Applied Seismology for Engineers

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Week – 07 Lecture - 02

Lecture – 15

Hello everyone, welcome to lecture 15 of the course Applied Seismology for Engineers, myself Dr. Abhishek Kumar. In lecture 14, we had discussed about one-dimensional equation of motion for primary waves. In earlier discussions, we have seen that the occurrence of earthquake will result in release of different kind of waves. These waves during the propagation starting from the focus as these are propagating away from the focus and interacting with the propagation medium that is the crystal medium depending upon the heterogeneity depends upon unconformity which are present in the medium the waves will interact with this particular medium and subsequently once these wave reach to a particular recording station depending upon the frequency content amplitude and duration of these waves these will be detected and recorded by a recording station.

So, in today's class that is lecture 15, we will be discussing about how the propagation of shear wave through a particular medium will be governed by an equation that is called as one-dimensional wave equation for shear waves. Before going to the equation for one-dimensional equation of motion for shear waves, we will be discussing again about what is the basic characteristics of shear waves and how these are relevant as far as the earthquake induced vibrations and sometimes the effect of earthquakes are also visible on the ground surface. As I mentioned in lecture 14 also it is not directly the earthquake which is responsible for loss of lives or damage to building and other infrastructure it is basically the response of the building to the vibration which are being transferred through different waves. So, if the buildings are designed properly such that the external loading because of seismic vibrations induced in the building can be taken up it, that particular load the building will remain safe otherwise the building can show minor cracks, moderate cracks and complete collapse.

It is applicable to other kind of infrastructure also like dams, bridges, even embankments also slope everywhere it is basically how the system is going to respond when it is subjected to seismic loading. When we say seismic loading that means some waves which at that particular site whether it is a slope, whether it is a dam, whether it is a bridge or any building at that particular site because of some earthquake which has happened maybe 50, 100, 150 kilometer away from your site what is the characteristics of vibration which has been transferred to that particular building that will define what is the vibration or external loading which is now available at the foundation level or to the superstructure corresponding to which the system has to offer resistance such that it should not undergo collapse or minor cracks or moderate cracks. And subsequently depending upon the response it will also define the safety of its intended user and which we always see in terms of fatalities, casualties. So, this is the complete picture of damage starting from the loading of or the vibration which is being generated from the part of the earthquake. So, this is again the summary of whatever I have just told it is not the earthquake which is directly causing the building damage as well as the casualties rather it is otherwise it is the loading is there depending upon the loading if you have designed the building properly if you have designed your other infrastructure properly that will define or ensure the safety of that particular infrastructure and secondly it will also ensure the safety of its intended users.

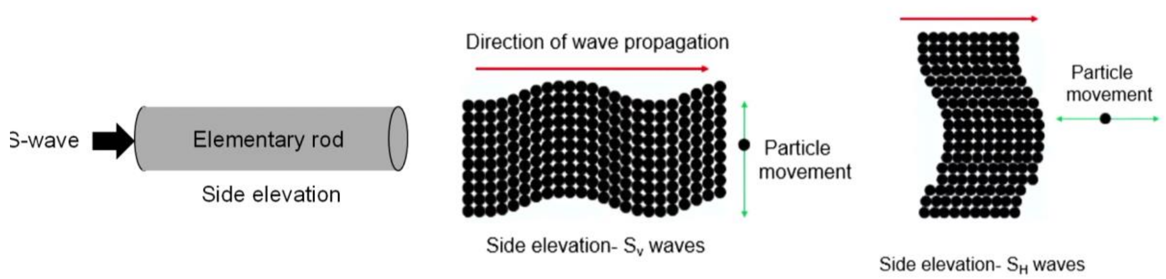
So, shear waves also known as secondary waves because these are the one which appear, or which are detected by a recording station after primary wave. So, if I have a recording station located at some epicentral distance away from your potential region of earthquake occurrence if that particular fault shows some signature of earthquake vibration will be released from that particular focus and will be propagating in the three-dimensional space. Now, depending upon the vibration which are propagating along the direction of recording stations will be getting detected at the recording station. Once you start analysing the ground vibration you can actually see what the frequency content is even you can differentiate between what is the content which is dominated by primary wave secondary wave and subsequently other content of the motions. This is very important because you must have an understanding about how much time required for shear wave to reach a particular recording station primary wave to reach a recording station.

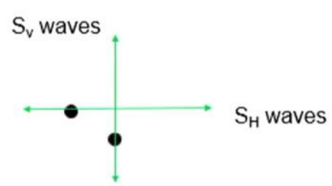
In lecture 14 also I told whenever it comes to primary wave arrival time it is very important as far as development of early warning systems such that once you are able to detect some primary wave has been detected at the recording station you can you can check and compare with some threshold value which is again determined by the designer and then you can decide whether and there is a need to issue a warning or not. Secondary wave as a name suggests these are also called a shear waves. So, whenever these waves are passing through particular medium the medium undergoes shearing. So, whenever we see most of the time damages mostly will be confined to shear failures, confinement of reinforcement and even in terms of ground many a times liquefaction also occurred which is again primarily because of loading or the vibration which is produced by the seismic waves. So, shear waves are also called as transverse waves because whenever the shear wave is passing through a particular medium it is going to induce vibration in transverse direction.

So, if this is the direction in which the wave is propagating it is going to cause particle motion in perpendicular to the direction of wave propagation. So, one perpendicular direction is in vertical direction as you can see on your screen and other direction is still perpendicular to the direction of wave propagation, but you can see it is parallel to your line of sight when you are looking at the screen. So, two directions in which the particle possibly can undergo shearing when a shear wave is incident on a particular medium that means in vertical direction that will be perpendicular to your direction of wave propagation and other perpendicular direction is in the line of sight when you are looking at the screen or if you see in front this is the direction in which the shear wave or the particle undergo movement when the shear wave is passing in the parallel to the line of sight. So, this propagation of shear wave through a particular medium cause shear deformation. Shear deformation because all the deformations are happening perpendicular to the direction of wave propagation.

So, wave propagation is happening in a particular medium as a result of a shearing in the medium is happening in perpendicular direction. These arrive second at a recording station for that reason we call it as secondary waves also. Usually, the velocity propagation velocity of shear wave is lower than primary wave velocity. Primary wave we have discussed in primary wave the particle will oscillate or there will be to and fro motion in the particle in the direction of wave propagation. In case of shear wave, it is in perpendicular direction with respect to the direction of wave propagation.

Again, very much similar to primary wave once the material is subjected to loading it will undergo shearing in horizontal as well as vertical direction, but once the wave propagates further the material will come back to its original position. So, shear wave these can travel through solids because solid offers stiffness against which the material can offer resistance, and the particle motion is also possible in perpendicular direction with respect to the wave propagation. However, whenever it comes to liquid or gases usually these do not offer resistance stiffness or shear modulus as a result of which whenever there is an incident shear wave on liquid or gas the wave will not propagate further because the medium is not there for wave through which it can actually propagate. Approximately shear waves travel at 3 to 4 kilometer per second in earth's crust and almost 4 to 4.5 kilometer per second in mantle. So, if you compare the propagation velocity of primary wave and shear wave you can see though it is lesser than primary wave velocity, but still shear waves are propagating quite fast in the propagation medium. This can also be correlated with respect to the fact that whenever there is an earthquake maybe 100 kilometer 150 kilometer away from your site suppose you are sitting at one particular location and then there was an earthquake maybe 100 kilometer 150 kilometer away from your site where you are sitting after couple of seconds depending upon the epicentral distance you may also experience some kind of vibration in the ground because the waves are propagating at that much particular high velocity. So, particle oscillation happens perpendicular to the direction of wave propagation depending upon the direction in which the particles are undergoing motion the shear waves can further be classified as SV as well as SH. So, SV is component of shearing which is happening in vertical direction and SH is the component leading to shearing of material in horizontal direction. So, both SV and SH are perpendicular to the direction of wave propagation. SV will cause upward and downward movement in the particle at the time the wave is propagating through a particular medium; SH will be left and right direction. So, in order to demonstrate the potential movement which can happen in an element when the secondary wave, or the shear wave, or transverse wave is passing through a medium, here you can see an animation.

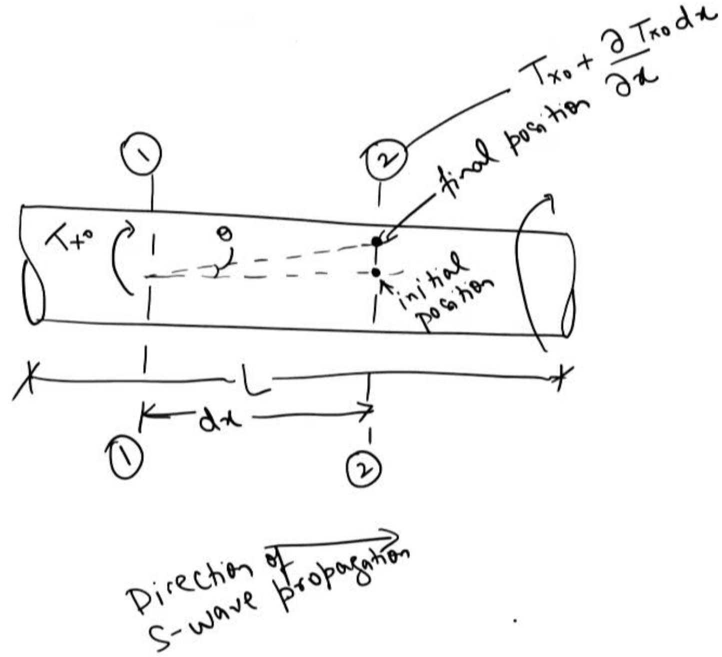




So, this is a shear wave which is passing through a particular medium. So, this is an elementary rod through which the shear wave is passing; you can take any medium through which shear wave is passing, and this is the direction in which the shear wave is propagating direction of S wave propagation. So, when the waves are propagating in this particular direction, you can see there is particle motion. Any particular cross-section you take along the length of this particular rod, the particles which are shown by these circles, you can see at any cross-section along the length of the rod, the particle is undergoing vertical motion, or there will be. So, you can see on that particular section, or in this particular section, at the interface between two particles which are adjacent to each other in horizontal direction, there is pure shearing which is happening in vertical direction. This is, you can see, this is the demonstration of S-V component. Similar way, the particle at the same time is also undergoing shearing in horizontal direction, which is shown over here. So, here again you can see the direction of wave propagation is. So, you can see here, this is the rod, elementary rod, which is shown in this particular figure. So, this is the elementary rod along which the shear wave is propagating, and then as a result of this propagation, S-H component, which is shown over here. That means in horizontal direction there is shearing happening; you can see also over here there is two flow motion in left and right direction, and then at the same time vertical motion is also happening over here, which is shown in the other animation. Now, collectively, when this shear wave is passing through a particular medium, you will see the particle at the same time is undergoing horizontal as well as vertical shearing. As a result, this information we will use when we start deriving the one-dimensional equation of motion for shear waves. So, let us discuss about the one-dimensional wave equation for shear wave, but keep in mind that the particle is undergoing rotation. So, if you say one particle, right now, I have shown over here two particles, but this shearing in horizontal as well as vertical direction, it is happening simultaneously in each particle, or the force is, the shear force is applicable in both the direction. As a result, you see one particle which is moving in this direction also, and in that direction also. So, collectively, you can see the particle is undergoing, or the cross-section in which the particles are located, it is basically undergoing some kind of rotation. So, that is another reason why many a time, when we discuss about one-dimensional equation of motion for shear wave, we will take into account as though rather than axial forces which were applicable for primary wave, in this particular case, for shear wave, the cross-section of the rod is actually experiencing torque. So, it is like the particle is undergoing motion here also and here also simultaneously, horizontal as well as vertical, as a result of which you are considering it is undergoing torque. So, using this information, we will be deriving the one-dimensional equation of motion for shear wave.

So, let us discuss about one-dimensional equation of motion for shear wave, or you can write it also as S wave, because shear wave we often refer to as S wave also. So, these will be coming; if you take a recording station, you will have firstly the component from P wave, second, you will have component from S wave. So, one-dimensional equation of motion for shear wave passing through an infinite medium, through an infinite medium. Now, in this particular case, we have to take into consideration what is the nature of motion which a particle is undergoing when the wave is propagating through the rod or through the medium. Now, whenever I say about medium, basically, I am trying to represent the medium which the wave is experiencing, or the wave is finding in order to propagate between the focus of an earthquake and before it is reaching to a recording station. So, any medium through which the wave is propagating between the source and the site, that is called as a medium. So, whenever I am talking about shear wave or primary wave and mentioning about the medium. So, I am basically mentioning about the medium through which the wave is propagating; you can call between the source and the site or, again, between the bedrock and the surface. So, every time and any medium through which the wave is propagating, you call that as the medium of propagation or the propagation medium. So, as mentioned earlier, as mentioned earlier, that the passage of shear wave through a medium causes particle motion in horizontal as well as vertical direction simultaneously.

As a result, the particle experiences motion upward, downward, as well as up and down. This nature of horizontal and vertical movement of particle, particle means the particles which are available in the propagation field. Vertical movement of the particle can be considered as application of torque about the axis of propagation medium, or you can call it as along the direction of wave propagation. So, if you consider an elementary rod, the direction in which the wave is propagating will be superimposed with respect to, or will be coinciding with respect to, the axis of the rod. So, along the direction of S wave propagation. For this reason, propagation of, or the determination of, S wave in a medium is approximated to application of torque. Now, when torque is applied to a medium, there will be, in case of primary wave, there was particle oscillation, or the particle displacement U value, which was taken in the direction of wave propagation. Now, here, because the cross-section is subjected to torque, there will be particle motion, but that will be angular displacement, that we will discuss. Application of torque in the direction perpendicular to, the wave propagation. Wave propagation with this information, the derivation of one-dimensional equation of motion for shear wave will be derived. So, that means considering the nature of particle motions happening in horizontal direction and vertical direction simultaneously, that is approximated with respect to the torque. So, now, when we are interested to develop the one-dimensional equation of motion for shear wave, we will take into account that there is a medium through which the wave is propagating, and this particular medium is subjected to torque which is approximated to the nature of movement in the particle when the shear wave is passing through the same medium.



So, let us discuss about it. Consider a cylindrical rod of length L as the propagation medium, or as the medium of propagation, as the medium of propagation as shown in figure. So, you can consider a particular rod of length L through which the shear wave is propagating. Again, as I mentioned earlier also, to understand the nature of particle motion, to understand the nature of particle motion, we will consider an elementary rod, and elementary portion of above rod will be targeted. Targeted means we will be interested to know what will happen if there is an elementary rod. So, consider that elementary rod to be existing between 1 1 and 2 2. So, there was medium which was basically the entire medium through which the wave is propagating; within that medium, I have considered again an elementary length of dx and try to understand what is happening when the shear wave is incident at section 1 1, and when the shear wave is leaving section 2 2. So, what we understand as a result of this particular torque which the material is subjected to, because of the propagation of shear wave, there will be angular displacement of any particular length. So, if you consider a particular length over here, or any particular element or the line in which you are basically trying to understand whether there is some change in the angular deformation. So, if you consider this particular line, which is basically parallel to the axis or perpendicular to your line of sight, you see when this will be subjected to some torque, initially this was the position of the particle, but once you start applying the torque, as a result, the position of the particle along the circumference of the rod changes. So, this is the initial position of the particle, and this is the final position. Consider, like this was the position when it entered 1, and the final position is when the wave left section 2. So, this is called the final position. So, this angular deformation, it is indicated by theta. We will use this particular value of theta whenever we further progress with the derivation part. Now, there, because of the application of torque and the stiffness which the material is offering, definitely there will be variation in the value of the torque at section 1-1 and at section 2-2. So, at section 1-1, if it is, I have considered as T x naught, at section 2-2, I am considering as incremental increase dou by dou x of T x naught dx. So, consider 2 sections considering an elementary rod where, which, which, where 2 sections are separated by dx. At section 1-1, the torque which is coming into picture is T suffix x naught. At section 2-2, the section which is coming into picture is T x naught plus dou by dou x T x naught, which is basically the rate at which, along the dx, at each unit length, there is an increase in the torque or change in the torque multiplied by dx. So, then you will get how much between section 1 and 2 the net change in the torque value, plus the initial torque which was actually available at section 1-1. That will give you how much is the value of torque 2. So, this is the position of the propagation medium, and this is the direction of S wave propagation. So, the S wave propagation direction is also given, and the direction in which the torque is applicable is also given to you.

Now, let us start. So, again, we, we know that once the wave leaves, the material comes back to its original position. So, in order to ensure the equilibrium condition of this particular elementary length of the propagation medium, that is dx, for equilibrium. So, we will try to find out the condition which is actually applicable for length dx subjected to external torque because of the propagation of the shear wave through a particular medium, and then we will try to find out further based on the equilibrium equation. So, for equilibrium of length dx, the difference in the torque, the difference in the torque between sections 1-1 and 2-2, should be balanced by the inertial torque. Now, inertial torque means resistance offered by the material, offered by the propagation medium against rotation, because we are considering the propagation of a shear wave as subjected to the particle or the cross-section undergoing rotation. Which indicates that if you are trying to find out the difference between the torque at section 1-1 and 2-2. So, that is the difference between the difference in torque between section 1-1 and 2-2 will be.

Difference in torque between section 1-1 & 2-2 will be;

$$\left(T_{x_0} + \frac{\partial}{\partial x} T_{x_0}\, dx\right)^{2-2} - T_{x_0} = \frac{\partial}{\partial x} T_{x_0}\, dx$$

So, we can find out T x naught plus dou by dou x T x naught dx, this is at section 2-2, minus T x naught, this is at section 1-1. So, this is going to be the total torque or the difference of the torque between section 1-1 and 2-2.

Again, based on this equilibrium, we will find out that in order to have this equilibrium, there will be a governing equation, which is given as mass moment of inertia into angular acceleration, mass moment of inertia into angular acceleration will be equal to summation of external moment about the same point in the direction of wave propagation. So, mass moment of inertia into angular acceleration, angular acceleration means the particle is undergoing angular displacement. So, corresponding to that, we can find out how much the angular displacement between section 1-1 and section 2-2, taking that into account and multiplying by mass moment of inertia. For equilibrium, this should be equal to the summation of the external moment, which has been also taken about the axis through which the wave is propagating, in the direction of wave propagation and about the axis. Here it should be highlighted that the mass moment of inertia, the mass moment of inertia is a measurement of the distribution of the mass of an object or the body related to a given axis. So, basically, mass moment of inertia means how the mass is distributed throughout the body with respect to a given axis. For solid rods, we have considered here an elementary rod, or the medium of propagation as a solid rod, cylindrical in nature.

For solid rods, mass moment of inertia is given as $\frac{1}{2} m R^2$

Hence, the left hand side of equation (1) can be written as

$$= \frac{1}{2} \underset{\text{mass}}{\underset{\downarrow}{m}} R^2 \times \frac{\partial^2 \theta}{\partial t^2}$$

$$= \frac{1}{2} \left(\rho \pi R^2 dx \right) R^2 \times \frac{\partial^2 \theta}{\partial t^2}$$

$$= \frac{\pi}{2} R^4 \rho \, dx \frac{\partial^2 \theta}{\partial t^2}$$

$$= \underbrace{\left(\frac{\pi}{32} D^4 \right)}_{J} \rho \, dx \frac{\partial^2 \theta}{\partial t^2}$$

$$= J \rho \, dx \, \partial^2 \theta / \partial t^2 \quad \text{— (2)}$$

$R \rightarrow$ radius of the rod

$m \rightarrow$ mass of the rod

$\theta \rightarrow$ angular displacement between sections 1-1 & 2-2

$$\text{mass} = \rho \times \text{volume} = \rho \times \pi R^2 \times dx$$

Consider D as the diameter of the rod $D = 2R$

$\frac{\pi}{32} D^4 =$ Polar moment of inertia of solid rod indicated by 'J'

So, the mass moment of inertia for solid rods is given as half m r square, where r is the radius of the rod, m is the mass of the rod, means the rod which is of our concern. So, the mass moment of inertia for our case, that is the rod through which the wave is propagating, this is the value of mass moment of inertia. Hence, in the previous equation, we had defined this particular part. This is considered as equation number 1, that is mass moment of inertia times angular acceleration. Hence, the left-hand side of equation 1, can be written as that is mass moment of inertia times angular acceleration. So, half times m r square and acceleration value or angular acceleration value equals dou square theta over dou t square, because theta is the angular displacement. I have already shown the value of theta in the previous slide, angular displacement between section 1-1 and 2-2. So, again, here this is the mass will be equal to rho times volume, considering this is a cylindrical rod. So, you will have a value of rho into pi r square, that is cross-sectional area, into the length dx. So, this is we can put over here also. This value will be equal to 1 by 2, m is rho pi r square dx into r square, which is over here, and then dou square theta over dou t square, which is basically the angular displacement, displacement angular acceleration. So, this you can write it as pi by 2 r 4 rho dx and then dou square theta over dou t square, where r is the radius of the rod. Consider d as the diameter of the rod, of the rod or the elementary rod which has been taken into consideration, that is d equals to 2 times r. So, you can write this equation as, again, pi by 32 d 4 rho dx dou square theta over dou t square. Now, pi by 34 d 4 is the polar moment of inertia, polar moment of inertia of the solid rod, indicated by, indicated by j. So, I am considering pi d 4 over 32 as the value of polar moment of inertia, which is indicated by j. So, I am going to replace this as by j, then you get j rho dx dou square theta over dou t square. Let this be equation number 2. So, equation number 1 is the basic one, equation which is controlling the equation equilibrium equation of the rod through which the shear wave is passing. As a result, the rod is subjected to torque. Equation 2 is giving you the product of the left-hand side of equation 1, that is mass moment of inertia times the angular acceleration, that is j times rho dx dou square theta over dou t square.

$$\frac{\partial T_{x0}}{\partial x} dx \quad - \quad (3)$$

The external moment, external moment which is basically given in the right-hand side of equation 1, along the axis. Axis of cylindrical rod we are referring here is estimated as the difference in the torque, which we have already estimated before writing equation number 1. So, the difference in torque between 1-1 and 2-2, which is already calculated as dou by dou x dx naught dx. Let call this particular equation as equation number 3.

equate (2) & (3) will give

$$\frac{\partial T_{x0}}{\partial x} dx = J \rho \, dx \, \frac{\partial^2 \theta}{\partial t^2} \quad - \quad (4)$$

As per equation number 1, the mass moment of inertia times angular acceleration will be equal to the external moment about the axis, which is given by equation number 3. So, equate equation number 2 and equation number 3 will give dou by dou x T x naught dx equals J rho dx dou square theta over dou t square. This is equation number 4.

Torsional equation

$$\frac{T}{J} = \frac{G\theta}{l}$$

Now, if we remember the torsional equation, from torsional equation, we can calculate t over j equals to G theta over l, which is the length. This is the angular displacement torque, and then this is polar moment of inertia. This is a general equation, and further, you have some more term, which is not useful here. So, I am not writing the further term. So, I am removing the other part of the equation. T over J for elementary rod of length dx dx subjected to angular displacement displacement dx d theta. Let me come correct this part. So, this will be equals to d theta in the length dx.

For elementary rod of length dx subjected to angular displacement $d\theta$

$$\frac{T}{J} = G\frac{\partial \theta}{\partial x}$$

$$\frac{T_{x0}}{J} = G\frac{\partial \theta}{\partial x} \qquad (T = T_{x0})$$

$$\frac{\partial}{\partial x}(T_{x0}) = GJ\frac{\partial^2 \theta}{\partial x^2} \quad \text{— (5)}$$

So, you can write accordingly for elementary length, the value of T over J can be written as g times dou theta over dou x each increment, not in the entire length of dx. This will be equals to dou theta over dou x. So, now, you get the value of T over J or, as for our equation, we have used t equals to t x naught; that is the symbol we have used. So, we can write it as T x naught over J equals to G dou theta over dou x, and we have to have an equation which is given in terms of dou by dou x because, if you remember equation 4, we require a value of dou over dou x of t x naught. So, from here, we can get dou over dou x of t x naught will be equals to g j dou square theta over dou x square. So, this equation which is given over here will be similar to what is required in equation number 4. So, now, we can compare equation number 4 with respect to this particular equation. I am writing this equation as equation number 5.

Comparing eq. (4) & (5) we get $$GJ\frac{\partial^2 \theta}{\partial x^2} = J\rho\, dx\frac{\partial^2 \theta}{\partial t^2}$$

So, comparing equation 4 and 5, comparing equation 4 and 5, we get what we get from equation 4 and 5 is G J dou square theta over dou x square equals to J rho dx dou square theta over dou t square. So, dx will also not be there because this is there on both sides. So, dx can be removed from this particular equation.

$$\frac{\partial^2 \theta}{\partial t^2} = \frac{G}{\rho} \frac{\partial^2 \theta}{\partial x^2}$$

Consider $\frac{G}{\rho} = V_s^2$

or

$$V_s = \sqrt{\frac{G}{\rho}}$$

So, again, this equation can be further refined as dou square theta over dou t square equals G over rho dou square theta over dou x square. So, again, here we can see, very much similar to the primary wave, a shear wave is passing through a particular medium. There is a change in the displacement of the particle with respect to space and time, which is correlated with respect to the stiffness and mass density of the medium as given by this particular equation. Remember, in the primary wave, it was linear displacement in the direction of wave propagation. In this particular case, it is a shear wave which actually induces angular displacement indicated by d theta over a length of dx, and the incremental value we have used is dou theta over a length of dou x. So, again, over here, consider the value G over rho equals to V s square, where V s is the shear wave velocity s wave propagation velocity propagation velocity through the propagation medium. Through the propagation medium having G as the shear modulus, that is what we got from our torsional equation, and rho as its mass density. So, a medium which is having g as the shear modulus and rho as the mass density, the ratio of G over rho will be equals to the square of shear wave velocity of a particular medium, or we can write it as V s equals to the square root of G over rho. So, if you know the value of shear modulus and mass density of the medium, take the ratio of the two, take the square root, and then you will get the value of shear velocity through a particular medium.

$$\boxed{\frac{\partial^2 \theta}{\partial t^2} = V_s^2 \frac{\partial^2 \theta}{\partial x^2}} \quad \text{--- (7)}$$

So, again, keeping this equation over here, you can get dou square theta over dou t square equals to V s square dou square theta over dou x square. So, this is basically the equation which I am calling equation number 7. Equation 7 is known as a one-dimensional equation of motion for a shear wave propagating through a medium propagating through a medium. Now, one thing to be observed here is we are interested in finding out the shear wave velocity of a medium, which is now defined as the ratio of g over rho and the square root of that. So, as far as the medium is offering, definitely the medium will be having some mass density, but any physical medium will be having some mass density, but whether the medium is having shear modulus or not, that will define whether a shear wave can propagate through that particular medium or not. For example, if any liquid such as water does not offer, if water does not offer shear modulus, that means the value of water, the value of G in water, is 0, then definitely V s will be equals to 0 for water. That means, while the primary wave velocity, if you remember the numerical which was solved in the last class corresponding to water as a propagation medium, we had determined the primary wave velocity. But in the case of a shear wave, if it is passing through water, it will not be able to propagate because the medium is not offering stiffness or shear modulus, and then the shear velocity cannot propagate through water. So, G equals to 0; that means shear wave cannot propagate through water. In the same way, if any other medium is there, any other liquid is there which is not offering any shear modulus, the shear wave will not be able to propagate through that particular medium. So, this is basically lecture 15, which focuses on the determination of how the shear wave velocity of a medium can be determined using the value of G and rho and how the variation in particle oscillation with respect to space and time can be correlated with respect to the shear wave velocity because it is the velocity with which the wave is propagating through the medium. Subsequently, later on, we will also try to find out how this equation can further be used when we are interested in finding out the response of the soil, which we call in-ground response analysis or local site effect.

Q 1: Determine S wave velocity for following materials;

Material	Specific Gravity	G (Pa)
Steel	7.82	7.093×10^{10}
Water	1.0	0
Rubber	1.28	1.15×10^{9}

So, now let us solve one numerical over here. So, the shear velocity required to be calculated. The value of G as well as the value of specific gravity is given. So, the material I have used is the same as what I used for primary wave velocity determination. So, let us solve this, and we have to determine the shear wave velocity.

Sol:

a) Steel

$$V_s^{steel} = \sqrt{\left(\frac{G}{\rho}\right)^{steel}} = \sqrt{\frac{7.093 \times 10^{10}}{7.82 \times 1000}} = 3184 \text{ m/s} \simeq 3.2 \text{ km/s}$$

So, the first one is for steel. V s for steel will be equals to the square root of G of steel over rho of steel. Again, be careful with the units. So, the value is given as 7.093 into 10 raised to the power of 10 over 7.82, that is the specific gravity of steel, divided, multiplied by 1000, that is the specific gravity of the mass density of water. So, this is going to give you the value of 3184 meters per second or approximately equals to 3.2 kilometers per second. This is the wave propagation velocity of a shear wave through steel.

b) water

$$G = 0$$

$$V_s^{water} = 0$$

So, in a similar way, we can also determine for the second one, that is for water. Now, for water, because g value equals to 0, we need not solve it further. We can simply say because this value is equals to 0, V s value for water will be equals to 0.

c) Rubber

$$G = 1.15 \times 10^9 \text{ Pascal}$$

$$\rho^{rubber} = 1.28 \times 1000 = 1280 \text{ kg/m}^3$$

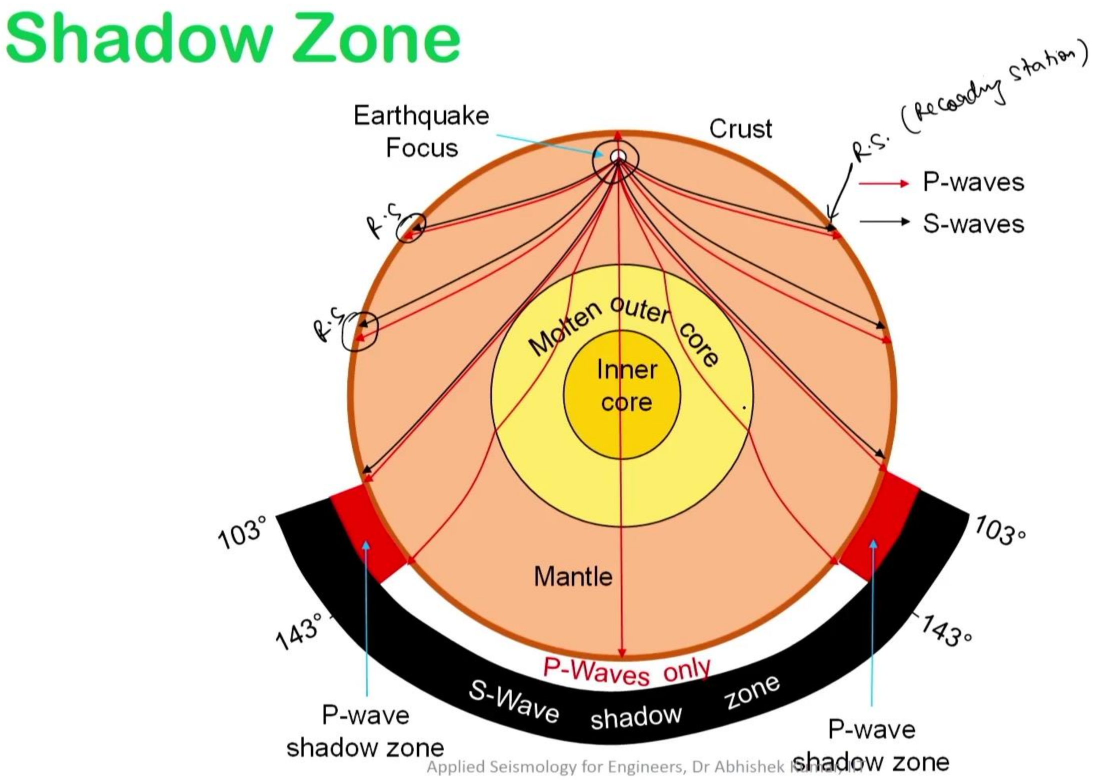
$$V_s^{rubber} = \sqrt{\frac{1.15 \times 10^9}{1.28 \times 1000}}$$

$$= 947.8 \text{ m/s} = 0.95 \text{ km/s}$$

The third one is for rubber. So, here, the g value is given as 1.15 into 10 raised to the power of 9 Pascals, and then specific gravity is given as 1.28. So, using these two values, we can determine the value of V s for rubber as the square root of 1.15 into 10 raised to the power of 9 over 1.28 into 1000. So, you can say rho of rubber equals to 1.28 into 1000 or 1280 kg per meter cube. So, coming over to this equation, we can determine the value as 947.8 meters per second or equals to 0.95 kilometers per second. So, this is the shear velocity or the propagation velocity of the shear wave. The first one is corresponding to steel, the second one corresponding to water, because water does not offer resistance to shearing, that is why the shear modulus is 0. So, the shear velocity of water is 0. The third one is the rubber. The value of shear modulus is given to us as we can see in this particular table. So, using this particular value of shear modulus and the value of mass density of the medium or the specific gravity which has been given over here, you can take the ratio of those two, and then the square root of this is going to give you the value of shear velocity. So, this is 0.95 kilometer per second. The previous one was 0, and before that, it was close to 3.2 kilometers per second.

Now, based on the derivation, what we have understood, based on collectively the derivation which we have done for the primary wave as well as the secondary wave, the primary wave can propagate to any medium, whether it is solid, liquid, or gas. However, whenever it comes to the shear wave, because it is inducing shearing in the propagation medium, as far as the medium is offering resistance or has shear modulus, the shear wave can propagate through it; otherwise, the shear wave will not be able to propagate.

Now, we know that in the Earth, there are different layers. We discussed in our earlier lectures also that Earth, you start from the surface, you have the crust, then you have the mantle, and then you have the core, and then further, it can be divided into the continental crust, oceanic crust, upper mantle, lower mantle, outer core, inner core. Depending upon the physical properties, some of these layers are in a liquid state, and some of these are in a solid state. Secondly, if you take into account the mass density of the medium, which is roughly available across the depths, that is also changing. As a result, what we see whenever there is an earthquake, and a wave has started propagating through different layers of the Earth, because of the change in the medium characteristics, the wave will undergo refraction. There will be reflection as well. But in addition to that, because some waves can propagate through liquid and some waves cannot propagate through liquid, even after reflection and refraction, once these waves reach the solid-liquid interface, primarily the shear wave will not be able to propagate further. However, the primary wave will continue propagating further and getting detected at different locations.



Let us see this particular figure. So, here we are trying to highlight the shadow zone. Shadow zone means though there is an earthquake, you can see over here, there is an earthquake focus shown over here. Because of this earthquake, seismic waves started propagating. Firstly, the primary wave, then the secondary wave started propagating. As far as the medium remains the same, the wave will continue without much change in the characteristics. If you take the medium as homogeneous and there is no unconformity or heterogeneity present in the medium, the wave will continue and will get detected at a recording station. So, there is a recording station. So, R.S., I am calling it as a recording station over here. Then you are having, again, R.S., R.S. But this is the position of the recording station. You see, because each of these recording stations is having sensors, depending upon, once the wave reaches a recording station, the sensors are able to sense the vibration, and we will get the ground vibration during that particular earthquake. Now, we also see that as you go deeper and deeper, there will be some molten state, and there will be some solid state.

As a result of which, whether you talk about the primary wave, which is indicated by the red line, you see over here, the primary wave is propagating in deeper layers, and then further, after undergoing refraction, there will be a slight change in the wave propagation path, or the incident wave and the wave leaving a particular layer. It is actually moving towards normal and away from normal because there is a significant change in the medium characteristics between these two interfaces, these two interfaces. As a result of this, the wave will not continue like this and get detected at the recording station. Rather, there will be some deflection from its continuous path as it started from the focus at two places minimum, wherever there is a significant change in the medium characteristics, and then further, it is getting detected at the recording station. So, if you see over here, this particular zone, which is highlighted over here, in this particular zone, with respect to the epicenter, there will be no P-wave detected, primarily because of the medium characteristics. The wave had undergone deflections. This is related to the P-wave. So, you can say any recording station located within an azimuth of 103 degrees to 143 degrees with respect to the epicenter—these are not, again, absolute values—these are with respect to the epicenter. So, with respect to the epicenter, this is 103 degrees, and the other one is 143 degrees. So, all the recording stations located between 103 degrees and 143 degrees with respect to the epicenter will not have any sign of a primary wave.

Now, coming over to the shear wave, because we see there are some molten state, liquid state, within the Earth. So, when the shear wave starts from the source or the focus of the earthquake, and when these waves, during their propagation path, encounter a material in a liquid state, the wave will not be able to propagate further. So, you will see there will not be any sign of a shear wave in this particular medium. So, all the shear waves which were actually propagating through solids or semi-solids, that will continue their path, may undergo some reflection depending upon the change in the medium characteristics over the distance. But once it encounters a liquid, the shear wave will be completely arrested; it will not propagate further. As a result, again, any recording station located between 103 degrees on either side of your focus—that means this is 103 degrees, 103 degrees on this side, and 103 degrees on the other side—with respect to the focus. So, if the focus changes from here to here, then with respect to this, whatever 103-degree recording stations are located, this entire stretch will not have any shear wave, no S-wave. So, this particular zone given over here from 103 degrees to 103 degrees, this entire zone is called the S-wave shadow zone. So, there is the shear wave completely shadowed. There will not be any shear wave detected at a recording station located at 103 degrees with respect to your focus. Similarly, from 103 degrees to 143 degrees, there will not be any primary wave. This is important because if there is a recording station and it is not detecting any primary or secondary wave, that does not mean there was no earthquake. It also does not mean that the recording station is not working. Simply, it means that because of the relative position of the recording station with respect to the focus, the recording station is not in a position to detect any shear wave or primary wave. These are called the primary and shear wave shadow zones.

So, with this, I have come to the end of this particular lecture, which is mainly focused on the one-dimensional equation of motion for the shear wave. In the next lecture, I will be discussing how the equation we have just derived, how the solution of this equation can be determined further. The solution of this equation will be used in later lectures. So, thank you, everyone, and we will catch you in the next lecture. Thank you.