. So, the first part of the question, route the flood hydrograph through the reservoir is over, we have determined the outflow hydrograph. Now, remaining parts are to determine the attenuation and the time lag, reservoir lag and also the maximum water surface elevation.

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Time (h)	Inflow (I) (m^3/s)	$\left(\frac{I_1+I_2}{2}\right)$ (m^3/s)	$\left(\frac{I_1+I_2}{2}\right)\Delta t$ (m^3/s)	$\left(S_2-\frac{Q_2\Delta t}{2}\right)$ (Mm^3)	$\left(S_2+\frac{Q_2\Delta t}{2}\right)$ (Mm^3)	h (m)	Outflow, Q (m^3/s)
0	13					51.0	27
4	27	20.00	0.288	5.0056	5.29	50.97	25.69
8	73	50.00	0.72	4.92	5.64	51.18	33.85
12	107	90.00	1.29	5.16	6.45	51.69	76.11
16	97	102.00	1.47	5.36	6.83	51.89	89.55
20	77	87.33	1.26	5.54	6.79	51.88	88.19
24	61	69.33	0.99	5.52	6.52	51.73	78.71
28	48	54.67	0.79	5.39	6.18	51.53	64.03
32	37	42.67	0.61	5.25	5.87	51.33	43.32
36	27	32.00	0.46	5.24	5.70	51.22	35.89
40	20	23.33	0.34	5.19	5.52	51.11	30.53
44	17	18.67	0.27	5.08	5.35	51.01	26.78
48	15	16.00	0.23	4.97	5.20	50.92	24.09
52	13	14.00	0.20	4.85	5.05	50.81	21.03

So, we can look into the table again. We are having the maximum value, peak of the outflow hydrograph is at 89.55-meter cube per second and when you look at the inflow value it is 107-meter cube per second. So, 107-meter cube per second is occurring at time t is equal to 12 hours and 89.55-meter cube per second is occurring at time t is equal to 16 hours, so the time lag we can find out from that and also attenuation also obtained by taking the difference between the peak values of inflow and outflow hydrographs. The water level elevation corresponding to this peak of outflow hydrograph is 51.89, maximum water level elevation is found out to be 51.89.

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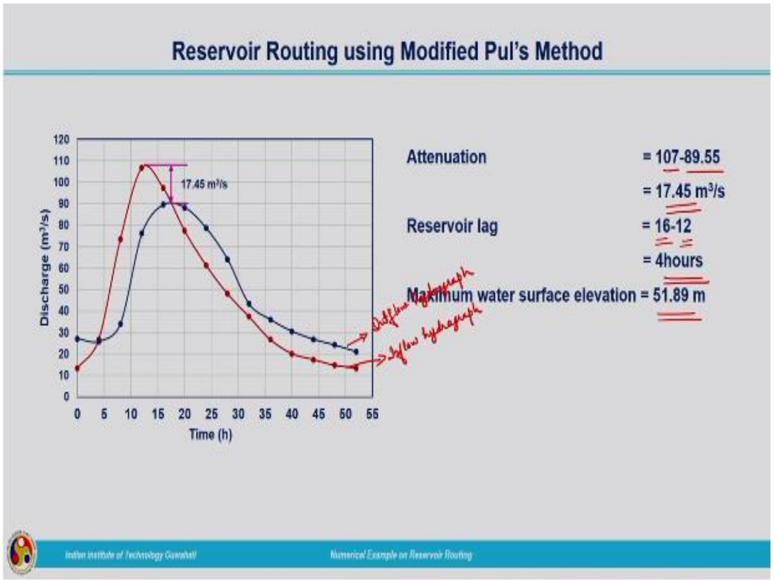

The outflow and inflow hydrographs can be plotted like this, red one is the inflow hydrograph and the blue one is the outflow hydrograph. While observing the inflow and outflow hydrographs it is very clear that the peak has reduced. You compare the values of peak of inflow hydrograph and outflow hydrograph that we have seen from the table and the same thing we can observe from the graphs also. The difference between these peaks will be giving us the attenuation, attenuation is the reduction in peak i.e.,

$$\text{Attenuation} = 107 - 89.55 = 17.45\ m^3/s$$

Attenuation is observed to be 17.45-meter cube per second in this case. So that can be marked like this, this is 17.45-meter cube per second that is the reduction in peak taken place as the flood hydrograph is routed through the reservoir.

Now, we need to determine the reservoir lag. Reservoir lag we can find out the outflow peak is occurred at 16th hour and inflow peak are at 12th hour. So, the reservoir lag is the difference in that time that is

$$\text{Reservoir lag} = 16 - 12 = 4\ hours$$

Now, coming to the maximum water surface elevation that we have observed from the table that is 51.89 meters.

So, whatever asked in the question we have computed over here. We had routed the inflow hydrograph through the reservoir, we have found out the attenuation and the reservoir lag and also corresponding maximum water surface elevation. This is the typical example related to reservoir routing. From this we can develop the water surface hydrograph, water surface which is obtained corresponding to the outflow hydrograph can be plotted.

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Various exercise problems you can get from these reference textbooks, try to solve different problems. So, here I am winding up the problem-solving session on reservoir routing. Thank you.