Applied Seismology for Engineers

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Week – 08 Lecture – 04

Lecture – 20

Hello everyone, welcome to lecture 20 of the course Applied Seismology for Engineers. Myself, Dr. Abhishek Kumar. This particular lecture is basically part 4 of the topic Local Side Effect and Ground Response Analysis. As we have discussed in lectures 17, 18, and 19, a prior information about the site where ground motion record is available, as well as the soil properties which are available beneath the ground surface, and above the bedrock, are required in order to find out how the properties of input motion and how the properties of bedrock motions are going to get altered because of the local soil. We have also discussed that, considering the properties of the soil which are going to offer resistance to external loading conditions, they are dynamic in nature. We have discussed in terms of KV solids that the resistance you are getting is primarily by means of shear modulus as well as damping ratio if you are talking about damped soil.

So, these two parameters, which are defining the dynamic soil properties, that is shear modulus and damping ratio, are actually not constant for a particular soil but are dynamic in nature, as the name suggests. Now, when we say dynamic, it means depending upon the level of shear strain a soil experiences, whether during a particular earthquake, whether corresponding to a particular confining pressure, or whether corresponding to a particular effective stress, whatever may be the controlling parameter. But depending upon the shear strain which is mobilized in a particular soil layer, that will define what particular value of shear modulus and what particular value of damping ratio will be used in order to control the response of that particular soil. If you remember the governing equation of motion which was given in terms of displacement correlated with respect to G value, that is shear modulus, as well as chi value, that is damping ratio. So, in that particular equation, when we are trying to solve that particular equation, then applying boundary conditions, what value of G and what value of damping ratio to be used, that will be depending upon the level of shear strain which is induced in a particular soil layer. Now, when we discussed in lecture 17 and lecture 18, we had highlighted very clearly that local side effect, particularly when you are going with a numerical approach, that is ground response analysis, you can have more than one method.

One is the linear method, which we have discussed in lectures 18 and 19. In the linear method, the properties, the dynamics of properties of the soil are assumed initially, and then accordingly, the solution of the particular equation is obtained. Firstly, we will try to find out what is the bedrock motion corresponding Fourier amplitude, then using the frequency content of input motion, shear wave velocity of medium, and thickness of the medium, we try finding out the transfer function. Product of these two, again whenever I am telling transfer function, the value of Vs is going to remain constant, the value of damping ratio is going to remain constant, and then we will multiply with respect to Fourier amplitude of bedrock motion. That will give you

Fourier amplitude at the top of that particular soil layer which is having a thickness of capital H if we are referring to lectures 18 as well as 19. That will give you the value of ground motion in the frequency domain at the top of the soil layer. Then, as we have seen in lecture 19, how we can separate and how we can convert this ground motion from the frequency domain to the time domain by means of inverse fast Fourier transformation. Once it is done, we will have a real part and an imaginary part. So, further, you will be taking only the real part to determine the acceleration time history at the ground surface, which is a representation of how much is the ground shaking in the time domain which has been transferred from a particular bedrock motion by virtue of a particular soil layer available at a particular site and then going to the surface. So, if we clearly observe the process which has been followed in lectures 18 as well as 19, clearly, we had defined the value of transfer function using some initial assumptions, and the value of shear modulus and the damping ratio was considered constant.

Now, in the beginning of this particular lecture and again in lecture 19, we have discussed that there are properties of the soil, that is shear modulus and damping ratio, which are not constant but rather keep on changing depending upon the shear strain induced in a particular soil layer. So, the actual soil response is non-linear, but we are trying to approximate it with respect to the linear approach as we have discussed in lectures 18 and 19. Please remember that the linear approach of ground response analysis is generally followed for lower values of shear strain, maybe in the range of 0.2%, 0.3% of shear strain, not higher than that. Because if we start understanding the dynamic soil properties, whether you talk about shear modulus or whether you talk about damping ratio, both of these properties do not show significant variation in the low value of shear strain.

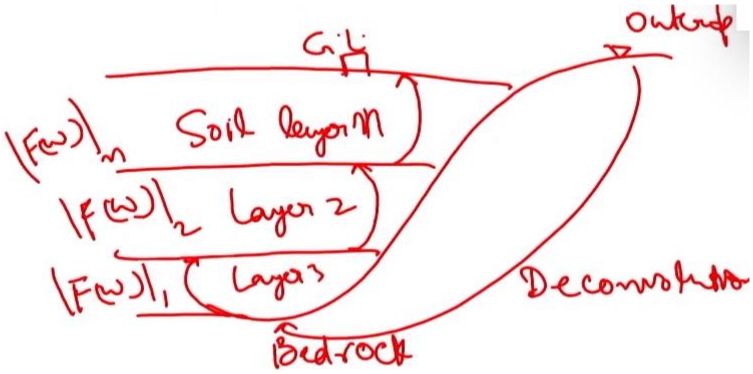
So, in case you are dealing with a ground motion which is inducing a very low value of shear strain, you can use the linear approach and try determining the response of a particular soil layer. That will give you reasonably accurate results because, though you are going with linear analysis, which primarily you will be using, very low strain corresponding shear modulus and damping ratio value of the soil, and using the transfer function and using the Fourier amplitude at the bedrock, you will be determining the Fourier spectra at the surface. However, one has to be very careful while using the linear approach of ground response analysis. You cannot use the linear approach which has been discussed in the previous two lectures for any value of shear strain because as you keep on going for intermediate or higher values of shear strain, there are more chances that the shear strain corresponding initial values which you have taken into account are actually not governing the response of the soil. What it means is if you go with local side effect assessment using the linear approach and the bedrock motion is such that it is inducing significantly higher values of strain such that the value of strain is not corresponding to or the value of dynamic soil properties are not corresponding to very low values of strain, but these have significantly changed from very low values. Let me show you with respect to the example. Let me start this particular topic and then we will go with whatever I am trying to highlight with respect to the issues with the linear approach, as a result of which we will go with other methods. So, lecture 20, which is part 4 of ground response analysis, that will be the last lecture on this particular topic of ground response analysis where we will try to understand the limitations of the linear approach and then some overview about equivalent linear and non-linear methods. Please understand that this particular lecture will only give you an overview of the equivalent linear and some information about the non-linear method. Non-linear ground response analysis itself is a big topic. So, covering the entire topic in one lecture will not be possible. We will be restricting it to a very brief introduction about the non-linear approach.

So, the topic which has been covered in the last class was how to determine the Fourier spectra or subsequently the acceleration time history at the top of a damped soil layer which is located above a rigid half-space. Now, if we continue that particular topic, one question will come to our mind: what if even the rock is an elastic half-space? So, in such a case, we will try again, and we will go back again to the step where we were trying to find out the solution of the one-dimensional equation of motion. Once we tried finding out the solution in terms of u, we had approximately considered that the strain compatibility at the rock and bottom-most soil layer interface is the same, or it is following the compatibility condition. The same equation will be applicable over here, but similar to the case of damped soil, there was the inclusion of damping ratio in the soil medium. When we go for elastic rock as well, there will be an additional component of damping ratio in the rock medium. So, that damping ratio component in the rock medium will also be useful when we are trying to satisfy the compatibility equation at the rock-soil interface, and this compatibility, because it is correlating some coefficients in the soil layer with respect to some coefficients in rock layers, and this particular rock layer is the layer in which the bedrock motions are available. Subsequently, this motion will be compared with respect to the free surface condition where A equals B condition we have seen in lectures 17 and 18. So, that is how we can correlate even the damping ratio of the elastic rock medium and the damping ratio of the soil medium with respect to coefficients, and subsequently, we can use it for determining the transfer function for this particular soil layer.

So, subsequently, there can be topics like we started with undamped soil over rigid half-space, we had some value of transfer function which was independent of any imaginary function because there was no damping ratio. So, simply, we transferred motion from the bottom of an undamped layer to the top of that particular layer by multiplying with the transfer function. That is the approach for linear ground response analysis. That means the solution which we have used, the solution which is, or the transfer function which we are using, is a function of G value or Vs value, as well as the damping ratio, but throughout the solution, these values are not changing. That is why this particular method, which we have been discussing in lecture 18, 19, and even some part in lecture 20, belongs to linear ground response analysis. Linear means you are considering that the properties of the soil are not at all changing. In general, in reality, the properties of soil, that means shear modulus and damping ratio, keep changing. But because, in the solution so far, whatever we have discussed, these are not changing, we are categorizing this part of the solution as linear ground response analysis, where we are actually not checking. Once we are getting the surface acceleration time history, we are not actually checking whether the value of Vs, the value of damping ratio which was utilized in the transfer function, has actually been mobilized or some different values of shear wave velocity, as well as damping ratio, are mobilized. So, in linear ground response analysis, we are not at all bothered about what the initial assumptions in terms of dynamic soil properties were and what the final results in terms of acceleration time history or shear strain you are getting as the output of your ground response analysis problem. So, this is one limitation with respect to linear ground response analysis. But, as I mentioned, though it is a limitation, considering the fact that for the majority of sites, the dynamic soil properties are not at all available covering the entire range of shear strain. So, still, if your site is not susceptible to a very high value of shear strain, then you can go with linear ground response analysis, and it will give you a reasonably accurate result.

So, we started with undamped soil, then we brought damped soil in lecture 19 over rigid half space. Then, subsequently, we can go to elastic half space. So, you will have an additional component of damping in the rock medium. Using that particular thing, compatibility equation, free surface condition, you tried solving those equations and came up with what will be the functional form of your transfer function. Multiply that transfer function corresponding to all the values of operating frequencies or omega values, and then you will get a transfer function throughout the frequency range. Multiply this value of the transfer function with respect to the Fourier amplitude of bedrock motion, and you will again get, very much similar to lecture 18 and lecture 19, the value of Fourier spectra at the top of the soil layer. Then, go for inverse fast Fourier transformation. You can go with acceleration time history determination. Same way, if you continue this particular problem, there can be bedrock, and above that particular bedrock, there can be n number of soil layers, which is most of the time available in physical conditions at the site. So, in such a case, what we will do is we will take bedrock condition into account, determine the transfer function between the bedrock and the bottom-most soil layer, and try determining how the values of coefficients of bedrock are correlated with respect to the A and B coefficients of the bottom-most layer, considering that at the bottom-most layer of soil and rock layer, the compatibility equation will be satisfied. Then, in the same way, you go to the second last layer or the second layer if you start moving from bottom to the top. So, the second soil layer, again, the compatibility equation between the bottom-most layer and the layer above it has to satisfy. So, that is how you can correlate the value of A and B coefficients of the bottom-most layer with respect to the second layer. If there are ten layers, you go with the tenth layer to the ninth layer, then ninth to eighth, and subsequently, you will reach the first layer. Why is the first layer important? Because the topmost part of the first layer will be indicating the free surface condition. And if you remember the free surface condition, the value of A and B were equal in that particular case. So, subsequently, you will get the value of A and B for the topmost layer. And since these have been correlated with respect to the A and B coefficients for each of the ten layers, you will be able to determine the value of coefficients A and B for all the layers with respect to the topmost layer.

Now, if you are interested to find out how the motion is getting transferred from the bedrock to each of these ten layers, you can determine the value of the transfer function at each of these layers. Repeat the same procedure which has been developed in lecture 18 and lecture 19. That is how bedrock motion can be transferred from the bedrock to the top of the bottom-most layer, then to the next layer top, then to the next layer top, and subsequently, you can get how much the modified ground motion is by each of these layers till the motion reaches the topmost layer. So, that can be another possibility when we go for ground response analysis. So, the approach will remain the same. The only thing is, in the linear approach, we will not be checking that whatever the final outcome you are getting as acceleration time history, whether that acceleration time history is also mobilizing the same value of shear strain corresponding to which the initial value of Vs, the initial value of damping ratio, has been used in determining the transfer function because this is a linear approach. So, the advantage of the linear approach is it gives you a direct solution. You just take two values of dynamic properties, one for shear modulus, one for damping ratio, and then try to determine the transfer function, and subsequently, you can go to the solution. You can go for deconvolution also. So, convolution means you are going from the bedrock to the surface. Deconvolution means, if you are having some value at the outcrop, using deconvolution, you can determine if the same material properties remain the same, what the bedrock ground motion will be if you are having in the same medium which is exposed to the ground surface. If you are having ground motions available, how that particular motion values. This is a place where motion is available, and this is n number of layers in the soil: soil layer 1, layer 2, layer 3. So, this was your outcrop motion which has to be transferred to the bedrock medium because we have been defining the transfer function with respect to the bedrock medium. We are also using the value of omega corresponding to the bedrock ground motion. So, this motion will be transferred. That will be called deconvolution.



So, in deconvolution, you transfer the motion from the outcrop to the bedrock or below the soil layers. Then, in direct, in linear ground response analysis, again, you will try to determine the f omega, that is, the transfer function at 1. That will help you in transferring the motion from the bedrock to the top of that particular soil layer. Again, determine the value of the transfer function for the second layer from the bottom. You will be able to transfer the motion from the bottom layer to the top of that particular second layer. Again, you will be having f omega. So, if you are having n number of layers, you can determine the transfer function corresponding to n number of layers, and subsequently, you will get, finally, what the value at the ground level is. So, depending upon your linear approach, you can use the same values without checking whether the strain compatibility condition based on the solution and initial assumptions are made or it's matching or not. Then, you can say it is like the approach which you are solving here is following linear ground response analysis.

The limitation is, it does not take into account soil nonlinearity. As I mentioned, if you are dealing with 0.5 percent, 1 percent, 2 percent strain, shear strain in the soil, which are mobilized because of external loading condition, certainly you will end up underestimating the nonlinear soil properties or local side effect. So, you will end up in you will not get actually whatever is supposed to be the modified ground motion because of the soil layer corresponding to particular ground motion because you have considered the initial assumption of dynamic soil properties, and the methodology which you have adopted is not letting you modify your dynamic soil properties at the later stage of your solution. So, depending upon the approach, you may or may not modify. So, the process of determining the ground local side effect or performing ground response analysis, the basic structure will remain the same as far as linear and equivalent linear are concerned. The only thing, there will be additional conditions or checks when you move from local side effect based on linear analysis to equivalent linear analysis, primarily because it is not taken into account. The linear approach is not taken into account the soil nonlinearity.

Now, the second part is the second method, which is called equivalent linear ground response analysis. The main advantage here is, you will be able to check whatever initial assumptions in terms of shear strain or whatever initial assumption in terms of shear modulus and damping ratio you have taken to start solving the equation whenever we are trying to find out the transfer function. So here, we will be finding out, in the end of the solution, whether the initial assumptions of shear strain were correct or if they need modification. So, if it is found correct, you can go ahead with the solution. If it requires any modification, you can go again with modification and solve the equation. So, it is like an iterative process; you go by the iterative process and find out until the initial assumed values and the values you are getting in the final form are within some threshold values. So, equivalent linear analysis, as it suggests, though it is not linear ground response analysis, it is not nonlinear analysis either. So, it is basically trying to find out, based on equivalent properties of soil which are available or which are known at a particular value of shear strain, based on which you can actually approximate to significant accuracy the dynamic properties or the dynamic behavior of the soil. The method involves modification of linear properties. Linear properties mean I am referring to linear ground response analysis.

$$\rho\frac{\partial^2 u}{\partial t^2} = G\frac{\partial^2 u}{\partial z^2} + \zeta\frac{\partial^3 u}{\partial z^2 \partial t}$$

So, if you refer to the governing equation of motion which is given over here for KV solids, you can see the displacement values or the external loading condition. Because you are having some value of shear stress also, it is a function of shear modulus and the damping ratio. What we did in linear analysis, we considered initial values and went for the calculation of the transfer function, then multiplied the transfer function with Fourier amplitude, got the value of the surface, and then determined the acceleration time history. In the end, when we are determining acceleration time history, there should be a way where we can check that particular acceleration time history you are getting at the ground surface. Whether corresponding to that acceleration time history, the shear strain which will be mobilized in the soil will also correspond to these values of shear modulus and damping ratio.

What I am trying to highlight here is you took some initial assumption in linear analysis. Now, in equivalent linear analysis, once you are finding out the acceleration time history, you will also be checking that, corresponding to that acceleration time history, how much is the shear strain mobilized in the soil layer, primarily at the center of that particular soil layer. Once that shear strain values, again, because it is continuous loading, you will be getting shear strain time history. So, from shear strain time history, you can find out peak strain and subsequently reference strain. That reference strain will give you an indication whether, corresponding to that reference strain, if you go with the dynamic soil properties of the soil layer corresponding to which these were the initial assumptions, whether those dynamic properties corresponding to reference shear strain are corresponding to these values or not. If these values are not corresponding to the same value of shear strain, subsequently, that indicates that the ground motion you are mobilizing or the ground motion which is getting generated in a particular soil layer because of some input motion is actually generating a different value of shear strain and not the value which you have used as the initial assumption. So, to start solving the problem, the initial assumption is fine. But whenever we are giving the final proposition of the results, we have to make sure that the dynamic soil properties which we are giving as a part of the solution are actually being mobilized in the soil layer.

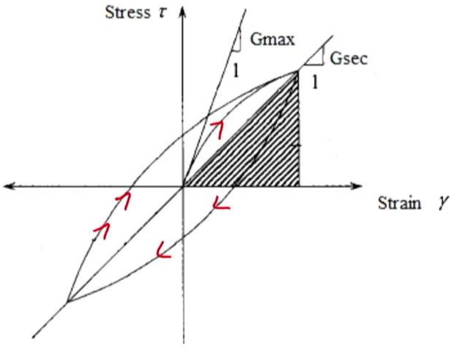
So, when we go for equivalent linear analysis, again, we are not dealing actually with the nonlinear part, but the nonlinear properties are actually approximated with respect to equivalent linear properties. So, we will be dealing with the nonlinear part, or we will be dealing with changes in dynamic soil properties, whether it is shear modulus or damping ratio, how it is changing as you keep on loading the soil sample, cycle after cycle. That means, when we are determining the dynamic soil properties in the laboratory, we keep on loading the sample, unloading the sample, loading, and unloading the sample, as a result of which, corresponding to each cycle of loading, you will be getting some value of the stress-strain curve. From that particular stress-strain curve, you can find out how much approximately the value of shear modulus is. You can find out based on the initial part of the stress-strain curve corresponding to the slope. Similarly, with respect to the area under the stress-strain curve, you can find out how much strain energy accumulation and subsequently the damping ratio. That means the stress-strain curve, at that time when the soil is subjected to cyclic loading, will, corresponding to whatever stress-strain curve is getting generated, give you an indication about how much resistance in terms of shear modulus, how much resistance in terms of strain energy accumulation or damping ratio, the soil is offering at that particular cycle of loading. Then, you will unload the sample, again reload the sample. As a result of this process, whatever the value of shear modulus and damping ratio was available from the soil in the first cycle of loading will change. You will go to the second cycle of loading. So, there will be some shift in the hysteresis loop which comes in dynamic soil properties. So, that means, certainly, you will have some value.

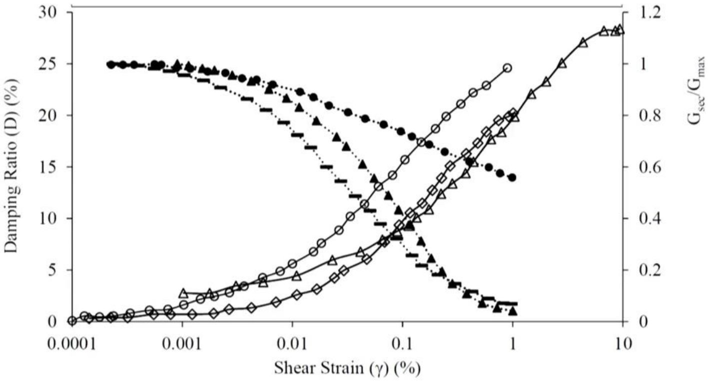
We will discuss some preliminary information about dynamic properties. So, what actually happens when you load a soil sample to cyclic loading or repeated loadings, loading, and unloading? There will be degradation in the material property. There will be a change in the damping properties of the particular soil layer. And why is it happening? Because of the inherent properties of the soil, there will be degradation in the material properties. So, you can see if the same soil is subjected to more numbers of repetition of cycles, there will be degradation in the material properties. There will be a change in the damping properties of the soil as you keep on continuing the loading. That means you are going from a very low value of strain to intermediate to high value of shear strain. So, when we go with equivalent linear analysis, we are actually trying to find out approximately, corresponding to each loading cycle, what are the approximate properties of the soil, which, though not taking actual nonlinear profiles into account, give you significant accuracy in terms of dynamic properties. Those dynamic properties you will be using over here in an iterative process. In the end, you will also check whether it is matching with the strain compatibility. That means the initial assumed shear strain and the value of strain corresponding to the final acceleration time history, if these two values are matching, you can say I have reached the final solution. If these are not matching, you will go for subsequent iterations.

So, the initial value in equivalent linear analysis is similar to linear analysis. The initial value of shear modulus and damping ratio, generally corresponding to a very low value of shear strain, is taken into account. Corresponding to that, you put the values in G and chi, and then start solving the equation and get the transfer function, very much similar to your linear approach. The only thing is, the same procedure will be checked again and again in order to ensure that the shear strain, which is there at the beginning of the iteration, and the system which you are getting at the end of the iteration, both are matching very closely with respect to each other. So, the initial value of shear modulus is generally referred to as secant modulus, and the damping ratio, again corresponding to the area under the stress-strain curve, can be assumed. These values start solving the equation. The equivalent linear shear modulus, which is going to be also defined as secant modulus, and the equivalent damping ratio, which will produce the same energy loss in a single cycle as happened in actual loading of the cycle. So, the hysteresis loop, which will be coming into the picture when you start loading and unloading the sample continuously for a higher number of cycles, will be represented in terms of the hysteresis loop, which will actually define the nature of the soil when it is subjected to cyclic loading, repeated loading, and unloading. That is defined by the hysteresis loop.

Now, the value of G and chi, which you have used as initial assumptions over here, you put in the solution, determine the transfer function, transfer the motion, determine the acceleration time history, and again check based on strain time history how much compatible these initial assumptions are. You can see over here; this is corresponding to the secant modulus. So, if we see the hysteresis loop, this value of secant modulus will be corresponding to some value of shear strain. This value of equivalent damping ratio will also be corresponding to the same value of shear strain. So, in the end, you are getting acceleration time history, and corresponding to that, you will be getting shear strain. You can check whether that value of shear strain and this value of shear strain are actually compatible or close with respect to each other. If it is not, the value of shear modulus and damping ratio, which are mentioned over here in the end, should be consistent. That means whatever the initial assumptions in terms of shear strain were there, that should be consistent with respect to the shear strain mobilized in a particular soil layer at the end of the iteration. If it is not, then you go for subsequent iterations and keep on modifying the value. So, input requires, generally, for equivalent analysis, are dynamic soil properties. When we say dynamic soil properties, that means how the shear modulus value is changing with respect to shear strain, how damping ratio is changing with respect to shear strain, which can be obtained based on cyclic tests in the laboratory. Other methods are also there based on which in situ measurements also can be done in order to determine the dynamic properties.

So, these are the properties for in situ soil one has to know when one is trying to attempt ground response analysis. So, the change in shear modulus with respect to shear strain is called the modulus reduction curve, which indicates how the shear modulus is changing with respect to shear strain or increase in shear strain. That is called the modulus reduction curve. Similarly, how the value of damping ratio is changing as you continue loading the same soil from very low to intermediate to high values of shear strain will come under the damping ratio curve. So, these two properties, if someone asks what the dynamic soil properties are, generally we can refer to the shear modulus reduction curve as well as the damping ratio curve of the soil, which is actually available at a particular site. That is called dynamic soil properties. Then, bedrock motion, corresponding to which this particular soil layer, whose dynamic properties we just spoke about, will undergo modification. Bedrock motion should be there, then subsoil properties, and water table depth, if you are taking into account the effective stress effect on shear strain also.



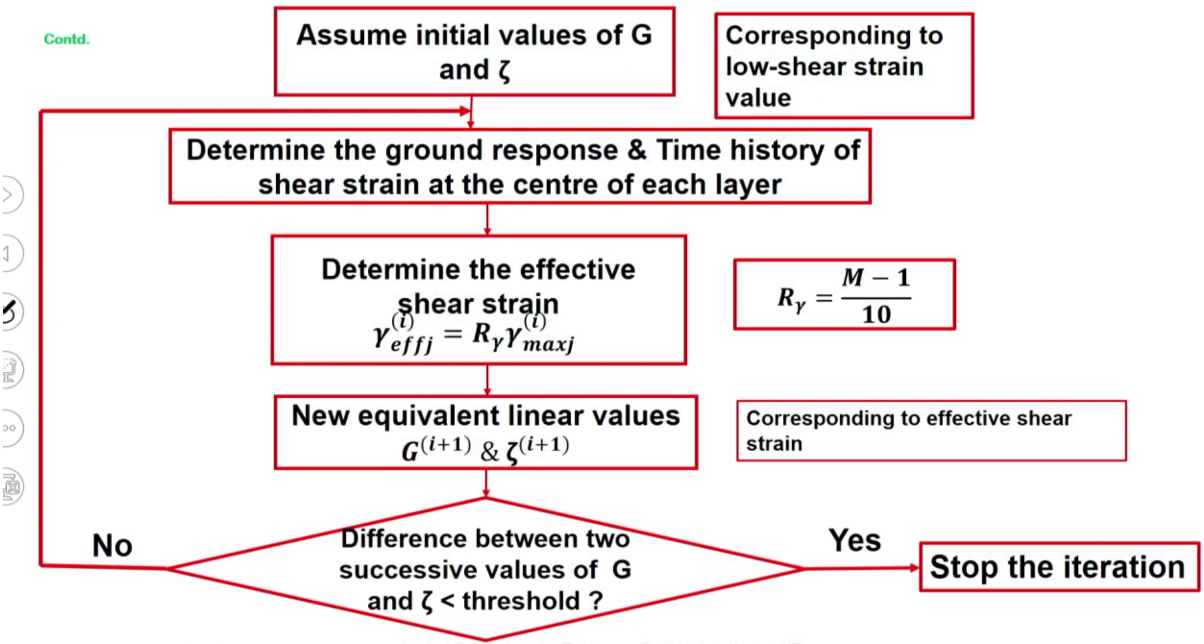


Now, we can see over here, this is one typical cycle of loading. You can see over here, you started loading the sample, and corresponding to the stress-strain curve, you started loading the sample. Once it reaches a particular value, you started unloading the sample. So, loading the sample and removing the load again, the sample will experience some kind of, you can see over here, and then again, you start loading the sample. Now, this is one complete cycle. If you continue this particular cycle, I mean, if you go for more number of cycles, you will see. Initially, if you see this particular loading part, approximately corresponding to the initial part, whatever is the value of the slope between the shear stress-strain curve, that is an indication of, initially, what is the value of shear modulus, which is called Gmax. It is corresponding to a very low value of shear strain. Again, you can see, based on the value shown over here, joining with respect to the origin, you can find out how much is the secant modulus of that particular first level of the cycle. If you continue this particular loading and unloading, you can see over here, this particular part is the actual indication of how the secant modulus normalized with respect to maximum shear modulus is changing if you keep on increasing the shear strain values. So, this is again for the same soil. I have shown multiple numbers of curves because for different soils. But if you take any particular curve, you can see, the same soil you keep on loading the soil sample, there is actually a reduction in the shear modulus of the soil. At the same time, if you keep on loading the soil for a higher number of cycles, there will be an increase in the damping ratio. As a result, you see on one side, when the shear strain is increasing, there is a reduction in the shear modulus of the soil, but at the same time, there is an increase in the damping ratio. The soil remains the same. If you talk about the soil corresponding to a low value of shear strain, you see the soil is offering a very high value of shear modulus and a very low value of damping ratio. These two values will go into your solution, determine the transfer function, and multiply the transfer function with Fourier amplitude. You will get some value of acceleration time history. In a linear approach, if you are going with an equivalent linear approach, these may be the initial assumptions. You try determining the acceleration time history, and you may get, okay, this may not be the value of shear strain that is actually mobilized. So, corresponding to the revised value, if this is the value of shear strain you are getting based on your surface acceleration time history, you take these two parameters into account, or modified values of shear modulus and damping ratio, put them again in your solution, try determining the transfer function, and try determining the acceleration time history at the surface again. Corresponding to acceleration time history or shear strain time history, determine the reference strain. Match if that reference strain corresponds to this value. If it is matching, you stop your process. If it is not matching, then take that particular reference strain, modify your dynamic properties again, and that's how, step by step, at some stage, what will happen. The initial assumed values, because these initial assumed values in the third step, are coming from the second step. In the second step, they are coming from the first step. So, the initial assumed value at the beginning of the iteration and the value of shear strain you are getting at the end of the iteration, at some stage, these two values will match. That will ensure that the value of shear strain which is mobilized in the soil layer, corresponding to the same value of shear strain, you are actually determining the transfer function and the response of the soil. That is basically the nature of or how you are differentiating with respect to linear ground response analysis and going into equivalent linear ground response analysis.

So, one-dimensional ground response analysis will be carried out by updating the level of shear strain through an iterative process. As I mentioned, at the end of this particular process, whatever you have done in linear analysis also, once you determine acceleration time history, you can also determine shear strain time history. If you remember lecture 17 and 18, we also determined shear strain value. Subsequently, you can determine shear strain time history. Corresponding to that value of shear strain time history, pick up the peak value, then determine reference strain from that. Compare this reference strain corresponding to how much is the value of shear strain which you have assumed initially, whether modification is required or not. That can be done, and subsequently, you can continue with the procedure. So, the procedure will be continued till the time the effective shear strain which was used as an initial assumption and what we are getting at the end of the iterative process. When these two values of shear strain are compatible with respect to each other, generally, the difference between these two should be less than 10^-2 to 10^-3 or as per the user-defined approach.

The effective strain, it is seen that the effective strain of the transient record, because this is not harmonic motion which we will be applying, so in the effective strain case of a transient record, may vary from 50% to 70%. So here, generally, the value of 65%, or 0.65 times the maximum shear strain, is referred to as effective strain, which will ensure that the rate of pore pressure generation in actual soil and here corresponding to harmonic loading and transient loading will be maintained. So, the output will yield acceleration time history at various depths. As I mentioned, for linear analysis also, in equivalent linear analysis also, wherever you are interested to find out acceleration time history, just keep on determining the transfer function or amplification factor, because amplification factor can be determined layer by layer also or between any two layers. Because you will be having the value of coefficients, it is up to the user to determine the value of the coefficients A and B with respect to the topmost layer or any other layer in between.

Accordingly, one can determine the acceleration time history at any value of any particular soil layer and subsequently the value of peak ground acceleration, which is the peak value of acceleration time history recorded at that particular soil layer. So, the same procedure, if you adopt for n number of layers, you can determine how the variation in peak ground acceleration with respect to depth at a particular soil layer subjected to a known earthquake loading and having maybe 10-15 layers of soil, will be there. So, if you look into the procedure of equivalent linear analysis, it is the same thing which we have mentioned in the previous slide.



Contd.

Assume initial values of G and ζ

Corresponding to low-shear strain value

Determine the ground response & Time history of shear strain at the centre of each layer

Determine the effective shear strain $\gamma^{(i)}_{effj} = R_\gamma \gamma^{(i)}_{maxj}$

$R_\gamma = \frac{M-1}{10}$

New equivalent linear values $G^{(i+1)}$ & $\zeta^{(i+1)}$

Corresponding to effective shear strain

Difference between two successive values of G and ζ < threshold ?

No

Yes

Stop the iteration

Applied Seismology for Engineers, Dr Abhishek Kumar, IIT Guwahati

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Figure.1. Flowchart of ELGRA

You go for the initial assumption of shear modulus and damping ratio value, try performing the ground response analysis and determine the acceleration time history. Corresponding to acceleration time history, you also determine the shear strain time history. Corresponding to the peak value from shear strain time history, determine which is given over here. Multiply with respect to your reference factor, which is given as 0.65 or generally this value of m you can refer to as 7.5. So, this is going to give you the value of reference factor as 0.65. Multiply this 0.65 with respect to the peak shear strain; you will get reference shear strain. Corresponding to reference shear strain, for the next set of iterations or the next cycle of iteration, determine or update your value of damping ratio and shear modulus for the next cycle. Remember, in this particular part, you have assumed some value, so this is the revised value. Again, if the shear strain is matching with respect to these particular values of shear modulus, you can stop the iteration. If the shear modulus and damping ratio, which you have used in the initial assumption, corresponding to these two values, if the shear strain was there at the beginning, if it is not matching with the shear strain coming over here, you can again go for it. You can again determine the value of shear modulus and damping ratio corresponding to this value of shear strain as the initial assumption. Repeat the same procedure. Determine the value of gamma effective. If you are getting the same value, it is fine. If you are getting a different value, again, go and update the value corresponding to gamma effective and keep on repeating the procedure until the value of shear strain, obtained in two subsequent stages as the initial assumption and the output, falls within the user-defined threshold value.

So, this is this particular procedure where the iterative procedure is coming into the picture or where this nothing is coming into the picture. That defines or differentiates equivalent linear ground response analysis with respect to linear ground response analysis. In the previous slide, we discussed equivalent linear ground response analysis, where, based on some initial approximation of shear modulus and damping ratio, the governing equation of motion is solved. In the end, we will get the value of shear strain, which will be compared with respect to the initially assumed value of shear strain. If the two shear strain values are matching, then we will consider the assumed value of shear modulus and damping ratio as the final dynamic soil properties. If this is not matching, then again, we will go for the next step of revising the value of shear modulus and damping ratio. So, generally, whenever we are going with equivalent linear ground response analysis, we will be having some discrete points which are defining the nonlinear dynamic soil properties, and every time we are revising these properties depending upon whether the assumed value of shear strain is matching with the value of shear strain you are getting at the end of your solution or not. However, the fact is, the soil behavior under dynamic loading conditions is nonlinear. That means there will be complete degradation in the material properties as the loading and unloading continue for a particular soil medium during earthquake loading conditions. So, though equivalent linear analysis is going to approximate the solution for ground response analysis, nonlinear analysis will give a more accurate solution. Primarily, whenever we are looking for the development of pore water pressure or degradation in the material characteristics, or when we are interested in finding out the response under extreme loading conditions, in such cases, nonlinear ground response analysis is going to approximate or give more accurate results as far as the local site effect or ground response analysis is concerned.

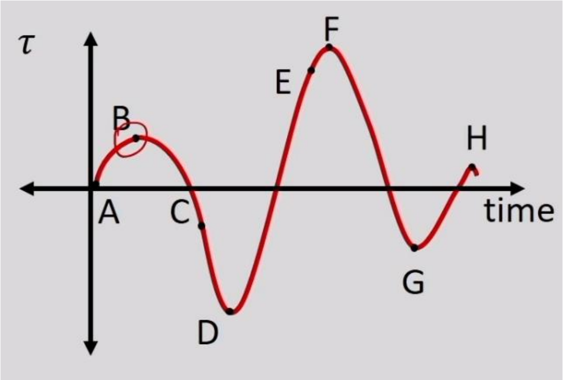
So, equivalent linear analysis was an approximation related to nonlinear analysis. An alternative way, where we can find out the nonlinear behavior of the soil, is one can go with nonlinear ground response analysis. So, in nonlinear ground response analysis, we will try to find out how, during the duration of earthquake loading, the soil sample is changing its response in terms of stress-strain behavior, which is approximated with respect to stress time history or strain time history values. So, mostly the loading and unloading characteristics, because we are talking about cyclic loading conditions, so there will be loading, there will be unloading, and this will be repeated over the duration of earthquake loading conditions. So, these loading and unloading characteristics, or the cyclic loading of the soil, will be approximated by means of the backbone curve. Usually, whenever we are discussing nonlinear response of the soil, we are interested in finding out or approximating how this backbone curve can be approximated by means of a suitable constitutive model.

So, the most important aspect of nonlinear analysis is what constitutive model of a particular soil one is assuming in order to understand how, during the loading characteristics of an earthquake on a particular soil layer, the soil is going to respond such that the model of the behavior of the soil approximated by a constitutive model is closely matching with the backbone curve of the soil, which is resembling the actual response. So, the integration is performed in small time steps. That means we will try to find out the particle velocity, and then, corresponding to those particle velocities at some moment in time, we will try to find out the ground motion characteristics. Again, we will try to revise it for the next increment of time, that is, the delta T value. Again, try to find out corresponding to that how much is the particle velocity, and going with the same procedure, we will try to approximate where the final value of shear strain and corresponding value of modulus and damping characteristics will be approximated.

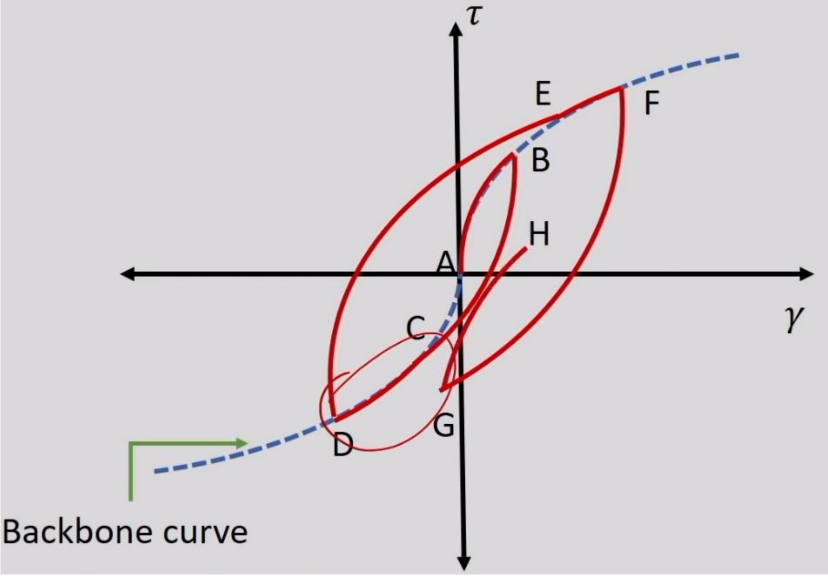
So, the nonlinear behavior is modeled using a hyperbolic function, which is how the stress and behavior of a cyclically loaded soil can be approximated. That is approximated by means of a hyperbolic function, which gives you a close approximation with respect to your backbone curve representing the nonlinear behavior of the soil. So, a widely adopted constitutive model you can say is Masing criteria and extended Masing criteria, which can be used to approximate how the stress-strain curve of the soil can be approximated using governing equations such that one can understand nonlinear soil behavior. So, certain rules are there under the Masing criteria. The first rule, because we will be approximating the stress-strain curve with respect to the backbone curve, is how the stress-strain curve can be approximated or can be closely matched with the actual response of the soil. That is described by the backbone curve. So, the first rule is, the stress-strain curve follows the backbone curve for initial loading. That means whenever soil will be subjected to an external loading condition, the very first cycle of loading will follow the backbone curve. That means the nature of the curve will be similar to your backbone curve of the material.

Rule 2 says that during stress reversal, because we are talking about cyclic loading conditions, there will be the application of load, and after a certain moment of time, you will see that the nature of loading, if it initially was compression, the nature of loading will change to tension, which can be indicated by means of upward and downward movement or acceleration values on either side of the axis in your ground motion record. So, during stress reversal, which is an indication like if the particle was moving in one direction because of cyclic loading, the particle is coming back to its mean position and then started moving in the other direction. Similarly, there can also be approximated with respect to unloading characteristics because now you are removing the loading. So, like, the particle will come back to its original position, and there will be stress reversal. Similarly, whenever it comes to reloading, that means again, there will be the next cycle of loading which will be applied from earthquake loading conditions because earthquake loading conditions are cyclic in nature, so there will be a reversal of stresses continuously. So, if we are interested to see a typical response, that is, with respect to time, the value of acceleration or similarly the velocity and displacement value, how these are changing with respect to the mean position. So, one time it is going up, the other time it is going down. That is called stress reversal or loading in the first cycle and then unloading in the second cycle, or once it crosses the mean position, it is going unloading part. So, this is approximated, and again, this can be, that will have the same shape as that of the backbone curve. So, initial loading will be approximated to the backbone curve, primarily the shape, and unloading and reloading because of the cyclic loading characteristics of the soil under earthquake loading will also have a similar shape as that of the backbone curve.

The next part is, if the unloading or reloading curve intersects, that means you are talking about actual loading, which you are estimating from stress time history. If that particular loading and unloading also intersect at some moment of time or at some level of effective stress to the backbone curve, that means it follows the backbone curve until the next level of stress or the second cycle of loading is applied to the soil sample. Rule 4 says if the loading, unloading, or reloading intersects with the unloading or reloading part from the previous cycle, that means the nature of strain or the value of strain from the previous cycle and the current cycle matches, then the curve follows the stress-strain path or trajectory of the previous loading cycle. That means the same nature of stress will keep on repeating if, at any moment of time, the unloading or reloading curve matches the stress-strain curve of the previous loading. So, this all can be approximated or can be understood based on a typical stress time history curve.

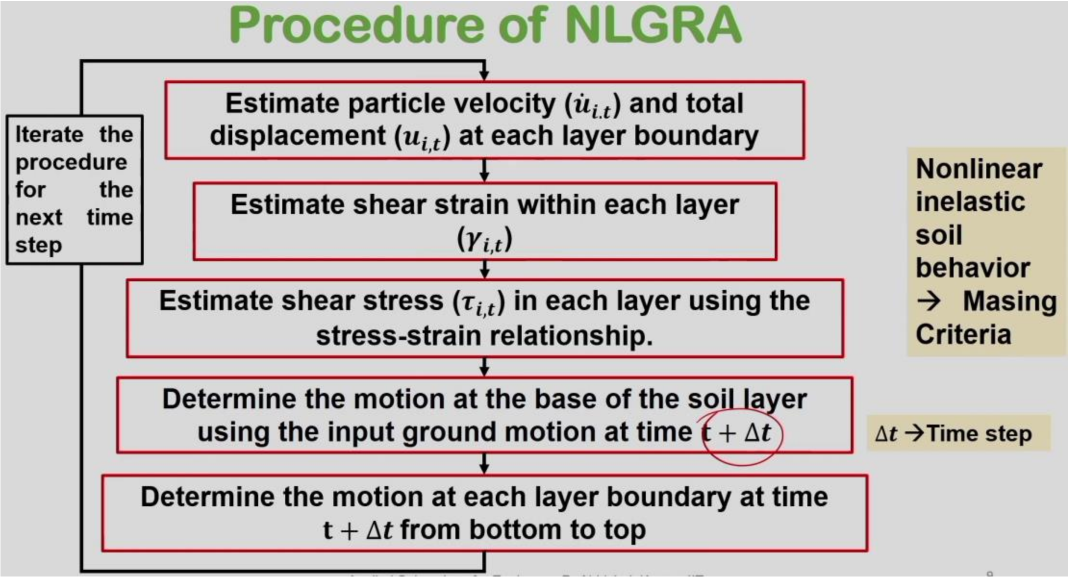


You can see over here how the shear stress varies with respect to time during earthquake loading conditions, which can be generated from the acceleration time history record from an earthquake record recorded by our ground motion recording instrument.



This is a stress time history, which will also suggest there will be a reversal of stresses, and then, at the same time, you can also find out, corresponding to different values of stresses or levels of loading, what is the applicable shear strain. This is the backbone curve, which is going to give you the response of the soil corresponding to cyclic loading conditions, which has to be approximated by means of assuming some suitable constitutive model. If you are going with this backbone curve, the first point is point A, which is the initiation of loading. That means your sample has just experienced external loading; this is point A. Then, once it reaches loading, as per Rule 1, it will follow the backbone curve shape. So, initial loading will follow the backbone curve shape; that was Rule 1. Then, stress reversal, which means once your loading reaches point B, again, there will be a reversal of stresses. It will come down, and while coming down, the unloading curve, in case it intersects your backbone curve, will continue following your backbone curve until there is a reversal of stresses again. That means the second cycle of loading, once it comes into the picture, until that moment after point C, the stress-strain curve will follow your backbone curve. That is why, even between point C and point D, the stress-strain curve is following your backbone curve. Again, there will be a second cycle of loading called stress reversal because initially, there was loading, then stress reversal. Again, stress reversal means the second cycle of loading will start from point D and continue until point E.

So, definitely, as the loading cycle continues, initially, there will be some traces of or some initial loading. Then, after some duration gap, you will have peak ground acceleration values. That means, in general, after some moment, it will be increased loading with respect to the initial loading condition, which is indicated by point E. Since point E is also located on the backbone curve, until it reaches the second cycle of unloading, it will follow the backbone curve. This nature, which is overall the soil behaviour, is following a backbone curve and a constitutive model defined by a Masing criterion and extended Masing criteria here. That is basically defining the nature in which you are approximating the loading and unloading cycles of a particular soil medium using a nonlinear curve. The governing equation for this particular Masing criterion can be referred to, and based on this, one can find out what is the approximate nonlinear behaviour. So, this will continue; the number of cycles will continue, and the reversal of stress and unloading will continue.



Procedure of NLGRA

Iterate the procedure for the next time step

Estimate particle velocity ($\dot{u}_{i,t}$) and total displacement ($u_{i,t}$) at each layer boundary

Estimate shear strain within each layer ($\gamma_{i,t}$)

Estimate shear stress ($\tau_{i,t}$) in each layer using the stress-strain relationship.

Determine the motion at the base of the soil layer using the input ground motion at time $t + \Delta t$

Determine the motion at each layer boundary at time $t + \Delta t$ from bottom to top

Nonlinear inelastic soil behavior → Masing Criteria

Δt →Time step

The procedure for nonlinear ground response analysis is: firstly, you will find out the value of particle oscillation velocity and displacement values at each of these layers. Then, estimate the value of shear strain within each layer, with respect to which you can find out the value of shear stress within each layer using the stress-strain curve as suggested by a constitutive model following a nonlinear response or hysteresis loop. Then, determine the value of ground motion at the base because now you have to go with incremental steps in terms of time. So, there will be an additional time step, that is, delta t. Accordingly, you can find out the level of ground motion in other layers also, from the top to bottom, and the procedure will be repeated for every time step. So, initially, you start at some moment of time t, repeat it for plus delta t, plus delta t, and so on, until you are able to cover the entire loading cycle.

So, collectively, nonlinear ground response analysis captures the nonlinear behaviour of the soil better, primarily when we are talking about strong ground motions. Lastly, it can also capture the behaviour in terms of pore water pressure. So, whenever there is significant development of pore water pressure, nonlinear analysis will be more effective in terms of understanding the governing mechanism at the site. So, that was all. Thank you, everyone. Thank you.