Applied Seismology for Engineers

Dr. Abhishek Kumar

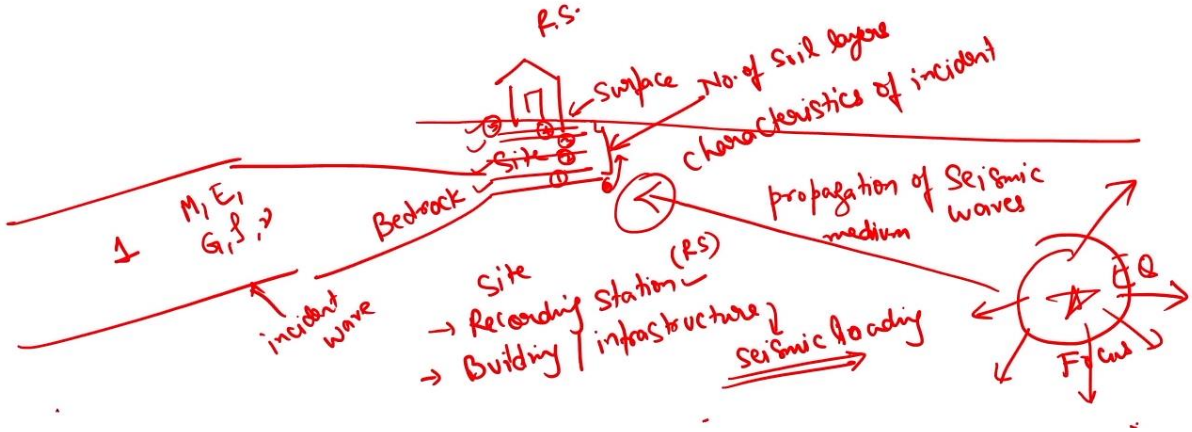
Department of Civil Engineering

Indian Institute of Technology Guwahati

Week – 07 Lecture - 03

Lecture – 16

Hello everyone, welcome to lecture 16 of the course Applied Seismology for Engineers. In lecture 14 and lecture 15, we have discussed primarily about the equations which govern the process of wave propagation. In lecture 14, we discussed about primary wave passing through an elementary rod. In lecture 15, we discussed about propagation of shear wave through a particular medium, how the propagation is governed by the equations which are actually representing the characteristics, primarily the mass density as well as the stiffness of the medium. Now, whenever it comes to using the ground motion record for design purposes, primarily we will be interested to know how much the characteristics of ground motions have been altered by a particular medium.



To elaborate further on this particular part, we can see over here. Suppose, somewhere here there was an earthquake. As a result of this particular earthquake, waves have been generated, including primary wave and secondary wave. Once these waves reach the recording station, so here is a recording station, you can see over here. So, this is your recording station. Recording station, you can have rotating drum kind of, you can have piezoelectric one which will sense based on the vibration, there will be movement in the piezoelectric component, and that will detect how much will be the vibration in the ground. So, what will happen because of this wave propagation, propagation of seismic waves, primarily we will be focusing about what is happening at a site. Now, when we say one particular terminology as site, we are interested where recording station is located, or a building is located, or any other infrastructure which is about to come or any other infrastructure. Such that we are interested either if we talk about recording station, we are interested to know what is the characteristics of ground motion which has been recorded by this particular recording station. When we are talking about the building, we will be discussing about whenever this kind of earthquake which is potential to happen in a particular region, what is the external loading, what is the seismic loading your building or any other infrastructure is going to experience. Now, in order to ensure that the structure remains safe during a particular earthquake loading, we have to make sure that this kind of seismic loading which is going to be implemented on a particular structure, the structure is going to respond such that the deflection and other parameter which are governing the response of the structure should remain within permissible limit. So, as far as recording station is concerned, it is going to sense the vibration, but if you see the overall understanding about the problem definition or the physical characteristics of the problem. So, here is the focus from where the seismic energies have been released. Seismic energy has been released from here, and then by means of waves which are actually propagating in three-dimensional space all around from your point of origin, wherever it is interacting, it will come into medium interaction which I have discussed in earlier lectures also.

So, between the focus and your site of interest, there will be lot of heterogeneity which are present in the medium. Some of these we have already discussed when we were discussing about attenuation of seismic waves between the source and the site including heterogeneity which are present in the medium, including energy which will be redistributed by means of heat, by scattering. So, finally, whatever energy seismic waves are carrying and reaching a particular site of interest, again at a particular site, there is bedrock which is present at which primarily the incident wave will be reaching. Now, between the bedrock and the surface, bedrock means it will be located at certain depth unless it is an outcrop motion. The bedrock will be located at certain depth, and between the bedrock and the surface, there will be n number of soil layers.

Most of the time when we are laying our foundation, we often lay our foundation in soil medium. This is generally encountered, but at times you can also encounter hard strata available at shallower depth. Primarily because the waves are incident over here at this particular location and here onward we have discussed whether it was primary wave or it was shear wave, we have discussed like depending upon the characteristics of each of these layers 1, 2, 3, 4, 5. So, total we are seeing in this particular example like total 5 layers are there depending upon the characteristics of each of these layers will govern how much particle oscillation between the bottom to the top of first layer will happen whether it is because of primary wave, if it is secondary wave how much shearing is happening or angular displacement is happening. Same way once it reaches to the top of layer 1, it will be the bottom of layer 2, and then again depending upon the stiffness and mass density of the medium you can again see how much change because considering the stiffness which each of these layers are offering, it is going to change the characteristics of incident waves.

So, if you are talking about layer number 1 which is the bottom most layer, there will be some incident wave, there will be some frequency content, duration, amplitude associated to that frequency content or the vibration which is incident over here depending upon the medium characteristics, the medium will offer some stiffness or resistance as a result of which between these particular layers. So, if you zoom in, this is layer number 1 having some value of constraint modulus, Young's modulus, shear modulus, mass density, Poisson's ratio that is going to define if this is my incident wave that means there is some frequency content or if I put some recording station related data some frequency content of the motion is there. When this wave is interacting this particular layer which is also composed of number of particles and overall, the material is also having some stiffness value and density values. What will happen? The material will start responding to this loading whether it is by means of to and fro motion by means of shearing as a result when this particular motion will be transferred from here to the top of the layer 1, remember this is not the top of overall stratification, this is not surface, this is top of this particular soil layer. The characteristics of the motion between these two stratifications between the bottom of the same layer and the top of the same layer that will significantly change. So, we will discuss about this particular part which is primarily change in the frequency content amplitude and duration or change in ground motion characteristics between the surface and the bottom of the same layer which is primarily a function of stiffness and other properties of the soil layer.

The reason why we are interested to know about the material properties that is constraint modulus, Young's modulus, shear modulus is because of inherent properties of each of the soil layer there will be change in the frequency content, there will be change in the duration, amplitude of the wave or the frequency content or the vibrations which is incident from propagation path. Now, same exercise of change in the ground motion characteristics will happen for layer number 2, then 3, 4 and subsequently for 5. So, if you are talking about designing a building or any other infrastructure, similarly if you are talking about the vibration which will be detected by recording station at the ground surface as per this particular picture vibration which was incident at this level and the vibration which are being detected by the recording station or are governing the response of your infrastructure will be significantly different. In other ways, if I am interested to design a particular building on the ground surface, I should be knowing how much change in the vibration which leads to my site but still at bedrock level has happened because of all these layers which are present between the bedrock and the surface as a result of which now I will be getting a modified ground motion, I will not be getting the same ground motion which was incident at the level at the bottom of this particular layer but some modified ground motion and if I am able to find out this particular modified ground motion that means I am able to get an idea about seismic loading with my structure, building, dam, bridge, slope is expected to experience because of this modification primarily because of local soil which are available near the ground surface. So, primarily one thing which will come into picture here is local site effect.

We will discuss about what is local site effect in the next class, but in a nutshell, you can see local site effect means modification in characteristics of incident motion by means of propagation medium characteristics. So, two properties are there, one is ground motion properties or characteristics which are actually incented, and the second one is the medium through which the wave is propagating as it is moving from the bottom to the top of each of these soil layers. Subsequently, after a lot of such phenomena getting repeated between bedrock and the surface, finally you will get modified ground motion. So, this modification, which is happening primarily at the site because of the soil which is available beneath the ground surface, is called as local site effect by means of properties of propagation medium.

In earlier lectures, we have discussed about when the wave is passing through a particular elementary rod, when primary wave is there, what is the equation which is governing the propagation of wave through that particular medium where the particle oscillation is also in the direction of wave propagation. But at the same time, when we are discussing about primary wave, we also discussed that, in general, primary wave is useful for detection of threshold values of seismic loading. It can be in terms of different parameters, but certainly, we are using primary waves so that one can understand a deadly earthquake-related seismic wave is about to hit an infrastructure. And when seismic wave, primarily the shear wave, is going to hit your infrastructure, there will be development of shear stresses. The material will undergo shear deformation; then there can be chances of failure, there can be chances of liquefaction, and so on and so forth.

So, the objective of deriving the primary wave equation was to find out how to get the value of primary wave velocity. Similarly, we also derived the governing equation for shear wave velocity. Now, considering loading point of view or from the objective like we are interested to quantify what is local site effect or what is the modification which is being done to the input ground motion, which is the motion at the bed of each layer to the top of that particular layer, we have to have an understanding about the governing equation. Which means at each of these soil layers, soil layer 1, how the displacement value or velocity or acceleration values are going to change as we move within the soil layer from one point to another point. This is with respect to space because we are talking about dynamic loading condition, so there will be change in terms of time as well. That means, right now, if I consider here itself a particular point of observation, how this particular point, the displacement is changing with respect to time when some incident wave is applied at the bottom of this particular soil layer. Same way, you can transfer to other point. Overall, for each particular point, how the motion with respect to space and with respect to time is changing, that is going to give me how much modification in my incident wave motion, which can also be considered as maybe some plot of displacement time history or acceleration time history. That means how, with respect to time, the displacement or the acceleration or the velocity values have changed because now the motion due to seismic loading has been applied to the bottom of that layer. So, this is incident motion, and then subsequently, this motion at each of these locations will keep on changing.

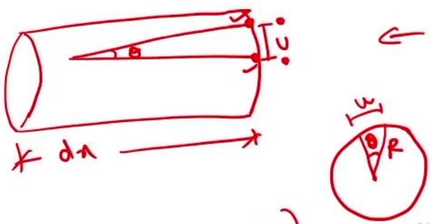
So, when we say about the solution of one-dimensional equation of motion, basically, we are interested to find out the governing equation based on which I will be able to determine what is the value of displacement at each of the positions at any point within the soil mass. That means, with respect to space and at any moment of time, that means before earthquake loading, during earthquake loading, or at the end of earthquake loading, how much are the displacement values or how much displacement in the particle has happened because of incident wave. Because when this incident wave reaches the bedrock, again some component will be travelling through these particular soil layers. So, in order to find out the solution, basically, we are interested to find out the response of each soil layer to external loading by seismic waves. More specifically, here we will be discussing about S waves because, primarily, it has been observed that shear waves cause more modification, which is important as far as the response of the building and other infrastructure is concerned. So, we will be discussing about the one-dimensional equation of motion primarily for shear wave. We have already determined one-dimensional equation of motion. Now, in today's class, we will be discussing about the solution of this one-dimensional equation of motion for shear wave.

So, let us put the heading as 'Solution to One-Dimensional Equation of Motion.' Here I want to repeat once more that when we say about the equation of motion, that means the equation which is correlating the displacement value with respect to space and time with respect to material properties. When we say about the solution, basically, we will be interested to know how much change in the displacement characteristics across your medium of interest has happened with respect to space, with respect to time, because it is dynamic loading condition. So, 'Solution to One-Dimensional Equation of Motion for Shear Wave.' So, here, if you remember, based on earlier discussion, based on earlier discussion, more specifically related to lecture 15, which was the lecture just before this particular lecture.

$$\boxed{\frac{\partial^2 \theta}{\partial t^2} = V_s^2 \frac{\partial^2 \theta}{\partial x^2}} \quad \text{—} (1)$$

The governing equation of motion for S wave is the governing equation of motion for S wave is dou square theta over dou t square equals V s square because this is for shear wave V s square and dou square theta over dou x square. Now, this is the equation which was given in terms of theta, which, if we remember lecture 15, it was angular displacement. By angular displacement, because when shear wave is passing through the medium, the medium will undergo horizontal as well as vertical displacement simultaneously, and that is how we have approximated it to the application of torque in the direction perpendicular to the wave propagation. Because of this torque, there will be angular displacement, though the particle will be moving along the circumference, but the measurement of movement of the particle we have done in terms of theta, which was angular deformation.

θ = angular displacement



$\left(\theta = u/R\right)$

u → particle displacement along the surface of the rod

R → radius of the rod

Again, if we want to see on that particular element, like if we consider maybe this particular length of the material, which was considered as dx elementary length. So, this is again still as u. That means the linear displacement along the circumference in which the position of the particle has changed from here to here, or you can say more specifically from here to here, the particle position or the initial position of the particle has shifted from position 1 to position 2 because the particle is subjected to torque. And this was given as theta, that is angular deformation. And if you see again in this particular view. So basically, we are trying to find out here. This is the value of theta, which is representation of the value of u happening along the circumference, and this is the value of R. So, in general, we can see over here that the value of theta. So, u is along the circumference, theta is the value, which is measured radially, and R is the value of radius with respect to the surface, the radius of this cylindrical rod. So, here, we can also see the value of theta can be defined over here as u over R, considering the value of torque is relatively smaller. So, you can approximate over here where u is particle displacement along the circumference or along the surface of the rod, R is the radius of the rod, and theta is angular displacement, which is already told over here. Now here, in this particular equation, which is given as equation number one, we can see the value of theta has been partially differentiated twice, once with respect to t, that is time, and the second time with respect to x also.

$$\theta = u/R$$

$$\frac{\partial^2 \theta}{\partial t^2} = \frac{1}{R}\frac{\partial^2 u}{\partial t^2}$$

Similarly

$$\frac{\partial^2 \theta}{\partial x^2} = \frac{1}{R}\frac{\partial^2 u}{\partial x^2}$$

So, based on this particular equation where theta is given as u over R (where u is the particle displacement measured along the circumference, and R is the radius of that elementary rod), we can get the value of dou square theta over dou t square as 1 over r dou square u over dou t square. Similarly, on one side we have dou square theta over dou t square, and on the other side, we have dou square theta over dou x square, which can also be determined as 1 over R dou square u over dou x square. Putting the values of dou square theta over dou t square and dou square theta over dou x square from above into equation 1, one can get.

$$\boxed{\frac{\partial^2 u}{\partial t^2} = V_s^2 \frac{\partial^2 u}{\partial x^2}} \rightarrow$$ 1D equation of motion for S- wave in terms of displacement. u

So, what you can get over here is dou square u over dou t square equals Vs square dou square u over dou x square. This is still called the one-dimensional equation of motion (1D equation of motion) for the S wave in terms of displacement u. So, this is the one-dimensional equation of motion. We are still on the equation of motion; we have not moved to the solution of this particular equation. So, when we are interested in finding the solution, we will use this particular equation: dou square u over dou t square equals vs square dou square u over dou x square.

Now, in order to solve the governing equation of motion, we can take help from the method of variable separable. Before going to this particular method, we can take help from u, which defines the motion of the particle with respect to space as well as time, because when loading is applied, the particle can undergo deformation, which changes with respect to space as well as time.

$$u(x,t) = \theta(x)\,T(t) \quad - (1')$$

$$\partial^2 u/\partial x^2 \;\&\; \partial^2 u/\partial t^2$$

So, I am considering u as a function with respect to space and a function with respect to time. So, Q is a function of u with respect to space, and t is a function of u with respect to time. I am considering that, though u was a combined function, it can be separated into two parts: one looking after the variation with respect to space, and the other with respect to time. Now, in our basic equation, we have dou square u over dou x square and dou square u over dou t square, which were correlated by means of Vs square.

$$\partial^2 u/\partial t^2 = \theta \frac{\partial^2 T}{\partial t^2} \quad - (2)$$

Using this particular equation, you can determine dou square u over dou t square equals Q dou square t over dou t square. This is the double differentiation of u, which is given in this particular equation with respect to t partially.

$$\partial^2 u/\partial x^2 = T\,\partial^2 \theta/\partial x^2 \quad - (3)$$

Similarly, dou square u over dou x square equals t dou square Q over dou x square. Now, keeping both, put the values of dou square u over dou x square—let this be equation 3—and dou square u over dou t square—let it be in the same order dou square x and then t. From equation 2 and 3, you can call this 1 prime because 1 was given, and now we have moved to equations 2 and 3.

$$Q \, \partial^2 T/\partial t^2 = V_s^2 \, T \, \partial^2 Q/\partial x^2$$

So, in 1 prime, what you will get is q times dou square t over dou t square equals Vs square t times dou square Q over dou x square. If you go with separation by the variable separable method, one can get. So, you can see this is with respect to time, and this is with respect to space.

$$\frac{1}{T} \, \partial^2 T/\partial t^2 = \frac{V_s^2}{Q} \, \partial^2 Q/\partial x^2 = -\omega^2 \quad (\text{assume})$$

$$\frac{\partial^2 T}{\partial t^2} + T\omega^2 = 0 \qquad \frac{\partial^2 Q}{\partial x^2} + \frac{Q\omega^2}{V_s^2} = 0$$

So, what we can do is 1 over T dou square t over dou t square equals vs square over Q dou square Q over dou x square. Assume this particular thing as minus omega square. That means, if you take this particular part with respect to this part, you will get dou square t over dou t square plus t times omega square equals 0. This is one equation, and the second equation is dou square Q over dou x square plus Q omega square over vs square equals 0. That means, in the first, you have taken these two things, and you will get this equation, and in the second, you will get these two things, and then you will get this particular equation. So, in total, you have two equations.

$$\frac{\partial^2 T}{\partial t^2} + T\omega^2 = 0$$

$$\lambda^2 + \omega^2 = 0$$

$$\Rightarrow \lambda = \pm i\omega$$

As far as the first equation is concerned, dou square t over dou t square plus T times omega square equals 0. From this equation, we can compare and find the characteristic equation of this linear equation. You can find that the characteristic equation will be lambda square plus omega square equals 0, indicating lambda equals plus minus iota omega.

$$T = C\cos(\omega t) + D\sin(\omega t) \quad (5)$$

Now, if that is the case, the solution will be the value of T, which can be determined as C times cosine omega t plus D times sine omega t. Let this particular equation be given as number 5. The equations given over here, these two equations, let these be part of equation number 4—both these equations. Equation 4 has basically one with respect to time and one with respect to space; both are linear equations. So, first, we have taken the linear equation with respect to time, tried finding the characteristic equation, and then, based on the characteristic equation's functional form, we tried finding the solution. Thus, we got lambda equals plus minus iota omega, and then the solution corresponding to t, which represents the part of u that varies with respect to time. This is the equation.

$$\frac{\partial^2 Q}{\partial x^2} + \frac{Q\omega^2}{V_s^2} = 0$$

Now, consider the second part of equation 4. That means dou square Q over dou x square plus Q omega square over vs square equals 0.

$$\delta = \pm i\left(\omega / V_s\right)$$

In this particular equation, again, one can determine, similar to the previous equation, the characteristic equation, which will be delta equals plus minus iota omega over Vs. Everywhere we write vs because we are trying to find the solution when the shear wave passes through a particular medium and how the particle displacement throughout your medium of interest happens with respect to space and time.

$$Q = A\cos\left(\frac{\omega}{V_s}x\right) + B\sin\left(\frac{\omega}{V_s}x\right) \quad \text{— } 6$$

Now, if we get the characteristic equation, the solution for this part—the solution for Q, which was the function with respect to space—will be A cosine omega over Vs times x plus B sine omega over vs x. This is one particular equation I have given as 6. Equation 5 gives you the particle displacement variation with respect to time. Equation 6 gives you the particle displacement variation with respect to space. Once you know the value of the components of u with respect to space (Q) and time (t), u can be determined. U is the particle motion. The actual solution of the entire equation depends on u. If you see the one-dimensional equation of motion for the shear wave, it was changed to the value of u. So, u equals Q times t. Now we have the value of Q, and we have the value of t. Let us write those equations separately in the next slide.

$$u = \left[A\cos\left(\frac{\omega}{V_s}x\right) + B\sin\left(\frac{\omega}{V_s}x\right)\right]\left[C\cos(\omega t) + D\sin(\omega t)\right]$$

$$= AC\cos\left(\frac{\omega}{V_s}x\right)\cos(\omega t) + AD\cos\left(\frac{\omega}{V_s}x\right)\sin(\omega t) + BC\sin\left(\frac{\omega}{V_s}x\right)\cos(\omega t)$$

$$+ BD\sin\left(\frac{\omega}{V_s}x\right)\sin(\omega t)$$

U can now be written as follows. We have two equations: the first is A cosine omega over Vs x plus B sine omega over Vs x, which is with respect to Q. A and B are the coefficient values for Q, and C and D are the coefficient values for t. Then, C cosine omega t plus D sine omega t. So, this is the complete solution for u. If solved further, you can get A cosine omega over Vs times x. C will also come over here and then cos omega t plus AD cosine omega over Vs times x and sine omega t plus BC sine omega over Vs x cosine omega t plus BD sine omega over Vs x sine omega t. Now, here we will make another assumption, that is, assume omega over Vs equals k, small k, which is representing wave number.

[Assume $\frac{\omega}{v_s} = k$
wave number,
k refers to number of complete
wave cycles in unit meter]

So, omega over v s, I am again modifying with respect to k, wave number, this refers to the number of complete waves or wave cycles in a unit meter. That means, within one meter, how many cycles, depending upon the operating frequency, are able to complete will define how much is your wave number. So, accordingly, this is one assumption. Second, you can extend it further.

Consider
$AC = p, \quad AD = q$

$BC = r, \quad BD = s$

Again, consider A C equals small p, A D equals q, B C equals r, and B D equals small s.

$$u = p\cos(kx)\cos(\omega t) + q\cos(kx)\sin(\omega t) + r\sin(kx)\cos(\omega t) + s\sin(kx)\sin(\omega t) \quad -(7)$$

Hence, this particular equation, which was given for u, can be modified as p cos k x cos omega t plus q cos k x sin omega t plus r sin k x cos omega t plus s sin k x sin omega t. This is the equation; I am considering this as equation number 7. Now, again in equation number 7, let us make some more assumptions.

Let us consider

$$\tan(\omega t) = m/q \Rightarrow q \sin(\omega t) = m \cos(\omega t)$$

&

$$\tan(kx) = n/r \Rightarrow r \sin(kx) = n \cos(kx)$$

Let us consider the tangent of omega t equals m over q, which indicates that q sin omega t will equal m cos omega t. Similarly, the tangent of k x is considered as n over r, which indicates r sin k x equals n cos k x. So again, let us use these values: q sin omega t, which is given here, and r sin k x, which is given here.

$$u = p \cos(kx)\cos(\omega t) + m \cos(\omega t)\cos(kx) + n \cos(kx)\cos(\omega t) + s \sin(kx)\sin(\omega t)$$

$$u = (p + m + n)\cos(kx)\cos(\omega t) + s \sin(kx)\sin(\omega t)$$

Replacing this with the updated terms, we will get p cos k x cos omega t plus m times cos omega t cos k x plus n times cos k x cos omega t plus s times sin k x sin omega t. So here, actually, if you see, each term earlier was having cos and sin components in two terms, which now have been converted to cos terms. So, if you see in term 1, term 2, and term 3, it is only consisting of cos k x cos omega t. So, if I say p plus m plus n cos k x cos omega t plus s sin k x sin omega t, this is the value of u, which indicates the particle displacement with respect to space and time.

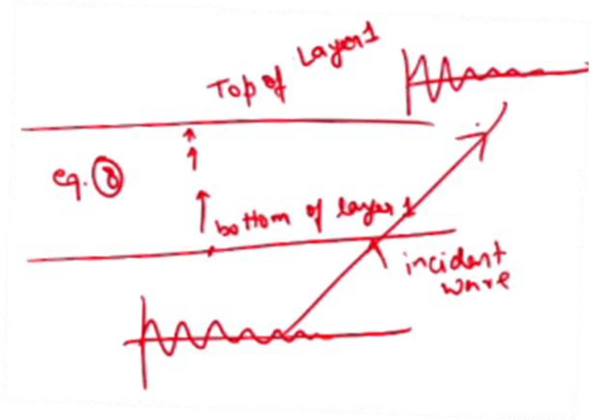
$$u = (E+F)\cos(kx)\cos(\omega t) + (E-F)\sin(kx)\sin(\omega t)$$

$$p+m+n = E+F$$

$$s = E-F$$

$$\boxed{\underbrace{u}_{\text{particle displacement}} = E\cos(\underbrace{\omega t}_{\text{time}} - \underbrace{kx}_{\text{space}}) + F\cos(\omega t + kx)} \quad (8)$$

And if you consider p plus m plus n equals E plus F and s equals E minus F, we can determine u equals E plus F times cos k x cos omega t plus E minus F sin k x sin omega t, which can further be simplified as u equals E times cos omega t minus k x plus F times cosine omega t plus k x. So now we are having the displacement value; this is particle displacement because of a shear wave propagating through the medium, given as a combined effect of variation with respect to time and space, both in terms of cosine terms. So, we can number this particular equation as equation number 8. Equation number 8, which is the solution of the one-dimensional equation of motion for a shear wave passing through the medium. What it means is if there is an incident wave, and I know the characteristics of the incident wave, using this particular equation, I should be able to determine, with respect to space, at h equals 0 or at x equals 0, at x equals the thickness of that particular layer, in that particular range, from time t equals 0 to the time of interest or the time up till which the duration of external loading is available with us, one can determine how much modification in the ground motion will happen by means of the medium through which the shear wave is propagating.



So, what I am trying to highlight here is, if this is the incident wave having some ground motion characteristics, using a particular equation, that is equation number 8, I will be able to determine how much vibration, which is incident at this particular level, that is, the bottom of layer 1, how much this vibration will propagate or how much modification in the same vibration will happen from the bottom to the top of layer 1. So, this is the modification in the vibration at layer 1. Definitely, the frequency content will be changing using this particular equation. Once we discuss the role of each soil layer and how to quantify it, as I discussed at the beginning, based on the quantification of the local site effect, we will be able to determine how much modification in the frequency content of the motion will happen between the bottom and the top of one soil layer with known properties and known input ground motion characteristics. Once that part is over, because this value is also a function of the strength or the stiffness properties of the medium, those properties will also come into play when we discuss the modification in the ground motion between the bottom to the top of that particular soil layer. If there is more than one layer, then the same exercise of modification of ground motion between the bottom to the top of that layer will keep on happening, such that the motion will transfer from the bottom of the lowermost layer or the bedrock to the topmost layer or to the ground surface, where one is going to construct a building-related foundation or any kind of other infrastructure. If a recording station is there, then the recording station will also detect that modified ground motion, not the motion which is actually being transferred at the bedrock level from the site of interest.

So, thank you, everyone. With this, I will close lecture 16 of this particular course. When we meet for lecture 17, we will continue this particular solution and try to find out how the modified ground motion can be evaluated for different characteristics of soil properties. Thank you.