Water Resources Engineering
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Lecture No. 17

In the previous lecture we had looked at the water cycle and various components of the water cycle. Precipitation is one of the major components of the cycle. So we started looking at the precipitation mechanism. We looked at different kinds of precipitation. For example we can have orographic precipitation, we can have cyclonic precipitation and similarly other forms of precipitations are also discussed. Then we started discussing the measurement of precipitation and we saw that two different types of rain gauges can be used recording rain gauges and non-recording rain gauges. The non-recording rain gauges don't record a continuous variation of rainfall. Once a day somebody goes and measures whatever was the rainfall during the previous 24 hours and therefore the record of a non-recording gauge would typically look like time verses depth of precipitation and this will be 1 day, 2 days, 3 days like this and there will be some depth measured during that day. This kind of precipitation record comes from a non-recording gauge. The drawback is that within a day we don't know when the rainfall occurred and sometimes during the day the rainfall may occur with a very large intensity and we will not be able to capture it. That's why recording gauges are used and we saw three different kinds; the tipping bucket, weighing type or weighing bucket and float or sometimes also called siphon. These recording rain gauges maintain a continuous record of the precipitation. So time verses depth of precipitation would be a continuous curve which will be a continuously increasing curve and different portions will give the intensity of rain. If there is no rain then it will be horizontal.

In the tipping bucket rain gauge we saw that there would be an arrangement like this where there will be two buckets and then the precipitation collected through a funnel will fall over one of the buckets and the bucket is filled up to a certain level. This level generally is about one hundredth of an inch or 0.01 inch or in millimeters, 0.25 millimeter. That much precipitation occurring over this area of the opening of the rain gauge will fill the bucket up to some point and as soon as it filled it will tip over and the second bucket will come under the funnel. In this way the record of the tipping bucket would look like time verses depth where depth really means one tip of the bucket. It will represent 0.25 millimeter or 0.01 inches. Depending on how fast or how intense is the rain we may get a record like one tip here, one more tip here. It will be discrete tips which will be measured here and the number of tips will show the intensity of the rainfall. So tipping bucket can record the variation of intensity with time. In weighing bucket instead of this tipping bucket here we have a bucket whose weight is continuously taken and that will give us a curve. The weight can be calibrated to give the depth and it will again give us a continuous record like this. Similarly a float or a siphon has again water collected through a funnel. There may be a chamber here in which there would be a float which would rise with water. This float will be connected to some measuring instrument which will record the location of the float and therefore the depth of rainfall continuously. The recording type of rain gauges give us a curve which we call the intensity or the continuous depth which means accumulated mass or accumulated depth which we call the mass curve.

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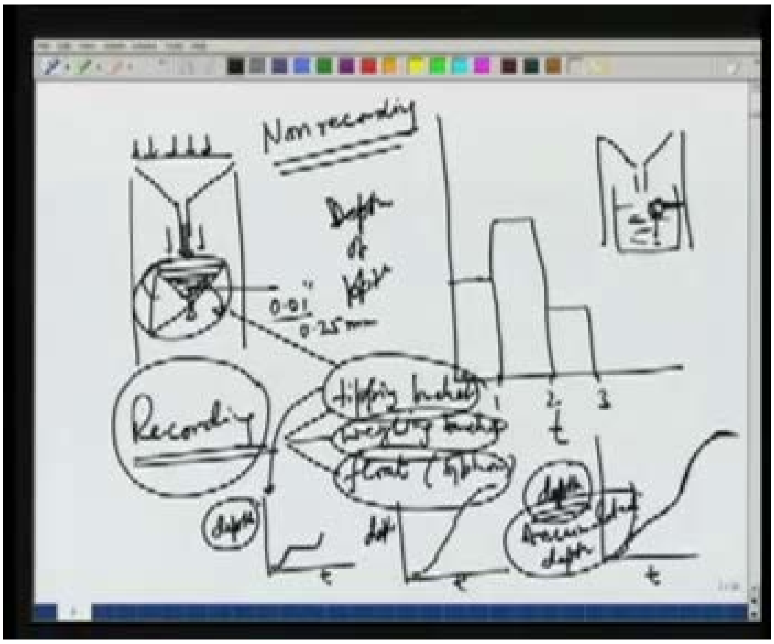

The slope of this line gives us the intensity at different times and that slope gives us what is known as a hyetograph; time verses intensity. Intensity we can measure in centimeter per hour or millimeter per hour or inch per hour. In India we use the SI units but in US inch per hour is more common. So time verses intensity graph can be plotted and we may get a discretized time verses intensity curve like this. For example for a storm the maximum intensity will occur somewhere in the middle portion. It will start with a low intensity and then achieve a maximum value and then again fall down. So this is known as a hyetograph. We looked at these two different methods of representing a data one in terms of mass curve; accumulated depth which we call the mass curve and the other the hyetograph which is the intensity verses time.

If we have a non-recording then typically it is done daily. We will get an idea about the daily accumulated mass but intensity for a period less than a day we will not be able to find. That’s why recording gauges are put. They will give us a fine discretization of time; every hour we can find the intensity of rainfall. Non-recording are less expensive and recording more expensive. The general factor which is followed is that 10% of the rain gauges should be recording. In an area if there are 10 rain gauges if one of them is recording then the non-recording data we can assume that the distribution of the intensity of rain within a storm will be similar at the recording station and we can use the same kind of distribution over the non-recording gauges to estimate some kind of variation within the day. The hyetograph and mass curve are two ways of representing the data.

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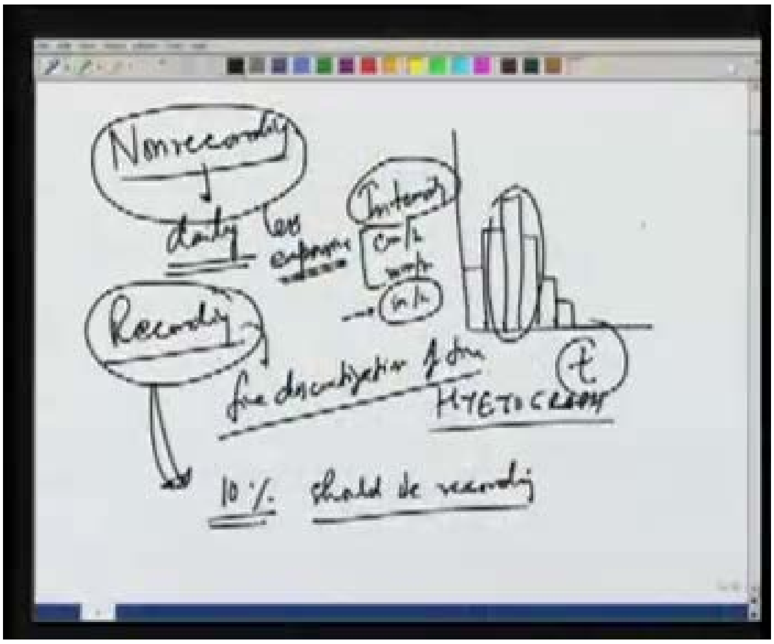

In the question which we saw about the data once we analyse the data the first thing which we saw was what happens if there is a missing data and what happens if there is inconsistent data? Missing data we saw that there may be various reasons of data being missed. But if there is missing data we can use the average of nearby rain gauges and assume that the missing data relates to those values. The value at the corresponding stations we can take the mean of that but if the rain gauges have very different normal precipitations then we can take the average of the precipitation divided by the normal values.

Inconsistent data again may be caused because of various factors and some of them may be moving the rain gauge from one location to the other or change in the environmental conditions near the gauge and we saw how to remove the inconsistency by plotting the precipitation at that station with the average precipitation of 5 or 10 or 20 nearby stations. Typically about 10 stations are used and what we do is we get a double mass curve which shows accumulated precipitation at the station at which we want to find out whether the data is consistent or not and this is accumulated precipitation at a few let’s say 10 nearby stations and ideally they should follow a straight line. If there is a variation from this straight line that means the data is not consistent and we will have to modify the record. This is more recent data this is the old data. This accumulated precipitation is an annual value. If suppose this is year 2001, 2000 and so on; we say that before 1990 there was some inconsistency in the data and we can modify those values so that they can be brought to this line and make them consistent. These two data analysis tools we have seen how to estimate missing data and how to make data consistent if it is found that it is inconsistent.

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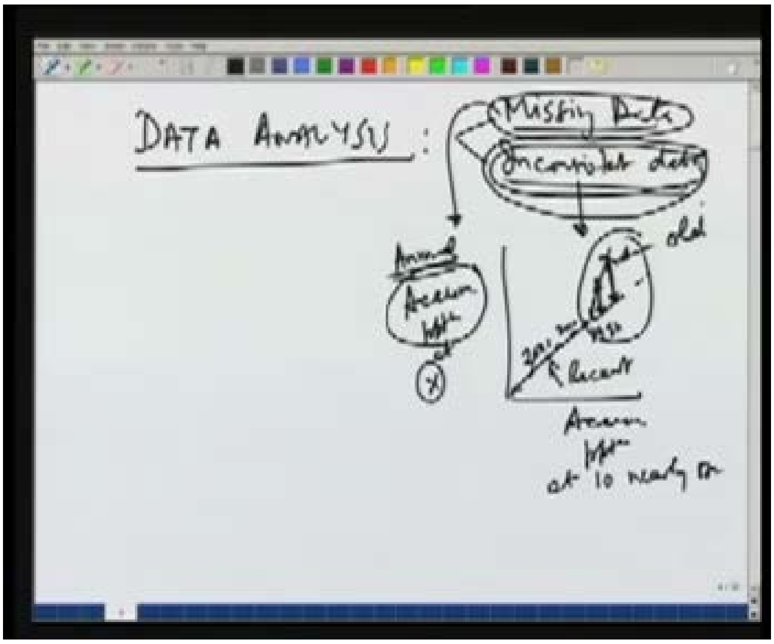

Today we will look at some other techniques the first of them being areal averaging. If we have an area over which we want to find out the precipitation depth and we have rain gauges located like this. Then the value we get for the depth of precipitation at a rain gauge really represents a point value and most of the times we would be interested in finding out the areal average. If there is a river flowing this may be the catchment area of the river or it may be some other area but we are interested in finding out what is the average depth of precipitation over the area so that we can get the volume of water which is generated by that precipitation. Suppose this area is A and these are let's say n rain gauges or measuring stations and for each of these n stations we have the value of precipitation P_i, i goes from 1 to n. From this data we want to estimate the average depth of precipitation over the entire area A so that we can multiply the area with that depth and get the volume of water generated from that precipitation event.

The simplest way is to find out the mean of the data and a simple arithmetic mean of the data may be thought of as being the average depth over the entire area. P bar can be thought of as …… where summation is i equal to 1 to n. Once we get this average depth we can multiply it with the area. So A into P bar will give us the volume of water generated by that precipitation event for which our point values are known from the rain gauges. It will work but the problem comes when the distribution of the rain gauges is not uniform over the area. For example we may have an area over which we may have let's say 3 gauges here and 1 gauge here. In that case giving equal weightage to these three and this gauge may not be proper and probably we should assign more weight to this station because it covers a lot of area and these three may be assigned smaller weights.

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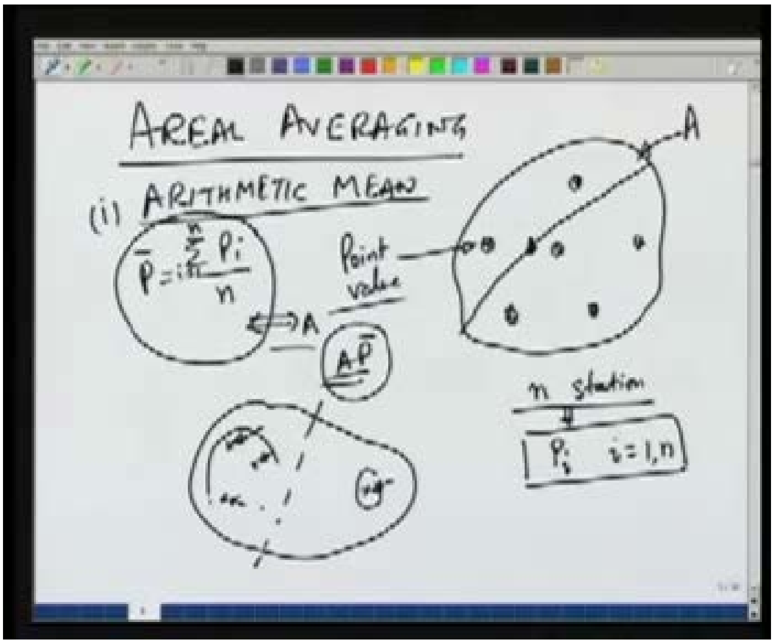

The next method which we see known as the Thiessen polygon method is based on this principle that different rain gauges may have different effective area and the rain gauge may even be out of the area still we may give it some weight. In the Thiessen polygon we join the stations by these lines and then we take the perpendicular bisector of these lines. These perpendicular bisectors represent the area which is closest to a particular rain gauge. If we take the example of these three gauges we can explain it. If there is a triangular network and we have these perpendicular bisectors then this entire area let's call this rain gauge 1, 2 and 3 then this entire area is closest to the rain gauge 1 compared to the other two gauges. Similarly this area is closer to 3 than any other gauge and this area is closer to 2 than the other two gauges. We find out this area let's call this A_1, A_2 and A_3 and assign the weight equal to this area divided by the total area. Similarly if have more than three gauges then the area surrounding a gauge would be the effective area for that gauge.

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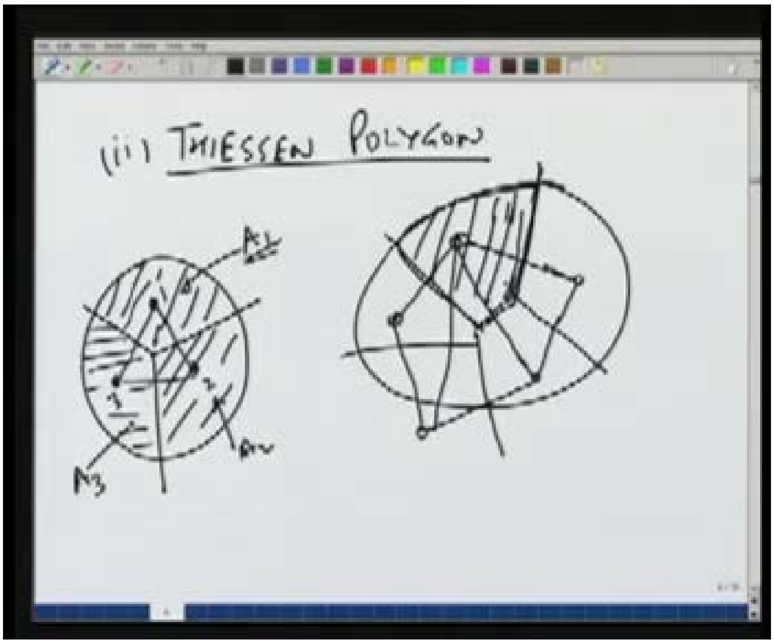

The equation which we use is P bar. Again it is a summation of all the precipitations with some weights. This A_i is the area of polygon containing i and i will go from 1 to n. A is the total area and summation of all the areas has to be equal to A. If there a station which is outside then its station polygon will only cover the area within the catchment boundary. It will not extend beyond so it will not be including this area. This term effectively means some weight which is assigned to the precipitation for that station and that weight will depend on how much area is effectively covered by that rain gauge. This Thiessen polygon method gives a better estimate than arithmetic mean.

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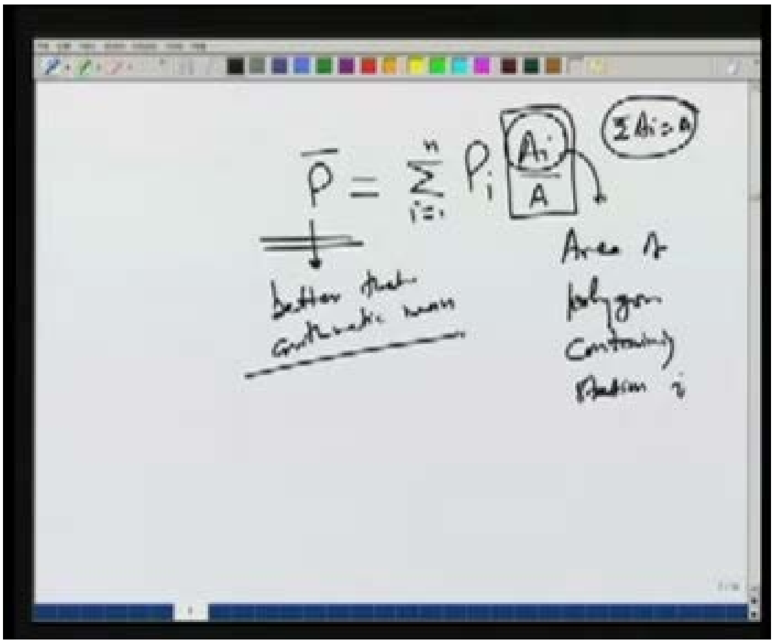

But still it does not account for variation in let’s say topography of the area. If we have an area where there may be higher elevation here. Suppose we say that this is at higher elevation and there is some rain gauge here; there may be some rain gauge here. It will not account for Thiessen polygon. It will divide the area like this. It will not account for the topographic variation and therefore rainfall variations within that area. The third method which we use accounts for this possibility also and it’s called Isohyetal method in which we use the rainfall values at each station also in dividing this area. Suppose we have the rain gauges at these locations. The values of the precipitation here are known. At all these stations we know the values of P_i.

Using that P_i similar to contour lines which show equal elevation we can draw lines which are known as Isohyetal lines. Isohyetal lines are lines representing equal precipitation depth. If this is showing let’s say 4 millimeters, this is showing 6 this is showing 5; this may be 6.5, 7, 7.5 and so on. Based on that we can draw some contours, which will show depths like this. This line represents the line on which the depth of precipitation is 4 millimeters. Similarly 5, 6 and so on and what we do is then we again assign a weight but this time we use P bar as some average precipitation into the area where P_1 and P_2 are the depth values or the isohyetal values at the two ends of the area. If we call this area as A_1 then the value here will be P_1 P_2. Similarly for this area P_2 and P_3; for this area we will have P_3 and P_4. For this area between 4 and 5 isohyetal values we will have $4+5/2$. We'll say that this area has an average depth equal to 4.5; multiply it by the area A_2. Similarly the first area has average depth $P_1 + P_2/2$ multiplied by the area and so we will add everything up, divide by total area and get our average precipitation.

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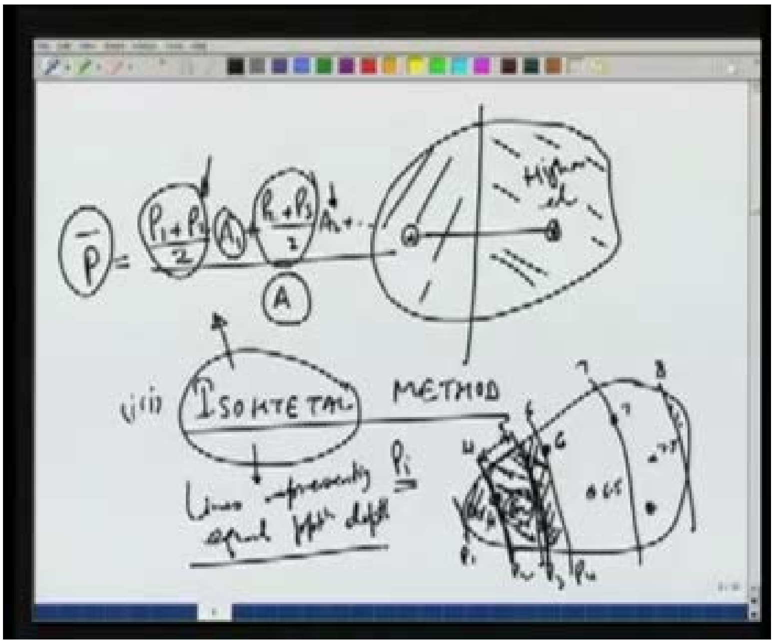

Isohyetal method is probably the best out of these methods but it requires a lot of data to be able to plot the isohyetal maps. Once we convert the point values from the rain gauge to the areal values we can analyse and present the data in different forms. One of the forms in which we can present the data is known as depth duration curve and it depends on the area. If we are talking about one catchment area suppose we have some storm occurring over this area, the storm will have highest intensity at the center. So near the center intensity will be very high and the depth of rainfall would be some high intensity into the duration into the area. As we move away from this storm center the area increases but the intensity decreases. In order to know how the depth is affected with the area we have these depth duration curve or depth area duration curves. It's commonly known as DAD curve; depth, area, duration and what it shows is a curve where area is in square kilometers verses some depth of rainfall.

If we take a particular duration of rainfall we can get an equation where P_0 is the depth at center of the storm, K and n are some coefficients and A is the area. As we move away from the center or the eye of a storm the intensity decreases. Depth will increase as we increase the area but the rate at which it will increase will become slower. P_0 is taken at the center of the storm. Suppose we have a rain gauge here. Then it is very unlikely that the eye of the storm will be exactly on the rain gauge. So the rain gauge value generally is not taken as P_0 and what we assume is that the point value at the rain gauge represents an area of about 25 square kilometres. So the P_i at the rain gauge corresponds to about 25 square kilometres. We can estimate the P_0 using this equation putting the area equal to 25 square kilometer and P equal to P_i. So P_0 can be estimated and then we can estimate the depth of rainfall for any given area using this curve.

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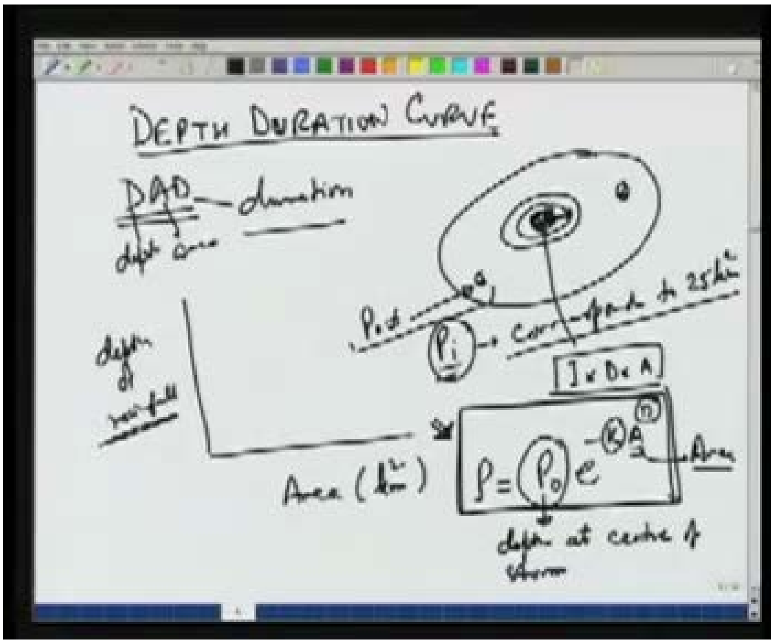

The values of K and n can be shown here. This is the duration of the storm, K and n. Duration will be like 1 day, 2 day, 3 day. K value is 0.0085. This data is shown in millimetres. If the P is in millimeters then the values of K and n would be given like this 0.66. Area is generally is in kilometer square and P in millimeter and P_0 also would be millimetre. Actually P and P_0 will have the same units. We can use both of them in centimeters or both of them in millimetres. Units of K will affect the constant K. Here the constants which we are showing 0.00085, they are based on area in square kilometres. Similarly for 2 days and for 3 days n is 0.66, 0.63 and 0.60. These values can be used to obtain the variation of depth of rainfall with area.

The other thing which we can do is plot a curve which shows area verses maximum average depth and this may be in centimetres, x-axis in square kilometers and this will depend on the duration of rainfall. For different durations we can have these different curves. This for example may be 18 hours this may be 6 hours, 1 hour and so on. The maximum average depth means that we will take some extreme storms occurring in that area and using those extreme storms we can estimate the depth for different areas. Near the storm center the intensity will be very large and as we move away the intensity becomes smaller. We can see that with increase in duration the average depth will increase but with increase in area the average depth will decrease because the intensity of the storm is decreasing as we increase the area. These curves are important because they give us extreme conditions. We can take a maximum or very severe storm. How much depth of rainfall will result under various conditions? For example if the storm is only 1 hour duration what will be the maximum depth which we can expect over a certain area which the storm is covering? Depth area duration curve and the maximum depth they are very important parameters to be estimated for given conditions.

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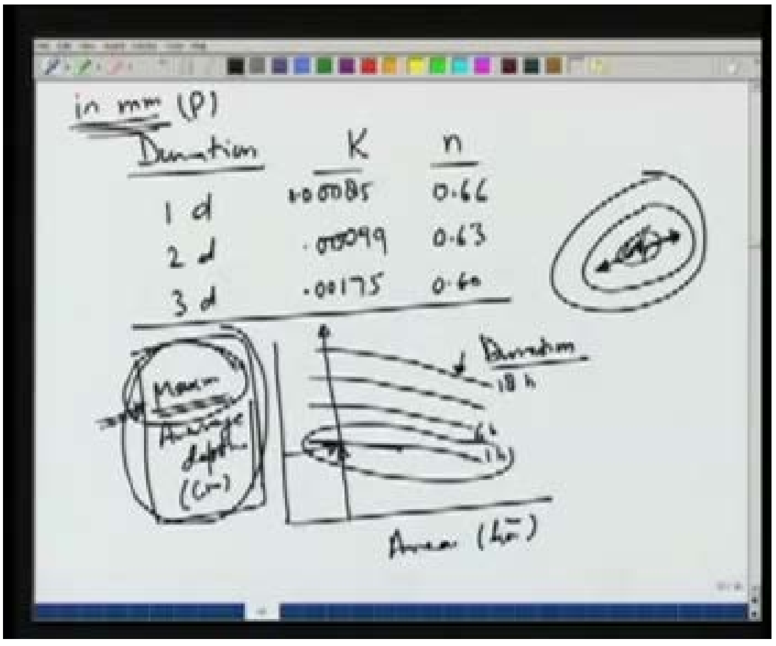

For example for north Indian conditions if we show the area in 10 to the power 4 square kilometers three typical values which are shown here are 0.13, 1.3 and 13. This is a 10 times difference, one order of magnitude difference. The duration is 1 day, 2 days and 3 days. This table shows the depth of precipitation and the values which are given 76.5, 47.2, 16.0, 97.5, 73.4 and 27.9. These are in centimeters and this is based on severe storms occurring in that area and you can see that if this storm occurs for 1 day over a small area of 0.13 into 10 to power 4 square kilometers we can expect a rainfall of depth 76.5 centimeters. We can look at this. As we increase the area the depth becomes smaller. We have seen that as we move away from the center of the storm the intensity decreases and as we increase the duration the depth will increase but not at the same rate. So the value for 2 days will not be twice the value of 1 day because again with the larger duration same intensity cannot be maintained. These kinds of data or curves are useful in deciding how much capacity or how much maximum rainfall can occur in an area. Depth area duration curve are quite critical from this point of view.

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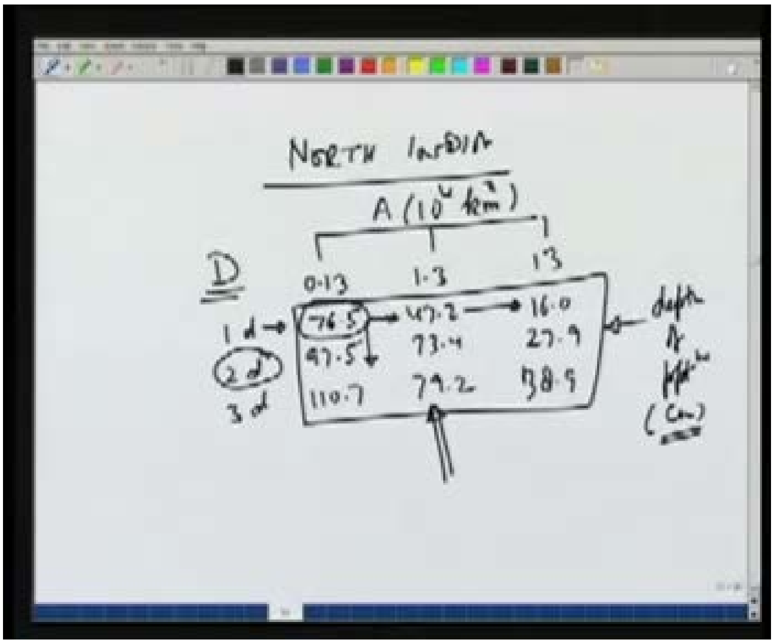

The other analysis which we can do from the rainfall data is based on statistics and is called frequency analysis. Not only are we interested in finding out the rainfall, how much it occurs, what is the maximum value but we should also be able to tell that within let's say a period of 10 years or 50 years or 100 years what is the maximum value of rain which we can expect and based on the importance of the project we can choose this what is known as the return period. In the frequency analysis we use the probability concept and the return period concept which we call T. On an average how much will be the period between the two events, average time period? For example we say that the rainfall of 100 millimeters in one day what will be the average expectation? We can say once in 10 years or once in 20 years. Generally we will talk about annual rainfall values and probability may be of occurring in 1 year. What is the probability that the event will occur in a certain period?

The way we obtain this frequency distribution is generally what is known as a plotting position method in which we obtain the values let's say annual rainfall and arrange it in an order. Let's say we have 50 years of data of annual maximum rain in 1 day. This may be in 1 day. It may be 1 hour, 6 hours, 18 hours whatever time period we choose we have the maximum value. Let's choose the period of 1 day. We can arrange it in the decreasing order. We have depth of precipitation and its order and this would be in decreasing order, the order or rank. The highest value will be rank 1 and suppose we have 50 years of data then the lowest will be ranked 50. This rank is denoted by m and the number of record years 50 would be n and the plotting position method says that this data of the rainfall depth P or let's use a different symbol here since we are using P for probability. Let's say Pr the precipitation. The precipitation, any probability of any value or the plotting position is generally given by m by n+1 and using this plotting position we can obtain the probability and the return period for different annual maximum values.

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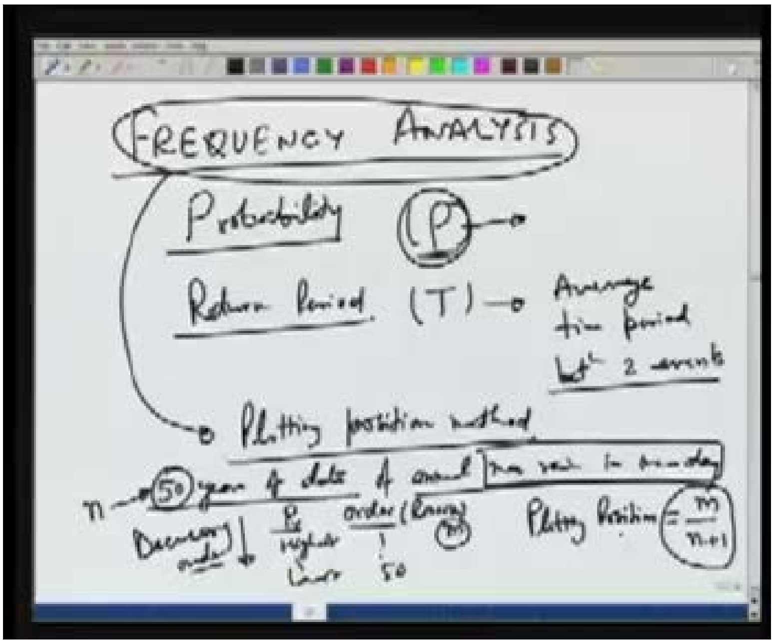

The plotting position as we have seen the largest value has a plotting position of 1. That means the probability that this value will be exceeded would be very small. 1 over n+1 or the return period which is 1 over probability would be given by n+1. This is the probability that the event will be equalled or exceeded in a year. The smallest value has rank m equal to n and therefore the return period will be n+1 over n. For large n it will come very close to 1.

There are some other methods also of the plotting position. There is a California method which uses a plotting position of m over N. There is Hazen method which uses plotting position of m-1/2 over N. Different methods have been used. We will use m over n+1 which is called the Weibell method. In m over n you can see that when m is equal to n this probability will become 1 for the California method. From the plotting position we can see that the largest value will have a large time period or return period and the smallest value will have a small return period.

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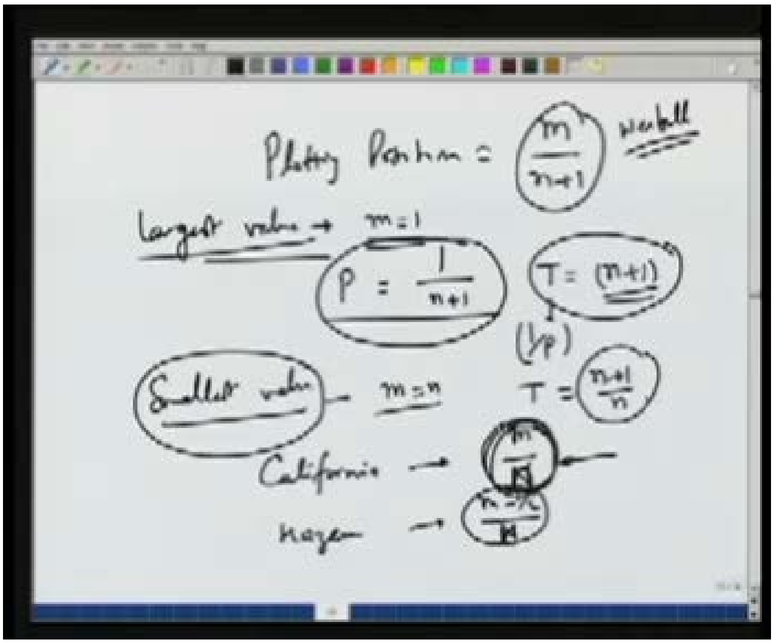

We can plot a curve which shows the recurrence interval or the return period which represents the average interval between occurrence of two events of some magnitude and then here we can plot let's say the annual maximum rainfall depth and this annual maximum may be in 1 day. Rainfall depth may be daily and if we plot the curve return period may be 1, 10, 100, 1000 years and the rainfall depth here may be in centimetres; we may plot 0, 10 and 20. We would get a curve like this and suppose we have 50 year data it will then stop near 50. But based on this available record we can extrapolate the data and obtain the 100 year rain or the 1000 year rain. This gives us some idea about how frequently a rainfall event will occur which is important to us because different projects will have different useful life, different importance and based on that we can decide what return period to use for an event used in the design of that project. Frequency analysis is important from this point of view.

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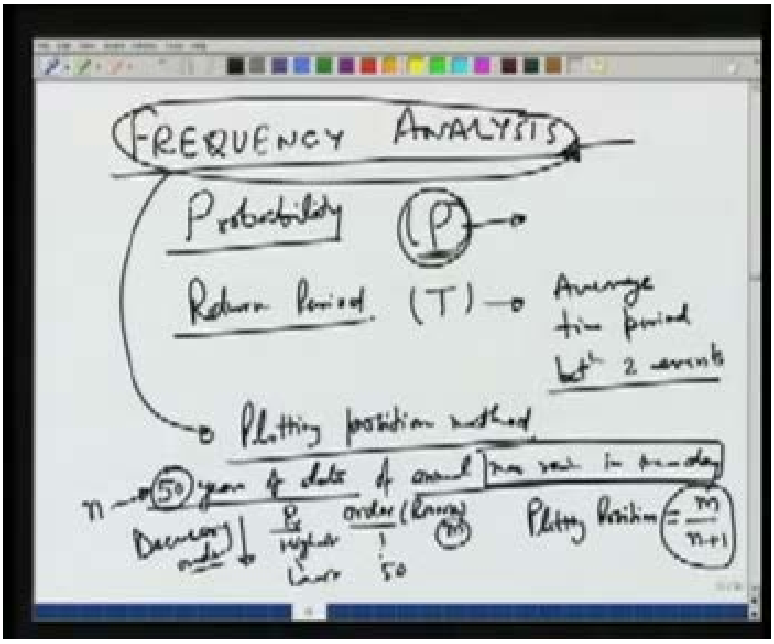

The result of this can be shown in terms of what are known as intensity duration frequency analysis. Intensity duration frequency analysis means we draw curves which show how intense rainfalls are common. The equation which we use In this case suppose i is the intensity of rain. As we move away or as we cover larger and larger area the return period affects the intensity the duration is also affecting the intensity. We can say that as the duration increases the intensity will decrease. As return period increases the intensity will increase. In other words higher intensity storms will have a higher return period. They will be less common and as the duration increases the intensity decreases because sustaining the intensity for a larger duration is not possible or for shorter duration storms we will have a very high intensity as the duration increases the intensity will decrease.

The values of K, x, n and A vary from area to area but some values again for Indian conditions can be written. There is a wide range of these values given in literature. Generally we would use some value about let's say 100; not exactly but close to 100 for this about 0.2. This is a; a is about 0.5 and n generally about 1 but there is a wide range for example K value may vary from 10 to 120. These values can be used and this will give us the intensity of rain in millimeter per hour time. Return period is in years and duration is generally in hours. These values will enable us to compute the intensity of rainfall for any return period and for a given duration of rainfall. Later on we will see that this duration will depend on the catchment area.

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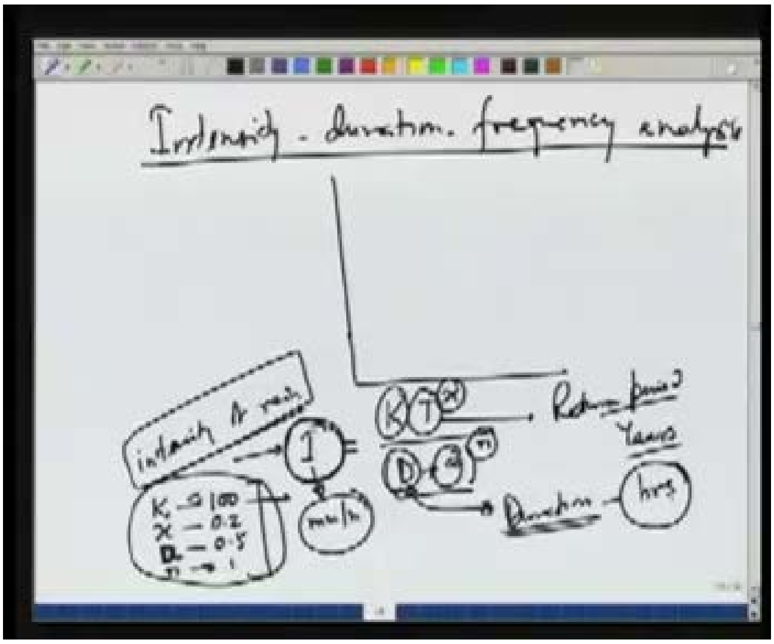

For example if there is some area and we are interested in finding out the flood here; flood volume or maximum flood discharge. What we would like to do is have a storm over this area which will cause the maximum runoff at this point and for that purpose as we have seen the intensity will decrease with the duration. We want a minimum duration and we don't want a duration which is very small also because then the entire area of the catchment may not contribute to the flow. If the duration is very small suppose the duration is such that only this much area is contributing to the flow at one time and then the flow from this area reaches after sometime then we will not be able to get the maximum flow discharge. There we use the concept of time of concentration which is the time taken by the farthest point in this catchment. Water drop falling on the farthest point to reach the channel and to reach the outlet, the time taken from the farthest point to the outlet is the time of concentration and as the duration increases the intensity decreases. If you want the maximum intensity duration should be at a minimum and t_c is that minimum duration.

Generally when we use the intensity duration frequency curves we take the duration as the time of concentration because that is the minimum duration which we should take. If suppose we take our duration less then t_c then what will happen is that the rainfall has already finished over this area. This water will take a time of t_c to reach the outlet. At this time t_c the entire area of the catchment is not contributing to the flow because the rain has already stopped and therefore this area may not be contributing to the flow. This area may still be contributing to the flow because different areas will take different time to reach the outlet and at the time t_c the entire area contributes to the flow because the rain drop falling here has also reached the outlet.

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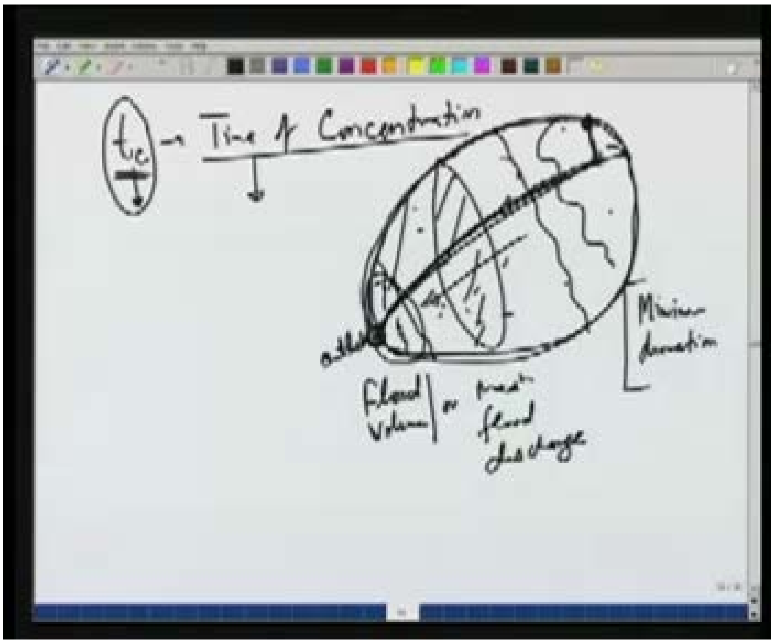

This t_c is important and in the intensity duration frequency curve this D can be taken as t_c to get the maximum flow possible at that location. So intensity duration frequency curves are important in deciding about what intensity of rainfall to take for designing a project. Duration depends on the catchment area and the time of concentration. Frequency depends on the importance of the project. For a more important project we should choose a return period which is of the order of may be 100 years or sometimes 1000 years. For not so important projects we can use 10 years or 20 years return period also. This equation enables us to obtain the intensity for different durations and frequencies. Values of K, x, n and D general average values are here but we will have to find out these values for the area under consideration and using that we can estimate the intensity.

The other thing which we are interested in is probable maximum precipitation. Some projects may be very important or it may cause a lot of life and property if the structure fails. That we have to find out what is the maximum precipitation which can occur in that area causing flood in that area. For a probable maximum precipitation generally we can obtain this based on climatic factors. If we have the ground level here the air above this can contain only a maximum limit of water vapour, maximum amount or the holding capacity is limited. There are methods using which we can estimate what is the amount of water vapour present in this air column and based on that you can estimate the probable maximum precipitation. But there are other methods which are based on the frequency analysis and what is done is that the probable maximum precipitation is given as some mean precipitation standard deviation multiplied by some constant k generally around 15.

As we saw intensity was also important but what is the probable maximum precipitation is also important. The intensity as we saw could be obtained from this curve. Let's say we have some soil erosion in an area. Intensity can be correlated or the erosion can be correlated with the intensity. For that purpose also intensity becomes important. Maximum flood we want to be very safe. We can use this kind of equation. The frequency distribution we have seen how to analyse and from that we can get some idea about what is the mean and what is the standard deviation of those values. Taking a very high multiplying constant about 15 we can obtain probable maximum precipitation. Some values for India; there are some empirical equations which are generally given but 1-d PMP. This is probable maximum precipitation which is generally written as PMP. 1 day PMP for India is generally taken to be about 40 to 100 centimeters and data from all over the world has been collected and an equation like this has been proposed for the PMP for different durations of rain. This is 1d. This is in hours. If this is 1 day then d would be 24 and the value for 1 day comes out to be about 190 centimetres. From the data from all over the world the probable maximum precipitation for 1 day is expected to be about 190 centimeters. For Indian conditions 40 to 100 centimeters is the value which is used.

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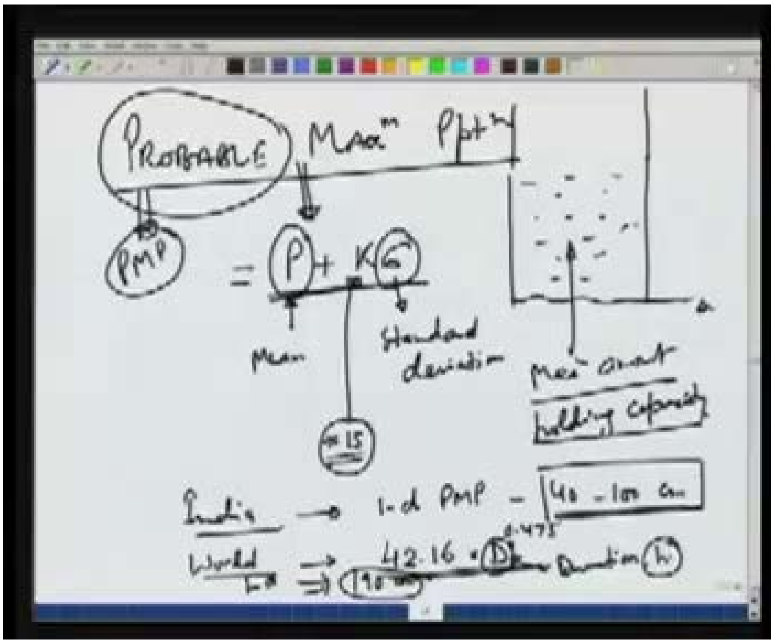

In today’s lecture we have seen some of the ways of analysing the data which is collected from the rain gauges. The data from the rain gauges can be recorded continuously or it can be recorded on a daily basis. The data can be analysed to find if there is any missing data or inconsistent data and that can be corrected. We have seen some methods of correcting that. Then some alternative ways of presenting the data for example the intensity curves, that is depth area duration curves have been studied and as we see that the intensity of the rainfall will depend on the frequency we adopt for the rain and the duration of the rainfall. Intensity duration frequency curve or the IDF curves have been studied and we have seen the use of those curves. The duration we have seen can be taken as the time of concentration if we are interested in finding the maximum flow at the outlet for a catchment. Then we can also find out the probable maximum precipitation which is the maximum value of precipitation which can occur at a given location for different duration of rainfalls and using those values we can estimate the ultimate maximum value which can be used for a very important projects although PMP is very high and it’s not really used for general design purposes.