Applied Seismology for Engineers

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Week – 07 Lecture - 01

Lecture – 14

Hello everyone, welcome to lecture 14 of the course Applied Seismology for Engineers. Myself, Dr. Abhishek Kumar. In earlier lectures, we have discussed different seismic waves which are generated during the process of earthquake occurrence as well as the interaction of the waves with the surficial layer of the earth. In today's lecture, we will be discussing in detail one specific wave, that is, P wave, also known as the primary wave or longitudinal wave or compressional wave. We will be discussing more about the nature of the wave; some portion we have already discussed in earlier classes.

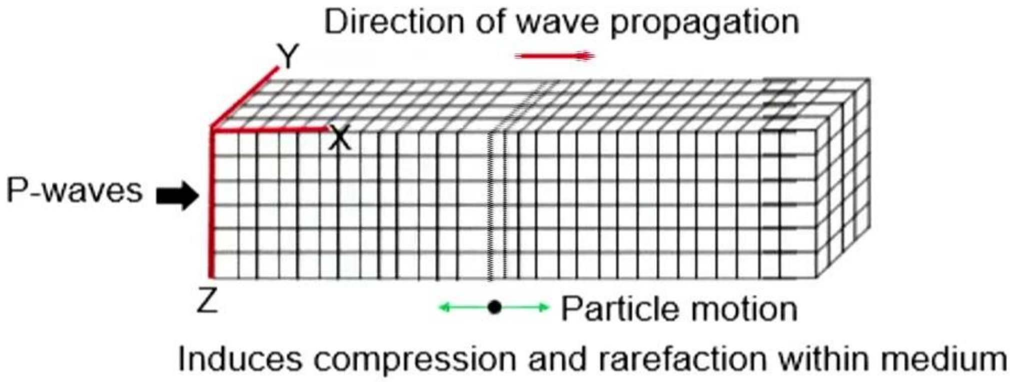
So, we will also be discussing in detail the governing equation which discusses the propagation of the P wave through a particular medium. This will be required in order to determine how much time a wave, which is primary wave in nature, will take between the epicenter and your recording station in order to reach and get detected at the recording station. Before going to the derivation for the one-dimensional equation of motion for the P wave, we will brush up on some of the basics of the primary wave, which we have discussed in an earlier lecture. As we know, earthquakes generally do not directly cause any kind of damage to the infrastructure and so on with respect to the fatalities.

So, most of the time, it is the response of the infrastructure, which can be a building, a slope, a dam, or a bridge, to how these infrastructures are going to respond to the loading induced by an earthquake. That means whenever there is an earthquake, different kinds of waves will generate from the epicenter and, depending upon the propagation medium, some waves will get attenuated, and some may get amplified. So, this modified frequency content of the wave, once it reaches the site where a structure is actually available, determines how the structure is going to respond to these external loading conditions. This will define whether the structure will remain intact, undergo cracks, partial damage, or complete collapse. Depending upon the response of the structure to these seismic loading conditions, it will also govern whether there will be any kind of fatalities to the intended user of that particular structure or not. So, finally, it is not the seismic wave which is directly responsible for the damage; it is basically the response of your infrastructure which is going to govern how a particular seismic wave and a building, soil, or dam is going to respond. So, that is going to collectively define whether it will be damaged or there will be fatalities.

Now, one thing we have discussed earlier is that depending upon the convection current, different plates will be moving in different directions. In this process, there will be the development of stresses within the plate itself. Subsequently, depending upon the type of motion dominating at a particular fault, whether it can be strike-slip faulting, dip-slip faulting, oblique faulting, or similarly, normal faulting or reverse faulting, each of these faulting types, when these kinds of movements happen along the fault, will result in the storage of strain energy.

When the strain energy storage exceeds or the building up of stresses exceeds the in-situ strength of the rock medium available at your fault plane, the medium will definitely undergo failure, including rupture. As a result, seismic waves will come into the picture. So, depending upon the type of wave, the material through which the waves are passing will respond in different ways. That means when we talk about primary waves, we will primarily be discussing the primary wave in today's lecture. So, let us discuss the primary wave, and then we will discuss the behavior of the propagation medium or the particles when the primary wave is propagating through a particular medium.

What we are trying to understand here is when, at the source, a primary wave was generated, as this wave propagates through a particular medium, what is happening within the medium is governing the propagation of the primary wave through that particular medium. Subsequently, depending upon the stiffness the medium is offering, there will be changes in the characteristics of the wave. When I say changes in the characteristics, that means primarily, I am focusing on the frequency content of the motion and its corresponding amplitude. So, let us discuss the primary wave. This is a kind of revision of the previous lecture in which we discussed seismic waves and their attenuation because we will be discussing seismic waves again after some gap in the lectures. So, here I am giving an overview of the primary wave.



As the name suggests, these are the waves that reach a recording station first. That means if there is a recording station and an earthquake has happened at some epicenter, when the waves start from the epicenter or focus, primarily, when these start propagating in all directions, the primary wave will be the first one to reach or get detected at the recording station. Since these waves result in compression and rarefaction in the medium through which they are propagating, these waves are also called compressional waves. Thirdly, because the motion of the particle through the medium in which the wave is propagating is also happening in the longitudinal direction. To understand more specifically about the nature of particle motion, here is an animation given at the bottom. So, you can see over here, depending upon the direction, if the wave propagation is happening in this particular direction, the direction of wave propagation is in the direction of this particular arrow, and subsequently, it is propagating through a medium. I have considered any particular element, and then you can pick up any particular element or one particular small part of this particular elementary rod. You can see whenever the wave passes through this particular element, initially, the material undergoes compression, and then the material undergoes rarefaction.

Finally, once the wave propagation is over through that particular element, the material comes back to its original position. That means there will not be any volume change in the material. So, the wave has propagated, but because of the nature, that is, compression and rarefaction, which the wave causes to the propagation medium, initially, there will be compression, followed by rarefaction, and then the material comes back to its original position. Subsequently, the wave will pass, or there will be the subsequent movement of particles adjacent to the initial particle, and that is how the wave will propagate to larger distances. The propagation of the primary wave is primarily identical to a sound wave passing through a liquid. So, there will be compression and rarefaction. These waves transmit the seismic energy, which they actually originated from the focus of the earthquake, causing compression and rarefaction in the material through which it moves. This can also be understood by the back-and-forth motion in the material through which the waves pass.

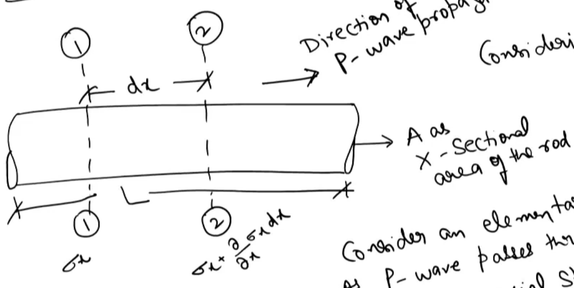
As I mentioned, these are the first waves primarily used or generated from the source and getting detected at the recording station. These are the fastest in terms of propagation velocity—5 to 8 kilometers per second in the Earth's crust, more than 8 kilometers per second in the mantle, 1.5 kilometers per second in water, and 0.3 kilometers per second in the air. So, this is roughly the propagation velocity of each of these waves. Generally, if you see the amplitude of the waves, it is relatively less in comparison to other waves, that means shear waves and surface waves. The amplitude of the wave will be lesser, but the propagation velocity of these waves will be much faster than other kinds of waves. Now, again you can see if this is the direction in which the wave is propagating, and this is the same direction in which the material is undergoing compression and rarefaction. So, the direction in which the particle is undergoing motion and the direction in which the wave is propagating are the same. In the end, once the wave passes through a particular medium, there will not be any volume change.

So, this property of how the material is interacting with the propagation of the wave is very important, that is, there is no deformation or volume change in the material through which the wave passes. One important characteristic is this can travel through solid as well as liquid. As far as the material is offering resistance to the propagation of these kinds of waves, the wave can propagate through that particular medium. So, this is to give you an idea about what a primary wave is. Now, based on our understanding so far about primary waves, we have understood that whenever there is an earthquake, these waves will be the fastest to reach a recording station. This is one primary reason why the primary wave's arrival time has been used in many countries across the globe while developing earthquake early warning systems.

Considering the nature of particle oscillation generated by the propagation of these waves, these are not actually causing a lot of damage primarily, but still, they can be used in order to detect, because once the primary wave reaches the site, it means somewhere there will also be a secondary wave which will be reaching the particular site and will actually induce shearing in the medium, leading to different kinds of failures. So, in order to make sure that some secondary wave is also on the way and about to reach a recording station, site, or building, you can utilize the characteristics and the arrival time of the primary wave. Depending upon your threshold value, which many of the early warning systems use, a person can decide whether it exceeds the threshold value such that an alarm needs to be issued or if it is still within the threshold value and there is no need to issue an alarm for the user or intended user in the region. So, if you release an alarm, definitely people will come to know that an earthquake's relative shear wave is going to hit, and then, particularly, they can move to a safer location, and your high-speed moving train can be put to a complete halt such that the damage can be minimized to a significant level.

Derivation of one dimensional equation of motion for P-wave

Considering a rod of length 'L' & having x-sectional area of A subjected to passage of P-wave.



Consider an elementary length of dx

As P-wave passes through elementary length of dx, there will be development of axial stresses along sections 1-1 & 2-2

Let σ_x be the axial stress at section 1-1, hence after elementary length of dx, axial stress at section 2-2 will $\sigma_x + \frac{\partial \sigma_x}{\partial x} dx$

[Assume that the stresses are uniformly distributed across the x-section]

Continuing with the topic, we will be discussing further the derivation of the one-dimensional equation of motion for the primary wave. That means we are interested in developing the governing equation which is going to describe the motion or propagation of the primary wave through a particular medium. As we discussed in the previous slide, in order to understand and derive the equation, we will take one elementary strip. Over here, I have taken an elementary strip of length L, and there are two cross-sections over here again within this particular elementary strip. So, again over here, I can mention A as the cross-sectional area of the rod. This is the cross-sectional area of a rod, considering a rod of length L and having a cross-sectional area of A, subjected to the passage of the P wave. That means the P wave is passing through this particular elementary rod, and you can also mark the direction of propagation as the direction of the P wave propagation.

Now, to further understand, we will consider an elementary length of dx. So, we had an element of length L, and within that particular element, I am considering a small length, dx, on which we will try to understand what the change in the medium characteristics will be when the wave is passing through this particular element. Let us consider these as two cross-sections: cross-section 1-1 and cross-section 2-2. So, two sections have actually been taken here, and I will be interested to know if this is the direction of wave propagation—that is, the wave is propagating from section 1-1 and enters the section of length dx and then leaves through section 2-2. So, considering an element length of dx as the P wave passes through the elementary length of dx, there will be development of axial stresses from sections 1-1 and 2-2. That means, when we have these two sections, 1-1, the material is offering resistance, and between 1-1 and 2-2, considering the length is dx and the material is also having some stiffness, there will be a change in the stress between section 1-1 and 2-2. Considering the nature of the primary wave, which causes compression and rarefaction, we will have axial stresses developed at sections 1-1 and 2-2.

Now, one thing we should observe here is that when the primary wave, which has the nature of compression and rarefaction, is incident at 1-1 and continues to 2-2, there will be some change in the stresses between 1-1 and 2-2 because of the length dx. As a result, if I consider sigma x is the axial stress developed at section 1-1. Let sigma x be the axial stress at section 1-1. Hence, after the elementary length dx, the axial stress at section 2-2 will be, there will be a change in the stress values between sections 1-1 and 2-2 because it is a material which is continuously offering resistance to propagation. So, this will be sigma x plus dou by doux (sigma x dx). So, this is along the length dx, if there is a change in stresses at the rate of dou by doux (sigma x), then this is at section 2-2. So, here it is sigma x plus dou by doux (sigma x dx), and at section 1-1, it is sigma x.

Now, for equilibrium, one thing we have to mention here or assumption that has to be highlighted is that we assume that these axial stresses are uniformly distributed across the cross-section. That means across the cross-section A, either you consider section 1 or section 2, the stresses are uniformly distributed throughout the section. So, there is no variation across the section in terms of stresses.

Following Newton's Second law of motion

$$F = ma \quad — (1)$$

Total force acting on elementary length dx will be;

$$\left[\sigma_x + \frac{\partial}{\partial x}\sigma_x \, dx\right]_{2-2} A - \left[\sigma_x\right]_{1-1} A$$

$$= \frac{\partial}{\partial x}\sigma_x \, dx \, A = ma \quad — (2)$$

Right hand side of eq.(2) i.e. ma can be written as:

$$ma = [\rho \times A \times dx] \underbrace{\frac{\partial^2 u}{\partial t^2}}_{a} \quad — (3)$$

[$\rho \rightarrow$ mass density of the rod medium]

$u \rightarrow$ particle motion/displacement

putting the value of ma in eq.(2) will give

$$\frac{\partial \sigma_x}{\partial x} dx \, A = \rho A dx \frac{\partial^2 u}{\partial t^2} \Rightarrow \frac{\partial}{\partial x}(\sigma_x) = \rho \frac{\partial^2 u}{\partial t^2} \quad — (4)$$

With this, you can go ahead with the value of stresses between the two sections, that is, section 1-1 and section 2-2, following Newton's second law of motion. We know that force equals mass times acceleration. Now, the total force acting on this elementary length or elementary rod, dx, can be determined based on taking the difference between the two because at section 1-1, you have some value of axial stresses which is assumed to be uniform throughout. Similarly, at section 2-2, whatever is the state of stress, it is uniform throughout the section. So, using these two—the cross-section uniform distribution and the value of stresses—one can determine the value of stress at section 2-2 because it is propagating from 1-1 to 2-2. So, you will determine that, and that will be the value sigma x plus dou by doux (sigma x dx) times A. This is the value of axial stress multiplied by the cross-sectional area. So, this axial force at section 2-2 minus the axial stress at section 1-1, we can also write it here at section 2-2: sigma x times A. So, this is at section 1-1, this is at section 2-2, and this is the total force. What we will get here is dou by doux (sigma x dx) times A. So, this is the total or net force which is applicable between section 1-1 and section 2-2, which resultant propagation of the primary wave through a particular medium. This will be equal to m times a. Let the earlier equation be numbered as 1, and this equation be numbered as 2. So, here we can write as the right-hand side of equation 2, that is, m times a. This can be written as, now, we have to make an assumption that rho is the mass density of the medium of the rod material. That means the rod which we have shown in the previous slide, that particular rod is having a mass density of rho. If that is the case, we can write m times a as m, the mass, which means mass density multiplied by the volume. So, volume is A, the cross-sectional area, and then dx, the length of that particular rod. So, the product of these three will give you the mass times acceleration. So, when these waves, the primary wave, propagate from 1-1 to 2-2, they are causing oscillation or change in the position of the particle with respect to the initial position along the direction of wave propagation. So, consider this particular change in the position of the particle represented by u. So, you can call it as dou square u by dou t square. You can say u is the particle motion or change in the position of the particle because of the propagation of the wave or particle displacement. So, we now have also understood there is a particle which is undergoing motion, and this particle is undergoing motion because the primary wave has a characteristic that whenever it propagates through a particular medium, it will cause oscillation in the particle in the direction of propagation, which, though it will result in compression and rarefaction, will finally cause the material to come back to its original position. So, this is going to give you this is basically the value of acceleration, dou square u over dou t square. Thus, combining or putting this particular value, putting the value of m a in equation 2, will give dou by dou x dx sigma x dx times A equals rho A dx dou square u over dou t square. Now, from here, we can understand this is going to give you the value as dou by dou x dx equals rho dou square u over dou t square. So, this is the same thing which has come over here. So, this particular equation I am giving as number 3, this particular equation I am giving as number 4. So, here it is basically dx and A were there on both the sides; the rest of the things will remain over here, just a correction: this will be sigma x. So, dou by dou x sigma x equals rho times dou square u over dou t square. Sigma x is the heuristic at section 1, and u is particle displacement with respect to the propagation of the primary wave through a particular medium.

we know that

$$\sigma_x = M \epsilon$$

where
M → Constrained modulus, which will ensure no volume change due to propagation of P-wave through the medium

$$\epsilon = \frac{\partial u}{\partial x}$$

$$\frac{\partial}{\partial x}(\sigma_x) = M \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right)$$

$$\frac{\partial}{\partial x}(\sigma_x) = M \frac{\partial^2 u}{\partial x^2} \quad - (5)$$

$$\rho \frac{\partial^2 u}{\partial t^2} = M \frac{\partial^2 u}{\partial x^2} \quad - (6)$$

Re arranging the terms of eq.(6), it can be again written as;

$$\frac{\partial^2 u}{\partial t^2} = \frac{M}{\rho} \partial^2 u / \partial x^2$$

$$\boxed{\frac{\partial^2 u}{\partial t^2} = V_p^2 \, \partial^2 u / \partial x^2} \quad - (7)$$

[Consider $\frac{M}{\rho} = V_p^2$ where V_p → Propagation velocity of P-wave through a medium having ρ as mass density & M as Constrained modul.
$V_p = \sqrt{\frac{M}{\rho}}$]

Eq. 7 is known as one dimensional wave equation for P-wave.

Now, let us further see to this particular equation. We can write sigma x equals to M times epsilon. Just I will mention over here is where M is the constraint modulus, which will ensure no volume change due to the propagation of the P wave through the medium. So, the constraint modulus we have used often; we correlate stress and strain by means of Young's modulus, shear modulus; here we are using the constraint modulus because, as we discussed, when the wave propagates, there will be some compression and rarefaction, but once the wave further propagates, the material comes back to its original volume. So, in order to ensure that there will not be any change in the volume, we will be using here compression modulus, constraint modulus. Further, epsilon value will be equals to dou u over dou x, that means change in the position of the particle over the distance dx. Remember, u is also in the same direction as we are measuring dx, therefore. So, in equation 4, we were having dou by dou x sigma x. So, from here, what we can get is M will be outside, dou over dou x of dou u over dou x, that is going to give you dou over this sigma x equals M dou square u over dou x square. So, you have now this particular equation, that is equation number 5 (I can name this) dou by dou x sigma x, which was there in the previous equation, what was equation number 4: dou by dou x sigma x equals rho times dou square u over dou t square. Now, the left-hand side of this particular equation has been determined in terms of u value because the right-hand side is also in terms of u value. So, we can write over here that is rho times dou square u over dou t square equals M times dou square u over dou x square. I can write it over here also, this as equation number 6, rearranging the terms of equation 6, it can be written as it can be again written as. So, equation 6, again we can write it as dou square u over dou t square equals M over rho dou square u over dou x square. Consider M over rho equals to Vp square, where Vp is the propagation velocity of the primary wave through a medium having rho as mass density and M as constraint modulus. So, any medium which is having mass density of rho and M as constraint modulus, we can determine the value of Vp, that is. So, here we can also write it as that means Vp equals to square root of M over rho, putting this value of Vp you can just put it in bracket. So, we can get dou square u over dou t square equals Vp square dou square u over dou x square. Now, here you can see on one side the particle displacement with respect to space and the particle motion with respect to time is correlated with the primary wave velocity. This particular equation, that is equation 7, is known as the one-dimensional wave equation for the primary wave or P wave. That is, the above equation is governing how the particle motion with respect to space and time will happen for a medium having Vp as primary wave velocity.

$$V_p = \sqrt{\frac{M}{\rho}}$$

M is Correlated with Young's modulus E & Poisson's ratio ν as

$$\boxed{M = \frac{(1-\nu)}{(1+\nu)(1-2\nu)} E}$$

Please note here that V_p is P-wave propagation velocity & is not the particle displacement corresponding velocity

$u \rightarrow$ particle displacement

$\frac{\partial u}{\partial t} \rightarrow$ particle velocity

$$\frac{\partial u}{\partial t} = \frac{\partial u}{\partial x}\frac{\partial x}{\partial t}$$

$$= \epsilon V_p$$

$$= \frac{\sigma_x}{\underbrace{M}_{\rho V_p^2}} V_p = \boxed{\frac{\sigma_x}{\rho V_p} = \frac{\partial u}{\partial t} = \dot{u}}$$

$\dot{u}$ is —— proportional to σ_x & the constant of proportional 'ρV_p' is called as specific impedence of the medium (ρV_p).

Now, we can write as Vp I have already mentioned that Vp is a function of constraint modulus of the medium and mass density of the medium. Again, M is correlated with Young's modulus and Young's modulus E, which is primarily used, and Poisson's ratio nu. So, if you know these two values because that is how you can correlate actually Young's modulus and constraint modulus. So, M will be equals to 1 over nu over 1 plus nu, 1 minus 2 nu times E or Young's modulus. So, constraint modulus and Young's modulus are correlated for a particular medium having Poisson's ratio of nu, Young's modulus denoted by E, and constraint modulus denoted by M. So, again using these three parameters, one can correlate how much will be the value of constraint modulus if Young's modulus is given or how much will be the value of Young's modulus if constraint modulus is given, or when these two values are given, what will be the Poisson's ratio of the medium. Now, one more information one can ask, because when the wave is propagating, it is not the particle velocity; it is the wave propagation velocity. Though the direction of wave propagation and particle motion is in the same direction, still the particle velocity as well as wave velocities are two different properties. So, please note here that Vp, that is P wave propagation velocity, and is not the particle velocity or the velocity with which the particles are undergoing to and fro motion, is not particle movement or particle displacement corresponding velocity. So, in order to determine it, u we discussed as u was particle displacement when the primary wave is passing through the medium, dou u over dou t is the particle velocity or the velocity with which the particles are moving. So, dou u by dou t one can write as dou u by dou x times dou x over dou t. Dou u by dou x is, we know that it is the value of epsilon, that is normal strain, and then dou x by dou t is the value of Vp for our primary velocity. This particular value of epsilon can be correlated with respect to constant modulus as sigma x over M, which can further be correlated as rho times Vp square and Vp. So, this can be written as sigma x over rho times Vp. This is the value of dou u over dou t, or which I can write as u dot. That means the particle velocity, or the velocity with which the particles are undergoing movement to and fro motion, that is directly proportional to the axial stress, and so u dot is directly proportional to sigma x, and the constant of proportionality, that is rho times Vp, is called as— not directly proportional, it is proportional to sigma x, and then rho times Vp is called as specific impedance of the material of the medium, which can be determined as once you know the mass density and wave propagation velocity, we can determine the value of rho times Vp, which is the specific impedance of that particular medium. Once this value is known to you, depending upon the axial stress generated at a particular section, one can determine how much is the particle oscillation velocity across that particular section, that is section 1 1 or section 1 2. So, this derivation, which we have discussed over here, is helping us to understand when the primary wave is incident on a particular medium, how the propagation velocity will be governed by means of constant modulus, by means of mass density of the medium, and how the change in particle position with respect to space and time is also governed by means of the one-dimensional equation of motion, which is given in the previous slides. So far, we have understood how the governing equation of motion can be determined for the primary wave. Secondly, the square root of the ratio of constant modulus over mass density of the medium will give you how much the primary wave velocity is. One thing is clear over here: as far as the medium is offering resistance in terms of constant modulus, the primary wave will be able to propagate through that particular medium. Secondly, once we know the primary wave velocity of a particular medium, we can also determine how much the particle motion will be through that particular medium. As far as the medium is offering stiffness in terms of constant modulus, the primary wave will be able to propagate through that particular medium. So, unlike shear waves, we generally do not pass through or generally do not travel through liquids. The primary wave can travel through liquids as far as it is propagating through a medium, and the medium is able to offer stiffness in terms of constant modulus, which is also available for liquids. The primary wave will be able to propagate through that particular medium. So, we can number this particular equation as equation number 8.

Q 1: Determine P wave velocity for following materials;

Material	Specific Gravity	M (Pa)
Steel	7.82	2.78×10^{11}
Water	1.0	2.34×10^{9}
Rubber	1.28	1.15×10^{12}

Now, let us discuss one numerical problem. Suppose, in general, you have been given the mass density of a particular medium, how much the constant modulus of a particular medium is. You can determine how much the primary wave velocity is. Depending upon the propagation medium, one can determine primary wave velocity. As we discussed at the beginning of this particular lecture, whether the primary wave is propagating through air, whether it is passing through water, whether it is propagating through rock, it is propagating through rubber, it is propagating to different mediums, the primary velocity keeps on changing. We will be using the information on primary wave velocity more frequently when we are trying to locate the epicenter of an earthquake. So, using the record, which is being detected, or which has been available at a recording station, we will try to find out how much the arrival time of primary wave velocity is, what is the arrival time of the primary wave at your recording stations. Once that arrival time is known to us, using that information, we can find out, taking that primary wave velocity into account and the time of arrival, we can find out what is the radial distance within which the epicenter of your earthquake is located. When this exercise is repeated for more than three recording stations, we will get a more accurate idea about the potential location in which the epicenter of an earthquake is located.

So, in today's lecture, we will also try to solve a numerical where we can determine how much the primary wave velocity is through different mediums. So, we will also get an understanding about depending upon the medium properties, that is, the constraint modulus, which is given over here. So, here, three mediums are given: steel, water, and rubber. The specific gravities are also given. Rather than mass density, we have been given the specific gravity of that particular medium, and we have been given the constraint modulus value. We could have also been given the Young's modulus as well as the Poisson's ratio. So, again, one can determine how much the constraint modulus is using the equation which was given during the derivation. Now, it is asked to determine how much the primary wave velocity is.

Sol: $V_p = \sqrt{\frac{M}{\rho}}$

a) Steel

$M = 2.78 \times 10^{11}\ N/m^2$

$\rho = 7.82 \times 1000 = 78200\ kg/m^3$

$V_p = \sqrt{\frac{2.78 \times 10^{11}}{78200}} = 5962\ m/s$

So, using this, we know the primary wave velocity is the ratio of constraint modulus over mass density of the medium. So, for case 1, that is for the case of steel, the constraint modulus value is given as 2.78 × 10^11 Newton per millimeter square. Specific gravity is given. So, 7.82 is the specific gravity of the medium, multiplied by 1000 grams per meter cube, that is going to give you how much will be the value of. So, 78200 kg per meter cube. One has to be very careful while dealing with the units because one is given in Newton per meter square, and the other one is given in kg per meter cube. So, using these two values, one can determine the value of Vp equals 2.78 × 10^11 over 78200. So, this is going to give you the mass density and constraint modulus based on the ratio one can determine. So, you will get the value equals 5962 meters per second. That is the velocity with which the primary wave will pass through the medium of steel, having a specific gravity of 7.82 and constraint modulus equals to 2.78 × 10^11 Pascal.

(b) Water

$$\rho = 1000 \, kg/m^3$$
$$M = 2.3 \times 10^9 \, Pa$$
$$V_p = \sqrt{\frac{2.3 \times 10^9}{1000}} = 1516 \, m/s \simeq 1.5 \, km/s$$

(c) Rubber

$$\rho = 1.28 \times 1000 = 1280 \, kg/m^3$$
$$M = 1.15 \times 10^{12} \, Pa$$
$$V_p = \sqrt{\frac{1.15 \times 10^{12}}{1280}} = 29973.9 \, m/s$$
$$\simeq 29.9 \, km/s$$

Go with the second part, that is, for water, having mass density of 1000 kg per meter cube and constraint modulus equals to 2.3 × 10^9 Pascals. One can determine the Vp value equals to square root of 2.3 × 10^9 over 1000, which is going to give you the primary wave velocity of 1516 meters per second, which is equal to 1.5 kilometers per second. So, this is the velocity with which the primary wave will propagate through a particular medium. C, that is given for rubber, the value of mass density is given as 1.28, the specific gravity is given, multiplied by 1000. So, 1280 kg per meter cube is the mass density of the medium, and the constraint modulus is given as 1.15 × 10^12 Pascals. Hence, the value of Vp, one can determine as 1.15 × 10^12 over 1280, which will result in 29973.9 meters per second or approximately 29.9 kilometers per second as the propagation velocity of the primary wave through the rubber medium.

So, overall, in this particular lecture, that is lecture 14, we try and understand how a primary wave passes through the medium. During the passage of the primary wave, there will be a change in the axial stresses because the particle is undergoing to and fro motion in the direction of wave propagation. Taking those variations in axial stresses, cross-sectional details, and stiffness of the medium, one can determine how much the change in the particle motion with respect to space as well as time, which is directly a function of primary wave velocity. Further, how primary velocity is correlated with respect to Young's modulus, we have also discussed. Then, if the primary velocity is known to you, how one can determine the particle oscillation or the particle motion or the displacement value. Using these, we tried solving one numerical problem where the mass density of the medium and constant modulus are given, or the specific gravity of the medium is given, and constant modulus is given. Converting each of these parameters into specific units, one can determine the value of primary wave velocity, as well as the primary velocity of all the mediums.

So, thank you, everyone. We will continue with respect to the derivation for the one-dimensional wave equation for the shear wave in lecture 15. Thank you.