Applied Seismology for Engineers

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Week – 08 Lecture - 02

Lecture – 18

Hello everyone, welcome to lecture 18 of the course Applied Seismology for Engineers, myself Dr. Abhishek Kumar. In previous lectures starting with lecture 14, 15, 16, we discussed about different kinds of seismic waves which are generated from the source and reaching to the recording station. At the same time if recording station is not there, building is there, then these seismic waves will interact with the structures and depending upon the resistance structure is going to offer to the loading induced by seismic waves will decide whether the structure or any particular kind of soil or any other structure whether it will be able to withstand the additional loading because of seismic activity or not. Then we discussed the governing equations which are going to control the primary wave propagation through a particular medium. In such a case the particle will undergo to and fro motion in the direction of wave propagation. Then we determined what will be the governing equation of motion for primary wave followed by that we discussed about what if secondary wave that is the wave which is followed by primary wave or reaching second at the recording station is propagating through a particular medium, what will be the governing equation? We discussed when these two waves, primary wave is propagating it is causing to and fro motion and shear wave is moving through a particular medium it is going to trigger shearing in the direction perpendicular to the wave propagation that means both in upward and downward direction as well as in horizontal direction. Both these directions will be perpendicular to the direction of wave propagation. Continued with that particular discussion we try to determine the governing equation of motion and then in lecture 17 we discuss about how this particular governing equation of motion can be solved using method of variable separable because there will be component of displacement, which is a function of space, there is a component of displacement which is a function of time.

So, when we take both these components into account we cannot bifurcate and get one solution we have to separate function with respect to space, function with respect to time and then try solving the equation. So both component will be now used to find out how a particular layer is going to respond if it is subjected to bedrock motion. In lecture 17 we also discussed about whenever there is an incident wave firstly, we discussed about local site effect which is primarily the modification in the ground motion characteristics at a site of interest primarily because of the local soil characteristics which are available above the bedrock and below the ground surface. At time there can be one layer, at time there can be more than one layer. So, depending upon the stiffness properties of these layers when we say about stiffness primarily, we are talking about dynamic soil properties that is damping ratio as well as the shear modulus of the material.

When we are talking about the soil the dynamic properties means these two properties that is shear modulus as well as damping ratio it changes with respect to the level of shear strain which a particular earthquake loading is going to induce in the medium of propagation. What it means if there is a layer of medium that is soil layer is there depending upon the shear strain which is mobilized in this particular layer because of propagation of shear wave through this particular medium that is going to define how much resistance this particular soil layer is going to offer whether in terms of modulus, shear modulus whether in terms of damping and collectively it is going to define how this particular soil layer having one value of damping as well as shear modulus is going to respond to external loading condition. It has been observed number of times across different earthquakes and in different parts of the globe that same site can show very high shear modulus, very low damping ratio or very low damping ratio very high shear modulus depending upon what is the level of shear strain which has been mobilized in that particular soil layer during a particular earthquake loading. In lecture 17 we also discussed that equally with respect to soil characteristics it is also important to know that the ground motion which has been given to a particular analysis whether that motion has been defined at bedrock level that means the rock level which is beneath the soil layers or it has been defined as outcrop level that means where the soil is where the rock is exposed to the ground surface or whether it is bedrock outcrop motion where bedrock is there but there is no soil above it. So even that particular bedrock is corresponding to free surface condition, or another case can arise where the ground motion which has been recorded and provided for ground response analysis that motion itself is available at the ground surface that means where one is going to construct a particular building or any other utility.

So depending upon where the motion has been recorded one should be careful in taking whether the particular motion ground motion signature can be directly taken into account for doing ground response analysis or the motion which has been provided whether it is corresponding to regional records whether it is synthetically generated motion or motion adopted from other region having similar system activity as that of the site of interest. Once a motion has been selected because it is how the soil layers are going to respond to that particular motion. So, one is the input soil properties which one can determine based on in-situ investigation as well as quantification of dynamic soil properties of soil. Input motion if it is bedrock motion you can directly use if it is bedrock outcrop motion you have to suitably modify that particular motion similarly if it is outcrop motion you have to transfer that motion to bedrock condition. If it is free surface motion either you can use it directly for quantification of seismic loading or if it has been recorded at different site then you have to transfer that motion to bedrock and then use that motion to other site as a same bedrock motion followed by ground response analysis.

So, this process of selecting the motion doing ground response analysis transferring the motion from bedrock to the surface transferring motion from surface to bedrock transferring motion from outcrop to bedrock or transferring motion from bedrock outcrop to outcrop depends upon the motion site characteristics and what is the objective whether it is ground response analysis or determination of bedrock level ground motion characteristics. So, depending upon the objectives the use of motion and the purpose of ground response analysis can be selected. Then we discussed primarily the motion can be of elastic half space or rigid half space that means the motion when it is incited on the bedrock medium depending upon the characteristics of the medium sometimes some portion of seismic energy which the wave is carrying will be contained in that medium and not the entire part of energy will be propagated further upward towards the surface that will be the part of elastic half space. If it is rigid half space a significant portion of the energy or you can say almost all the energy with seismic wave at the time of incident on the bedrock level will be transferred to the surface and subsequently it will control the response of the soil which is available above bedrock to the different soil layers. So, continuing with that part now we have understood based on our understanding so far that firstly is the input motion secondly the bedrock characteristics thirdly the soil characteristics primarily these are the three important informations which one will be interested to know before starting ground response analysis.

So, lecture 17 was part one of the topic local site effect and ground response analysis this is continuation of the same topic here we will try to solve one typical field problem. So, we will also discuss how to go about the solution if we recollect the discussion from lecture 17, we will try to remember that at the time of developing the solution for one dimensional ground response analysis that too based on linear approach we have to have one is transfer function second is Fourier spectra of input motion. So, Fourier spectra of input motion means whenever a motion is given to you generally because of earthquake motion it will be random in nature. So, when we say Fourier spectra of a given earthquake induced motion, we are interested to find out how many cycles of harmonic motions are present in that particular given motion. Harmonic cycles means the operating frequency of the motion as well as the amplitude of each harmonic motion is going to remain constant. So, when we say we are interested to find out the frequency content of the motion we are interested to find out that for a given earthquake induced motion how many samples of harmonic frequency motions are existing.

So, that will be on the x-axis; you can say that is the frequency content of all harmonic motions. Suppose if your ground motion has n number of harmonic motions, so corresponding to each of those harmonic motions, there will be some operating frequency; that is the value mentioned on the x-axis. Now, being harmonic motion, one is the operating frequency, and the second is the amplitude at which the motion is operating. So, we can again take into account that this is a harmonic motion which is getting repeated cycle after cycle. So, you can see from here what the operating frequency is, and from here, we can also find out how much the amplitude is. So, this is the amplitude of the motion and based on the time taken by the wave in each cycle, this is one half-cycle; this is another half-cycle. So, how much time it has taken, from which you can find out what the frequency is in which the oscillation of the motion is completing. That is going to give you how much the operating frequency is, or you can find out from here the time duration, and one by the time duration, you can find out how much the frequency of this particular harmonic motion is. So here, you can take the frequency value f1, and then corresponding to f1, this is the amplitude of the motion, that is A1. So, the next time when we see Fourier spectra of any particular ground motion, basically, it is going to give you combined information or the summary of all harmonic motions present in the motion, and corresponding to each point, you will have some plot like this. So, if you take any particular motion, this is the frequency content of the harmonic motion which is present in this particular random motion, and corresponding to this is the amplitude of that motion. So, based on Fourier spectra or Fourier analysis, you can find out how many frequency contents of harmonic motions, or how many harmonic motions with different operating frequencies, are existing in a ground motion. If we recollect the topic where we discussed the ground motion characteristics primarily, we were looking at three characteristics: one is the duration of the motion, again significant duration, bracketed duration we can take into account, frequency content again, natural frequency there, or predominant frequency is there.

Similarly, the amplitude of the motion, again depending upon what is the period of interest, you can go with displacement, velocity, or acceleration values. So here, when we say about Fourier spectra, we are basically interested in finding out the frequency content of your motion. This motion is, remember, the complete Fourier spectra. So, you can write also over here Fourier spectra. So, it is going to give me a complete picture of how many harmonic motions are present in your motion. This is the motion which, if I am going to use for ground response analysis, based on the discussion given in lecture 17, this motion, if you remember, this was the motion which was actually given to you as some random motion generated during a particular earthquake, which is basically the response of your point of observation with respect to time.

So, you can see with respect to time how the position of this motion is changing, whether it is in terms of acceleration, velocity, or displacement values. So, this is the input motion, and from where you had tried determining the frequency content of the motion. So, remember, this is the motion which we are taking into account to perform ground response analysis. When we are talking about motion over here in terms of variation with respect to time, it is generally referred to as in the time domain. When we are taking specifically towards the frequency content of the motion as given in terms of Fourier spectra, we often refer to the term in the frequency domain. So, dealing with the ground motions or performing ground response analysis, we often talk about two terms: one is time domain, and the other is frequency domain.

So, in this particular lecture, lecture 18, we will be discussing further about ground response analysis. The topics which were covered in the previous class that I have already spoken about, the introduction, the significance of why ground response analysis is significant, that was discussed in lecture 17 with the citation of different earthquakes and induced damages which were primarily not confined to only epicentral distance but even at larger distances. There, subsequently, gave an indication about the importance of going for site-specific ground response analysis. Then, we discussed what is the local site effect and the method based on which one can quantify the local site effect, starting with empirical methods, semi-empirical methods, theoretical methods, and hybrid methods. So, all these we had already had an understanding in a very brief.

Further, we discussed more about theoretical methods or numerical methods that can be used to quantify local site effects, which take into account the equation of motion as well as the characteristics of the input motion. We also discussed that primarily a particular medium will offer resistance in terms of stiffness value, or you can say in terms of shear modulus, because we will be talking about elastic material, as well as the damping ratio, which was indicated by D. So, these are the two properties of the medium, based on which the material will offer resistance to loading, to deformation when external loading is applied to the particular material. Transfer function, when we talk about transfer function, so that means the transfer function was a correlation of the filter, based on which one can quantify how much modification in the ground motion characteristics will happen between the base of that particular layer and the top of that particular layer. So, if that means if there are more than one layer, every time you try to determine the transfer function in order to find out how much change in the ground motion characteristics between the bottom of that particular layer and the top of that particular soil layer is happening. Then we discussed in the light of the one-dimensional equation of motion, which was given at the end of the solution of shear wave propagating through a particular medium, that was redefined in terms of exponential form.

So, now we will be talking about homogeneous undamped soil. When we say homogeneous, so now onward when we will be discussing about ground response analysis, we will be focusing primarily on two things: one is what is the characteristic of the medium which you will be using to quantify local site effect, medium means soil medium, and secondly, what is the characteristic of the rock medium. Rock medium means the medium on which incident wave is there, so we will be talking about what is the soil medium and what is the rock medium as discussed earlier. Rigid means whatever incident energy is there, that will be progressing further towards the surface; if it is elastic, some portion of energy will contain there or propagate downward, and the rest of the energy, or a significant portion of the energy, will be propagating upward. Now again, in soil, as we discussed, it is going to offer resistance in terms of shear modulus as well as damping ratio, referring to the example of KV solids given in lecture 17. So here, we will be talking about undamped soil; that means we will be taking a relatively easier example to understand how to solve a particular equation, whether the damping is there, or it is not there.

So, first is homogeneous soil; I am not taking into account that there is a change in the properties of the medium within one particular soil, the material properties are not changing, it is an undamped system; that means there is no damping associated with the system. Thirdly, the material is associated, or the input motion has been transferred from propagation path to rigid rock. Now, let us see with that, this is the general nomenclature and geometry of the problem.

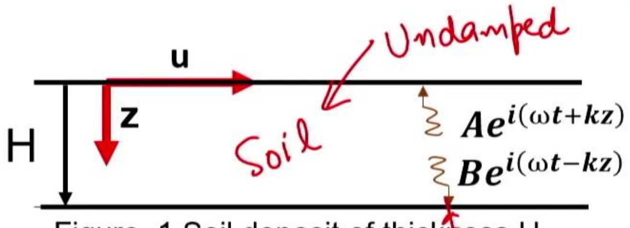

Figure. 1 Soil deposit of thickness H

So, you are having this soil layer in which you are interested to find out how much alteration in the medium characteristic will take place because of this particular soil layer, which is called as undamped. So, the soil is undamped, which has been defined by the user or in the beginning of this particular discussion, and the energy or the waves which were propagating through the site are basically to the rigid rock. So, the rock medium is rigid, whatever incident energy is there, a component of that will be propagated towards the movement of the soil layer.

Right now, we are discussing about a uniform layer of isotropic linear elastic soil. So, if you remember the discussion with respect to KV solids, the material is undamped, so the only resistance the material is going to offer to external loading will be in terms of a linear elastic spring. That means, corresponding to the loading, the material is only going to offer spring stiffness, which is going to define how much the resistance or modification in the soil properties will be.

$$U(z) = Ae^{i(\omega t+kz)} + Be^{i(\omega t-kz)}$$

Now, this was the equation of motion; that means the equation which we derived at the end of the one-dimensional equation of motion related solution has been reconverted in terms of A exponential iota omega t plus k z. z, I have used here as the thickness of this particular soil layer, which is given over here. So, this is the value of H, which is the value of thickness, and z is measured in the direction towards the thickness. That means at any point from z equals to 0, which means at the ground surface, and as you move downward, that means you are increasing the value of z, and the value of z can maximum reach up to H. That means the bottom of that particular soil layer, and this equation is going to define how much is the oscillation or how much shearing corresponding displacement is happening in the direction of U in the soil medium when it is subjected to vertically propagating SH waves that we have discussed in lecture 17. So, this is a vertically propagating SH wave, which is going to induce shearing in the horizontal direction in both directions. So, if the wave is propagating like this, shearing will be happening in this direction as well as in this direction, and this is the stratification in the soil layer, and in our ground response analysis, there is an assumption that the soil layer is extending infinitely and is horizontal. So, with that information in the background, U, the displacement value, which is a function of space as well as time, is defined as A and B are the values of the coefficients exponential iota omega t plus kz. So, remember, this is the thickness of the medium; z is the thickness of the medium, k is the wave number, that means how many cycles in unit length are going to get completed. That will be defined by the wave number k. Omega is the operating frequency of the external loading condition. So, if you take any harmonic motion, the operating frequency of that motion in terms of radians per second, so you can define the value of omega in general in circular frequency, and then, so all these terms are there.

Now, the first term, that is A exponential iota omega t plus kx or kz, is basically giving you the displacement component which is coming from vertically upward propagating. Secondly, the second component is there because there will be a redistribution of energy continuously, so again B exponential iota omega t minus kx. It is basically going to give you the component of displacement because of downward propagating waves from here. So, this particular equation, U (z,t), that means the displacement values in the horizontal direction varying with respect to space and time, is a function of two components. One component is coming because of the vertically upward propagating wave, and the second component is coming from the vertically downward propagating wave, and these two are coming because of the operating frequency of the motion as well as there is stiffness in the medium. So, if you remember, the value of k will be again equal to omega over vs.

$$k = \frac{\omega}{V_s}$$

So, this is the value of wave number k, omega over Vs, so omega is the operating frequency and Vs is going to define how much the stiffness in the medium collectively based on this equation. You can define in unit length how many complete number of cycles of the given motion can be completed. That will be defined by the wave number.

So, continuing with this particular discussion, this is the equation of motion, the general equation of motion. The only thing, if we clearly observe this particular equation, we will see that the value of k is a function of the stiffness of the medium, which is an indication of linear elastic soil. That means we are only taking into account that the correlation between shear stress and shear strain is uniform throughout the loading, or in other words, shear stress and shear strain are directly proportional to each other, and the constant of proportionality is called shear modulus.

$$\tau = G\gamma$$

That means the value of tau equals G times gamma, that is shear strain. So, this is solely governing the shear stress mobilized due to external loading condition in the medium of interest. Referring to the problem definition again, we are talking about an undamped system, so you see over here, we can recollect KV solids; some component of stress was coming from shear modulus, and some component of stress was coming from damping or damper, but here in this particular case, it is only the elastic spring which is going to offer resistance or mobilize the stresses in a particular medium, which is given in terms of elastic spring or the value of shear modulus.

So, one thing is clear: we are talking about a linear elastic spring where the only resistance is from shear modulus. Secondly, the k value is given over here; it is only a function of shear modulus or shear wave velocity. However, if you take both parameters, that is shear modulus as well as the damping ratio, into account, you will have an additional component over here, and at that particular time, this particular wave number will also become a complex wave number. That means if we take into account, rather than an undamped system, if we take into account a damped system, that means now the resistance the medium is going to offer will have an additional component which is a function of damping ratio, which is not applicable in this particular case, so I am going to cut it here, but that is how you can distinguish between one type of medium of interest and another type of medium of interest. So, when an undamped system, as mentioned over here, is available, you have this as the governing equation of motion. When the system becomes damped, the k value will be replaced by the complex wave number. Similarly, if the medium subjected to or the incident wave is applied to an elastic half-space, again, there will be some damping in the bedrock medium itself. So, again, the governing equation corresponding to rigid bedrock conditions will also be modified, taking the damping characteristics of the bedrock medium into the solution.

$$\boldsymbol{U}(\boldsymbol{z}) = \boldsymbol{A}\boldsymbol{e}^{\boldsymbol{i}(\omega \boldsymbol{t}+\boldsymbol{k}\boldsymbol{z})} + \boldsymbol{B}\boldsymbol{e}^{\boldsymbol{i}(\omega \boldsymbol{t}-\boldsymbol{k}\boldsymbol{z})}$$

So, this was the equation U (z, t) = a exponential iota omega t + k z + B exponential iota omega t - k z. All the terms have already been explained. Now, in order to find out the solution, that means this is the equation which is given to you, when we say about the solution, we are interested in finding out the modified ground motion when some incident motion characteristics are given to you. Incident motion characteristics means how, at your observation point (which may be this particular bedrock condition), what is the ground motion characteristics, and how much change in the acceleration value at unit time intervals. It can be 0.005 seconds; it can be 0.005 seconds, depending upon the rate at which every time the sensor is detecting the change in displacement, velocity, or acceleration values. That is going to give me the complete record, which is basically the ground motion of interest.

So, taking that ground motion into account and the medium characteristics, which are defined by the k value (which is basically an important component correlating with the operating frequency and stiffness of the medium), we will try to find out how the motion which has been incident at bedrock level will be changed from the bottom of a given soil layer to the top of the soil layer. So, the solution of the one-dimensional wave equation means finding out the functional form of U. The solution of this particular problem is to find out how much the modified ground motion will be at the top of the particular given soil medium. Now, omega is the circular frequency, k is the shear stress number, and A and B are the amplitudes of the motions traveling in the upward and downward directions if you compare these with respect to the general equation of motion.

$$\tau = G\gamma = G\frac{\partial u}{\partial z}$$

As I mentioned, the shear stress for an undamped system is directly proportional to shear strain, and that is the only component which is going to offer resistance to the external loading condition. So, the shear strain, we have discussed this earlier also, that can be defined as dou u over dou z. You have the value of u given in this particular equation. So, once that value of u is known to us from this particular equation, we can partially differentiate this equation with respect to z to get the value of shear strain.

$$\tau = G\left[ikAe^{i(\omega t+kz)} - ikBe^{i(\omega t-kz)}\right]$$

So, this equation, which is given, the term inside this particular bracket is basically obtained by differentiating this entire equation with respect to z. That means this is the value of u, differentiating dou with respect to z partially, and you are going to get iota k times A exponential iota omega t + kz - iota k times B exponential iota omega t - kz. So, that is the governing equation correlating the shear strain and a particular medium. Multiply that with the shear modulus of the medium, which is again constant because we are taking into account that it is linear elastic, and we are also dealing with linear ground response analysis. So, you will get the value of shear stress from here.

Now, again, there are some boundary conditions. Remember, in this particular case, we are not taking into account n number of layers available above the bedrock. We are only taking into account that there is a rigid bedrock medium on which incident waves are coming, and above this particular bedrock medium, only one layer of undamped soil is available. We are not right now concerned about n number of layers above the bedrock. In practical situations, you often encounter more than one layer between the bedrock and the surface. So, we will go step by step and see how that particular solution can be obtained when the number of layers is more than one. In this particular case, there are some boundary conditions. That means, at the free surface, only one soil layer is there. Below that particular soil layer, the bedrock is there, and above that particular soil layer, it is the ground surface, which is the free surface. So, when it is the free surface, it is not offering any kind of resistance to deformation. So, this particular free surface, that means the ground surface, the shear stress, and subsequently the shear strain are zero. That is a boundary condition. At the free surface means at z equals to zero. Remember, z is increasing with respect to depth starting from the top of the soil layer, so it is increasing downward.

$$0 = Gik(A - B)e^{i\omega t} \rightarrow A = B$$

Apply this particular equation condition to the previous equation. That is, equation number three. Zero will be on the right-hand side because this is the value of tau stress at the free surface. So, this is the value of tau stress under free condition on the ground surface. It is free to move, and it is not offering resistance, so zero equals to the other part. Now, in this, if you see, the entire equation can only be equal to zero if A equals B. That means the amplitude of the downward and upward propagating wave in this particular case are equal. When these two things are equal, you can put this in the governing equation of motion. Using this in equation number three, you will get the equation which was defining the displacement at any space and time.

$$U(z,t) = 2A\left(\frac{e^{ikz} + e^{-ikz}}{2}\right)e^{i\overset{\omega}{\cancel{w}}t} = 2Acos(kz)e^{i\omega t}$$

U(z, t) = 2 times a because now A and B are equal, so I have replaced B by 2A, and then the term within the bracket, exponential iota omega t, and this can be further written as 2A cosine of k z exponential iota omega t. Because this term, which is given within the bracket, can be replaced by means of cosine of k times z, where k is the wave number and z is the displacement or the position of the point of observation, starting from the surface of the ground toward the downward direction.

$$F(\omega) = \frac{U_{\max}(0,T)}{U_{max}(H,T)} = \frac{2Acos(0)e^{i\omega t}}{2Acos(kH)e^{i\omega t}}$$

So, again, we define a function called a transfer function. If you remember lecture 17, the transfer function was basically used to transfer the motion from the bedrock to the surface, or more precisely, from the base of a particular soil layer to the top of that particular soil layer. In this particular case, we are having just a single soil layer, so the transfer function here has been defined as the displacement maximum at the ground surface at some point of time, capital T, divided by the maximum displacement at the bottom of that particular soil layer because H is the thickness of that particular soil layer at the same moment of time, capital T. That means, at the same moment of time, when a particular soil layer is subjected to a loading condition, how much is the displacement at the surface and how much is the displacement at the bottom? That is basically going to define my transfer function.

How you are going to get the values of U(x), u at zero, and U at H (z equals to H), we have the governing equation, which is just deriving the previous equation. So, this equation you can use, which is given over here, to determine the value at the top surface (that means z equals to zero) and z equals to H. You will get these values. z I have simply replaced by means of H and equals to z also in the denominator and numerator, so the right-hand side of this particular equation, equation six, is going to define the ratio of displacements at the same moment of time between the ground surface or the top of that particular soil layer and the bottom of that particular soil layer. In other words, I am interested in finding out how much the displacement at the top of this particular soil layer will be if I know the displacement at the bottom of that soil layer based on recorded ground motion. So, in actuality, we will be interested to find out the displacement at the surface but taking the boundary condition (that is the free surface condition) into account, I have defined a filter which can be used to transfer my motion, the recorded motion, from the bottom to the top of the particular soil layer.

So, in actual, we will be interested to find out the displacement at the surface, but taking the boundary condition, that is, the free surface condition, into account, I have defined a filter which can be used to transfer my recorded motion from the bottom to the top of the particular soil layer. So, if you see the right-hand side of the equation, everything is given in terms of one over cosine of wave number multiplied by the thickness of the soil layer because other terms will get canceled out. So, the rest of the things will get canceled out. Cos zero is one. One over cosine of k times H is going to define my transfer function, which is given over here.

$$F(\omega)=1/\cos(kH)$$

Now, this is the transfer function that is going to tell me how much modification is there between the bedrock and the surface.

$$k = \frac{\omega}{V_s}$$

Remember, the value of k, which is given in equation number seven, is basically the ratio of operating frequency in radians per second divided by the shear wave velocity of the medium of interest. So, using this particular ratio, omega over Vs, I can define how much is the value of the wave number. Once that wave number is known to you, or you can directly put the value of k equals omega over Vs times H in this particular equation, you will be able to determine. See, here we did not require any value of time; we need not require any value of intermediate thickness. Simply, what we require is the value of H. That means when we are going with linear ground response analysis, we are also assuming that one particular soil layer, whatever is the thickness of that particular soil layer, at any moment of time, the soil layer will remain constant. So, any kind of motion within the particular soil layer is not going to change significantly within that; it is going to remain constant. So, omega over Vs, is basically defining the value of the wave number, and based on the previous understanding, we are saying that, throughout, at any moment of time, the entire layer is going to respond the same.

So, the modulus of the modulus of the transfer function, that means this modulus, is going to give us how much is the amplification between the bedrock and the surface. It is also notable that, based on this particular equation, the maximum value of the surface displacement can be anything. If the denominator of F omega, that is, the transfer function, becomes 0, that transfer function becomes very high. In such a case, the soil medium will experience resonance. That means whatever is the input motion, the surface motion will be remarkably high. Anything which is less than 0 means the maximum value of the denominator can be 1. In such a case, the transfer function value itself will be 1, or there will not be any change in motion properties between the base and the top of the particular soil layer. So, that means it says that the maximum value of the transfer function can be as high as infinity. The minimum value should be at least one for the transfer function based on the equation given over here because H will always have some value, and the wave number will always be there. So, the minimum value of the transfer function can be 1, and the maximum value of the transfer function can be as high as infinity. Keep on changing the value of omega because vs value is going to remain constant. Keep on changing the value of omega—0, pi by 4, 3 pi by 4, pi, and so on and so forth—and every time you put the value over here, you will get the value of the transfer function. Always keep in mind the value of omega you are going to use is in radians per second.

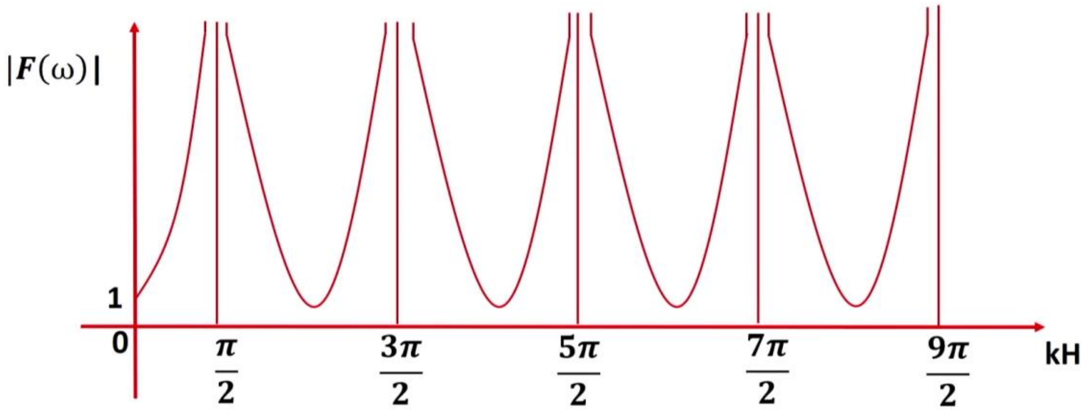

Figure.2. Graph of Transfer function versus frequency

So, if you use those things over here, you are going to define the variation in the transfer function. This is going to give you the mode of transfer function f omega, which is defined in equation 6 in the previous slide. Keep on changing the value of kH, or, more precisely, the value of omega, because h and vs are not going to change. So, the only value of omega you are going to change over here. You see over here, when the value of kH becomes pi by 2, the value of the transfer function becomes infinity. That means, at the omega corresponding value where k H equals pi by 2, your system is experiencing resonance, and this characteristic of resonance is getting repeated at every pi interval. So, I can say here any value of n pi by 2 plus pi is getting repeated. That means pi by 2, 3 pi by 2, 5 pi by 2, 7 pi by 2, 9 pi by 2—every time when the value of kH reaches these values, the transfer function becomes infinite. In between, this has some finite value, and the minimum value of the transfer function is 1. So, it is not possible to have any value of the transfer function lesser than 1.

Now, in order to understand this particular problem more specifically, we will try to solve a numerical. So, what we are trying to understand here is if there is a rigid rock on which incident ground motion is available, that means bedrock motion beneath the soil layer is available, and above that particular motion, above that particular bedrock, there is undamped soil of thickness H available, how this particular soil layer of thickness H is going to experience a change in motion characteristics between the bottom and the surface of that particular soil layer. So, what we try determining is the value of displacements at the free surface and at the bottom of that particular soil layer, and the ratio of these two displacements I am defining as a transfer function. So, this transfer function is going to basically tell me how the displacement is going to change between the bottom and the top of that particular soil layer of interest. Once we know this value of the transfer function, we have seen that the minimum value of the transfer function is 1, and the maximum value of the transfer function is infinity. Infinity means it is indicating resonance. So, how to solve the equation? So far, we are doing the derivation part; we have not taken any ground motion into account, and we have not taken any specific soil layer into account. In order to demonstrate how the methodology, which we have just now discussed, is a part of linear ground response analysis, that too in a medium that is undamped, with rock being rigid, here is an example.

Qn.01. For a site having an average shear wave velocity of 340m/s, Compute the time history of acceleration at the surface of the linear elastic soil deposit of 4m thick overlying a rigid bedrock considering the deposit for undamped condition.

For a site having an average shear velocity of 340 meters per second, compute the acceleration time history or time history of acceleration at the surface of the soil layer, having 4 meters thickness and overlying a rigid bedrock, considering that a deposit for undamped condition means the soil layer is, firstly, linear elastic, having thickness H, capital H equals 4 meters, and it is located above a rigid bedrock.

So, we are interested to find out if a motion is given to you, and it is also defined that the bedrock is rigid, how much amplification this motion will have because of a 4-meter soil layer which is linear elastic, undamped soil. How are we going to solve it?

First of all, write down what is given to you in the problem. Again, one thing that was not defined here is which motion one has to take into account. So, either you can be given the motion, or you can adopt some motion and just say, "I am going to perform ground response analysis considering a particular motion."

As far as motion selection is concerned, you can go with seismic hazard analysis, find out what is the expected level of bedrock motion or site class A motion expected at your site of interest. Corresponding to that value of P ground acceleration or PGA, select a motion from a region having comparable seismic activity, or generate synthetic ground motion. If a regional ground motion record is available, take that into account. If none of these are available, and if you are doing ground response analysis, then you have to be very precise: "I have done ground response analysis considering this particular motion."

Generally, people use Loma Prieta earthquake, Bhuj earthquake, Sikkim earthquake, Nepal earthquake, Tōhoku earthquake, Chile earthquake. One has to refer to which particular earthquake and the year in which the earthquake has happened because not every place experiences earthquakes just once. In regions where earthquakes are frequent, people can get confused about which year's earthquake one is referring to. So, every time you mention an earthquake, also mention the year in which the earthquake happened.

So, in this particular case, I am referring to another earthquake which happened in 1985 in Chile. The Peak ground acceleration of that particular earthquake, that means corresponding to the acceleration time, is to record the peak value. After all the corrections, whatever is the peak value of acceleration, that is called as Peak ground acceleration. That value is given as 0.12 g. Now, in order to solve this, as I mentioned earlier, you can take any motion into account. The steps to be followed remain the same; the only thing is, depending upon the motion characteristics, the value of omega will change, and subsequently, the value of Fourier spectra at the bedrock will change. So, any motion one can use. Firstly, one has to do Fourier transformation because the record for the Chile earthquake or any other earthquake will be given in terms of acceleration time history or velocity or displacement time history. So, we have to find out the Fourier spectra, which can be done using fast Fourier transformation. You can use different tools to do fast Fourier transformation. Here, I will be using the Excel tool to determine how the fast Fourier transformation will be done. So, to do that, ensure that

Microsoft Excel has activated data analysis. If you go to the data analysis option, you have to actually activate that; then only this particular option for Fourier transformation can be used.

So, again, when we go for fast Fourier transformation, the maximum number of data points is limited to 4096 in your acceleration time history record. So, the procedure for computing ground response analysis for linear elastic undamped soil located over a rigid half-space remains the same, but the governing equation will change if you move from undamped to damped soil, or if you move from elastic to rigid elastic half-space. The generation of Fourier series at bedrock level, that means whatever recorded motions are there, those have to be converted from time domain to frequency domain using fast Fourier transformation or Fourier analysis. So, here, you can use data analysis and then Fourier analysis in Excel. Whatever input motions are there in terms of acceleration values, you can give. It will give you the amplitude, and based on the frequency of time recording during your earthquake, that is going to define how much is the maximum and minimum frequency content available in your particular motion. So, the result corresponding to the initial 10 points of acceleration is shown over here. So, firstly, your ground motion records will be converted from acceleration time history to Fourier spectra using data. You go to data and then data analyzer, analysis, and then Fourier analysis.

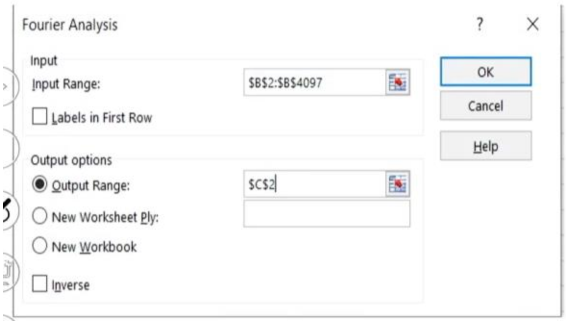

Fig.3.1 – Input and output ranges for computation

Time(s)	Acc(g)	Fourier Amplitude
0	-0.00045	1.23514217851749
0.005	-0.00034	0.509959898345385+8.29304769247872E-002i
0.01	-0.00015	0.505800021257452+0.190868940909305i
0.015	8.85E-05	0.412057826504709+0.445870885262561i
0.02	0.00033	0.340700518550535+0.946706698469754i
0.025	0.000543	-0.233045415360519-0.710943091104957i
0.03	0.000697	1.02621793770719+0.718114502849389i
0.035	0.000767	1.25176744404983-0.326332601786579i
0.04	0.000739	0.180365680132803+0.672497849516099i
0.045	0.00061	1.0282212141-0.479995344323163i

Fig.3.2 – Fourier Amplitude corresponding to initial 10 data points

You can see over here, once you go to Fourier analysis, it is going to tell you how much the input range is. So, here, in order to decide about the amplitude of motion, you will take into account the column in which. So, this is the column base; basically, your ground motion record was having, and you have acceleration versus time record. Often, you will be given the value of acceleration, the unit of that, and the frequency or the time interval in which each value of acceleration has been recorded or sensed by the sensor. If it is given, this is a value of acceleration recorded at 0.005 seconds. That means at every 0.005 increment, the next value of acceleration has been recorded by your recording station. So, put all those values of time at an interval of 0.005 seconds or whatever has been given by your recording station or corresponding agency. That will be in the first column. The major value of acceleration will be in the second column. Now, here, when you go for Fourier analysis, you have to basically tell what the range of your acceleration values is because your amplitude will be getting from there. So, it is given over here. This is column A, this is column B, and this is column C. Then the output range, that means the Fourier amplitude, you have to also define where the Fourier amplitude has to be put. As I mentioned, the maximum number of points you can give is 4096. It is starting from the second row and going up to 4097. That means the total number of data points you can use is 4096, and the output of Fourier amplitude is going to be in column C.

Now, in order to find out the frequency content, we know that each point has been recorded at an interval of 0.005 seconds. So, based on that, you can find out how much is the Nyquist frequency of the motion. That is 1 over delta t. Delta t is the time interval within which each value of acceleration is recorded. For this particular record, it is given as 0.005 seconds. So, 1 over 0.005, that is 200 hertz, is the highest frequency available in your record. Acceleration time history recorded if you talk in terms of frequency domain. Now, the minimum frequencies, you know, 4096 points are there, recorded at a 0.005-second interval. So, the product of these two is going to tell you the duration. 1 over the duration is going to give you the minimum frequency available in your record frequency interval, or successive frequency, which will be known to you as the first frequency f1 plus the time interval into the total number of data points and inverse of that.

The minimum frequency, f1 = 1/(0.005*4096)

The successive frequency , f2 = f1 + (1/(0.005*4096))

So, f1 plus 1 over 0.005 into 4096 is going to give you the successive interval of frequencies f1, f2, and then up to 200 hertz. Now, as we know, depending upon the Nyquist frequency, that means that is the highest frequency in your Fourier spectra. But if you say in terms of the amplitude, whatever motion you are getting, up to 1/2 of Nyquist frequency, that motion in the rest half cycle, that means if it is having Nyquist frequency of 200 hertz. So, from 0 to 100 hertz, the amplitude of the motion you can find out, but from 100 to 200 hertz, it will be the conjugate of the motion which is available from 0 to 100, which can be evidenced from this particular plot also.

1	Time(s)	Acc(g)	Fourier Amplitude	Freq(Hz)
2045	10.215	0.259481	-3.48134496086477E-002+1.15999688430846E-004i	99.8046875
2046	10.22	0.26737	-3.48134234236079E-002+9.27995812676774E-005i	99.85351563
2047	10.225	0.262088	-3.48134030576072E-002+6.95995870821919E-005i	99.90234375
2048	10.23	0.242977	-3.48133885105274E-002+4.63996776531306E-005i	99.95117188
2049	10.235	0.21045	-3.48133797822999E-002+2.31998246953344E-005i	100
2050	10.24	0.166052	-3.48133768729195E-002	
2051	10.245	0.112403	-3.48133797823003E-002-2.31998246961046E-005i	
2052	10.25	0.053022	-3.48133885105281E-002-4.63996776515208E-005i	
2053	10.255	-0.00796	-3.48134030576072E-002-6.95995870820254E-005i	
2054	10.26	-0.06618	-3.48134234236093E-002-9.27995812669558E-005i	
2055	10.265	-0.11747	-3.48134496086462E-002-1.15999688431401E-004i	

Fig.3.3. Frequency content and complex conjugate nature of Fourier amplitude (highlights)

You can see over here the frequency amplitude of the motion from 0 to 100. It is coming like this as shown over here. So, in the last column, you can see, up to 100, this is the amplitude of the motion, but after 100, if you go to the next motion, it is basically getting repeated over here. So, you can see this particular motion is getting repeated over here. This motion is getting repeated over here. This motion is getting repeated over here, and subsequently, so it is like the cycle is getting repeated after 100 hertz, which is one half of the Nyquist frequency of the maximum frequency contained, which has been indicated by the recording frequency. So, this motion, that means the amplitude of the motion is getting conjugate.

$$F(\omega) \quad = \frac{1}{\cos(\omega * \frac{H}{Vs})}$$

Now, the generation of the transfer function, that means F omega value, F omega was cosine of 1 over cosine of k times h. k is wave number, and h is the thickness of the soil layer, which in this particular case is given as 4 meters. So, this is already defined. Omega value, h value is the thickness, and vs is 340 meters per second, which is given in this particular solution. Omega is the circular frequency content of the motion, which can be defined. Now, just now, we have defined the frequency content of the motion multiplied by 2 pi, so omega will be equal to 2 pi f. f is the frequency content you have defined, multiplied by 2 times pi into the frequency content. You will get circular frequency or frequency in radians per second. H is the thickness, given as 4 meters. Shear velocity of the medium is given as 340 meters per second. So, the number of data points again will be considered the same as the number of points in frequency content, that is, up to 100 hertz. Again, in the next slide, only the frequency, the transfer function values for the first 10 terms corresponding to which the acceleration values and later on the Fourier amplitude values are also given. So, in the next slide, we will be also getting the transfer function, and the mode of the transfer function is going to give you how much the amplification function.

Freq(Hz)	Transfer function
0.048828125	1.000006514
0.09765625	1.000026056
0.146484375	1.000058627
0.1953125	1.000104229
0.244140625	1.000162866
0.29296875	1.000234541
0.341796875	1.000319259
0.390625	1.000417026
0.439453125	1.000527847
0.48828125	1.00065173

Fig.3.4. Frequency content and the transfer function for the first 10 points

So, this is the frequency content, the first 10 frequency contents, and corresponding to that, the transfer function values, the Fourier spectra of the ground surface, as discussed in lecture 17 and here also in the beginning. So, when we are interested to find out the Fourier spectra at the ground surface, you have to take the Fourier spectra at the bottom of that particular soil layer, multiply by the transfer function. That is going to give you the Fourier spectra at the surface. So, F A is going to tell you how much amplitude is multiplied by the transfer function at each frequency content. The amplitude is also known at each frequency content, and corresponding to that frequency, you have the value of omega. Put the value of omega and try to determine the value of the transfer function.

So, both the Fourier amplitude as well as F omega transfer function are both functions of the frequency content of the motion. The multiplication of these two values will determine how much the Fourier amplitude at the ground surface will be. The only thing is there will be complex quantities also, so you can use the IMPRODUCT function, and it will give you the compatible inbuilt real values. So, the Fourier series from 100 to 200 hertz basically will have conjugate components of 0 to 100 hertz. The frequencies of the Fourier series from 0 to 100 to 200 hertz will conjugate complex values of the values which are available from 0 to 100 hertz. So, again, using IMCONJUGATE, you can basically find out what the Fourier amplitude at the ground surface corresponding to 100 to 200 is, which can be put over here either manually or using IMCONJUGATE.

Freq(Hz)	Transfer function	F(ω)	FA x F(ω)
99.951172	2.227338082	2.227338082	-0.0775411860038569+0.000103347769043867i
100	2.243468589	2.243468589	-0.0781027240109282+0.0000520480779692317i
			-0.0781027240109282-0.0000520480779692317i
			-0.0775411860038569-0.000103347769043867i

Figure 3.5. Fourier Amplitude at the ground surface of points above and beyond 100Hz.
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So, you can see over here, up to 100 hertz, what are the values there. Once you go to 101, the value is getting repeated, and the value which is given in the first column is getting repeated in the fourth column. So, that's how it keeps on repeating. It's like up to 100 hertz values are there, and after 100 hertz, similar to the Fourier amplitude, because this is also going to give you the Fourier amplitude at the ground surface, that will also be the conjugate value from 0 to 100.

Now, based on this process, you have determined the Fourier amplitude at the ground surface, which now has to be transferred from the frequency domain to the time domain. So, if you remember, the actual ground motion was given in terms of acceleration time history at the base of the soil column. Now, based on this product of Fourier spectra and the transfer function, the Fourier amplitude or transfer function, that is going to give you the Fourier spectra at the ground surface, which has to be converted to acceleration time history. So, that can be done again using inverse fast Fourier transformation. You can go to data, data science, analysis, and then Fourier analysis. There, you can provide the input, that is, Fourier amplitude at the ground surface, and choose inverse, which is given in the options. Then press okay, and it is going to give you the acceleration time history values.

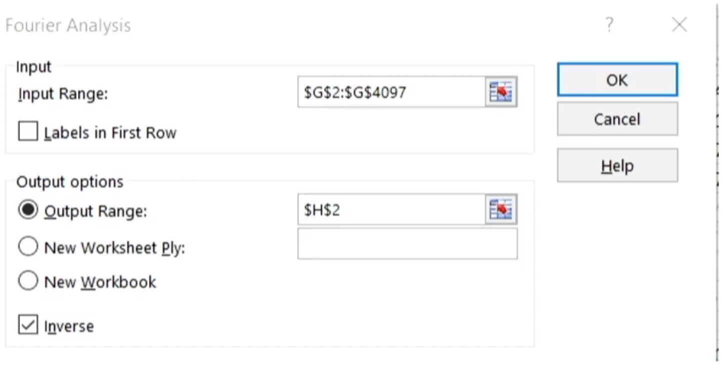

Figure 3.6. Input and Output ranges for Inverse Fast Fourier Transform
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So, here you can see, you are basically selecting how much the input value is. That means how much the Fourier amplitude at the ground surface is and then check inverse. It is going to do inverse fast Fourier transformation and give you the value in the time domain, that is, acceleration time history values at the ground surface. So, you started with the acceleration time history at the bottom, converted it to fast Fourier transformation, and converted that to Fourier spectra or frequency domain, multiplied this value with respect to the transfer function, which was again based on the operating frequency, converted to circular frequency, the thickness of the soil layer, and shear velocity of the medium, estimated the transfer function, took the amplitude of the transfer function, multiplied with the successive value of Fourier amplitude at the same frequency. You will get the Fourier spectra at the ground surface, then performing inverse fast Fourier transformation. You will be able to convert this from the frequency domain to the time domain, and subsequently, this is going to give you the acceleration time history at the ground surface.

Plot 1 - Bedrock Acceleration vs Time - Figure. 3.7

Plot 2 - Fourier Amplitude of the bedrock motion vs Frequency content (use IMABS to get the absolute value) - Figure 3.8

Plot 3 -Amplification function vs Frequency content (Amplification function = ABS of Transfer function) - Figure 3.9

Plot 4 -Fourier Amplitude of the ground surface vs Frequency – Figure 3.10

Plot 5 -Ground surface Acceleration vs Time - Figure 3.11

The absolute value of surface acceleration inclusive of the nature of amplitude

=IMABS(Amplitude)*IMREAL(Amplitude)/ABS(IMREAL(Amplitude))

Abs of the first amplitude of acceleration in cell H2

= IMABS(H2)*IMREAL(H2)/ABS(IMREAL(H2))

So, some typical outputs which are given over here are bedrock acceleration versus time, which was the input taken from the 1985 Chile earthquake, then Fourier plot of that particular motion, then amplification versus frequency content or the transfer function values over the entire frequency range, then Fourier amplitude at the ground surface, and then acceleration time history at the ground surface. So, all these plots or the processes which have been done have also been shown in terms of these plots given over here.

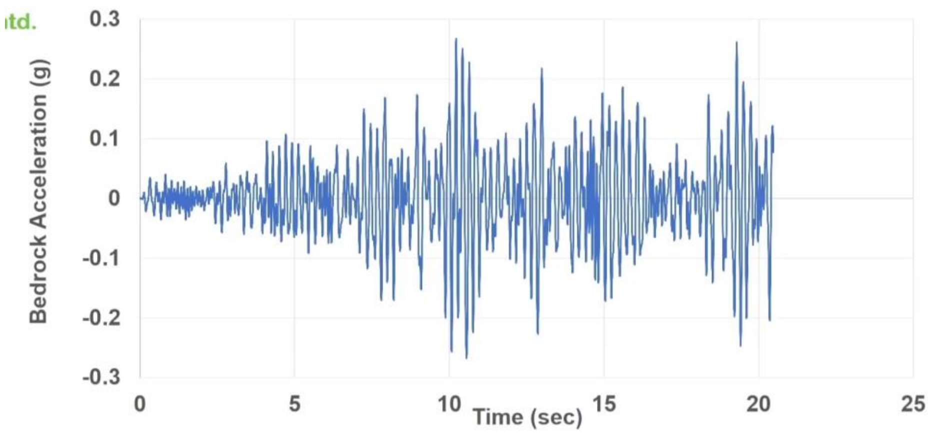

Figure 3.7 – Bedrock Acceleration Vs Time

So, this is the bedrock corresponding to acceleration time history, which has been taken in this particular example, converted to frequency domain using Fourier spectra, Fourier analysis, which is discussed in the previous slides.

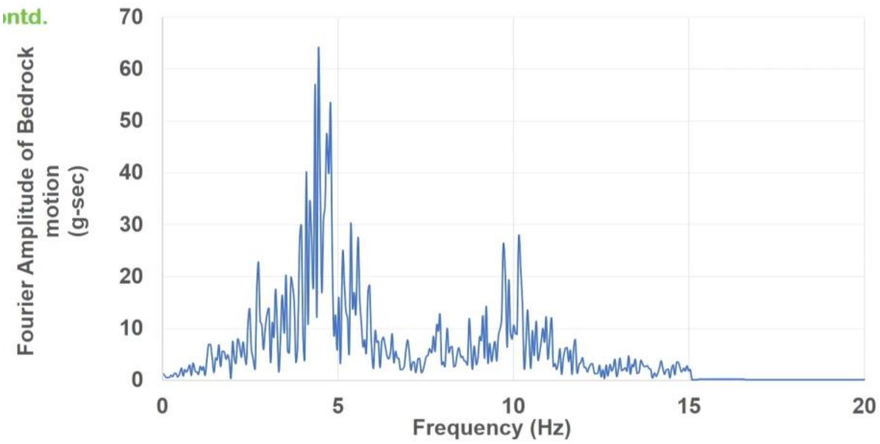

Figure 3.8.–Fourier Amplitude of Bedrock motion Vs Frequency

From here, you determine amplitude versus frequency content.

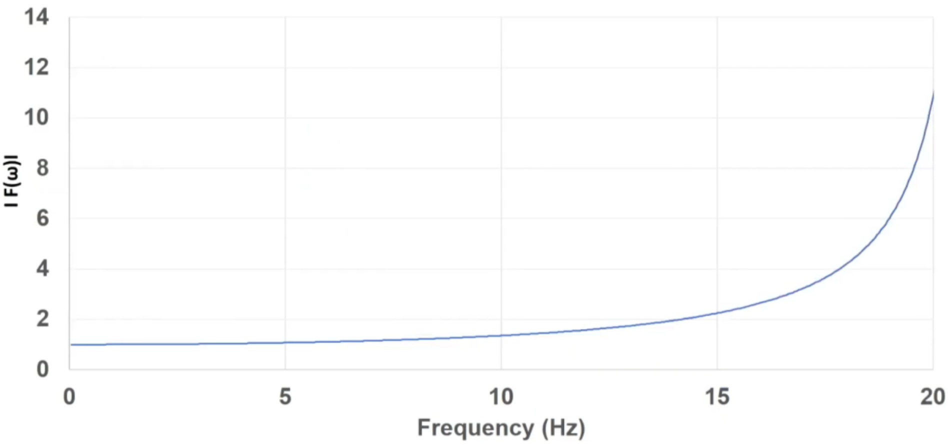

Figure 3.9.–Amplification due to soil properties Vs Frequency Applied Seismology for Engineers, Dr Abhishek Kumar, IIT

This is the transfer function variation over the frequencies of interest.

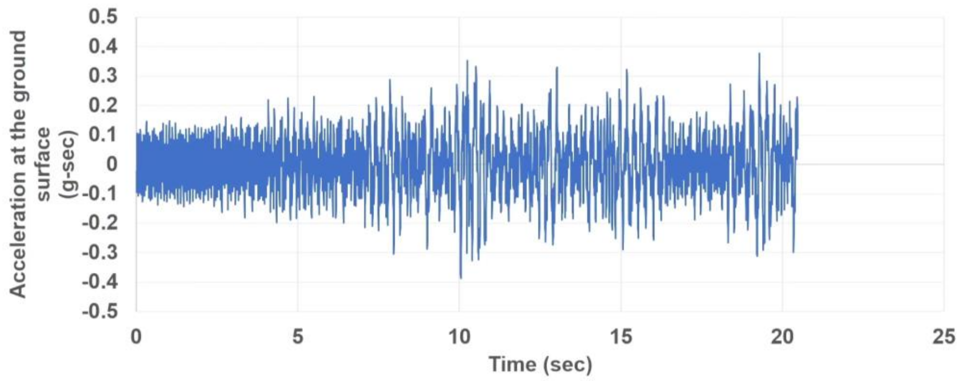

Figure 3.11.– Acceleration of the Ground motion Vs Time

This is again acceleration time history at the ground surface, which was given based on the product of the transfer function and Fourier amplitude at the bedrock, which is going to give you the Fourier amplitude at the surface and converting it from the time domain to frequency domain to time domain. This is the acceleration time history at the surface.

So, that's how one can quantify how much change in the motion, if you are able to compare this particular motion, which is available at the ground surface with respect to the motion available at the bedrock, you can compare and see that because of the local soil, there is a significant change in the motion characteristics. But the only thing to remember is that this is corresponding to linear undamped soil. In reality, there will be damped soil, and there will be an elastic half space also. So, this is one simplest example which has been taken to demonstrate how one can perform local site effect or ground response analysis using a linear approach.

So, thank you. With this, we close this particular topic of undamped soil located over a rigid half space. We will try to solve one or two more examples related to damped soil over a rigid half space and elastic half space, and then we will move to other methods. Thank you, everyone.