Surveying
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Module - 2
Lecture - 5

Basic Concepts of Surveying

Welcome again to these video series on basic surveying, and today, in this video lecture, we are going to talk about, again, module 2 and lecture number 5.

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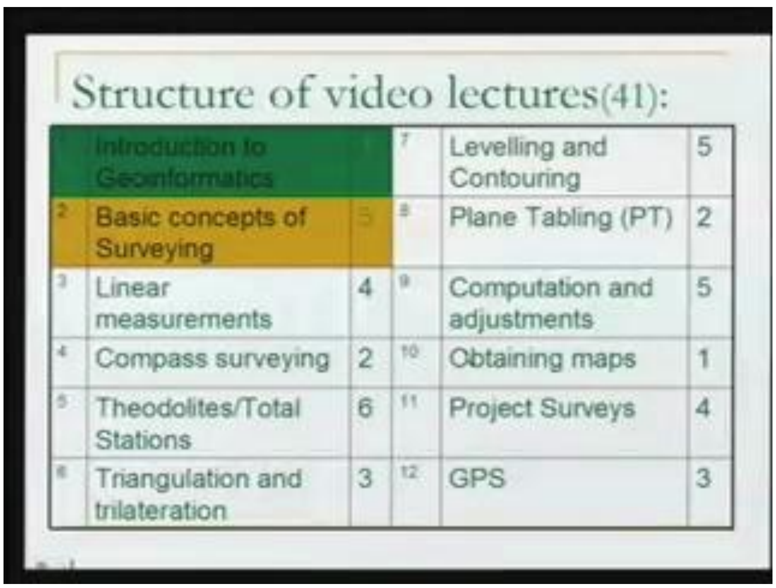

If you look at our structure of the video lectures, this is the lecture number 5 though it was - our writing number 4 here. I was planning that we will have only 4 lectures, but in order to accommodate some material which is very, very important, I am inserting one extra lecture here.

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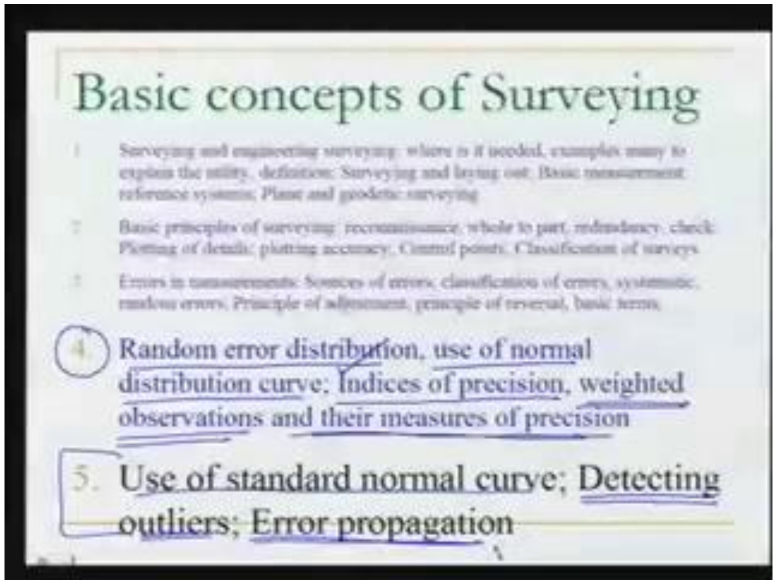

What we have done so far in last 4 lectures, particularly this last one - the fourth one: we talked about the distribution of random errors, we saw what kind of distribution it follows - the normal distribution, we saw the equation also for the normal distribution. Then we looked into - because once we know our observations, one: we remove the blunders and outliers from the observations, also, from the observations we have removed the systematic errors. Then, the observations still are not the true values - why they are not the true values? Because still there are some errors in the observations - something which we collect from the field - still, some errors are there, and those errors are the random errors. The random errors, as we have seen, will be always there. We do not know the sources of those random errors. Even if you know the sources, it is difficult to model them. And generally, maybe a single random error behaves to be in one direction like the systematic error, but if all the random errors are put together, the behaviour is as we have seen in the normal distribution curve - towards the negative side equally, and towards the positive side equally. The likelihood of large errors occurring is very less. And the likelihood of smaller errors - smaller random errors - occurring is very more. So, this is what we have seen. And we saw that the random errors are distributed like that.

Once we know this, what to do of that? Well, we try to make use of this, because we know it has been also observed that the observations which you take in field do always - or mostly, I will say, unless there is some bias - behave in this normal distribution. So, what we will do, we will start making use of this concept, this theory, because we know our observations are normally distributed; they follow a mathematical law - so we will start making use of that. Well, so this is what we saw in the use of normal distribution curve. While we are talking about this use of normal distribution curve, we talked about various things. One important thing was 'standard normal curve', because in order to determine anything within the normal curve, for any variable x, we cannot determine directly. So what we need to do, we need to make use of the standard normal curve for which we know the probability; cumulative probability density function or, that is also called distribution function. So, making use of standard normal curve, we can determine the probabilities of a variable occurring between any values - any two values - as we have seen between x_1 and x_2. What are the probabilities that our variable X occurs between two values, x_1 and x_2. So, for these kind of answers, we can make use of the standard normal curve.

Then, we are looking into some indices of precision - 'precision' means the closeness of observation. We saw, using some examples also, that there may be some observations for which the spread of the observations may be big, while there are some observations in which the spread is very small. So the spread, when it is small we say highly precise observation. So, how to quantify it? We required a method to quantify, so what we did, we made use of standard deviation, because once we form a table of data - the observations which have been taken - once we compute the standard deviation, standard deviation tells us the spread of the data; that what is the deviation from mean value where 68.3 percent of the data do occur. If the standard deviation is large, we say the observations are less precise. If the standard deviation is small - the deviation of observations for the mean value is less - so the observations are more precise. But just standard deviation does not help, because we are not talking about how many observations are there. When we want to be ensured about the confidence, we want to associate to a mean computed from a set of observations which also includes how many observations were there. So, to make use of this, we made use of standard error.

Well, next, we looked in to the weighted observations - this is very important. Important, in the sense that the reliability of the observations there in the field is not same. We saw many cases, many examples, why the reliability is not same. If the reliability is not same, should we give them same treatment, same values? When we are deriving any result - we have some results from the field - we want to make a prediction; we want to, you know, plan a road using those results. Should we keep same weightage to all those observations? No; the answer was ‘no’ - we discussed it. Then, how to quantify the weight? We saw some methods: number one, your personal judgement. You know, this particular observation has got less likelihood to be more reliable. Then, number of observations - if a particular quantity has beeen measured many times, the chances are, the errors are less in that. So, that is the number of observations. The weight is proportional to number of observations ‘n’. Then the other thing: precision. We know precision is an indicator of the accuracy, provided we eliminate systematic errors. So precision becomes an indicator of accuracy; if the precision of the data is good, we say our observations have got higher weight; they are better. So what we did? We did, the weight was inversely proportional to the variance, and variance is an indicator of the precision. Then, what we saw, we saw some measures of precision also, for the weighted data. So, this is all what we have done so far in our last lecture.

What we will do today in this extra lecture - the fifth lecture in module 2 - we are going to use the standard normal curve. I will solve a little problem so that the way in which we make use of standard normal curve becomes clear to us. Then we will talk about one very interesting area, which is the ‘outliers’ in observations, and how to detect the outliers by a quantitative method. Then, we will see another interesting area, which is the ‘error propagation’. So, we will talk about all these things today. So today, we will start with one little problem, one little numerical.

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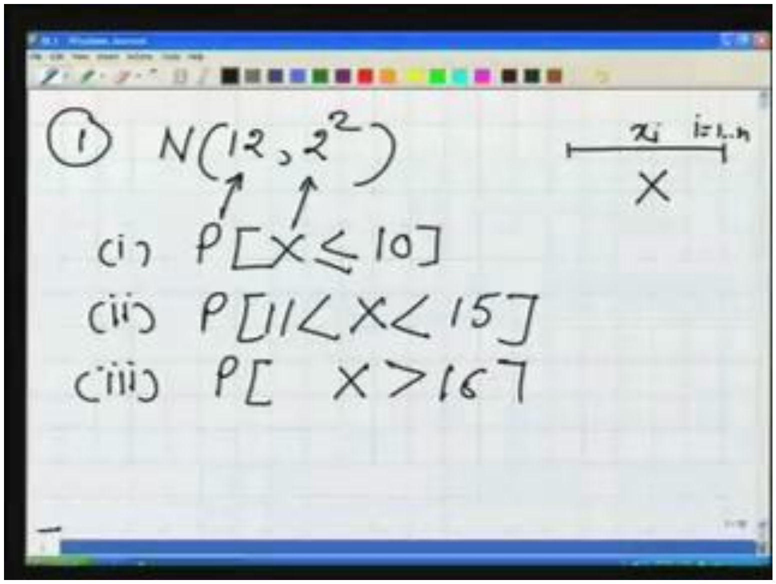

The problem is: we are given - for example, for a length - a set of data (Refer Slide Time 08:23). The data means, this length x was measured many times, 'i' is 1 to n (Refer Slide Time 08:29), and from the observations, the outliers have been eliminated, as well as the systematic errors have been eliminated. So, in our observations, only the random errors are there, and they follow the normal distribution. So, the data is ((by it)) (08:53) 12, 2^2 (Refer Slide Time 08:49) - what is the meaning of this? The meaning is, the mean of the data is 12, and the standard deviation of the data is 2. Well, the answer that we have to provide - we have to find the probability that our variable X - that is our variable X (Refer Slide Time 09:22) - X will have a value, is less than or equal to 10 (Refer Slide Time 09:27). Also - number two - we have to find that the probability of X being more than and less than 11 and 15 (Refer Slide Time 09:36). Also - the third one - because the treatment in this third case is slightly different, so this is why I am giving this third one case also. We have to find the probability that our X is more than 16 (Refer Slide Time 10:02). So, what we have been given? We have been given a data, and the data is found to be normally distributed with these parameters. Now, how to proceed in this? Definitely, we make use of the standard normal distribution, but how to proceed?

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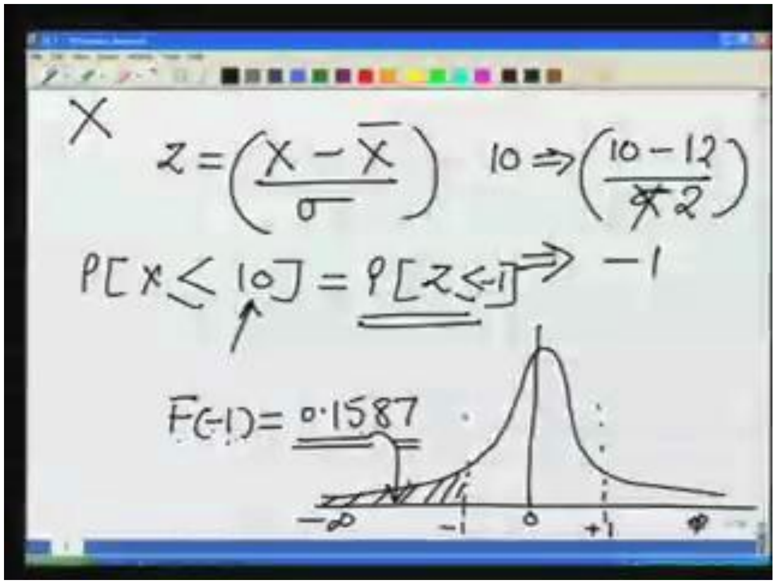

Well, to proceed in this, first of all, the value X which was given to us, we will transform it into the standard variable Z, and the standard variable Z, in this case, becomes 'X minus X bar by sigma' (Refer Slide Time 10:37). So, this is - we are transforming our variable; we are making it standard, and using this, we can make our variable standard - we have seen this before also. Well, in the first case - case number 1 - probability is less than or equal to 10; of X being less than or equal to 10 (Refer Slide Time 11:03). We can write it as - if we convert our variable now; transform our variable - so what is this value? If you transform this value, so 10 will be equivalent to, X - is 10 - minus X bar - as you can see here (Refer Slide Time 11:31), the mean is 12 - divided by sigma, which is 2; this value is 2. So, we can write, the equivalent value is minus 1 (Refer Slide Time 11:44). So, what we can write this as? This probability is same as - in the case of our standard curve - Z being less than or equal to minus 1 (Refer Slide Time 11:59). Now, what is this, 'Z being less or than equal to minus 1'? If I draw a standard normal curve - this is how it will be (Refer Slide Time 12:08) - from minus infinite to plus infinite, and the value - the mean - here is 0, the standard deviation is minus 1 and plus 1 here, so plus -minus one is the standard deviation. This is what is the standard normal curve is. What we are looking for, we are looking for the probability that the Z is less than or equal to minus 1 (Refer Slide Time 12:43). So, you can see here, that is equal to the area shown by the shaded region (Refer Slide Time 12:49). Now, how we find this? This area, as we have seen, is equal to - in terms of the distribution function - Z minus 1 (Refer Slide Time 12:56). What is the value of the distribution function for minus 1? That is what we need, and for that, we make - we need to make use of the table.

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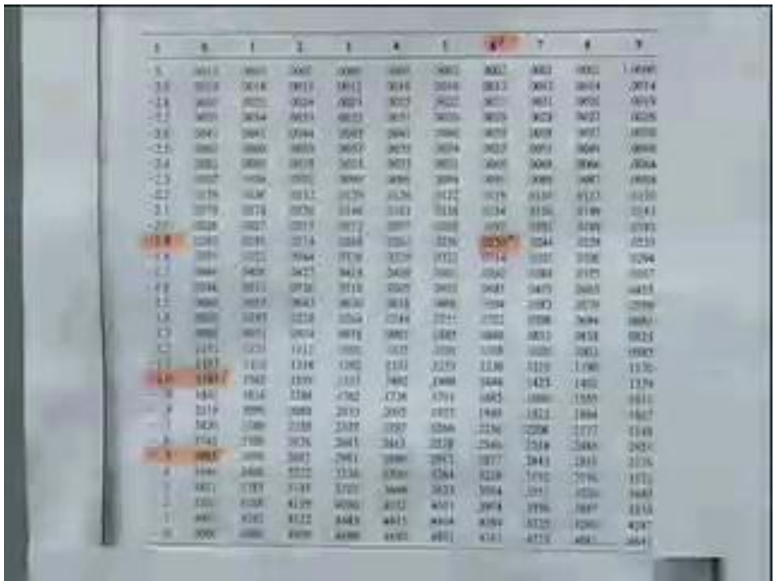

So, we have to find the values from the table, and here is the table. This is the table for standard normal distribution function. Now, in this table, there are the values of Z for which we are writing the distribution function f(Z). And what is the value of that? So we see, along this, it starts from minus 3 here (Refer Slide Time 13:40), in this table, though it should start from minus infinite to plus infinite. So, minus 3 - it will go up to plus 3, and over here, if you follow, here is the highlighted one, minus 1.0 (Refer Slide Time 13:55). We need a value of X minus 1.0, so what we see, in the corresponding value here, the value is 0.1587. So this is the value - 0.1587 is the value of X minus 1.0. So, as we have seen from the table, we are writing here now. The value of the distribution function for standard normal curve, for corresponding to minus 1, is 0.1587. What is the meaning of this? The meaning of this is, the probability - as it comes from here (Refer Slide Time 14:52) - the probability that Z will have a value less than or equal to minus 1 - this area (Refer Slide Time 15:00) - is this value. So, we can say again that the probability our original value X - the probability that X is less than or equal to ten is 0.1587. So, this is how we make use of the normal curve in order to determine the values.

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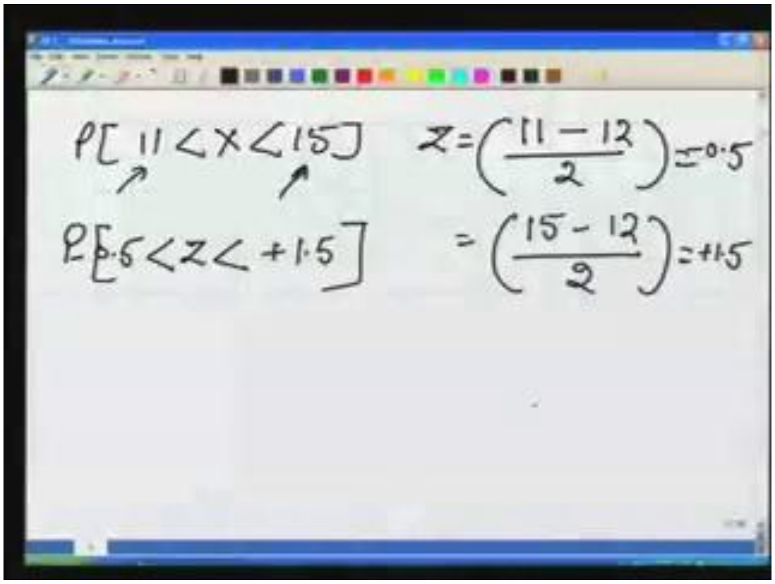

Now, what we will do, we will go back to the problem number two (Refer Slide Time 15:26). It is slightly different; slightly different in the sense that we have to find for this X, which is more than 11 and less than 15. Well, we will try to go very fast here: 11… X less than 15 (Refer Slide Time 15:42). What it is equivalent to? What we will do, first we will transfer - transform these values into their corresponding standard variable or standard values. How do we do that? We have seen that already, as you saw here (Refer Slide Time 16:00) : ‘X minus X bar divided by sigma’. The value of X here is 11 minus 12 by 2, and in the other case, that is 15 minus 12 by 2 (Refer Slide Time 16:11). So, these corresponding values are, as here, minus 0.5 and here, plus 1.5. So, what we are looking for? This particular probability is same as probability that our standard normal variable - it is more than 0.5 - minus, of course - minus 0.5, and less than plus 1.5 (Refer Slide Time 16:37).

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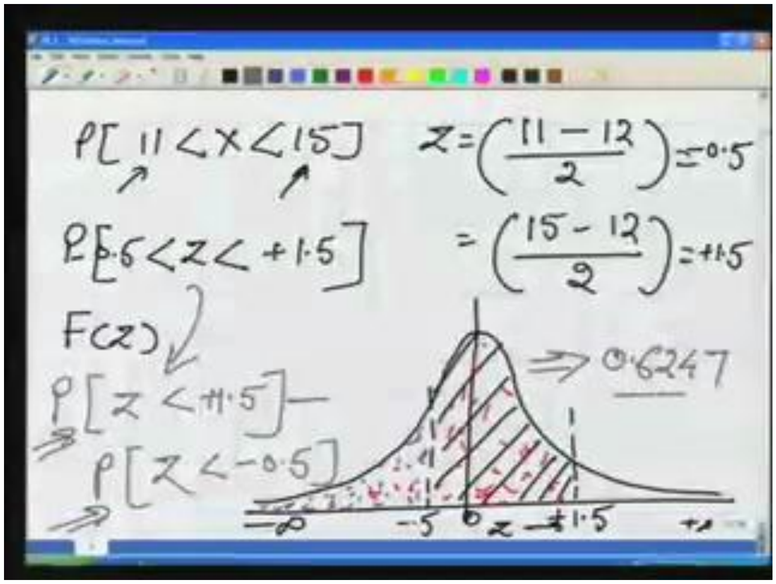

So, what we need to do, we need to find from our standard normal curve, this particular value. Well, what I will do, I will draw this curve for you again - that is zero, this is standard normal curve, minus infinite to plus infinite, and that is the 'Z' - our variable (Refer Slide Time 17:05). Now, what we are looking for, we are looking for the area of this curve within minus 0.5, which may be somewhere here - minus 0.5 and plus 1.5 (Refer Slide Time 17:29) -plus 1.5. So, we are interested in this shaded area (Refer Slide Time 17:43). How do we find it; how do we find it by making use of the table that we have just seen? In the table, the values that are given to you are only F(Z). Well, what we will do, in order to make use of this, we can write this (Refer Slide Time 18:08) as probability that this shaded area is the total area. The total area, if I mark by red dots (Refer Slide Time 18:21) - that is the total area - minus - if I mark the area here by green dots (Refer Slide Time 18:35). So, what these areas are? These areas are: the area which is marked by the red dots - that is, Z is less than plus 1.5 minus, probability that Z is less than minus 0.5 (Refer Slide Time 18:09). So, what we need to do, we need to now collect from our table the corresponding values, and then where we will do it is the same; we will go back to our table and we will find these answers. Well, I am not looking into the table now; I am leaving it to you to go to the table and find the answer. The answer in this case, of course, when you find the distances, will be 0.6247. So, please check your answer also - do it yourself.

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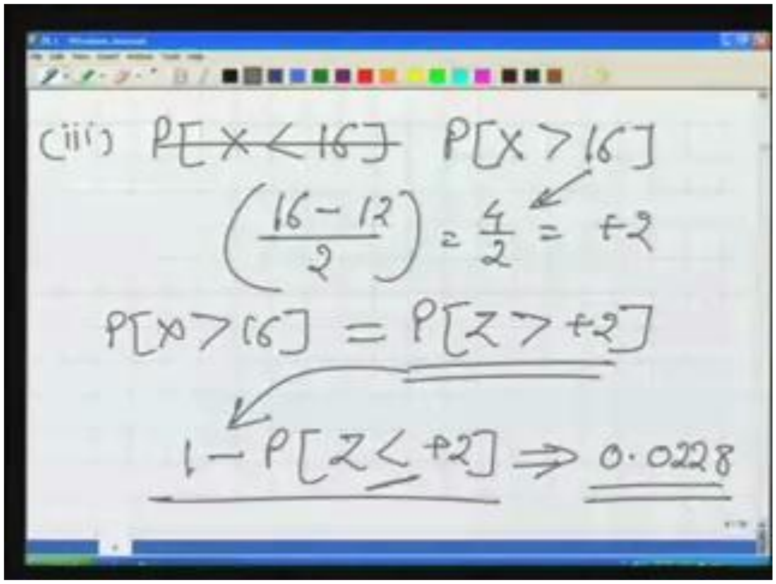

Well, now, we will go for the third part. It is slightly different; we have to handle it in a different way, so we should know how to handle this also. Well, the third part is, we have to find the probability that our X the variable is less than 16 - I am sorry, more than 16. X is more than 16. Now, how do you find it? Initially, we will find the corresponding values of the 16, and that will be 16 minus 12 divided by 2 -as the usual case - so 4 by 2, plus 2 (Refer Slide Time 20:33). So, that is the standard normal value corresponding to the 16. Well, the probability that X is more than 16 - it is same as probability that our standard normal variable is more than plus 2 (Refer Slide Time 20:45).

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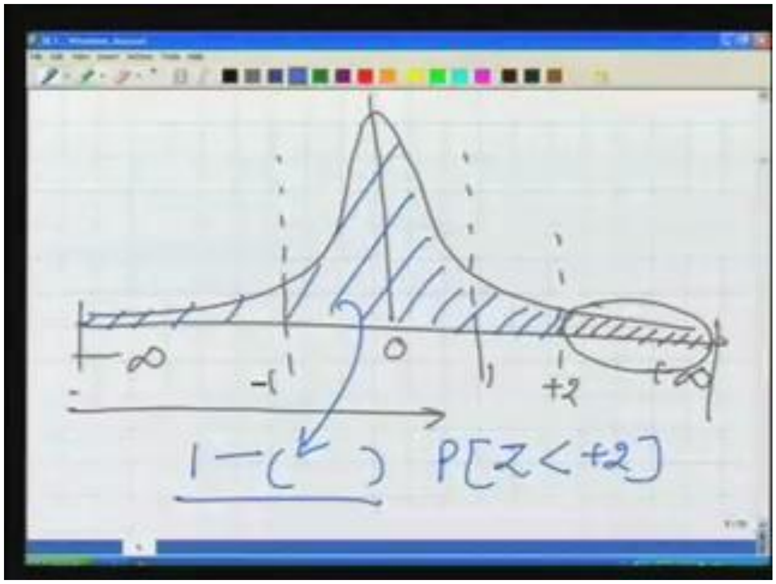

Now, what is the meaning of this? If you look into the standard normal curve - I will draw the curve again for you - this is the standard normal curve - minus infinite to plus infinite; this value is zero; if this is the standard deviation, it is 1 and 1 (Refer Slide Time 21:11). We are interested in Z more than being plus 2. So, plus 2 is somewhere here so which area we are interested in we are interested in finding this area (Refer Slide Time 21:33). The tables, they give you the values starting from here (Refer Slide Time 21:46) - the way your variable increases. So, how to find it? Well, this (Refer Slide Time 21:56), you can write it is same as - as you can see here (Refer Slide Time 22:02) - this area (Refer Slide Time 22:07) is same as the total area, which is 1 minus this white area here - if I do blue line, so shaded by blue colour - so, 1 minus the area which is shaded by blue colour (Refer Slide Time 22:18). What is that? The area which is shaded by blue colour is probability that Z is less than plus 2 (Refer Slide Time 22:36). So, what we are looking for? We are looking for - this is same as - 1 minus probability that Z is less than plus 2 (Refer Slide Time 22:48). So, we will now find, from the table, the value corresponding to this (Refer Slide Time 23:01), and this value, from the table, you please determine it and find the answer. The answer for this problem should be - I am going to write this one - so, the answer for this is 0.0228. So, you try to find this answer using this table - you have seen use of the table. One more thing here, because we are writing here Z is more than plus 2, so we should write here less than or equal to (Refer Slide Time 23:29), so that mathematically, this is okay. And, please find this answer. So, this gives us how we can use normal curve or the standard normal curve in order to answer the questions like these.

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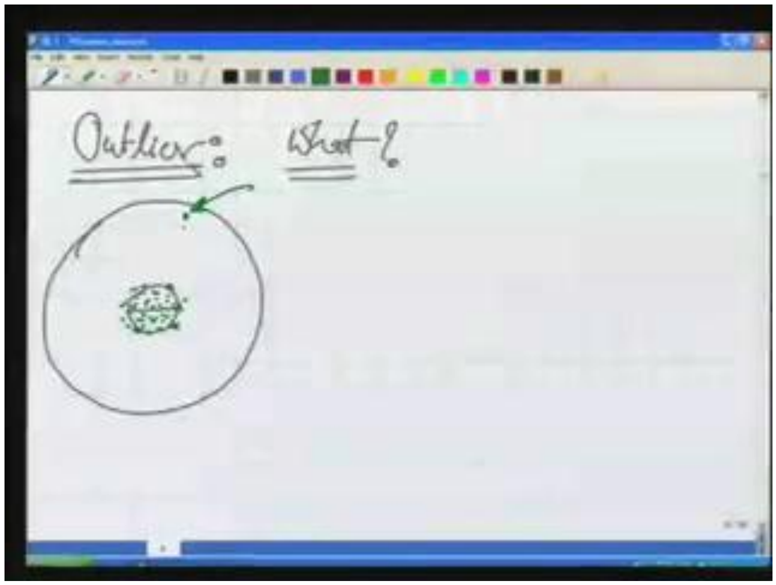

Well, having said all this, now we will go further in our discussion. We are going to talk about a very interesting area today which is called ‘outlier’. So first, we will talk about why and what are outliers. You remember that old example where - this is the bulls eye and our target (Refer Slide Time 24:13), and you are firing in the target. So, while you are firing in the target - you are a very good fireman, you are using a very good rifle - most of your bullets are in this area (Refer Slide Time 24:29); very precise. But somehow, one of the bullets is here also (Refer Slide Time 24:42). Now, your firing instructor will say that you are very good in your firing; how come you fired a bullet here -the moment he looks at your target. You yourself - the moment you look at your target, you yourself will feel, ‘How come I could fire such a wrong?’ though mostly, your observations are here. So what it is, it is definitely because of some problem - something went wrong. While you are firing, maybe you look somewhere else, something went wrong, and this is why you have a value here.

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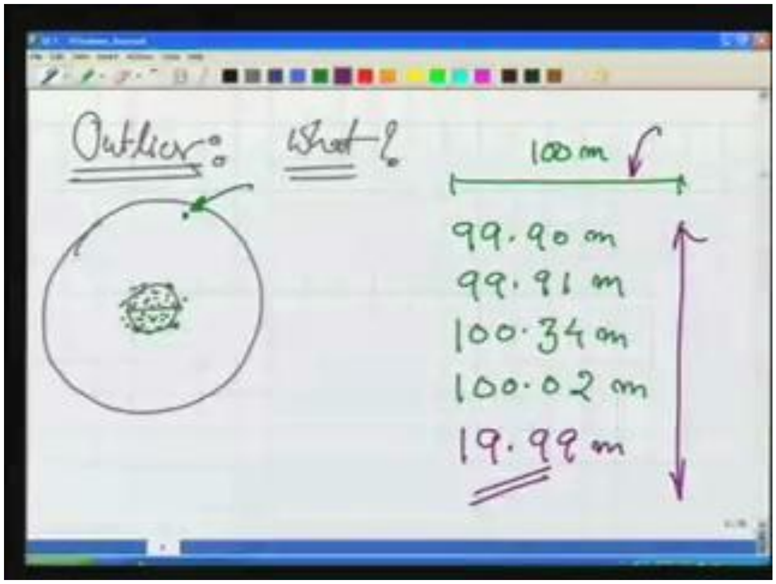

So, same thing happens - for example, let us say our line; you want to measure this line, and here in this line - this line is 100 metre (Refer Slide Time 25:25). Well, we are taking the observations, so the observations may be 99.90 metre, 99.91 metre, 100.34 metre, 100.02 metre - similarly, you will have many observations. If you end up with an observation which is, let us say, 19.99 metre - so, what you are doing? Just by looking at your observations, you are able to say that this observation - though it is recorded in your notebook - it does not belong to this set. Now, how this observation came in to this set of observations? Maybe you are making measurements of this length while your friend is recording the readings. Somehow, he misses something - no, he misses writing 99.99, rather, he writes 19.99. It is a kind of blunder, a mistake. So, this kind of observation, we say, also, outlier. If there are outliers in our observations, we need to reject them; we need to eliminate them so that our observations are more clearer.

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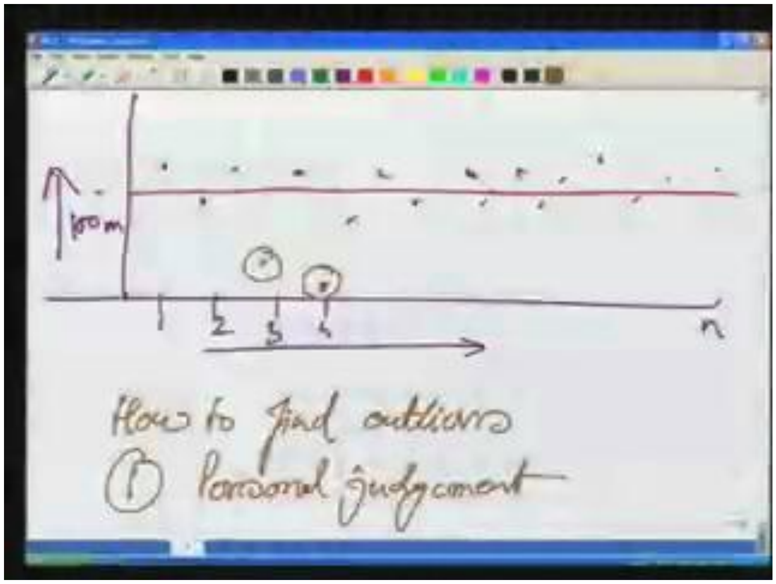

I will give you another example here, that how you can see the outlier. For the same length - if I plot the length here - let us say, along this ordinate we have the values, and 100 metre is around here (Refer Slide Time 27:18). Well, the values which you are taking - and these are the observations - the observation number 1, 2, 3, 4, and so on, up to n (Refer Slide Time 27:24). These observations will be plotted something like this (Refer Slide Time 27:34), because they will be around the value of 100; the true value of 100 - they will be here and there around the value. So, this is actually - the spread of the point, we have seen, is the precision. Now, let us say one of your observations is lying here, it is lying here (Refer Slide Time 28:05) - then we suspect, 'No, it cannot belong to this set of observations'. So obviously, this is going to be the outlier (Refer Slide Time 28:13). Another question: how to find outliers; how to determine them; how to locate them? Well, the method number one should be your personal judgement, as in the previous case. Looking at these observations (Refer Slide Time 28:42), you can straight away, right away, you can delete this set of - this observation. You know it cannot belong to my set of observations; it has to be an outlier. So, this is a personal judgement which we are doing.

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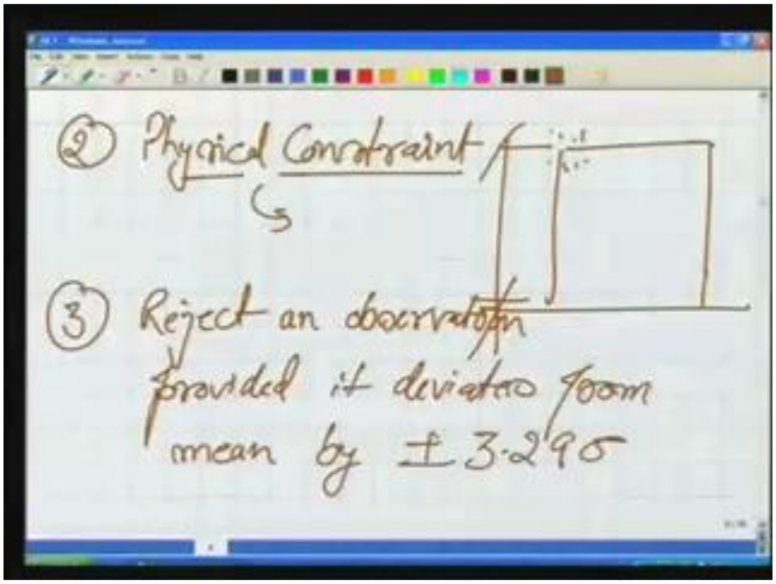

Many times, the method number two could be the physical constraint. Let us say you are measuring the height of a building (Refer Slide Time 29:13). You are measuring the height of the building, and the height of the building - you know it cannot be more than a certain value, in no case. So, if a height is needed like that, so most of the heights you have measured will be here and there of the actual height (Refer Slide Time 29:29), but if there is a value which is physically not possible - you know, physically, there is no buildings in the town which are more than this height - you can easily eliminate those observations. Physically, it is not possible to have those values. Well, the method number three: because these first two methods are based on your intuition - our intuition is really not very, very helpful, because we know we are trying to make some subjective judgement about the observations. So, the third method - it says you reject an observation, provided it deviates from 'mean by plus-minus 3.29 sigma (Refer Slide Time 30:11). Now, we explain the meaning of this. What is the meaning of this, what I am writing here? What do we mean, and why it is so? We will try to explain this.

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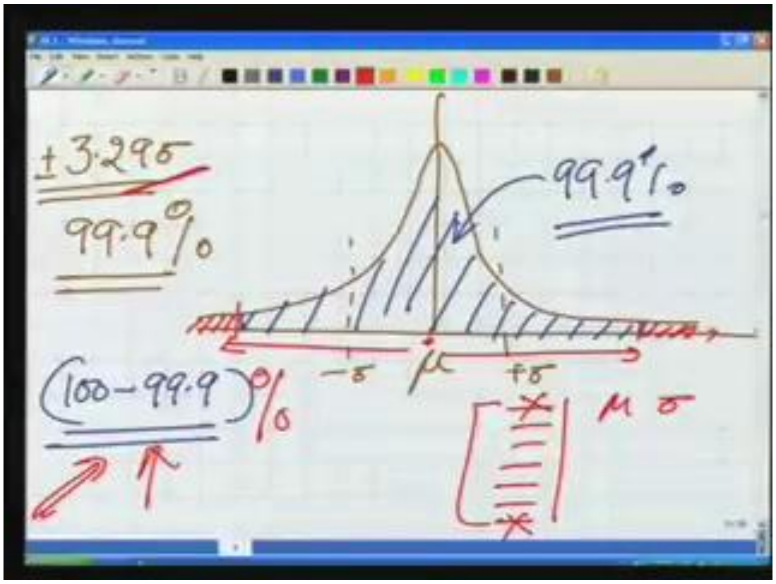

Why 3.29 sigma? Well, if you look at the normal distribution curve - the normal distribution curve is like this (Refer Slide Time 31:00), where this is the mean, and we will have the standard deviation from here - minus sigma and plus sigma. Now, this 3.29 sigma - plus-minus - this corresponds to 99.9 percent of area of this standard normal curve. What is the meaning of this? The meaning is, if I draw it with a different colour here (Refer Slide Time 31:41), this entire shaded part of the curve will have 99.9 percent of the data. Now, the rest of the data - 100 minus 99.9 - this data will stay outside, here (Refer Slide Time 32:12), because this is now a very feeble possibility, this percentage. The occurrence of the data which is here is of less probability. So, what this will say - if the data deviates from the mean by more than 3.29 sigma - that is it lies - it has the probability, very small probability - is this value only, then reject it. So for our set of observations, if these are the observations, what we do, we find the mean, we find the sigma, and we find if at all there are any data which are more than - which deviate more than - this value. We outright reject them.

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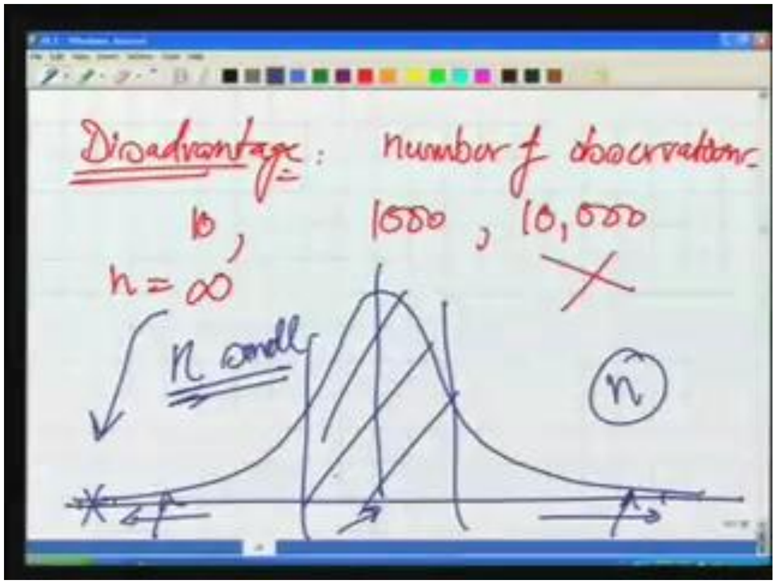

Now, there is one disadvantage of this method - what is the disadvantage? The disadvantage is, we are not considering number of observations. There may be 10 observations in a set of data, there are 1000 observations in a set of data, there are, for example, 10,000 observations in a set of data, and our treatment is same in all the cases. This is wrong; we should not do it. Now, I will explain why. For example, let us say n is equal to infinite. Now, for n equal to infinite in our normal curve (Refer Slide Time 34:03). Because n equal to infinite means we have got infinite observations, so even if the values are here, even if this value, this value, this value - however small the probability is, it will occur. It will definitely occur when we are taking the observations. Also, on this side, however small the probability of these values are, they will occur, because there are infinite number of observations. So, if the infinite number of observations are there, we cannot reject these values, because these values will occur, however big they are. Now, if n is very, very small - what happens if the n is small? If n is small, as we know, in our normal distribution curve also, the way our random errors behave - we know it that the probability of occurrence of values which are nearer to the mean is more, is higher, than those values which are on these sides (Refer Slide Time 35:21). We know it - this is how the random errors behave. So if our n is small, we are more sure about the values which are on somewhere in the middle part (Refer Slide Time 35:35) than on these parts (Refer

Slide Time 35:39). So, there is a rule, and that rule is based on number of observations, and which is based on the concept what we have just discussed.

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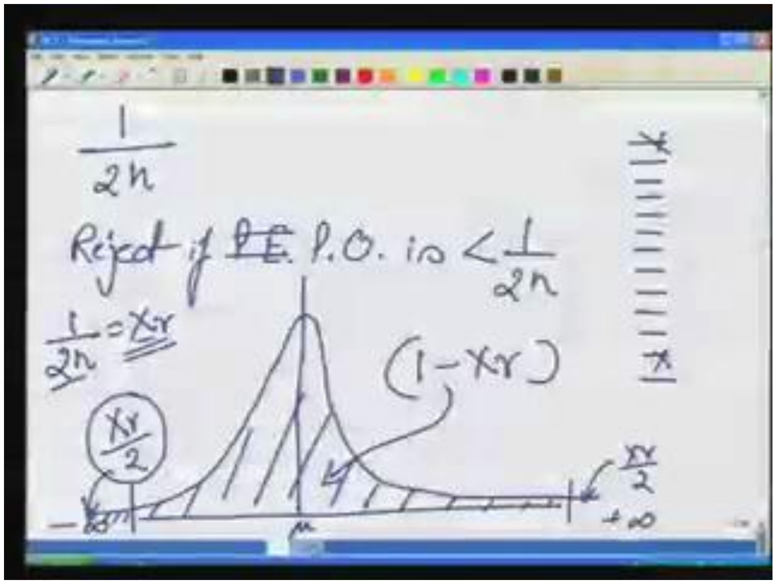

What that rule is? The rule is, you reject an observation - because what do we have, we have a set of observations - we may have 10, 100, you know, as many as number of values - we have a set of observations. You want to reject some of these, because we said they are the outliers. How do we say that they are the outliers? So, the rule in this case is, reject an observation provided if probability of occurrence is less than 1 by 2n (Refer Slide Time 36:28) - I will explain it. It says, reject if probability of occurrence - sorry - probability of occurrence is less than 1 by 2n. What is the meaning of this? We will try to see this in, again, the normal curve. I am drawing the normal curve again - this is important and we must understand this, what we are doing. From minus infinite to plus infinite and that is the mean (Refer Slide Time 37:08). Let us say 1 by 2n is a value 'X_r' - the value of the variable, X_r is the value of the variable - it is Xr. Now, we are saying we have to reject an observation if its probability of occurrence is less than this. So, we are computing 1 by 2n - let us say 1 by 2n is there (Refer Slide Time 37:46). Now, if you find '1 minus X_r', what it will be? What I am doing, this side is an area which is X_r by 2 (Refer Slide Time 37:59). Do not confuse why Xr is the variable at the moment; this is an area ‘X_r by 2’, and here is also the area ‘X_r by 2’ (Refer Slide Time 38:15), while the area in between, which is shaded, is corresponding to here. Now, what we are saying, we are saying we will try to reject those values which have the probability of occurrence which is less than ‘X_r by 2’.

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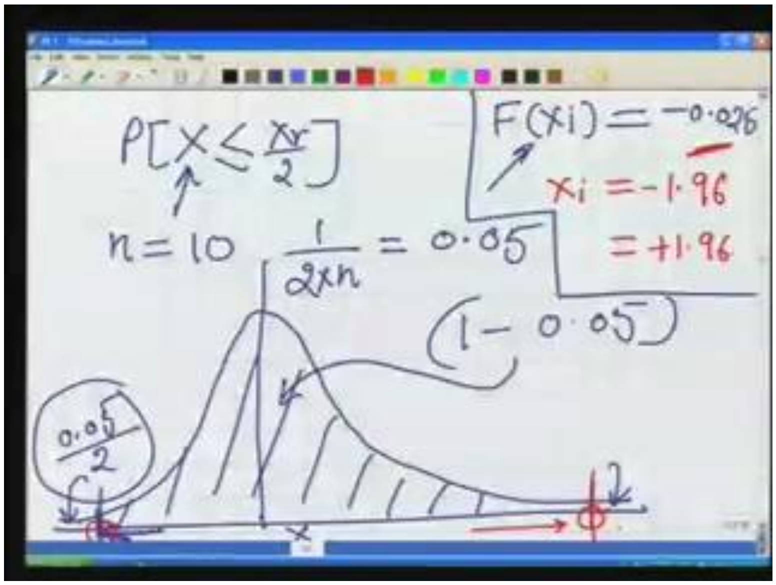

If I write it the other way round, it means we are looking for the variable X so that probability of occurrence is less than ‘X_r by 2’. All those variables, all those values, we will remove. So, the same thing will be on the other side also - here (Refer Slide Time 39:06). Well, what we will do, we will try to do it by an example. Let us say n, in our case, is 10. So ‘1 by 2 into n’ - 1 by 2n - that is corresponding to 0.05. Now, what we are looking for? Again, I will go back to our curve. We are going to reject observations which are on the tails of the curve, and having a probability of occurrence - all those variables having probability of occurrence less than this value (Refer Slide Time 40:03). So now, in this case, what is the meaning of this, and why we are doing it, actually? We are doing it because the area in between here (Refer Slide Time 40:17), which is 1 minus 0.05 - we assume all these values are closest to the mean - obviously - this is the area which is closest to the mean and the area which are on the tails, we want to reject it as per the rule. So, in order to reject it, how do we find this value? Again, we will make use of the normal curve. So, in this case, what we will do, we will look in this normal curve. If I write here (Refer Slide Time 40:53), for F - let us say the variable here is X - so we are looking for a value $F(X_i)$ for which the value is minus 0.025 - corresponding to this value here. So, what is the meaning? All those variables - because what does this say; you have seen this before also - all those variables for which the probability of occurrence is less than this value here. You want to know this value, actually - the idea is, we want to determine this value, what this value is. That is what our concern is, because in order to reject the observations, we want to know what this value is, and as well as what this value is (Refer Slide Time 41:46). If I know this value in terms of my sigma, so I know this deviates from my mean by this much, and I reject all those values which are larger than this, and here, which are smaller than this. So I am interested in knowing this particular value and this particular value here (Refer Slide Time 42:05), and our rule says that below this value, the probability of occurrence of all the variables should be this (Refer Slide Time 42:17). Why I am doing half, because half is on this side and half is on this side - our outlier could be on negative side, could be on the positive side. Now the corresponding value of X_i, if we go back to our table - you can make use of the standard normal curve table - and this value is 1.96. Over here, it is minus 1.96, and over here, it is plus 1.96.

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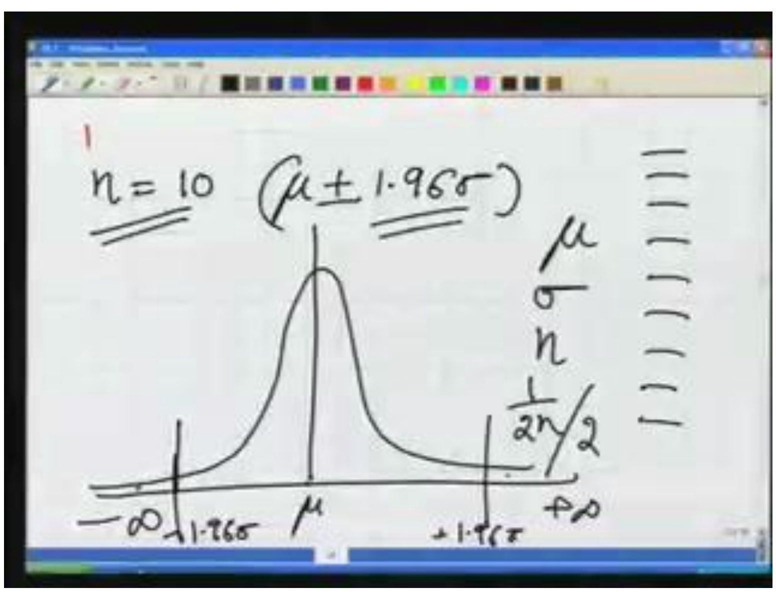

So, what does the rule say? Now, if n is equal to 10 as in this case, our rule says, from our set of observations we reject all those observations which has variation -if I am writing mean by this plus-minus 1.96 sigma. In our standard - in our normal curve, the normal curve, that is our mean (Refer Slide Time 43:40); corresponding to 10 observations, the rejection criteria is this (Refer Slide Time 43:45). That means, we remove the observations which are beyond 1.96 sigma. So, how to attack this problem? If we have got the data - we have this data - how to answer this question? Well, first of all, we find the mean of the data. Let us say I am writing the mean again as 'mu' (Refer Slide Time 44:06), so you can find the mean - no problem about that; also, we can find the sigma - you know how to find this standard deviation. Then, how many observations are there? You know it - the value of n. Once you know the value of n, you find 1 by 2n. Again, whatever is 1 by 2n, half of that - half of that, because it is on this side as well as this side - so that is the value of our $F(X_i)$. $F(X_i)$, and corresponding to this value, we want to determine the value of X_i from the standard normal table.

And once we have found the values, as in this case, we have found the values to be 1.96 - minus here, plus here - so in your set of observations now, you can find - now you can find 1.96 sigma, whatever it is, you know it. So start looking - are there any values which are less than this value 'mu minus 1.96 sigma' or more than 'mu plus 1.96 sigma'. If it is so, reject them. The only thing is, we will do this exercise only once, not in iteration; only once we do it with these observations, and that is it.

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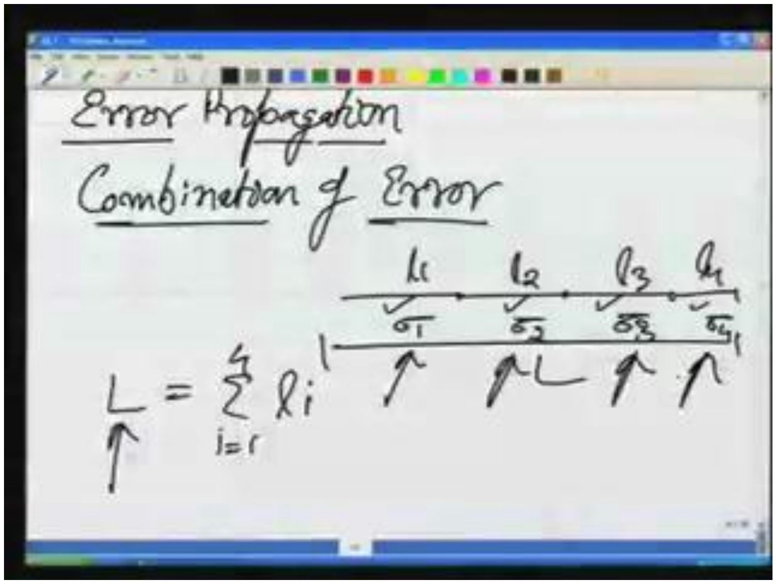

Having discussed this, the last part of it - that is about ‘error propagation’. Also, this is said ‘combination of errors’. Now what is this many times? We are taking the measurement, let us say, one length, another length, another length, another length - l_1, l_2, l_3 and l_4 (Refer Slide Time 46:15). Your job was to find this full length. So what you do, you find this full length by sum of these individual lengths (Refer Slide Time 46:25). So what you did? There, in the field, it is not possible to measure the entire length in one go. So, what you did, you measured the length in parts, and in each of these, there are sigma 1, sigma 2, sigma 3 and sigma 4 kinds of errors (Refer Slide Time 46:42). What we are interested in, when we are talking, finally, we are interested in what will be the error in L, while we know the standard deviations of these individual ones, or what is the variance of this final computed value (Refer Slide Time 47:04).

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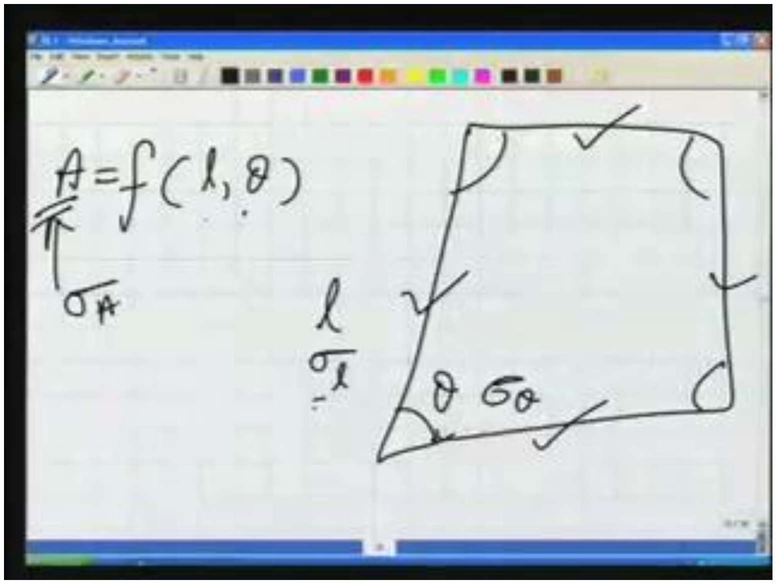

Another example: you have a field. You find these angles (Refer Slide Time 47:13), as well as these lengths (Refer Slide Time 47:17) - all these are measured. While you are measuring any length, you also know the corresponding standard deviation, or we can say, the variance. Similarly, for all the angles also - you know the value of the angles, as well as, for that angle which you have measured, what is the standard deviation. Now the area of this land parcel is a function of ‘l’ and ‘theta’ (Refer Slide Time 47:43). In l and theta, for all these lengths and for all these angles, we know their individual standard deviation. What we are interested in? We are interested in the area which we are finding - how confident we are about that area. Because, what is the standard deviation? Standard deviation is a kind of measure of confidence - confidence in our result; the precision. So, we are interested here, how confident we are about the A. For that we should know the standard deviation of A.

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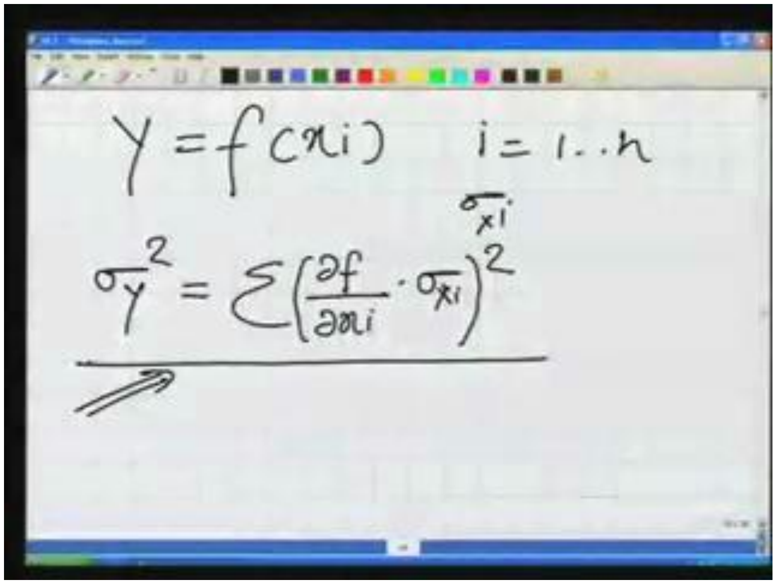

So, a very simple rule is there, and this rule says, for any function, if I write it now, in general form, Y is f(x_i), where i varies from 1 to n (Refer Slide Time 48:32). The meaning is, we are using n number of variables in order to arrive at Y through some manipulation of these variables, and for each variable, we know its corresponding standard deviation. So, in that case, the standard deviation of our final output y - I am not going to the derivation of this, because this is beyond the scope of this video lecture, but you can see in any good book the derivation also. So, this is given by, as we write here, sigma, partial derivative of f, with x_i multiplied by the standard deviation of that - if we are writing it as x_i here - of that variable, and square of these, and here also it is square (Refer Slide Time 49:14). so this is how it is given. So, what you can do now, you can make use of this and compute the standard deviation of your final value. So, we will give a little example over here, and we will see something very interesting out of that example when we make use of this.

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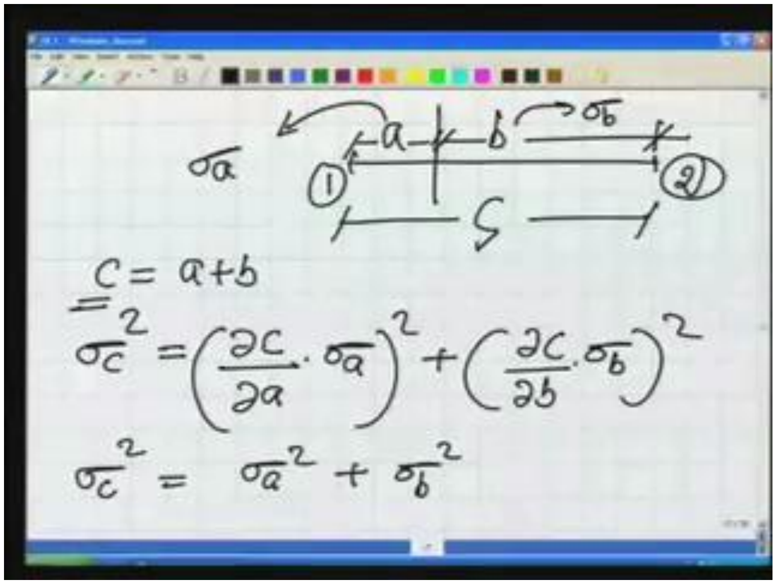

The example is, let us say there is a length C, and this length was measured in two parts, a and b (Refer Slide Time 50:09). This is what we do very often in surveying - we have to do it. Many times, we will measure the individual things and then finally, come out with the answer. A simple example is, you know, area, or maybe any complex example. You want to measure the tilt of a tower. The tilt of the tower will not be given to you directly by any instrument, rather, what you will do, you will take couple of measurements. Maybe you are measuring some angles, some lengths - there on the ground, in the vertical - and all these values, all these measurements which you have taken, they will pass through a trigonometric relationship. And once they pass through that trigonometric relationship, you will find your answer; the tilt of the tower. So you want to be sure about that tilt of the tower; that the answer that you have given - how confident we are about that. What is the precision of that answer; what is the standard deviation there? So, it is a very common problem, so we should always - whenever we are computing, whenever we are doing any kind of computation - we should know during these things, for our formulation for this tilt of tower, we should know how to arrive at the standard deviation of the final value; you should be able to compute that. Well, a little example here - the example is, we want to measure C, we want to determine the length between point - here, number one (Refer Slide Time 51:48), and point two, which we did by measuring a and

b. So, the length will be, straight away, written as ‘a plus b’ (Refer Slide Time 51:56). So, a length is given to you, but that is not enough. ‘a’ has a standard deviation of ‘sigma a’. Similarly, b has standard deviation of ‘sigma b’. How do we know these standard deviations? For example, this b was measured, let us say, 10 number of times. You have 10 observations for b; you can compute b, you can compute the standard deviation. Similarly, for ‘a’, well, we know the length, but we do not know what is the confidence here. So what we try to do, we try to find ‘sigma c’, which we have seen as - will be, now - partial derivative of c, with ‘a’ into - square of this - standard deviation of ‘a’, square, plus partial derivative of c with b, into sigma b (Refer Slide Time 52:44). Now, if you compute this value - ‘sigma c square’ - now, standard deviation of ‘c’ with ‘a’ will be 1, so that becomes ‘sigma a square’. Similarly, here, ‘sigma b square’ (Refer Slide Time 53:17).

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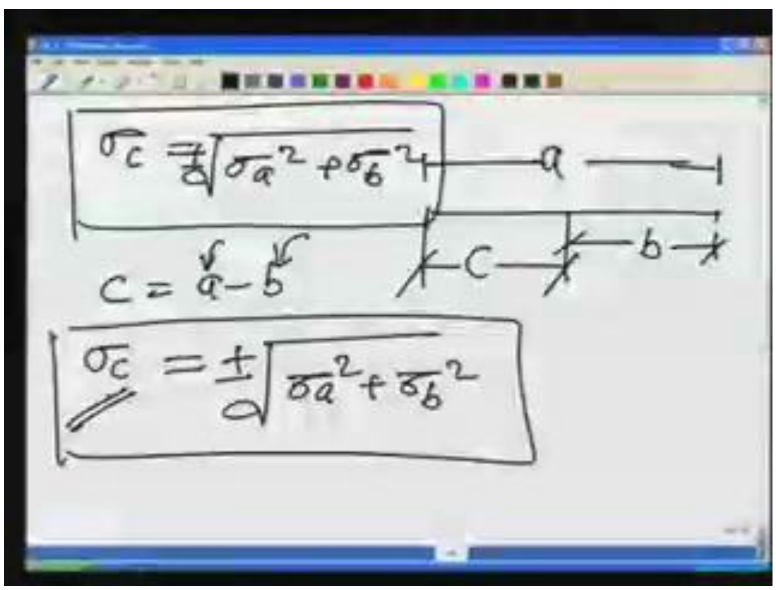

Well, so, by that, we can write - we will write in this next slide - ‘sigma c’ is the root of ‘sigma a square’ plus ‘sigma b square’ (Refer Slide Time 53:33) - the answer is there. Of course, we must write this plus-minus (Refer Slide Time 53:44). One more important thing: we saw here, in the previous case, that the sum ‘c’ is ‘a plus b’. Let us say we have a different thing - the different thing is, we are interested in finding our ‘c’ - this is this length (Refer Slide Time 53:56), while the thing that we measure is ‘a’ - the full length - and as well as we measure ‘b’ here (Refer Slide Time 54:04). That is also possible; you can arrive at ‘c’. So ‘c’ becomes, in this case, ‘a minus b’. Now, can you think of what will happen to ‘sigma c’? Actually, if you do the derivation, you will find ‘sigma c’ will again come out to be ‘sigma a square’ plus ‘sigma b square’ - you must do it. Now, what do we observe from this? In both the cases, whether we have the sum like this, ‘a plus b’, or ‘a minus b’, our value of the standard deviation for final output is same - obviously. What is happening? There is some error which is being introduced by ‘a’ variable, some errors being introduced by ‘b’ variable - it does not matter whether these variables are subtracting or adding. It does not matter, because our errors are being added; our errors are accumulating. So, final error will be more. That is why, in both the cases, it is so.

So, what you can do, you can solve many more numericals, many more examples on this line. I am not taking - I am not solving the examples here, because the time is the limitation. But you can make use of the books - all the books which you have referred to - I have given you many books. Go through those books - you will find many examples - and solve them. That will give you a confidence in order to arrive at the answer. So, what we have seen so far - today and as well as in this module 5 - because we are going to end the module 5 today. We started with, you know, what the surveying is - the basic surveying. We saw many examples. Then, we saw about the, you know, the scaling thing, the plotting accuracy - many principles - whole to part, reconnaissance - we have seen all those things. Then, we talked about the measurements; the basic thing. It will be affected by the error; we must start treating the errors. Outliers - well, we have seen we can detect them. Systematic error - we will see it later also. If we have any systematic error, you know the physical rule behind it; you can eliminate that also. Now, the errors which cannot be eliminated are the random errors. But we have to still treat for them. We should know what kind of observations are there with us. So, whenever we are writing an observation or a measurement -we are reporting it - we should report it as ‘l’ plus-minus something of sigma. This is how we are communicating our measurements with the kind of the confidence which is there with the observations. So all these things, these basic concepts of surveying, are very, very important. Henceforth, we will start talking about the instrument; the measurements. But we must keep all these things in our mind. So, we will finish our video today, at this point. Thank you very much.