Surveying
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Module - 6
Lecture - 2
Triangulation and Trilateration

Welcome to this another lecture on basic surveying. Now, we are in module 6, and today we will be talking about lecture number 2 in module 6, and that is also about triangulation and trilateration. This is the structure what we are doing in all the modules.

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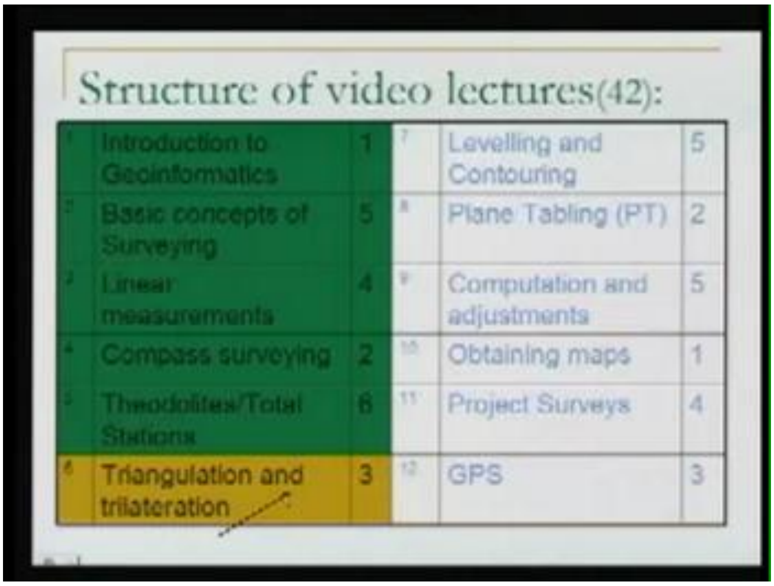

These green ones we have done already. This is the one we are doing at the moment.

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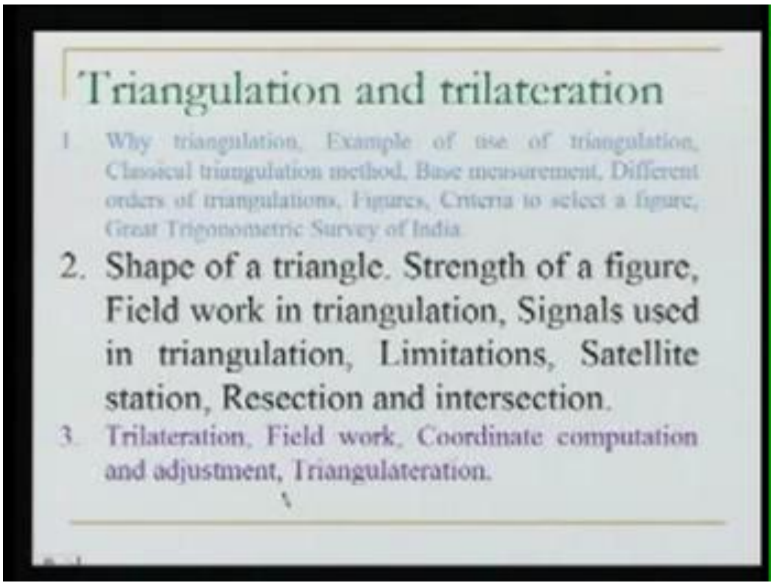

Now, in this module triangulation and trilateration, what we have seen so far? We saw that what triangulation is. Now, what do you mean by that? What it is? We saw that it is a way of generating the control network by means of network of triangles. We do it by basically we measure only one length which is the base length, and then all other angles in the network of triangles. Now, using the known length and all the known angles, computations can be done, so that starting from this known length, all other lengths in that network of triangles can be computed.

Now, this measurement of length which is the base length is very important. We saw a method for that. We can measure a small length, we can extend it into a bigger one which we said as extension of base length, and then later on we saw the types of the figures which we can use in triangulation. It could be a chain of triangles, a chain of quadrilaterals, also centered figures.

In practice actually we will be using a combination of these various figures depending on what the terrain is, what is your application, what is the area which we need to map. So, accordingly we will decide the area. Then, the other thing that we have seen is what the criteria by which this triangulation figure will be selected. It should cover the entire area. We also show that there should be multiple roads for calculation or computation because we are starting from one known line; we are computing the other lengths. So, there should be multiple roads for the computation. We have seen that concept.

So, then finally towards end of our last lecture, we are looking at the great trigonometric survey of India. Now, we will give you a little bit more information on that GTS in India.

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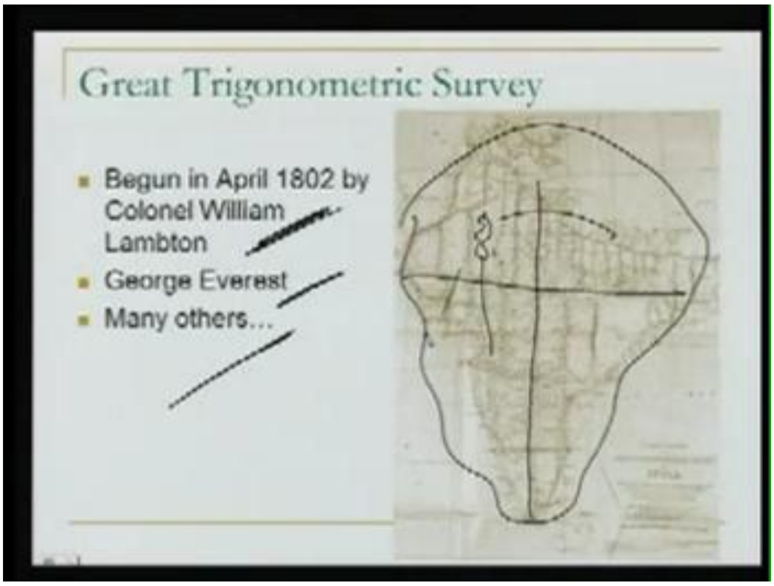

It began in 1802, the April of 1802 and the very first person who started that was Colonel William Lambton. Then, later on George Everest also joined it. You know the highest peak in the world the Mount Everest is given the name on the basis of the name of George Everest because he was the person who measured, who did this great trigonometrical survey. Then, there are many others who joined in this effort. So, this was an effort which was done some 200 years back, but it helped to measure the arc in the South-West, South-North direction in India as well as in East-West.

It helped to know the shape of the earth. In this part, it helped to lay down a reference system for India. It was a wonderful effort by these people, and ultimately at the end of the day we have a network of triangles. As we have already discussed, this is the great iron pattern all over the India in which for each point, we have the coordinates known. So, we can make use of these coordinates in order to do the survey in other areas and relate our survey to rest of the survey of the country.

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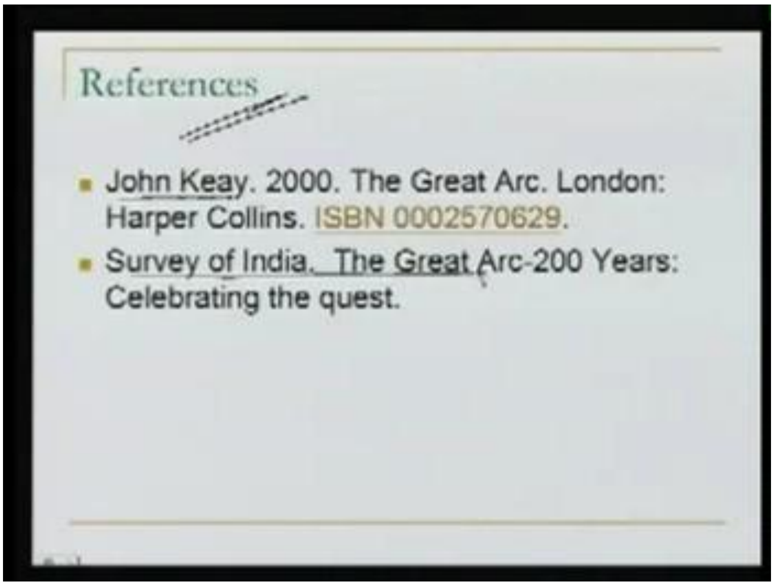

There are some references where you can get information about the GTS. There is a book by John Keay, a wonderful book as well as the survey of India has also come with some publications. So, you can go to those in order to know more about this great arc or the GTS.

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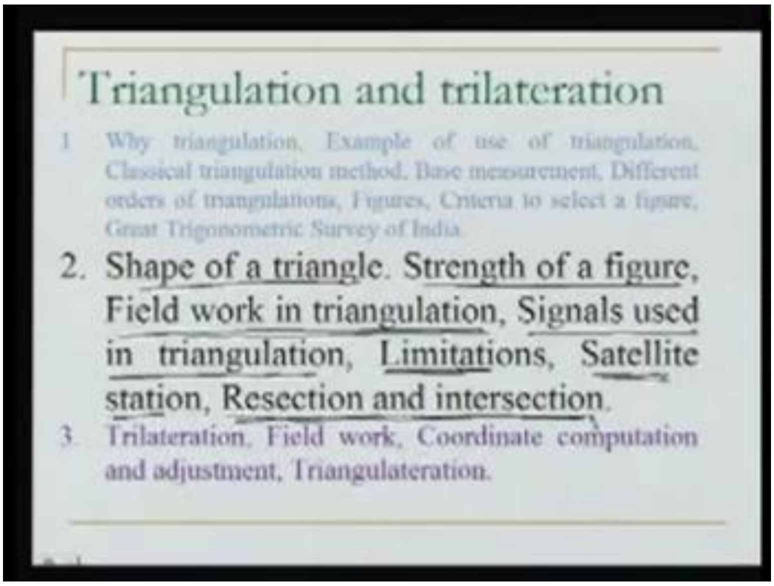

What we will do today? We will start talking with the shape of a triangle, then strength of a figure, then further what all field work is required in the triangulation, what are the steps, what we do actually in the field, then something about the signals and towers which we use in the triangulation process, some limitations of these signals, then a little bit about satellite station and something more on resection and intersection. So, we will try to cover all these in the video lecture today.

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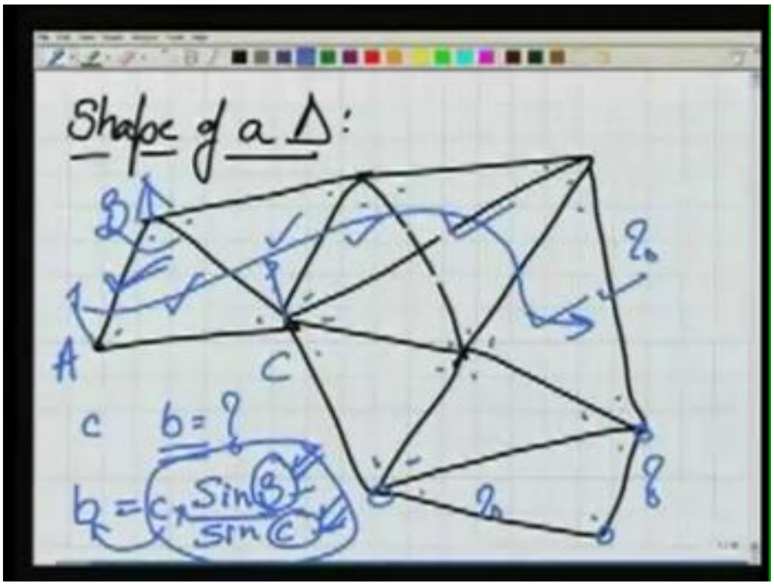

So, we will start with the shape of a triangle. Well, how we do the triangulation we have seen that we start from a known line which is the base line, we measure this accurately and then, later on we lay a network of triangles in the entire area. So, our triangulation figure could be anything. It may have the centered figures, may have triangles or the chain of triangles. So, starting from this known line and all the measured angles all, these angles are measured here. All these blue dots they indicate that the angle is being measured here.

Now, over here I would like to delete this because this figure is a quadrilateral, a base quadrilateral and these angles will not be measured here in order to compute starting from here any other length. For example, we want to compute this length, or we want to compute this one or this one because in order to know the coordinates of these points, we need to know these lengths. So, what we are doing? We are following a route. That route means because we are basically making use of the sin rule. So, we are working first in this triangle, and then this triangle, then here, then here and then finally, we are reaching here.

So, what we are doing? We are involving the angles. For example, here if it is A, B and C, then length c is known. It is known, because it has been measured. We want to compute length let us say b because this is unknown. So, in order to compute b, what we do? We make use of sin rule. So, we write here b is sin of B divided by sin of C into the length C which is known to us. So, if there is an error in angle B and C which is there because we have measured these angles in the field, and we know now once we are measuring the angle in the field, we are making use of the theodolite. What do we do? We bisect the target.

For example, let us say to measure angle at B. We are bisecting the ranging rod at A and ranging rod at C. We have seen ranging rod. It could be the signal, the big tripod signal which we will see later on. So, by doing that we are measuring angle ABC. So, in measuring this angle, we know there are various sources of errors and those errors will finally give us an angle which is not the actual true angle, rather an observation to estimate of that angle. So, we are working with the angle here, but in this angle, there is some error. Similarly in C also there is some error.

Now, with these errors which are there in these angles, these errors will through this computation propagate in the value which we are computing of B. So, what is happening here in any of these lengths which we are computing is, there is an inference of the errors in the observation of the angles. So, we should have an idea that how these affects us. So, in order to achieve a condition where the effect of this error is minimum, we look for a shape of the triangle which we say is the best shape. So, the question now is what is that best shape of the triangle? We have seen that graphically.

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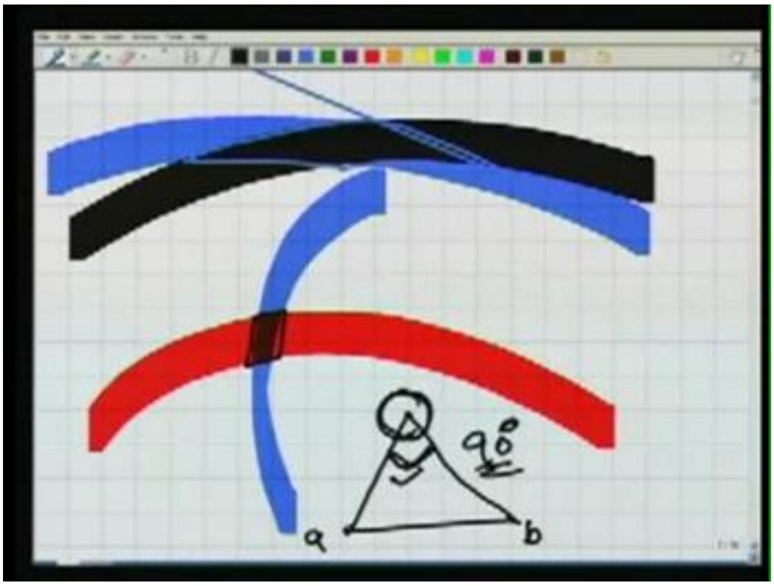

We saw that graphically, previously also that if two arcs, they intersect at very small angle here, then the zone of where the point is that zone is a very small area here. Sorry, a very large area. So, the uncertainty is very large while if these two arcs, they intersect now at 90 degree angle, let us say that is the one arc and the second arc it intersects at 90 degrees. The zone of uncertainty is very small. So, we solve graphically previously also in our last lecture.

So, what we found from there? We found that in order to locate with respect to two known points a and b or third point c by this graphical method of drawing, the arcs, this point c will be located with good accuracy if this angle is 90 degrees. If this angle is very small or this angle is very large, this point will be located now with more error. So, what we are doing? We are going to find the same thing, the graphical thing. What we are doing, because we came with a conclusion that this angle should be 90 degrees there. Similarly, we will try to do it now mathematically.

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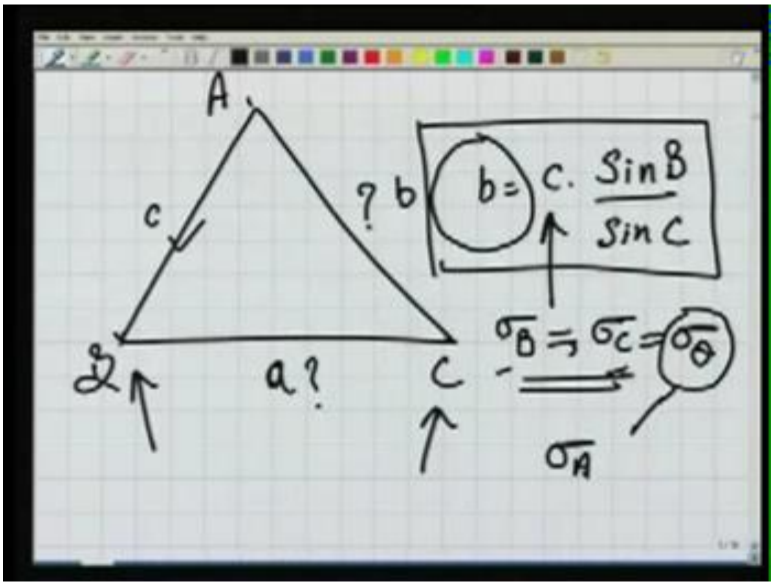

Well, we have a triangle and this triangle is A, B and C, where length c is known. It is known because this is the length which is either coming from the previous computations or is the base length which is being measured here. So, we know this length. Now, the job is to determine length b and length c, sorry length a. What these are? We want to compute these. So, for length b, we have seen previously also and we can write it as c sin of B and sin of C.

Now, in the angle B and C when we measure it, let us say there is an error and the standard error is delta B and delta, sorry sigma C. So, sigma B and sigma C is the standard error there. We are using the theodolite in the field, let us say the same theodolite was used in measuring B as well as in measuring angle at C. If you are using the same theodolite, the weather conditions or other conditions are same and as well as the observer is same. So, we will expect that these two errors will be same, and we can write this error as sigma theta. So, this is the error in any angle measurement in my triangle, same will be the error also in measurement of angle A.

Well, what we are trying to do? We are trying to see now the effect of this error in computation of length b. As far as small c is concerned, I can assume it to be constant because this length is coming from some previous computation or is being measured here. I will assume it to be without any error right at this moment because I am only going to consider the errors, which are because of the angles we know. Now, in a case like this how the error will propagate? So, to write this error propagation, we can write it as now.

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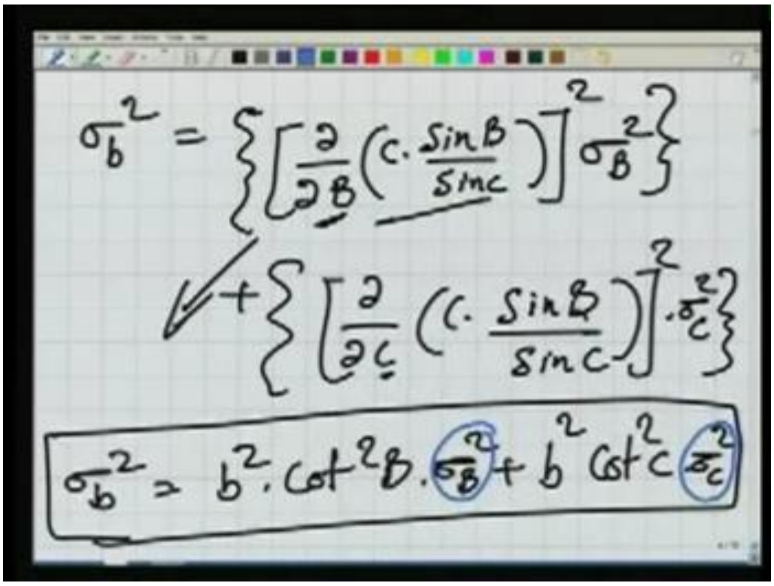

That is the error in computation of length B. We can write this as by del, first with this is the partial derivative with all the variables which are coming there of my function. Now, the function is c sin B by sin C and square of this plus, sorry multiplication by this is multiplied by square. That is first and then, for the second one by delta C that the function now c sin B by sin C and square of this and sigma C square.

Now, this is how we know it. How I am writing it, we have already discussed this when we are talking about in our first two lectures, the error propagation. If there is error in these individual measurements and we are computing a quantity using these two, how the error will propagate. So, we have already seen this. So, please go back to your lecture notes and learn about this and do it yourself. Now, this can be simplified and this can be further written as, I can write it as b square cot square B square plus b square cot square C sigma c square. So, you can do it yourself. I am just finding the partial derivative of this with angle B and similarly, here with C and then simplifying this. So, it can be written now like this. Now, in this also we know that these values and these values are same as sigma theta square.

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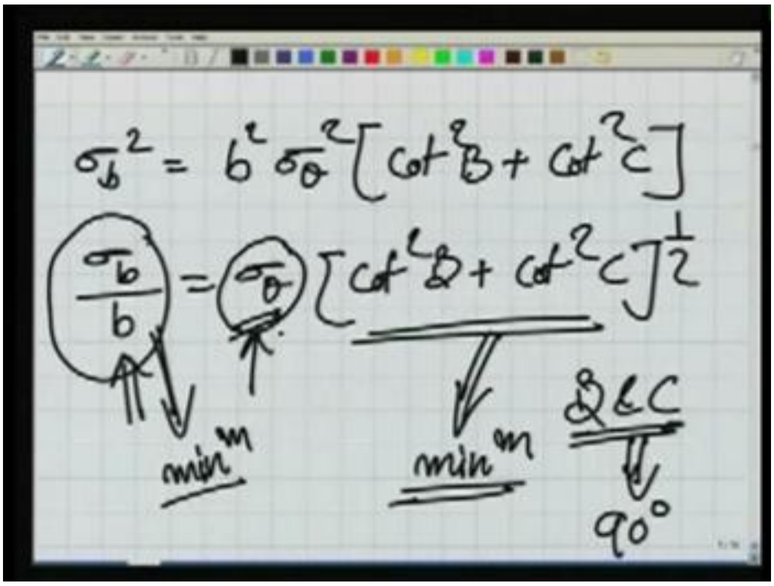

So, I can write this further as I am taking this b square sigma theta square out. So, cot square B plus cot square C. This is how this can be written. Well, I further simplify it. This is now sigma theta and cot square B plus cot square C raised to the power half. So, please do it yourself.

Now, what is this? This is the relative error in computation of length b. Isn't it? This is what how we start it. So, it depends upon a standard error of angle measurement because we are using some instrument, some person involved depending on the conditions of whether and other things. This will be controlled and this is generally known we can have an estimate of this. So, what we can say the relative error in computation of length b depends upon of course this value which is nearly constant here. So, in order to make this minimum, because this is constant, this has to be minimum now.

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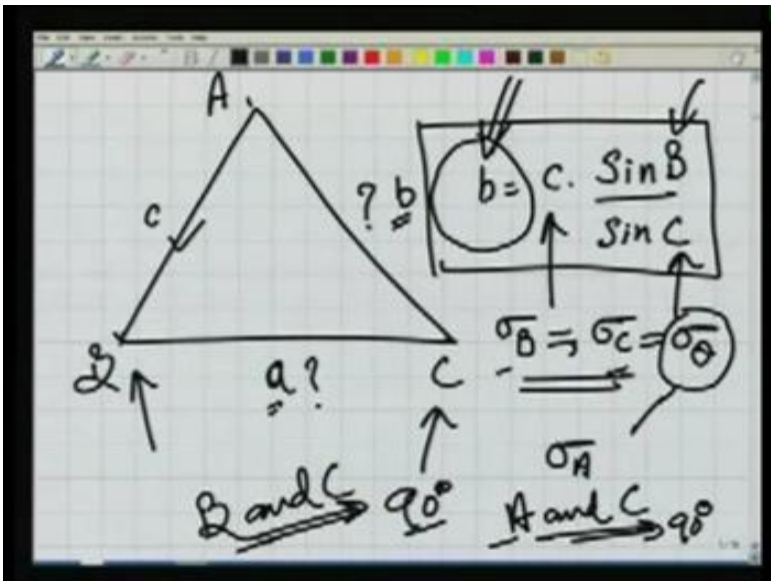

When the cot square B plus cot square C will be minimum, this will be minimum when both B and C, they approach 90 degrees. So, what we end up with? We end up with a situation where if we look at our triangle, we end up with a situation here now and we want now both B and C to approach 90 degrees, so that the error in computation of B will be least. Well, that is really not possible practically.

The other way on right now we are computing B. If you compute A, will end with a situation that A, angle A and angle C both should approach 90 degrees. So, we are looking for a case, so that in computation of lengths of sights, the error will be minimum. If this is satisfied and this is satisfied well, they cannot be satisfied. So, what is the solution?

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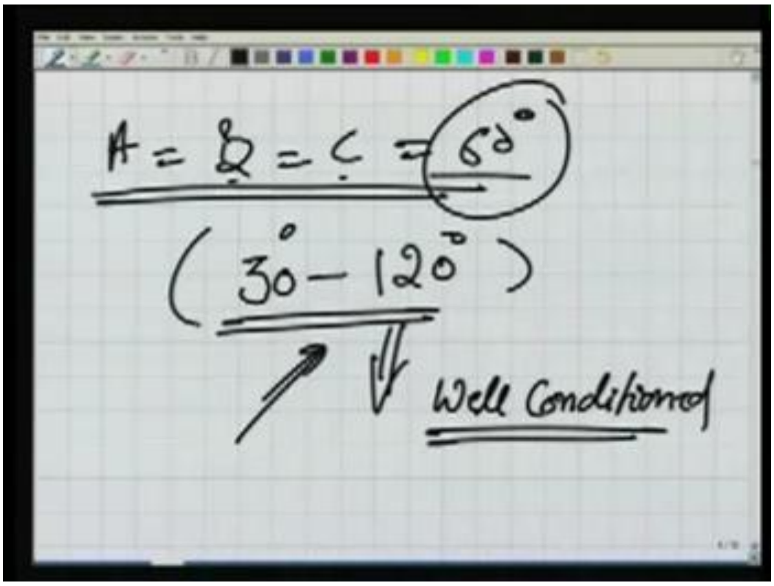

The solution is when all A, B and C should be 60 degrees, so this is also in kind of ideal situation we are trying to maximize the values of B and C, so that the cot value which was there you know it approaches towards 0. So, this is also kind of ideal situation which is not possible practically. You cannot lay down the triangles always with the three angles being 60 degrees. So, what is the practical thing? The practical thing that we say is generally in the field that the angles should be within 30 and 120.

In our network you know that was our network here. For example, here in this network if all the angles are being 30 and 60, we will say our network or the triangles in the network to be well conditioned. So, this is what we try to achieve. We try to when we are working in the field, when we are setting up the control stations, we will see that our angles do not go beyond this because we know now the reason that if they go beyond this, there will be error in computation of the lengths. So, we want to minimize that by maintaining this. Now, next we will see another interesting thing that is called strength of a figure.

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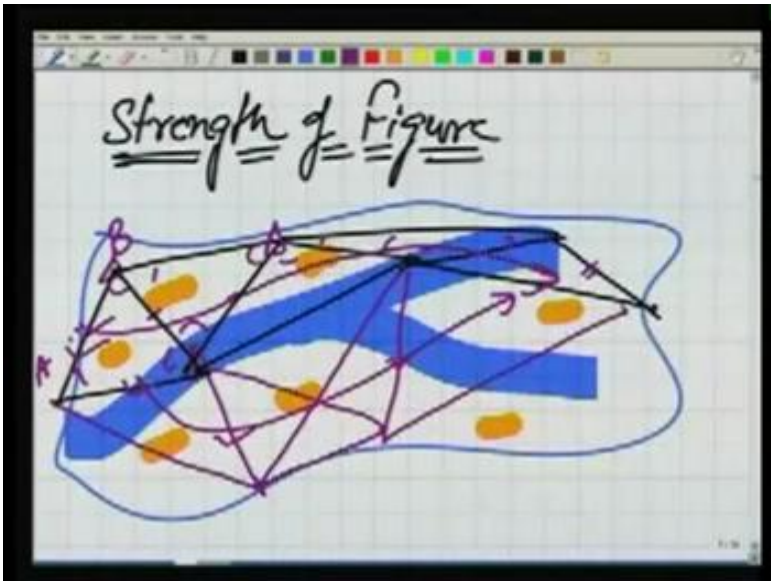

First, I will explain the concept of what is the concept of strength of figure. Let us say we have a job and the job is if this is my terrain here in this, we have some another roads, some features are there and we also have some houses you know anything and we want to make a map of this. We want to map this. So, what we decide? We decide that we will go for triangulation and we have a triangulation network let us say our triangulation network is like this number 1, and in the second case, I again go for a triangulation network, but now the triangulation network, which I choose is a slightly different one. So, now, we have one is this route see that is our base line if I say A B. So, AB is the base length which has been measured.

Starting from this AB, I want to establish these control networks. So, what I am doing, I am computing the length, all these lengths using the angles which are measured in sight, all these angles and then, finally I can compute these lengths also. So, this is you know we are doing one job by two sets of triangulation system, or two sets of triangulation figures. So, this is one root by which I can establish the control and the second root which is possible here is I start from this known length. Then, I compute this. Then, this and then in this quadrilateral I compute this and then, finally this again. So, I have got a second root. So, to do one job computation of this length, I can do it from two different triangulation networks, two different figures.

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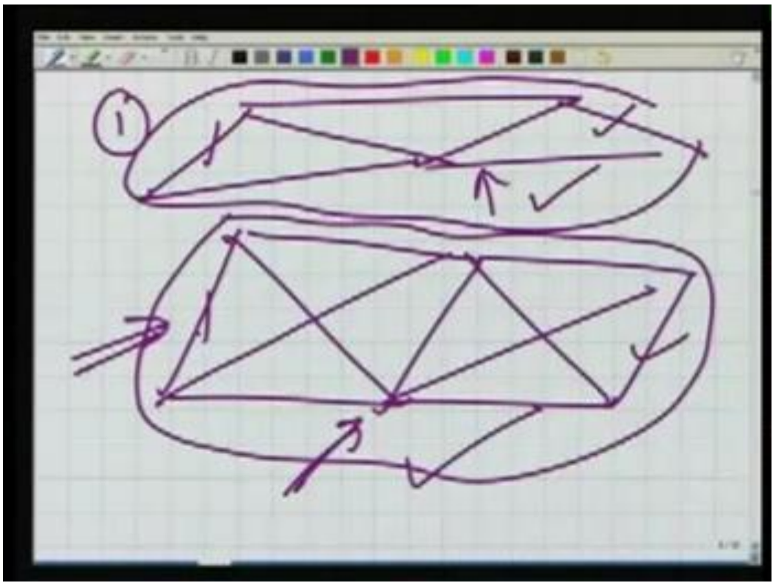

So, what we are talking here? We are talking here that in order to complete a job, in order to lay down a network, a triangulation network or we can say the control network in an area, we can go for a figure, which is made of you know very poorly well conditioned network or maybe we can have another figure which is very well laid down. All the angles are well conditioned here, while angles are not well conditioned here though they are covering the same area.

So, this network as it looks here should be better for computation of the lengths, while this network should not be better for computation of the length if we started from the same base length. Now, we want to have a proof of this as well as many times if we have a figure like this and figure like this, we want to differentiate. We want to say yes, this figure is better than this figure is there. How can we say that this figure is poorer than the figure at the bottom? We should have a way out in order to say that. Well, here is another example. The example is as you can see in the slide.

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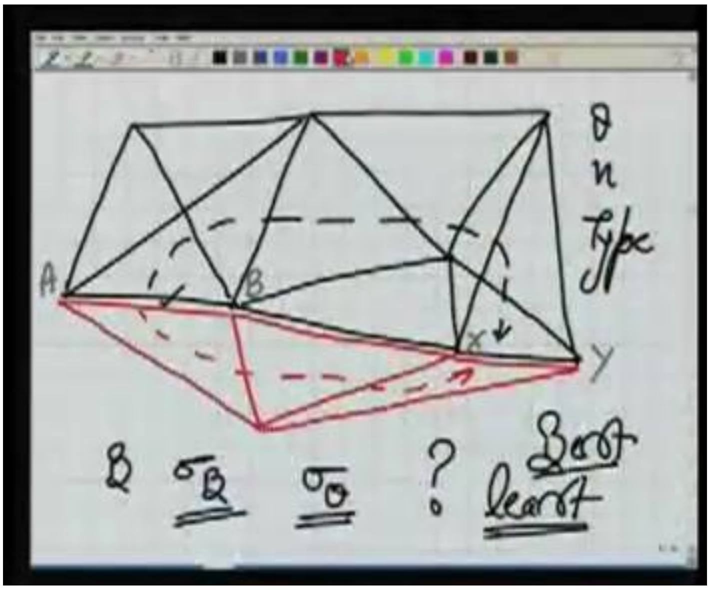

In a video screen, we are starting from a base length AB. So, AB is measured and we want to determine XY. Let us say that is the job we want to determine XY. So, starting from AB to determine XY, I can go by a root here which is shown by this dot if I highlight this. I can go through this route. In this route I have well conditioned triangles which are forming a very good figure which is the base quadrilateral and finally, I compute this XY. Similarly, I can also do, I can start from the same AB and then, go through this root where we have only 3 triangles and again I can compute XY.

Now, the question is if the angles have errors as we have seen the angle at A or angle at B, they have the errors or for all the angles if the same theodolite is used, we have the error sigma theta. We have seen it. So, if we are computing in the route here, in this route we are computing let us say or in this route we are computing. So, in this route because of this error, there will be computation of length error propagation. We have seen that.

So, the question is now in which of these two routes if they are errors in the angles, the error propagation will be least? Which of these two routes is better or we can say the best which of these two? I will say not even routes. Now, rather I will say these two networks. They are entirely two different triangulation figures, one the red one, and other the black one. Which out of these two is better? Now, in order to answer questions like this we go for the concept of strength of figure.

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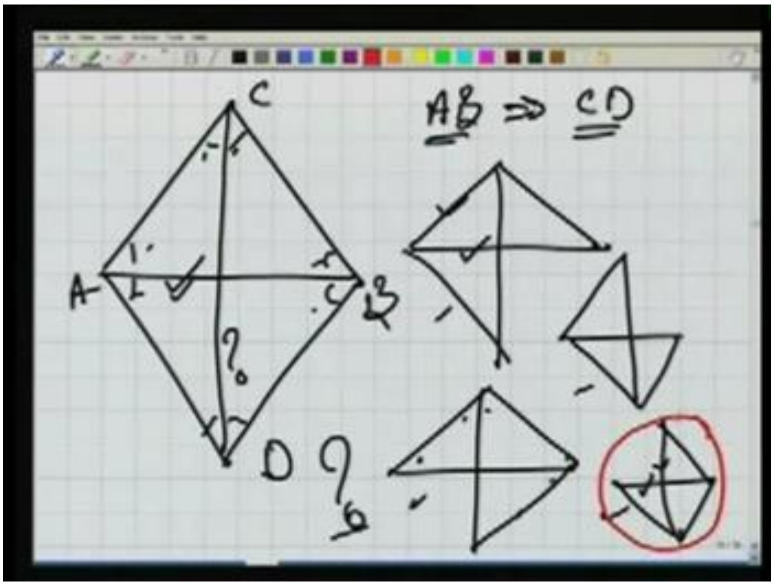

There is one more concept, which we can answer by the strength of figure and that is if we have let us say a triangulation figure like this, and in this A B is known while C D is unknown and this is known, all these angles are measured. As in the case of the triangles, it is starting from AB. You want to compute CD. Now, it is only one triangulation figure, one triangulation network is not like here.

In this case, the black one was entirely different network than the red one, but here in this case, it is one triangulation figure and within that figure also. What I can do? To start from AB to reach CD, I can do the computations number one in a route like this. I start with AB. So, I know this length, and then I compute this CD. So, this is route number one. Similarly, I can also do another route and also, I can do one more route and also one more route. So, what we see here starting from known AB to compute CD, we can do our computations by 4 routes.

If there are errors in all of these angles, we know the error will be there because this is how the observations have been taken. It is not possible to measure the true value. So, the errors are there if these angles have errors which of these routes are supposed to be the best?

So, we have to again answer this thing because we would like to compute from this known length AB, the unknown length CD through the route which is best may be, let us say this route is best. So, we would like to find out that which one is the best route for us, and do our computations from that route. So, how to do this thing? We can do it by this concept of strength of figure.

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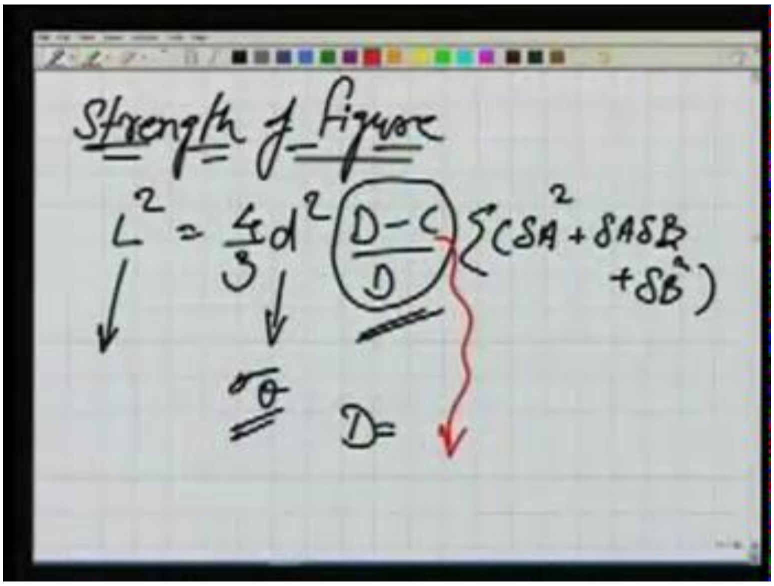

Now, what we see here couple of things whether the problem is this or in this case. In both the cases, the strength of figure means a figure in which the computational errors are least. It will depend upon number 1, how many angles are measured? You know this figure is better because there are only 3 angles measured. There are more numbers of angles which we have measured. The chances of error are more because finally it will accumulate. How many triangles are involved in the computation? Over here only 3 triangles are involved in the computation while here you have got more.

So, how many numbers of triangles, which are involved in the computation? More important what is the value of angle? As I said all the angles in this black route are well conditioned while in the red route, the angles are not well conditioned. You know we can see the type of the angles or what their types are. Are they good angles or not? Are they poor angles? So, that will also control it. Similarly, it may be possible in some situations. For example, let us say it was not possible to occupy this point. If this point was not possible to be occupied, these points still we can derive these angles. So, the angles which are here are the derived angles or not. They are actually observed angles. So, we should give less weight to those angles which are derived while we should give more weight to those angles which are actually observed.

Then, in order to measure for example the angle here, and the angle here, this particular line has been bisected twice. Number one in order to measure this angle, then to one observe this angle. So, wherever there are multiple observations like this, we should give more weight to that. So, what we end up with? Well, how many angles, how many triangles, how many lines, what are the types of the angles, the sizes of the angles, all these things needs to be put together in a formulation and that formula should be able to give us now the strength of figure.

Now, what that formula is? I am straight away coming to the formula. It is given as L square is 4 by 3 d square D minus C by D sigma del A square plus del A del B plus del B square. Now, what these terms are I am going to explain. Now, this sigma, sorry this L square is the probable error in computation in a chain of triangles. What we are doing in? Anything we are starting from a known length AB was known length here. As you can see this is known length. We are starting from the known one and then, we are passing through a chain of triangles in order to compute this.

So, this L square is the probable error in computation. When we are going through a chain of triangles, this d square is the probable error in angle measurement. So, this is something what we have similar to our sigma theta which is constant throughout the process. If you are using the same theodolite, if you are using the same observer, the weather condition and the things are same. So, this is a kind of constant D minus C by D. It talks about; it gives the difference in these two figures whether this black figure is different than the red figure. It gives us that difference. So, we will come to each and every part of this D minus C by D.

Now, here D is total number of directions observed except along known lines. Now, what is the meaning of this? In order to help to understand this, I am going to draw another figure here. So, in this figure now we will see what we have shown in this figure.

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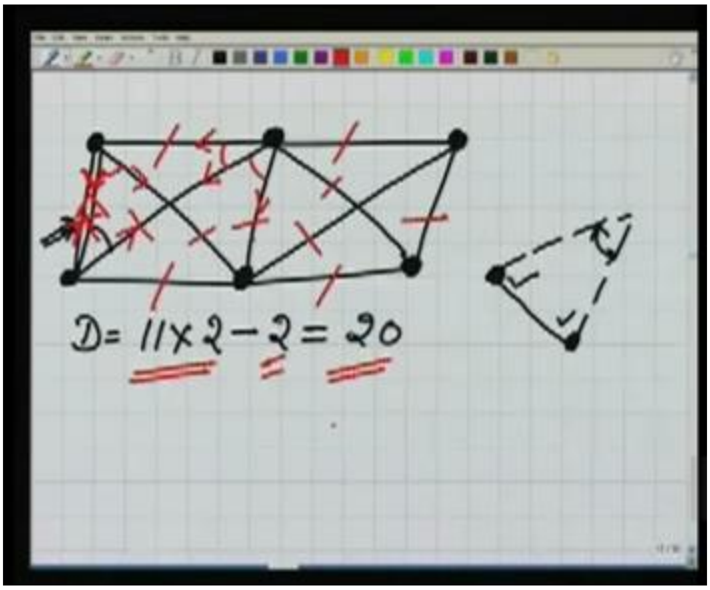

We have again a triangulation network and in that triangulation network these points are the stations, and as is conventionally shown if this station is occupied, I am making a big black dot there. If it is not occupied, for example let us say there is a station, which is not occupied. I will just draw the dotted line. I will not make a dot there. So, here in this case, there could be a thing that in this triangle only these two points are occupied, but this is not occupied. The meaning is the angle here is not measured while this angle is measured, this angle is measured.

Now, D in our formula here, what it stands for? D stands for total number of directions which are observed in both forward and backward direction, except along the known lines. Now, what is the meaning of this? The meaning is we have here only one length which is known. This length is known while I am observing the angle here; I am taking one direction this way and one direction this way, so two directions. Then, when I am observing the angle here, again I am taking one direction this way, one direction this way. When I am observing this angle, I am taking a direction here, another direction here. For this angle, one direction here and already direction is there.

So, in order to observe these angles, what we are doing? We are sighting along these lines. So, how many directions we have actually observed? If you see that there are total numbers of 11 lines here. 1 2 3 4 5 6 7 8 9 10 and 11 and each line has been observed in both the directions, forward and backward. So, total numbers of directions which have been observed are 22, but out of these there are two directions which are along the known line. This is the known line. The base length I had observed like this and like this. So, minus 2, so D here for this figure tends out to be 20. Now, what is the value of C? To see that we look at this figure again.

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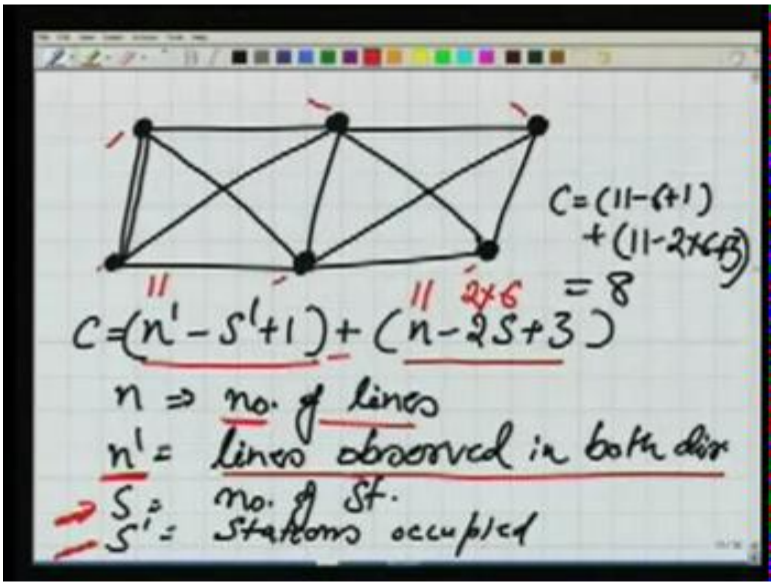

Now, the C is given as n dash minus s dash plus 1 plus n minus 2 s plus 3, where n is number of lines. How many numbers of lines are there? Total number of lines which are 11 n dash numbers of lines observed in both directions. So, n dash total n dash is number of lines which are observed in both directions, n is number of lines. The total number of lines here are 11. I will write 11 here, and n dash lines observed in both the directions. Again it is 11. S is number of stations. How many stations are there? Here in this case, 1 2 3 4 5 and 6, there are 6 stations. So, s is 6 multiplied by 6. Here 2 s s dash stations occupied.

There could be a situation when there are stations which are not occupied. As in this case, there is ranging rod. We are bisecting this. We are observing that direction here in this direction, but not in this way. So, this is station C is not occupied while, A and B are occupied. So, in this case what we see because all our stations are occupied. So, our s dash is also 6. So, both are 6 and 6. So, if you compute it here, now if you put these values, the value of C comes out to be 8. So, using this, what we can do?

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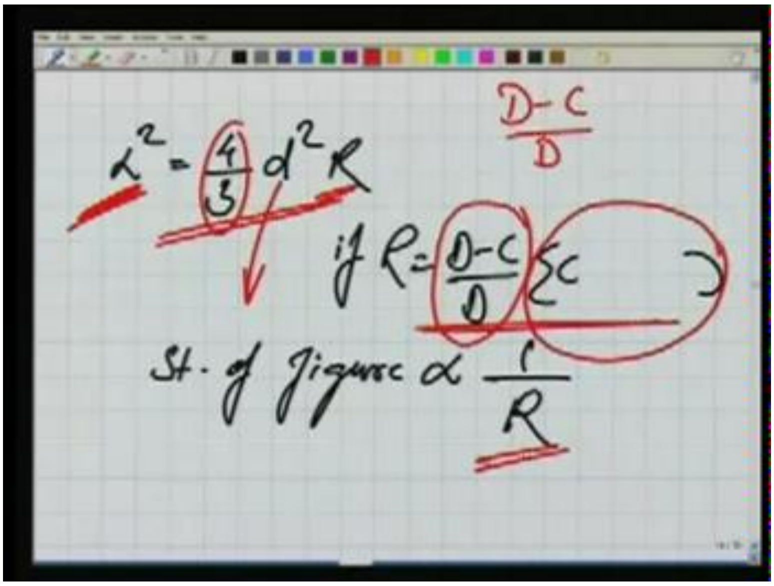

We can now determine D minus C by D. So, what we say this D minus C by D should be constant for a figure. If we are talking of the black figure here, this entire black figure is that D minus C by D will be constant for this figure.

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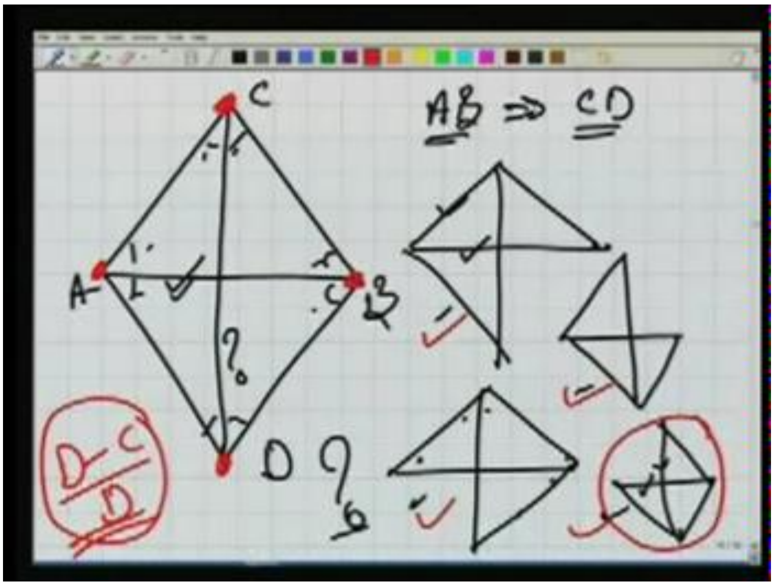

Now, whichever route we compute in this figure from let us say here in this example, we can compute for this. If this is occupied, this is occupied, this is occupied and this is occupied. We can compute the value of D minus C by D. Now, whichever root I am doing my computation from this value will be same. So, this value is actually useful in order to help us separate two different figures.

You know there is black figure, and then the red one. So, it gives us the strength of the figure in which case it is better, either in the red or in the black. So, this thing is coming out from this value of D minus C by D as you can see also. What are the parameters in D minus C by D? We are talking about the number of the stations, which are occupied. How many lines are observed? How many times the lines are observed? We are taking account of all those things. So, it is basically comparing these two figures here, the black one and the red one. Then which one is better?

Having seen that, so now we know that the strength of the figure L square or the probable error in computation along one chain of triangles you can write it as this way, where R is D minus C by D and sigma within these delta A del A square del A del B plus del B square. Now, why I am writing this way? It is because d is a constant because we are using the same theodolite. So, if this is constant 4 by 3 is constant, the strength of the figure because this error, now this error is proportional to R. So, error in computation is proportional to R. So, we can say strength of figure is inversely proportional to R, and this R is also made up two parts D minus C by D and this part. Now, so far by making use of this R, now in this we will see the role of this part. What is the role of this? We have seen the role of D minus C by D, and now this part.

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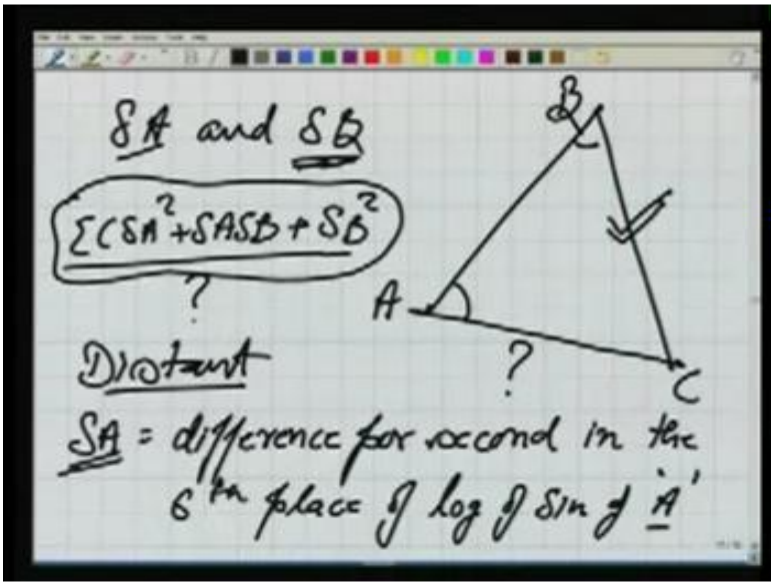

As we can see is del A square del A del B and del B square, what these are del A and del B. What these are in a triangle? Basically we are doing the computation within a triangle. Now, A is the angle which is measured and which is in front of the known length. Let us say BC is the known length. Known means it is either the base measurement or the base length, or it has come from some previous computation. So, this is the known length and we want to compute AC. So, the length which is or the angle in front of the known length is angle A, angle in front of the unknown or the length which is to be computed. For example, AC and angle B here is the angle B. So, this angle A and B are also called distant angle. These are the names.

Now, what is del A and del B? Then, del A is difference per second in the sixth place of logarithmic of sin of angle A. Now, what is the meaning of this? Why we are putting it this way? How it is going to help us in our computation? How it is going to help us in knowing whether a particular route is good or not? Now, we are going to look into that as we have seen before also in a figure like this.

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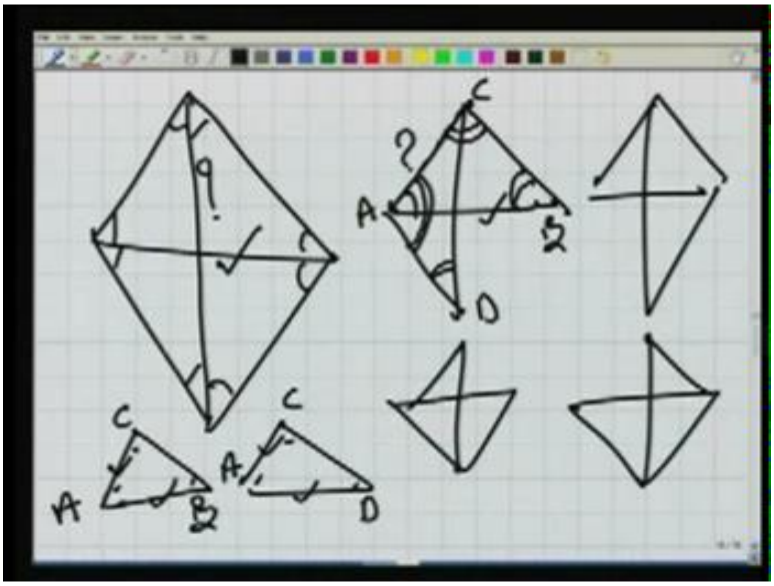

We have 4 routes of computations starting from a known one. We want to compute the unknown, and of course all this we are talking if there are errors in our angles which we have observed, they had the errors. Now, in that case out of these 4 routes which one is better? Now, when we start computation, what we are doing is, starting from the known length, we are making use of sin rule. So, we are including some angles here. Let us say we want to compute now this length. What is this length? So, we are including this angle and this angle in the computation. Then, once we know this length, this was AB and CD.

So, once we know AC, starting from AC, I want to compute again CD. So, again I am including the angles at D, this ADC as well as the angle CAD. So, what we have done?

We have now two triangles for computation. First, we are computing within triangle A, C and B. Then, we are doing the computation within C, A and D and in both the cases, the error will propagate. We have seen it because we are starting from here. First, we compute this and then, starting from here, we compute this and in all our angles, we have the errors. So, error will propagate.

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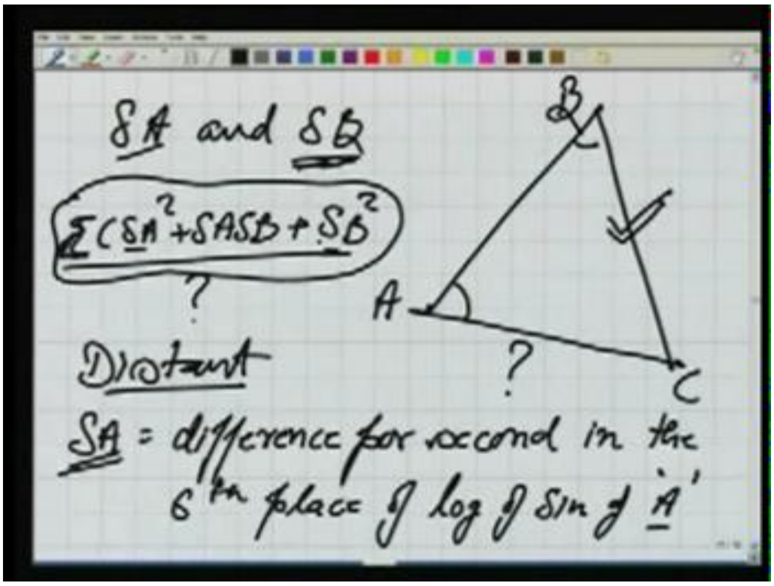

So, this particular term that is why we are writing sigma. Here sigma is because del A and del B are computation within one triangle. So, once we are computing in triangle ACB, we have some value of del A and del B. These values of del A and del B can be taken from the tables which are available in the text books. So, we have one set of del A and del B. Similarly, once we are in this ACD, triangle will again have del A and del B the distant angles. This is in front of the unknown; this is in front of the known. So, this is why we are writing sigma.

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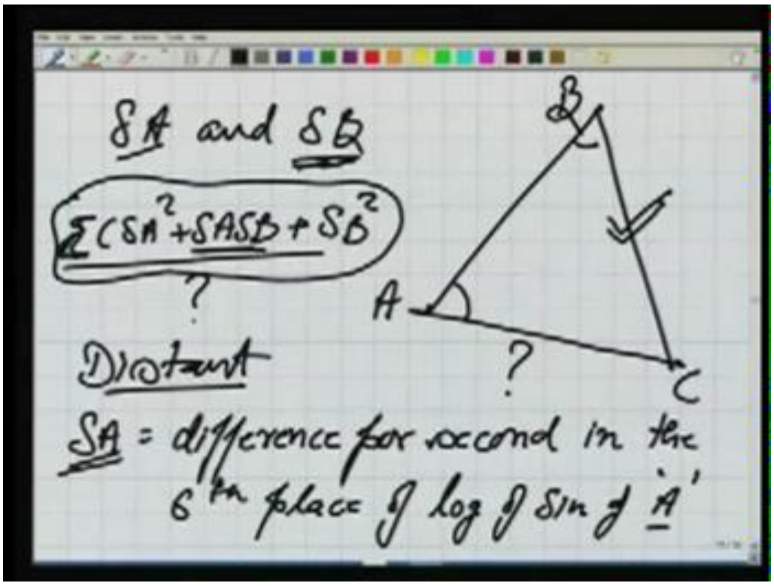

So, what this is actually indicating? It indicates del A as well as del B.

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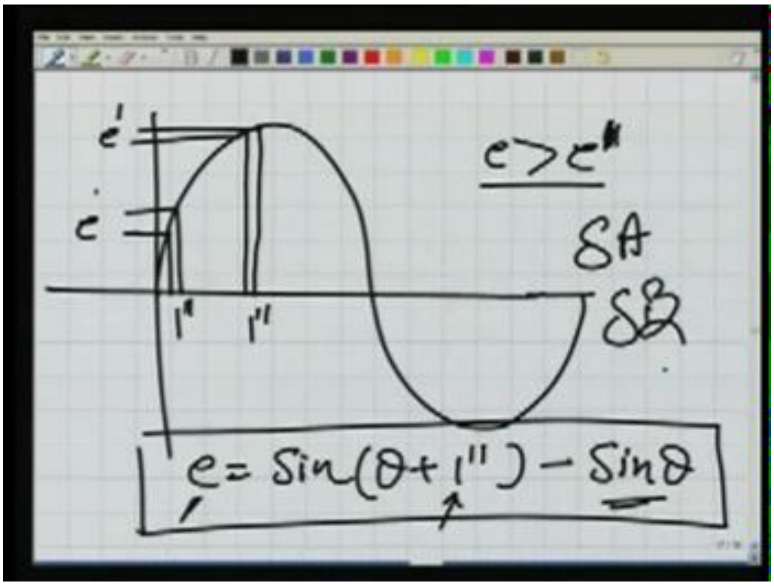

It indicates that for example, here if we know this is how the sin varies, the sin theta for a very small angle here. Let us say theta we are writing the error e as sin of theta plus 1 second minus sin of theta. I am just trying to show it here in order to explain what we are trying to do, or how it is going to help us. What I am doing? I am computing the difference in this sin for angle theta, where I am adding one second here, that is the error had there been no error would have got zero error. Here the e value would have been zero sin theta minus sin theta will be 0, but if there is some error, what is the influence of that error from the actual value? Actual value is sin theta. What is the influence of the error? This is what I am trying to estimate here.

So, if you look here, there may be two situations. There is one situation here and one here for the same one second, and one second. The error e will be large and will be small. So, if I write it as e dash and e, so e will be larger than e dash. This we know because of the proper to the sin. So, what we are looking at here? We are looking at if it depends upon the value of the angle that how much error will be propagated. We have seen this before also. I am trying to do the same thing again by this simplified way, and the same thing we are trying to take into account. When we are computing del A and del B this del A and del B in that formula are an indication of the error which will be coming into our computation because of the value of the angle. If the larger value, angle value is large, 90, the error will be less or if it is very small 0 1 2 3, that kind of angles the error will be more. So, this is what we are taking into account when we are writing our this thing.

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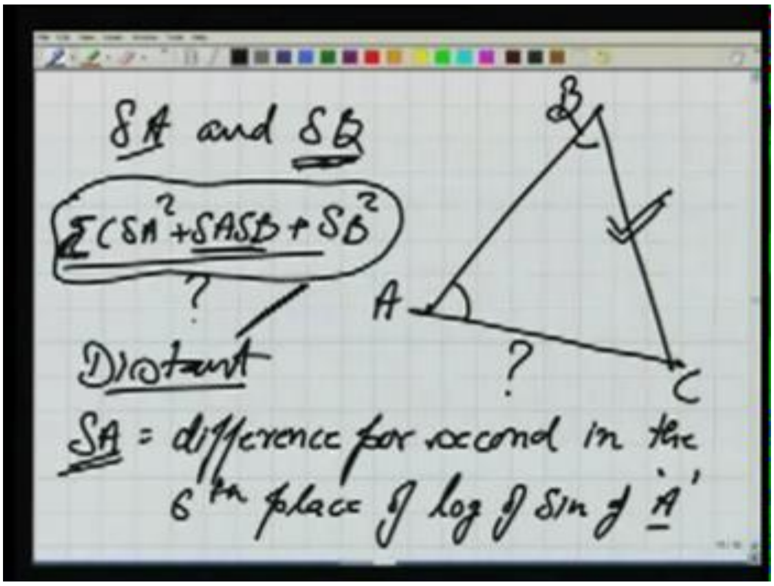

So, in our formula, this as the strength of figure we are writing it as 1 is inversely proportional to R.

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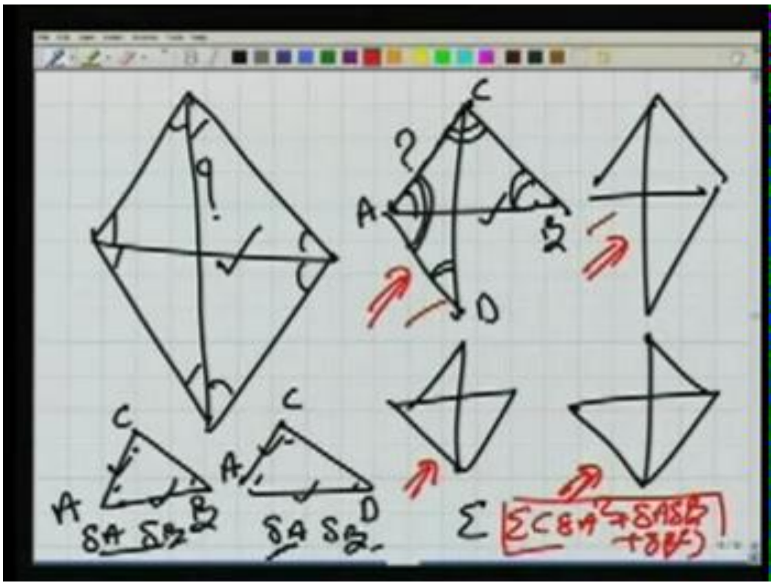

So, now what we can do? We can start comparing these different figures over here. Over here for each of these I can compute this particular value sigma del A square plus del A del B plus del B square. I can compute this and whichever the route, this route, this route, this route or this route, in whichever the route this value is least, that route will the best. That will be a better root for the computation. So, this is what the strength of figure indicates, and it helps us to do the computations through the best route, through the best network when our angles have got error.

Now, we will see that what the steps are when we go to the field in order to do the triangulation. Well, the very first thing that we know, very first thing is reconnaissance. We will make a very rough map of the area. We will observe the field, we will go to the various places there because our idea is we want to you know establish a controlled network there, and that controlled network through triangulation. So, what we are doing? We are establishing initially some stations, which will form our triangulation stations.

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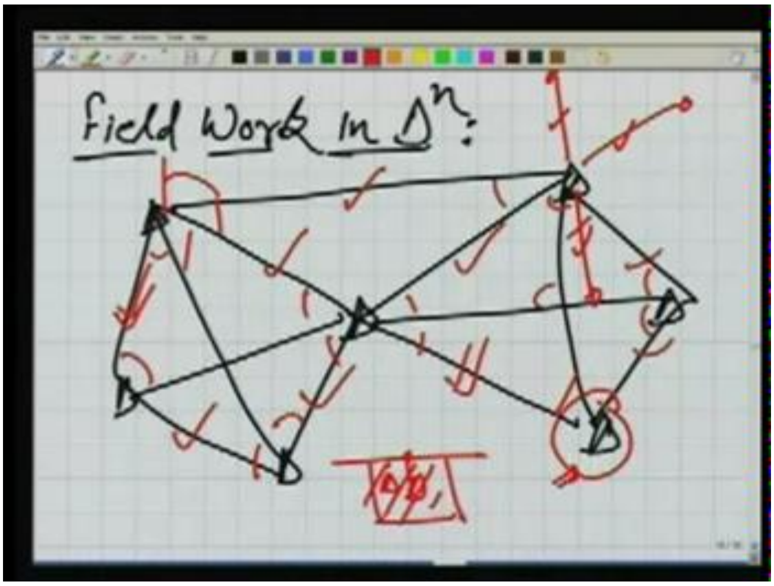

So, basically the moment I establish these stations as here in this figure by visiting the field, I am establishing these stations. So, that means, I am deciding what will be the form of my triangulation network, how it will look like. Well, we have taken a decision that our triangulation network will look like this. Now, why it should cover the entire area? Our triangles should be well conditioned. There should be multiple route of computation. All these things need to be taken into account.

Similarly, for example, if we are talking of this station, this may be on a hill top. If it is accessible, then only we can locate it. Otherwise, we will be in trouble. So, all these things need to be taken into account when we are establishing these triangulation stations. Once you have decided this point is going to be a triangulation station, what we do? We put a concrete block there in the ground, and then on top of this by a nail we will mark that this is the station. We will center our theodolite here; we will put our signals here. So, we mark these points there in all the places. So, somehow we marked these places as well as in order to locate these stations.

Later on we would like to take some witness marks from some permanent objects. They around will try to take these distances, so that if you visit this area after 2 years, after 1 year, after 10 years, we can again locate our station because this station is imbedded in the ground, or it may get lost because of some reasons. So, we should be able to locate it again. So, all these things are required to be done at this case of reconnaissance. Once the reconnaissance is over, we do the base measurement; any lengths as well as we carry out for any line in the Laplace station, the bearing measurement. Similarly, we also like to carry out one length as the check length or the check base.

Then, the next step is we are measuring all these angles, whatever the process you want to use, but all the angles are required to be measured. Once the angles have been measured, we go for you know an angle adjustment procedure. We will see it later how to adjust these angles. So, we adjust the angles in this network, so that this satisfies the geometric conditions which are there in the triangle. Once we have adjusted the angles starting from the known length, we do the computations for lengths of other lines. Having known all these lengths and their bearings, we can compute the coordinates, and then all these coordinates are plotted on a sheet.

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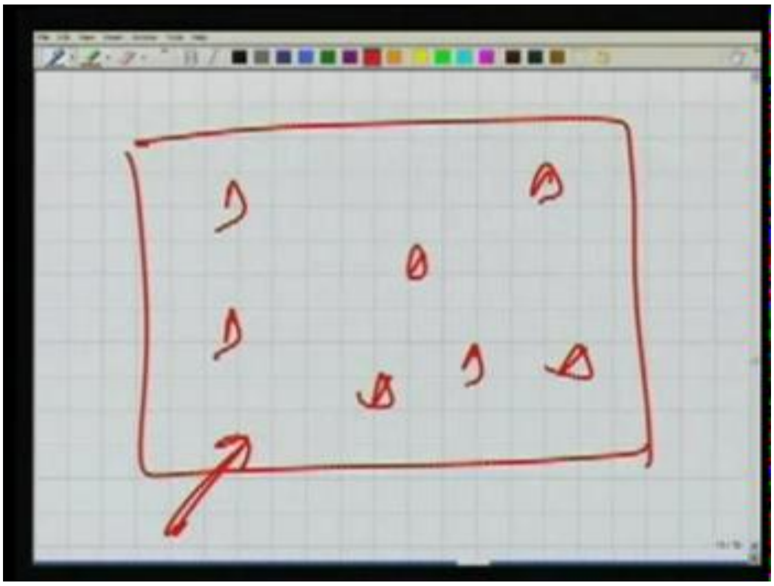

So, now we will have a sheet, a drawing sheet. It may be in auto cad or whatever we are plotting these like this. So, now, we have a sheet or rather we can say we have brought that network or the skeleton of the ground onto our sheet, and now we can start using this for some other purpose, for some planning or for some detailed mapping or something else. So, these are the steps which are in any triangulation process. Some more things which we need to know now about this triangulation or this triangulation process is about the signals and towers.

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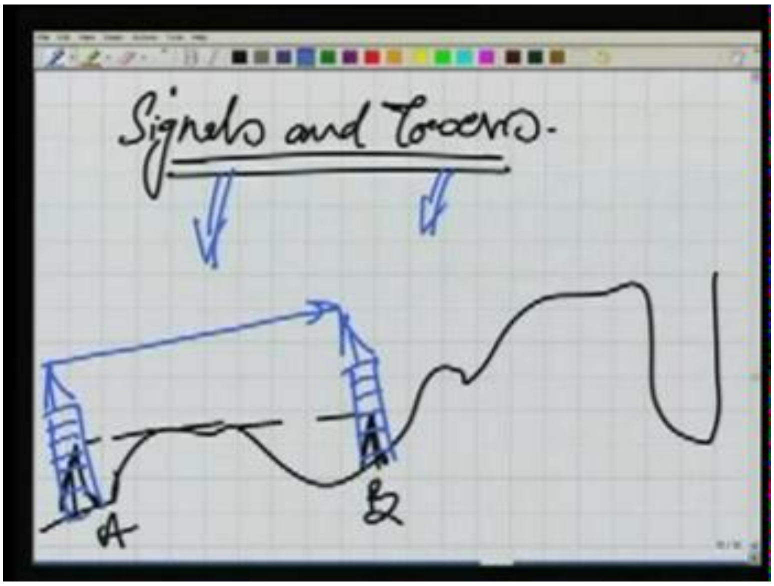

Many times for example, you have taken a decision that you are taking this point as a triangulation station, and this point also as a triangulation station point A and point B. Now, if these A and B are triangulation stations, they should be intervisible. Also, if they are not intervisible, what we would try to do? We will try to erect a tower here because we want to have these two definitely as triangulation stations because our triangulation figure which is formed is very good. The angles are well conditioned as well as the requirement of the field, but this line is not intervisible because of the intervening ground.

So, what we would like to do? We will like to erect the towers, so that I can keep my instruments or the theodolite on the tower, and I can then take the measurements. So, there are various kinds of signals which we use in triangulation as well as the towers. Now, the signals, all this is this stuff you know all this material is given in any text book. So, please go through any textbook on triangulation and you will find a lot of this running material in these text books which talks about the signals. Signals could be like the ranging rod. We have seen the ranging rod. It uses the sunlight. The sunlight will get reflected and then, it works as a signal.

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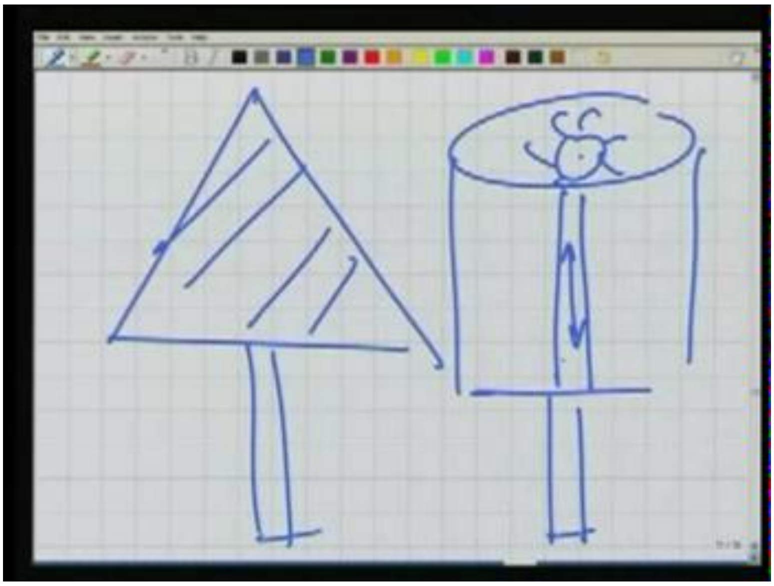

Similarly, we can have a thicker ranging rod. I can say you know a cylinder, a wooden cylinder and it is painted white and black or red and white, and we are using that as a signal something to bisect or we may have the signals which are of you know a kind of cone. Then, we have the stand of that. We can use it for the signal because we can see it from a distance very far. We can have instead of cone, a cylinder, a big cylinder and again we can see it from a distance.

So, there is variety of these signals. Some signals are there which make use of the light because you want to do that triangulation at night. If you want to do at night, your sigma should have some mechanism by which let us say, there is an electric bulb and here is just a slit. So, this slit can be seen if this bulb is switched on, or there is some lantern or something is kept there, which is pursuing the light. So, you can see this light from a distance. So, these kinds of signals are called night signals.

So, we have day signals and night signals. So, all these varieties of stuffs are written in the text books. So, please read those. I will show you only some diagrams of these signals now. Now, here is a picture of a signal as you can see here.

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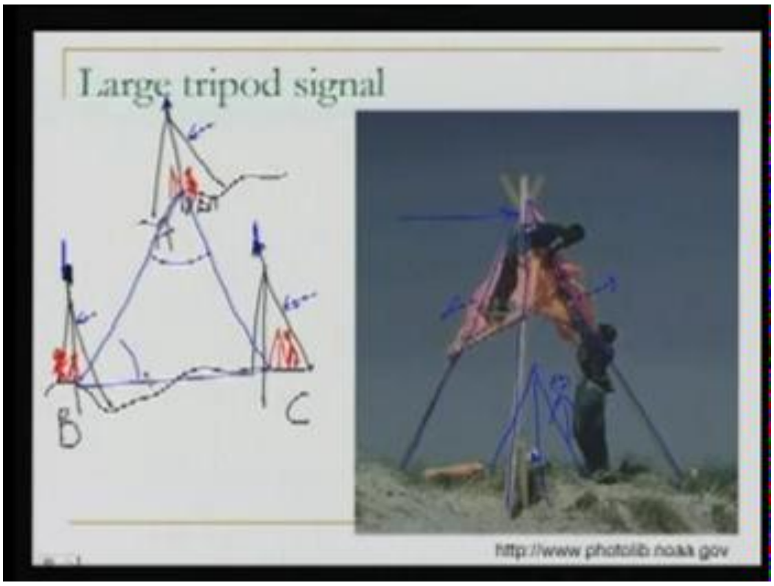

Many times we will need to erect signals like it is a huge tripod, and this is being erected here and this cloth is put here, so that this tripod can be seen from a large distance. As we know the distances are of order of 2 kilometers, 3 kilometers, 5 kilometers and more than that. So, we want to see this signal from a distance. So, we can erect a tripod like this. Now, the thing is under this tripod there is somewhere the point that station and we will erect our theodolite in such a way, or the total station in such a way that its vertical axis is passing through the point here.

So, the facility which this kind of tripods, they give someone from the other station can bisect this tripod. This tripod is also centered on this point while at the same time someone is taking the observations here. There is a person who is standing and taking the observations as in this figure. There is a triangle A, B and C. All these black ones or the tripod signals and the red one, these are the observers which are trying to observe the angles and this red line is the theodolite. So, the observer here can bisect the signal here, and the signal here in order to observe this angle.

Similarly, the observer here will bisect the signal here, and the signal here to observe this angle. So, we have a situation where under the tripod, there is an observer with the instrument with the theodolite, and this is how you know this is how the triangulation is carried out or the angles are measured in the field. In this case, as we saw we need to use the bilby towers.

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What this bilby towers are? Here is a figure of the bilby tower. It is a huge tower which is erected using the stresses and that is the figure on top of the bilby tower. We have the human being, all the observers are there with the theodolite and with other things by which they can you know sign to others about their locations. They can send the signals to others about their location. So, this is how the triangulation is actually carried out in the field. If your lines of sights are very large, if your lines of sights are smaller, you are working only within an institute or university campus or small town. You may not need these things also. So, if you require you may go for things like this, or if it is not required, you can avoid using this.

Now, some more things about the triangulation like this satellite station or the resection and intersection will see in our next lecture. So, what we covered today? We covered about the shape of the triangle, why the shape is important, why those angles are important. We saw it graphically also, we saw it mathematically also. How the value of angle if there is error in the angle is going to propagate this error finally in the land computation, and what should be the ideal angle. What should be the practical angle that we try to achieve in the field? The angle should be within 30 and 120, and this is what we say well conditioned.

Then, we saw there could be various networks in order to cover one area. Out of all these networks which one is the best? So, we came out with a criterion called strength of figure. Then, within one network also it is possible that starting from the base length, we can compute the other lengths to various routes. Out of all these routes which route is the better? So, we again found that the strength of figure criterion is useful there, and we can take a decision that yes, this particular route is better in order to compute starting from the known to the unknown, because that extent a figure criterion, it takes into account the values of the angles, are they smaller or are they larger.

At the end of it, we also saw that what we do actually in the field. When we do the triangulation, we start from reconnaissance establishing the station seeing that what kind of network we are going to do establishing the towers, and then taking the angle measurements, measuring the base lengths and then, finally adjusting the angles in the entire network. Then, we do the computation for the bearing and for the length, and finally, we found the coordinates of our control network.

Thank you.