Water Resources Engineering
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Lecture No. 22

We have looked at the process of runoff and we have seen that out of the total precipitation some part is abstracted through interception deficient storage, evaporation, transpiration and infiltration and the rest goes as runoff. This runoff is the component in which we are very much interested as water resources engineer because we want to know how much water is available at a certain point in a river so that we can use it to supply water or so that we can design some flood protection works if the magnitude of the water available is very high. In order to look at this we have to analyse the quantity of water which is available at a certain point and for that purpose we sometimes put gauging stations which measure the height of water or the depth of water in the river and correlate that with a discharge so that if we measure the depth or what is known as the stage then we can obtain the discharge from the stage discharge curve.

This gauging station may not be available at all rivers but let's say that some river has a gauging station. The record at the gauging station is in terms of stage which is converted to discharge and a typical record will look like this. This is the year and this denotes the date or the day of the year. First represents the first January and in this case 365 will represent 31st December. For the whole year we will have data available for the discharge through the river and what is done is that discharge is averaged over a day. This is the daily mean flow passing at that point. You have a catchment and there may be a river and there is a gauging station here at point A. Then measuring the stage at point A we can convert that into discharge and for the daily discharge data we can obtain what is the mean daily discharge at that point A for different times of the year. On 1st January we can see that the mean discharge was 10.2 metre cube per second. On 2nd it was 11 and similarly we have data for all other day. At day 200 the discharge is quite high at 121 meter cube per second and similarly at the end of the year it is 17.1 meter cube per second.

This record is available for a number of years. For example here we have shown 5 year data; 2002, 2003 and 2004 we will have 366 days. This corresponds to 31st of December and then again 2005 we have 365 days.

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We have a lot of data; 5 times 365 plus 1. In order to analyse all these data we need to reduce the data in some form so that we can analyse it more quickly and accurately. For that purpose what is done is we divide into range of flow. Instead of taking individual values of the flow we can say that we will divide the discharge into certain ranges. For example here we have divided them into discharges of 10 for high discharges and 5 for lower discharges. This represents the discharge range again in meter cube per second. This is the daily average discharge in metre cube per second and what we do is we look at each year how many days are there on which the average falls in this range.

In 2001 there is no data which is more than 140 or in the range of 140 to 150. In 2002 on 4 days the mean daily flow was between 140 and 150. When we talk about the range we would include the higher range. So this will be included and 140 would be excluded. If the discharge is exactly 140 then it will go into this range. If it is 140.001 then it will go into this range. Similarly in all other ranges and for each year based on the available data we can obtain the number of times or number of days on which the discharge is within that range. For example here we can see that 35 to 40 there are a large number of days with that range and then finally our aim is to obtain a flow duration curve which tells us what is the probability that a particular discharge will be equalled or exceeded and how much percentage of time?

For that we need to obtain the cumulative or the total number of days for this 5 year period on which the discharge was within 140 to 150 or 130 to 140 and so on. We can see that total number of days out of these 5 year data there are 6 days in which this discharge is within the range of 140 to 150. Similarly there are 26 days on which discharge is within the range of 130 to 140 and then we will do a cumulative discharge. This will be 6. Then 6 plus 26 will give us 32. This 6 represents number of days for which discharge is higher than 140 and 32 represents number of days for which discharge is higher than 130. Because 130 to 140 is 26 and 140 to 150 is 6 so total of these two will give us the number of days for which discharge is more than

130 meter cube per second. Similarly we do all this and ultimately this 1827 represents 5 year data. So 5 into 365 plus 1; It should be 1826.

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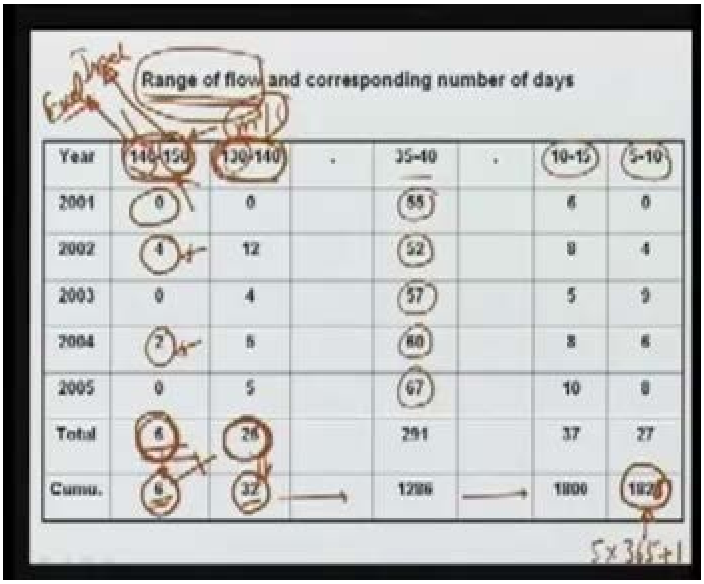

This represents the cumulative number of days for which discharge is exceeded. Now we can do a frequency analysis to analyse the probability of exceedance. We write Q as we have done in the previous slide and then we write the cumulative again this is from the previous slide. This probability P is m over N plus 1 where m represents the cumulative total and N represents the total number of records in this case 5 years. Dividing this cumulative number of days by the total number of records plus 1 we get a probability of 0.33% for discharge to be within the range of 140 to 150. Similarly we can look at discharge of 5 to 10 and the probability is 99.95%.

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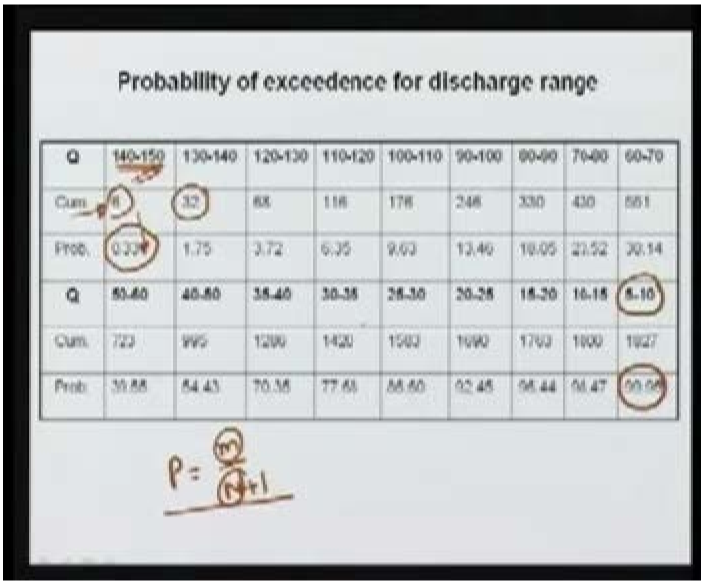

This tells us that almost 100% of the time discharge will be more than 5 metre cube per second. If we look at the flow duration curve which expresses this data in graphical form here we have a probability in percent for a given Q and this probability is of exceedance which means that if you look at any particular Q for example let's look at Q equal to 70. Then this gives us a probability almost 25% that a Q of 70 metre cube per second would be equalled or exceeded or in other words we have a value of let's say 140 here which has a very small probability of exceedance and then we have a value of 5 here which has almost 100% probability of exceedance. What it means is that if we go to the river and measure every discharge over a day, for any day there is a very high chance that it will be more than 5 meter cube per second and there is very little chance that it will be more than 140 metre cube per second. The average discharge throughout the day is likely to be smaller than 140 and if we look at this point then the probability that discharge will be higher than 70 metre cube per second is about 25%. This gives us a flow duration curve and we can use this curve to obtain the reliability of certain discharge. When we design a power plant we want some discharge which will be available 85% of the time. We can look at this and say that our 85% dependable discharge is about 25 metre cube per second.

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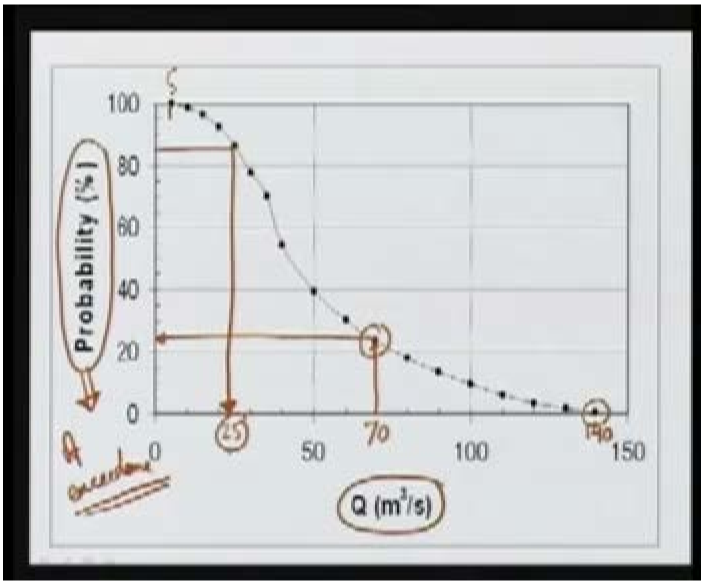

If we design something based on a design value of 25 metre cube per second then it is likely that 85% of the time the discharge will be more than that. This gives us an idea about how much flow is available in a river.

Based on this availability of flow we can design a storage system which can store water and then supply it when the flow is smaller. For this purpose we analyse it as what is known as a mass curve. A mass curve is nothing but how much accumulated mass of water is available throughout the year and to analyse the mass curve we can look at in terms of monthly rates. Instead of analysing daily data it is better to combine data for a month and plot it versus average monthly rate. Let's say that we have certain point; again we have a catchment and then at point ‘a’ we are measuring the stage and their whole discharge and the mean monthly flow at the point a for different months is obtained as these values.

We can see that the mean flow rate is very high in September just after the monsoons and it is still higher in August and October but then again it decrease as we go away here and decreases as we go away here and January February again there is a reasonable amount of flow 45 and 48 metre cube per second. This represents overall and average flow rate throughout the month and based on this we can find out the volume. For example in January there will be 31 days multiplied by 24 hours; then 3600 seconds. That gives us a volume which we have shown here in terms of million metre cube. This value of 120.5 represents the volume of flow in the river in the month of January. For the entire month of January 120.5 million metre cube of water will pass through this point a. Similarly for February, March and so on till December and September has the highest amount of volume available for the entire month.

Now we can add them up and find out the cumulative volume. The 120.5 is here then 120.5 plus 116.1 will give us 236.6. Then 236.6 plus 101.8 will give us 338.4. In this way we can obtain an accumulated mass which gives us 1682.6 million metre cube for the entire year and based on this value we can obtain an average rate throughout the year. If we have this much volume available we can convert this into an average rate which in this case turns out to be 53.4 metre cube per second. This means that the entire water available to us at point a can be represented by an average rate of 53.4 metre cube per second throughout the year. This rate is the maximum supply rate which we can maintain in the river throughout the year but again we can see here that the flow available is only 45. If we have to maintain a rate of 53.4 we must build some storage so that during these years water can be taken out of this storage because the supply rate is less than the demand of 53.4.

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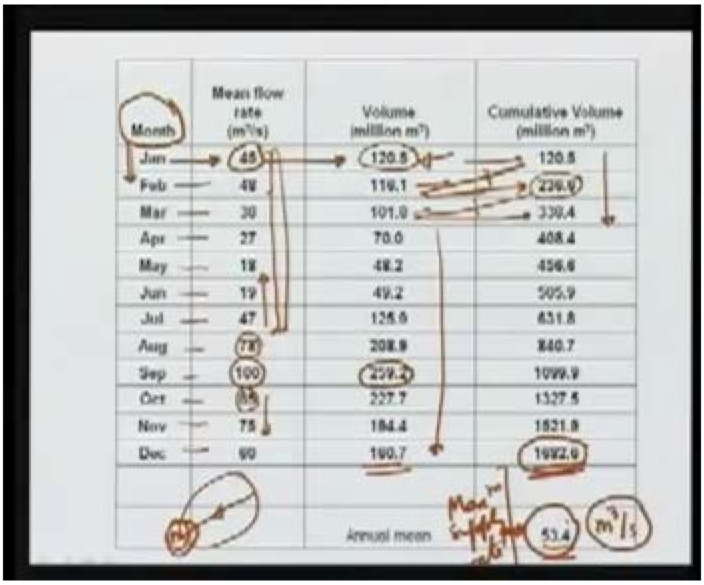

Month	Mean flow rate (m^3/s)	Volume (million m^3)	Cumulative Volume (million m^3)
Jan	45	120.5	120.5
Feb	48	116.1	236.6
Mar	38	101.8	338.4
Apr	27	70.0	408.4
May	18	48.2	456.6
Jun	19	49.2	505.9
Jul	47	125.9	631.8
Aug	78	208.9	840.7
Sep	100	259.2	1099.9
Oct	85	227.7	1327.6
Nov	75	194.4	1521.9
Dec	60	160.7	1682.6
		Annual mean	53.4

In order to analyse how much storage capacity we would need, we plot the mass curve as shown here. This is for different months the cumulative volume; this is again million cubic meters. This shows for the whole year what is the accumulated volume for a certain time? In January we have so much accumulated volume and at the end of December we have so much of accumulated volume which we have already seen as

1682.6. This value is 1682.6 and if we join these two points this will give us the maximum demand which we can satisfy which can be met. We would not use the demand as 53.4. This value is the maximum value but let's say our demand is smaller than this. For a given demand for example let's take the demand as 45 metre cube per second. For this given demand if we want to find out what is the storage required then what we need to do is we need to draw the line which has a slope of 45 metre cube per second. This line will give us an idea about how much storage we would need for a demand of 45 metre cube per second and in this case suppose this is the 45 metre cube per second line we draw a line tangential to this curve and parallel to this and then the difference between these two will give us the storage in terms of million metre cube.

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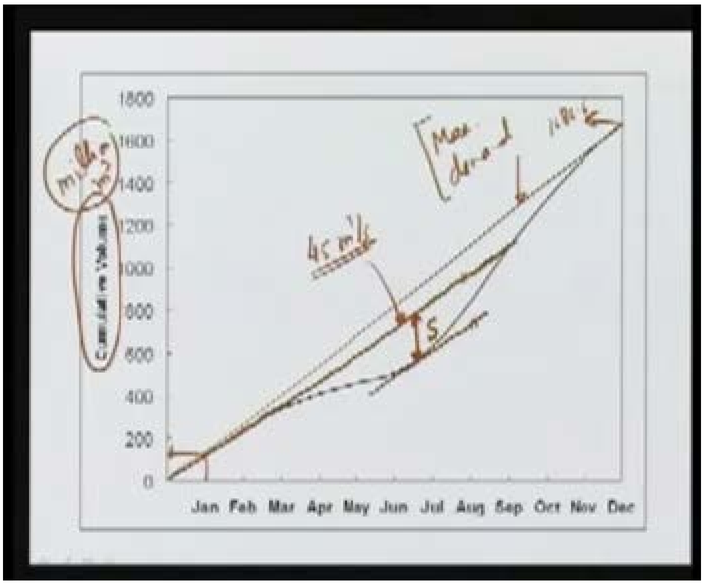

If we look at the calculation what we can do is, in tabular form we can find out what is the difference between the available mean flow rate in the river and the demand which we want to maintain. In this case the demand is 45. In January the mean flow rate is also 45 so there is no difference. In February the availability is 48 metre cube per second while the demand is 45. So we have a surplus of 3 metre cube per second. So till this point we don't need any storage. Now this 3 metre cube surplus can be converted into a volume in terms of million cubic meters. Again 3 metre cube per second into 28 days, 24 hours, 3600 seconds will give us metre cube and then we divide it by 1 million to get this value in million cubic meters.

After February if you look at the March data, in March we have a supply of only 38 cubic metre per second while the demand is 45. That means we have a deficit of 7 metre cube per second which translates into a deficit of 18.7 million cubic metre for the month of March. In the next month again we have a deficit till we reach July in which case again we get a surplus. These four deficit months they are the critical months for the storage capacity and our storage should be sufficient to take care of these deficit values and if we add all these volumes we get -205.1 which really corresponds to this. In the graphical terms this storage numerically would be equal to 205.1. This tells us that if we want to have a storage reservoir built at this point ‘a’ we can build a reservoir with the capacity of 205 million cubic meters and we would be able to satisfy a demand of 45 metre cube per second throughout the year. This is the required storage capacity S.

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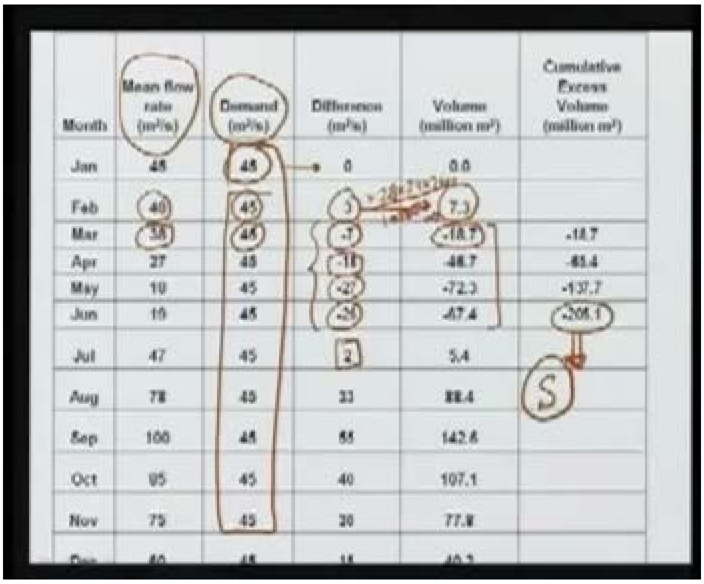

Month	Mean flow rate (m^3/s)	Demand (m^3/s)	Difference (m^3/s)	Volume (million m^3)	Cumulative Excess Volume (million m^3)
Jan	45	45	0	0.0	
Feb	40	45	5	7.3	
Mar	38	45	-7	-18.7	-18.7
Apr	27	45	-18	-46.7	-65.4
May	18	45	-27	-72.3	-137.7
Jun	19	45	-26	-67.4	-205.1
Jul	47	45	2	5.4	
Aug	78	45	33	88.4	
Sep	100	45	55	142.6	
Oct	85	45	40	107.1	
Nov	75	45	30	77.8	
Dec	60	45	15	40.2	

Similarly if we change the demand let's say now we want to have a demand of 50 cubic metre per second. In this case we can see that from January itself we have a deficit and then from August to December we have surplus. In this case following the same philosophy we can add up all these volumes and we get 284.1. In graphical terms this would simply mean that drawing a line at 50 and then parallel to that and taking NS which would in this case turn out to be 284 million cubic meters. This procedure gives us the required storage for a given demand. Sometimes our storage may be limited. For example we may not have the 284 million cubic metre of available storage. The maximum we have available is only 200 million metre cube.

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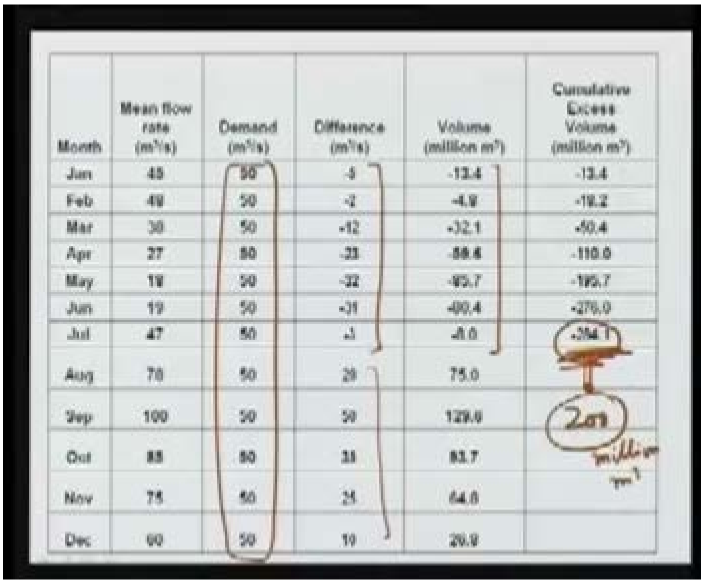

Month	Mean flow rate (m^3/s)	Demand (m^3/s)	Difference (m^3/s)	Volume (million m^3)	Cumulative Excess Volume (million m^3)
Jan	45	50	-5	-13.4	-13.4
Feb	48	50	-2	-4.8	-18.2
Mar	38	50	-12	-32.1	-50.4
Apr	27	50	-23	-59.6	-110.0
May	18	50	-32	-85.7	-195.7
Jun	19	50	-31	-80.4	-276.0
Jul	47	50	-3	-8.0	-284.1
Aug	78	50	28	75.0	
Sep	100	50	50	129.6	
Oct	83	50	33	83.7	
Nov	75	50	25	64.8	
Dec	60	50	10	26.8	

The problem that we now have is suppose we have the storage available to us 200 million metre cube what is the maximum demand which we can meet with this storage? For that purpose we can do a number of things we can do figuratively in the graphical method also. We have this S as 200. We can take this difference and draw tangential lines here and find out what is the rate which gives us S exactly equal to 200 or we can do it. These days computers are easily available and there are lot of facilities with which we can give any value here find out this S and then we can say that make this value equal to 200 and find out what is this Q? In that case we start with any value. Suppose we have started with this 50 and we have obtained -284.1 or we have started with 45 and we have obtained this 205.1. One option would be to do some kind of interpolation between 45 and 50. At 45 we have 205.1 at fifty we have 284.1. So little extrapolation will give us the value for 200 but using the spread sheets we can put any value here. This value will be different from 200 and then we ask the computer what should be this value to make this 200 and in this case it turns out that this value is 44.5. This is the demand which can be satisfied for 200 million cubic metre of storage.

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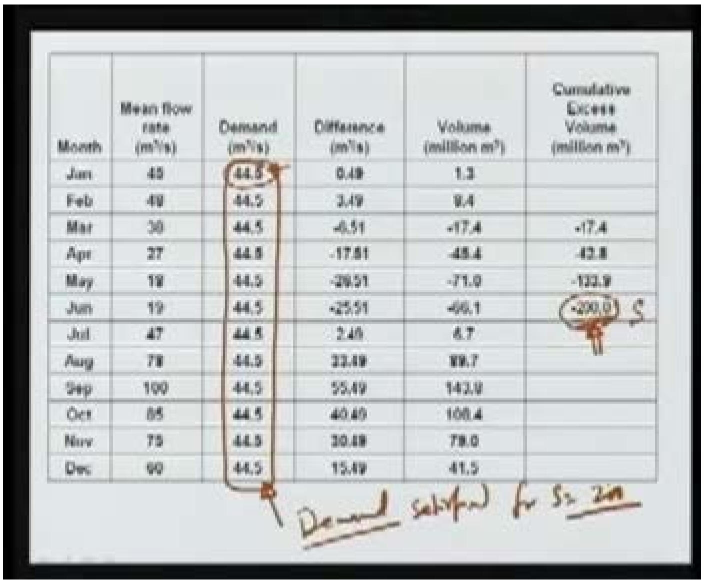

Month	Mean flow rate (m^3/s)	Demand (m^3/s)	Difference (m^3/s)	Volume (million m^3)	Cumulative Excess Volume (million m^3)
Jan	45	44.5	0.49	1.3	
Feb	48	44.5	3.49	9.4	
Mar	38	44.5	-6.51	-17.4	-17.4
Apr	27	44.5	-17.51	-45.4	-62.8
May	15	44.5	-29.51	-71.0	-133.9
Jun	19	44.5	-25.51	-66.1	-200.0
Jul	47	44.5	2.49	6.7	
Aug	78	44.5	33.49	86.7	
Sep	100	44.5	55.49	143.9	
Oct	85	44.5	40.49	105.4	
Nov	75	44.5	30.49	79.0	
Dec	60	44.5	15.49	41.5	

Using these techniques we can determine either what is the storage required for a given demand or what is the demand which can be satisfied for a given storage. We have looked at the problem of finding out the storage for supplying required rate of water or we can find out what is the maximum rate which we can supply for a given storage.

Now we move on to discussion of hydrographs which we have seen are record of the discharge in a river with time and suppose we have a data given like this. This is time in hours and for a particular rain the intensity is given here. We obtain the runoff as Q. This is the total runoff which includes the base flow and the direct runoff. The intensity of rainfall is given here. Let's look at 0, 4, 8 up to level 12. From 0 to 4 we can see that there is no rain. Then the intensity is 8 millimetre per hour. If you look at this time t = 5 the intensity is 8; 6 it is 8 and 8 it is 8. Then at 10 hours and 12 hours the intensity is 6. By plotting at 4 it is 0. But if you look at the runoff we see that the runoff starts increasing at t equal to 4. From 4 it goes up to 5. That means a rainfall has started roughly at about 4 hours.

Instead of showing it at 5 we say that from 4 to 8 we have the intensity of 8 millimetre per hour and then from 8 to 12 we can idealise this as intensity of 6 millimetre per hour. This is our rainfall which is causing this runoff. Rainfall will cause the direct runoff and base flow will come from the ground water contribution.

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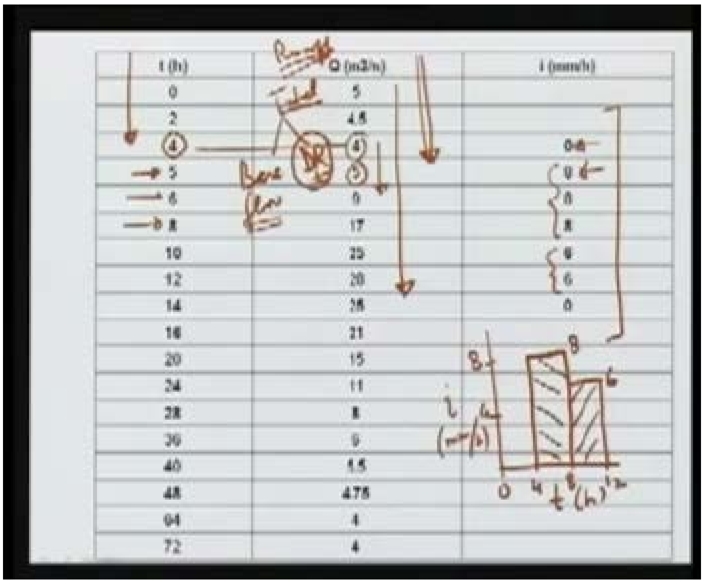

If we plot the total runoff, this is the runoff. It starts at 5; then it starts to go down. If we look at these values 5, then 4.5 and 4, it goes down till the rainfall starts. This is the point at which the rainfall has started and then the runoff starts rising. If we look at the data we can see that at 12 hours it reaches a peak of 28 metre cube per second which is shown here and then again it starts to decrease because the rain has stopped and it will continue till it becomes almost constant at about 4 after 64 hours. We get the runoff or the hydrograph; the runoff hydrograph has this typical curve. Our aim is to derive the direct runoff hydrograph which means that we have to somehow separate the base flow component and we also have to find out the effective rain.

In this hydrograph which we have shown here this is the total rainfall. A part of it will go as infiltration or evaporation. That loss as we have seen earlier is given by the phi index in a very simple average loss function. We will see how to obtain the phi index, how to separate the base flow and obtain the DRH. The starting point is quite clear that wherever it starts rising we can say that this is the start of the DRH. As we have seen here the base flow contribution is decreasing at a certain rate. One of the methods of separating the base flow is just taking a constant value of base flow, for example this and saying that everything above this line is the direct runoff. But that method is not very accurate. So we will be using a method in which from the peak we take a distance of N where N depends on the catchment area. In this case let's say that the catchment area A is 30 square kilometres.

The value of N based on various observations has been given. If A is in kilometre square then N comes out to be in days. So for an area of 30 square kilometres we get N about 40 hours. That means the direct runoff will stop 40 hours after the peak. In this case we can get some point here which will be 40 hours from the peak. The method of base flow separation is that the groundwater contribution is continued till the peak; just below the peak we will continue it and then we will join the point after N days directly with this point.

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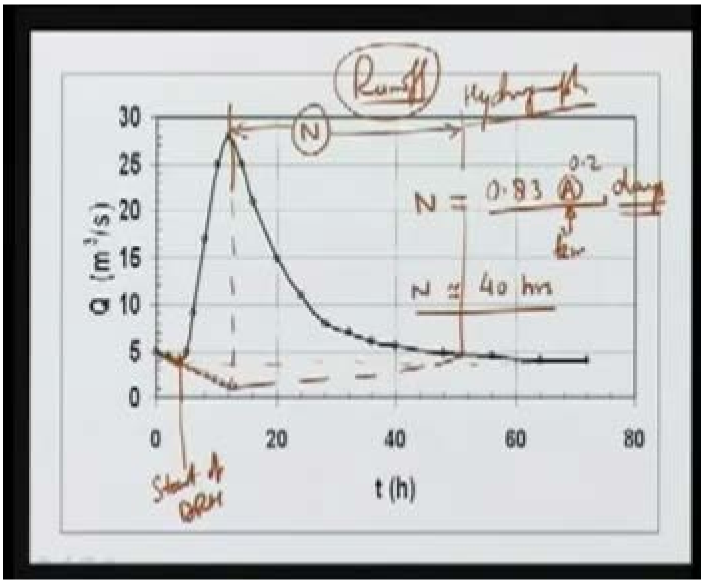

Using this method of base flow separation we can obtain the DRH by taking this Q, total runoff and subtracting the base flow which is done here. Corresponding to total runoff 4 hour ordinate we get zero because this is the start of the DRH; DRH starts here; corresponds to t equal to zero, the ordinate is zero. Then at 1 hour which is this point we take the ordinate which is 5 metre cube per second and subtract the base flow contribution which is computed from here and we get a DRH of 4.75 saying that the base flow contribution at this point is 0.25.

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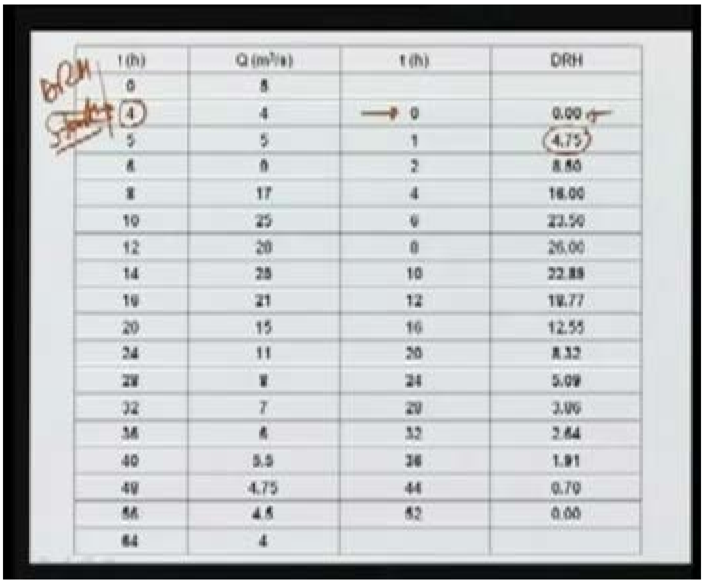

t (h)	Q (m³/s)	t (h)	DRH
0	5		
4	4	0	0.00
5	5	1	4.75
6	6	2	8.50
8	17	4	16.00
10	25	6	23.50
12	28	8	26.00
14	28	10	22.89
16	21	12	18.77
20	15	16	12.55
24	11	20	8.32
28	8	24	5.09
32	7	28	3.06
36	6	32	2.64
40	5.5	36	1.91
48	4.75	44	0.70
56	4.5	52	0.00
64	4		

These ordinates give us the direct runoff and we can see that the total discharge, maximum discharge in this case is 26 which is 28-2. In this way we can obtain the DRH and plot it and this figure shows the DRH.

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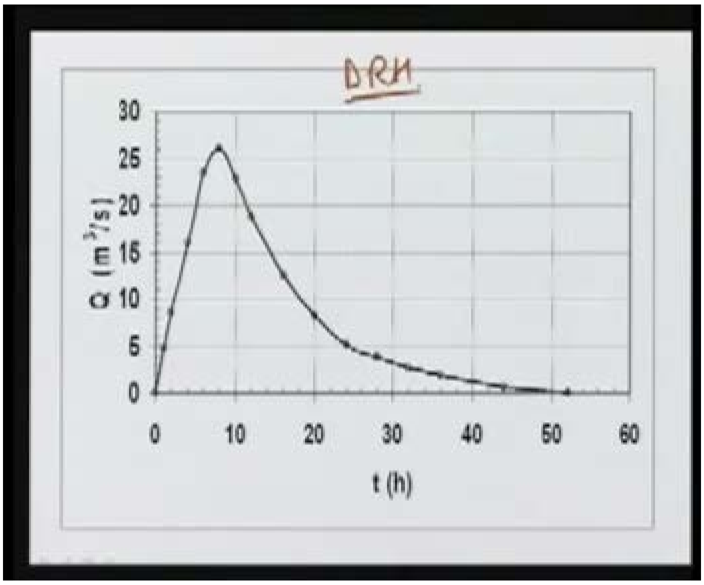

This DRH is because of rainfall which is given by two 4 hour rain of intensity 8 millimetre per hour and 6 millimetre per hour. Now we have to find out the phi index and to do that we need to find out what is the volume of direct runoff and that computation can be done based on the DRH ordinates. Here we have shown the ordinates of the DRH at t equal to zero. We have 0; then at 1, 4.75 and at 2, 8.5 and at 52 it stops here. So for a 4 hour rain the runoff will continue for about 52 hours. We can find out the volume which is accumulated at different times. This is in 1000 metre cube. At time equal to zero we have zero volume; at 1 hour we can say that this corresponds to the mean between zero and 4.75 for 1 hour duration. If we do that we get 8.5 5000 metre cube and here we have cumulative from the beginning of the rain. Doing this for each time step you will notice here that the time steps are not uniform. Here we have 0, 1, 2 and then it becomes 4, 6, 8, 10 and 12. We have a difference of 1 here; then 2 and then it becomes the difference of 4 up to here. Last point we can interpolate 148 also here but in this case it is 8 hours here. So 8 hour difference; then this is 4 hours, 2 and 1. Generally the computations are much easier if we have equal spacing of the coordinates. But in this case since the rising limp is very steep we have decided to take small intervals on this and therefore it’s not uniformly spaced. Computations are little more difficult in this case but we can still do it using the spreadsheets.

Let's look at this volume for each time step and cumulative volume. For the entire direct runoff hydrograph the volume is given as 1.43 million cubic meters. This 1.43 million cubic meters on an area, area is given as 30 and the volume. We want to find out the losses so we can find out what is the excess rain volume? In this case the volume of the rain will be same as the volume of the DRH and therefore we can convert into depth. Depth would be 1.43 into 10 to the power 6 divided by 30 kilometre square. We can convert into metre square; so, so many meters and this turns out to be 47.7 millimetres. This means that the DRH which is shown in this figure, this DRH is a result of 47.7 millimetre of rain over the entire catchment area of 30 square kilometre. The assumption is that the rainfall occurs uniformly over the entire area. In that case it should have a depth of 47.7. If you look at the rainfall hydrograph the depth, total depth is 8 millimetre per hour into 4 hours plus 6 millimetre per hour into 4 hours equal to 56 millimetre. Out of total rain of 56 millimetres the effective rain or the excess rain is only 47.7 millimetres. That means we have a loss of 56 minus 47.7; so 8.3 millimetre.

This 8.3 millimetre loss is occurring over a time period of 8 hours. Therefore we can say that phi index will be 8.3 divided by 8 which is 1.04 millimetre per hour. So if our rainfall is like this, 8 and 6 then we have a phi index of 1.04. That means our effective rainfall intensity for 4 hours this would be 8 minus 1.04 and this would be 6 minus 1.04. This is the effective rainfall, 4 hour of intensity 6.96 another 4 hour of intensity 4.96.

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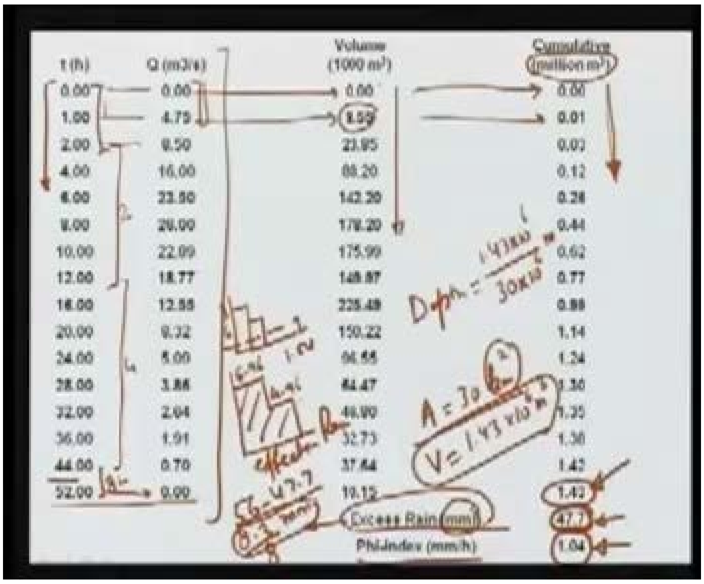

t (h)	Q (m3/s)	Volume (1000 m³)	Cumulative (million m³)
0.00	0.00	0.00	0.00
1.00	4.75	8.55	0.01
2.00	9.50	23.95	0.03
4.00	16.00	88.20	0.12
6.00	23.50	142.20	0.26
8.00	26.00	178.20	0.44
10.00	22.09	175.99	0.62
12.00	18.77	146.87	0.77
16.00	12.55	225.48	0.99
20.00	9.32	150.22	1.14
24.00	5.00	96.55	1.24
28.00	3.86	64.47	1.30
32.00	2.04	46.90	1.35
36.00	1.91	32.73	1.38
44.00	0.70	37.44	1.42
52.00	0.00	10.15	1.43
		Excess Rain (mm)	47.7
		Phi Index (mm/h)	1.04

In this way we have obtained the phi index which tells us that effective rainfall intensity for the first hour period is 6.96 or almost 7 millimetres per hour and for the 2nd, 4 hour period it is almost 5 millimetre per hour. Suppose we have a unit hydrograph available for the catchment. We can use that unit hydrograph to compute the direct runoff for storms which have duration of 4 hours. For these 2 storms which are consecutive storms of 4 hour duration we can use a 4 hour unit hydrograph to predict what will be the direct runoff. Let's assume that a 4 hour unit hydrograph is given to us. This is given. A little later we will see how to obtain or how to derive this unit hydrograph but let's for the time being assume that 4 hour unit hydrograph is given.

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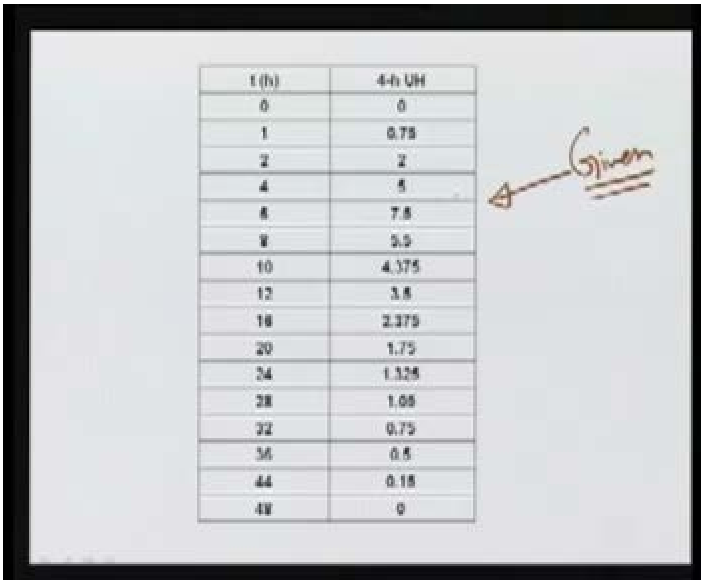

t (h)	4-h UH
0	0
1	0.75
2	2
4	5
6	7.5
8	5.5
10	4.375
12	3.5
16	2.375
20	1.75
24	1.325
28	1.05
32	0.75
36	0.5
44	0.15
48	0

You can look at the ordinates and this is the figure of 4 hour UH. As we have seen already 4 hour UH means this is due to 1 centimetre rain over a period of 4 hours which indicates that the intensity should be 1 by 4 centimetre per hour or we can get 2.5 millimetre per hour. This hydrograph is a result of intensity 2.5 millimetre per hour continuing for 4 hours. If we draw the effective rainfall this is 4 hour duration and this is 0.25 centimetre per hour or 2.5 millimetres per hour. Because of this assumption of linearity we can now obtain the direct runoff for any rainfall of duration 4 hours all of a different intensity.

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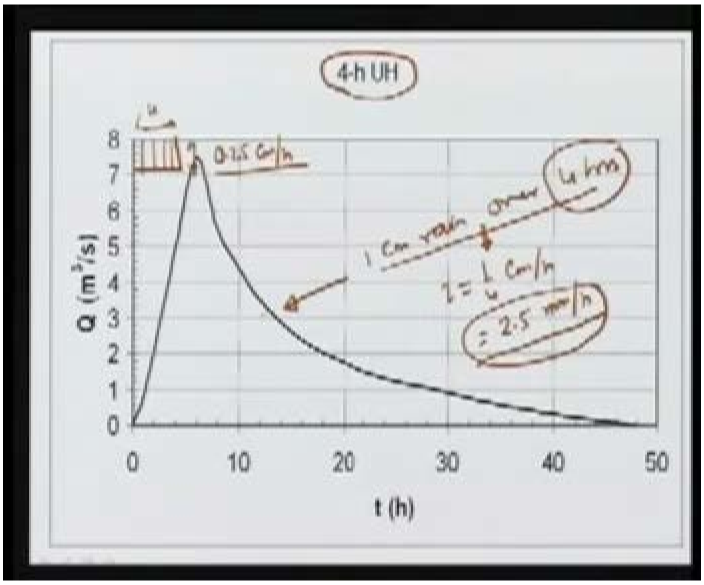

We have seen that the first 4 hour rain which we had, had the intensity of 6.96. The second had intensity of 4.96. If you want to use the 4 hour unit hydrograph to compute the DRH for this effective rainfall of 6.96 and 4.96 duration for 4 hours we can make a table like this where the first two columns show the 2 hour UH; the next column Q_1. Q_1 corresponds to a rain of intensity 6.96 millimetre per hour while 4 hour UH, we have already seen 4 hour unit hydrograph corresponds to 2.5 millimetre per hour. Q_1 can be obtained by the 4 hour ordinate multiplied by this factor of 6.96 over 2.5. This 2.1 is nothing but 0.75 times 6.96 divided by 2.5. This will give us 2.1.

Similarly for two if we replace this by 2 we would get 5.6. In this way the first column Q_1 shows the result, the direct runoff because of a rainfall of 6.96 millimetre per hour for 4 hour duration. The second rainfall is again 4 hour duration but the intensity now is 4.96. Now not only we have to multiply with this factor 4.96 upon 2.5; so Q_2 has a multiplying factor of 4.96 over 2.5 but it is also shifted. Because the rainfall is starting after 4 hours we have to shift the ordinates by 4 hour. So this zero corresponds to this zero; at 6 hours the value which we get will correspond to 6 minus 4 which is 2 hours for the 4 hour unit hydrograph. So 2 times 4.96 over 2.5 will give us 4.0. This 4.0 is the value at 6 hours. So Q_2 at 6 hours will be equal to 4 hour ordinate at 6 minus 4 hours into the factor 4.96 over 2.5. We can take any value; we can take this t equal to 10. The value here 15 corresponds to 4 hour unit hydrograph at 10 minus 4 which is 6; so 7.5 times almost factor of 2 which will give us 50. In this way we find out Q_2, lag it by 4 hours and then we add these two to get the total direct runoff because of this entire rain. Because of principle of superposition we can add these two and get this value of Q.

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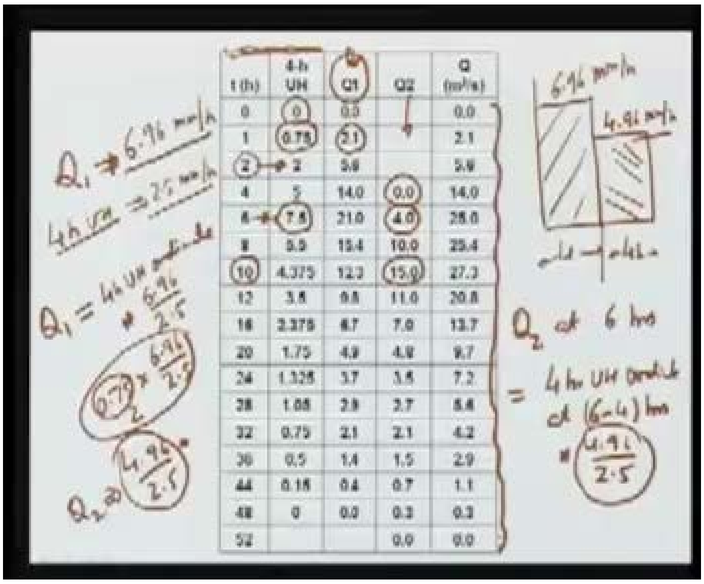

In this way we can superimpose any number of rains provided they are all 4 hour duration and we can plot this. This is from 4 hour; this is the DRH, the solid line shows the value, this value which we have obtained from the 4 hour unit hydrograph and these symbols, this we have shown as the observed from the earlier lessons of measured data at the outlet.

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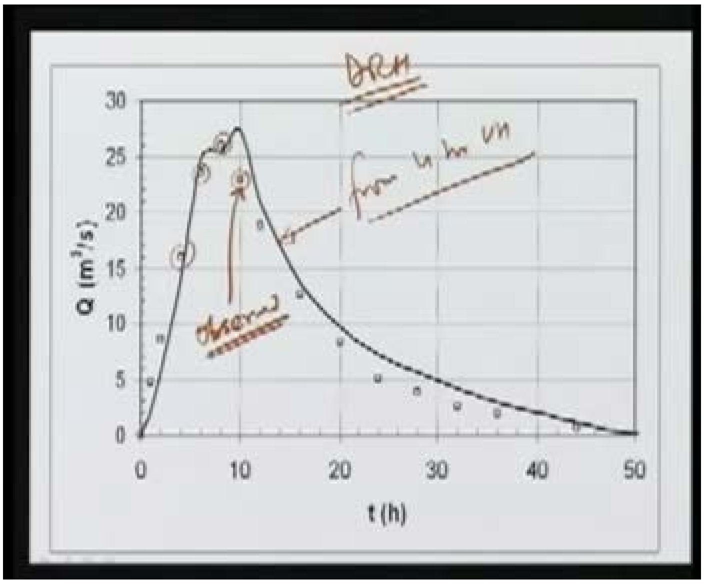

We can see that they are quite close but there are lot of assumptions in the unit hydrograph theory and therefore they are not exactly matching. But still it's a fairly good match and therefore a 4 hour UH can be used to obtain DRH for storms which are of 4 hour duration. We can derive unit hydrograph for any other duration which is a multiple of 4 hour by this principle of superposition. For example if we want to obtain 8 hour UH; for 8 hour unit hydrograph what we can say is that the 8 hour UH can be divided into two 4 hour segments. 8 hour UH corresponds to a rainfall which has intensity of 1 by 8 centimetre per hour or i is 1.25 millimetre per hour which is exactly half of the intensity which we have used for our 4 hour UH. 4 hour UH has i equal to 2.5 millimetre per hour. We can add two 4 hours UH. This is first 4 hour unit hydrograph shifted by 4 hours. If you look at these two columns, 2.5 millimetre per hour; this is 4 hour and 4 hour. This column represents the DRH because of rain of 2.5 millimetre per hour for 4 hours. This represents same thing but shifted by 4 hours because the rainfall is now starting after 4 hours. So we have 4 hour unit hydrograph ordinate shifted by 4 hours. This value corresponds to zero. The value at 6 corresponds to 2; 8 to 4 and so on. This shifting then allows us to sum them up and after summing we can get total Q and divide it by 2 because as we have seen the 4 hour UH has intensity of 2.5, 8 hour UH has intensity of 1.25. After summing them up we get an effective rain like this and then we divide it by 2 to get effective rainfall same as the 8 hour UH.

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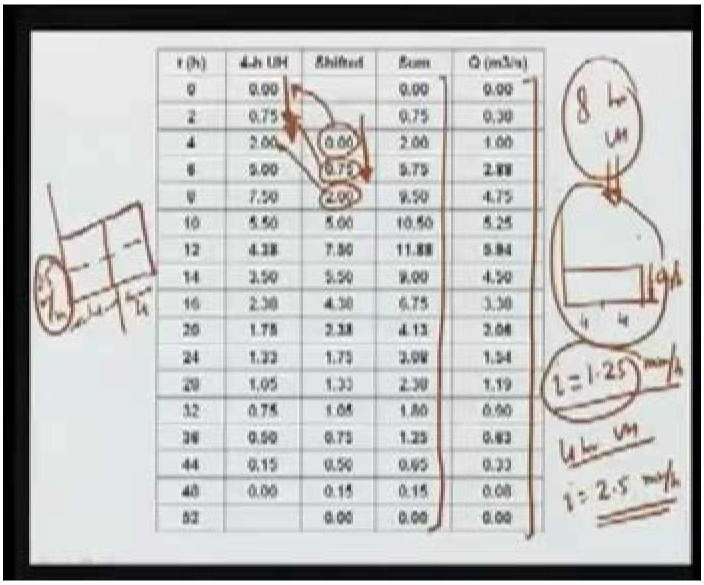

t (h)	4-h UH	Shifted	Sum	Q (m3/s)
0	0.00		0.00	0.00
2	0.75		0.75	0.38
4	2.00	0.00	2.00	1.00
6	5.00	0.75	5.75	2.88
8	7.50	2.00	9.50	4.75
10	5.50	5.00	10.50	5.25
12	4.38	7.50	11.88	5.94
14	3.50	5.50	9.00	4.50
16	2.38	4.38	6.75	3.38
20	1.75	2.38	4.13	2.06
24	1.33	1.75	3.08	1.54
28	1.05	1.33	2.38	1.19
32	0.75	1.05	1.80	0.90
36	0.50	0.75	1.25	0.63
44	0.15	0.50	0.65	0.33
48	0.00	0.15	0.15	0.08
52		0.00	0.00	0.00

In this way we can obtain an 8 hour unit hydrograph which plots this last column versus time. At different times we can get the discharge for 8 hour UH which means due to a rain of duration 8 hours and intensity of 1 by 8 centimetre per hour. So getting 8 hour unit hydrograph or let's say 12 hour unit hydrograph; if you want 12 hours then it's the same procedure because a 12 hour UH corresponds to a rainfall which is duration of 12 and intensity of 1 by 12 centimetre per hour so that the total volume is 1 centimetre. If we add 3, 4 hour UH ordinates by shifting them by 4 hours we get effective rainfall 12 hour duration but intensity of 1 by 4. After summing them up we have to divide it by 3 to get the corresponding 1 by 12.

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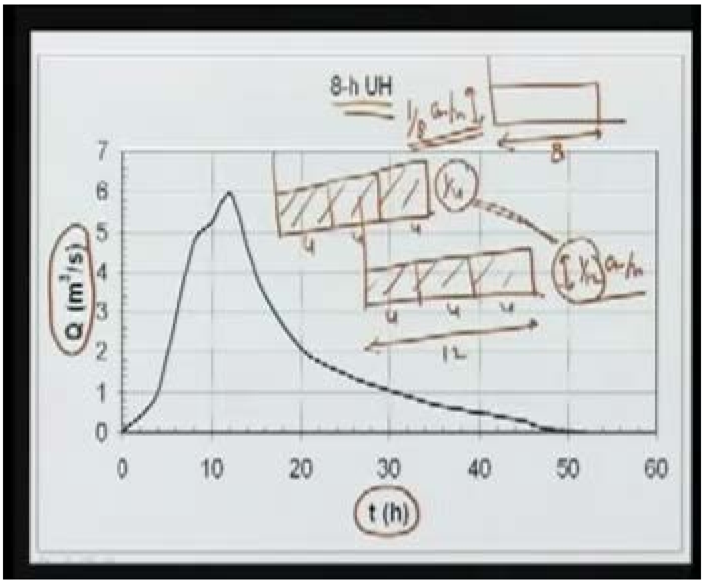

Any multiple of 4 hours we can obtain the unit hydrograph directly from the 4 hour unit hydrograph. But for any other duration which is not a multiple of 4 hours let's say we want unit hydrograph for 6 hour duration or we want 3 hour duration or 2 hour duration then we cannot directly use this principle and we have to use what is known as the S curve. The S curve indicates a rainfall of infinite duration. Suppose we are talking about a 4 hour S curve. The intensity would be 1 by 4 centimetre per hour but the duration will be infinite. What it means is that we have a rainfall which is continuous with intensity of 1 by 4 centimetre per hour or in terms of millimetre, 2.5 millimetre per hour.

We want to find out the direct runoff because of this rain, the excess rain. We again do the same thing. We assume that this is a lot of consecutive rainfalls of 4 hour duration. If we have a lot of 4 hour consecutive rainfall we can use the shifting method again. The first column shows the 4 hour unit hydrograph. We have omitted some data here and here. Then the second column shows the same data but shifted by 4 hours. This is a shift, 4 hour shift. This is 8 hour shift, 12 hour and so on. This is again the same data repeated but shifted by 4 hours every time and then we sum them up. This 21.9 is the sum of all these columns; 21.25 this is again the sum of all these columns and so on. In this way the last column here Q represents the effect of a continuous rainfall of infinite duration with intensity 2.5 millimetre per hour.

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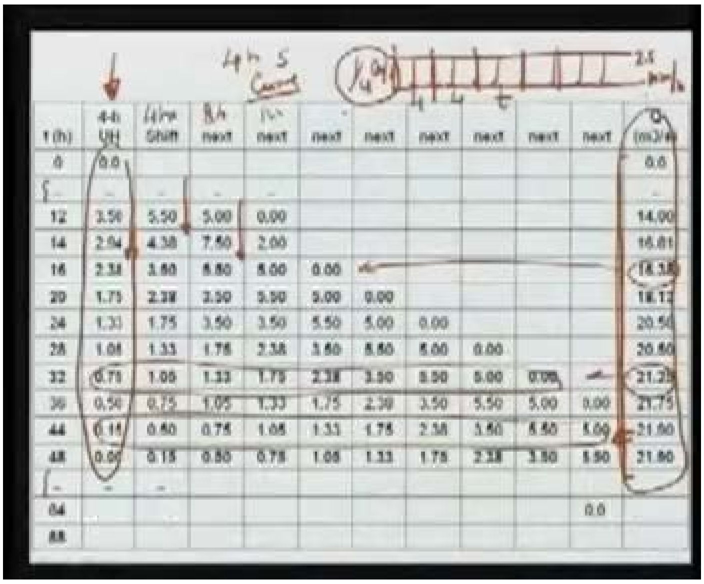

t (h)	4-h UH	4 hr Shift	8 h next	12 h next	next	next	next	next	next	next	Q (m3/s)
0	0.0										0.0
-	-	-	-	-							-
12	3.50	5.50	5.00	0.00							14.00
14	2.94	4.38	7.50	2.00							16.81
16	2.38	3.50	5.50	5.00	0.00						18.38
20	1.75	2.38	3.50	5.50	5.00	0.00					18.13
24	1.33	1.75	3.50	3.50	5.50	5.00	0.00				20.58
28	1.08	1.33	1.75	2.38	3.50	5.50	5.00	0.00			20.60
32	0.75	1.08	1.33	1.75	2.38	3.50	5.50	5.00	0.00		21.29
36	0.50	0.75	1.05	1.33	1.75	2.38	3.50	5.50	5.00	0.00	21.75
44	0.14	0.50	0.75	1.08	1.33	1.75	2.38	3.50	5.50	5.00	21.80
48	0.00	0.15	0.50	0.75	1.08	1.33	1.75	2.38	3.50	5.50	21.80
-	-	-									
84										0.0	
88											

As we can see in the end here the values will become almost constant and if we plot this S curve it would look like this. The constant value here corresponds to a rainfall of intensity 1 by 4 centimetre per hour occurring over an area of 30; in this case we have taken the catchment area as 30 square kilometre. We can convert into metre cube per second and the value turns out to be about 21 metre cube per second. This means that if a rainfall of intensity 1 by 4 centimetre per hour occurs continuously over the entire area of 30 kilometre square, at steady state it will give us discharge of about 21 meter cube per second.

If you look at this shape of the curve it is not a very smooth shape. There are lot of wriggles here. This is because our 4 hour unit hydrograph is not exactly correct. We have some approximations there and therefore in computations we can see these wriggles. If you want to make it smooth we would like to fit some kind of smooth curve like this and use that as our S curve. This S curve is because of a rainfall of infinite duration. Suppose we want the unit hydrograph for a 2 hour duration rainfall. In that case what we can do is we can shift this S curve by 2 hours which means that the other S curve is because of a rainfall again of infinite duration but shifted by 2 hour and then if we take a difference of those DRH's the difference will be caused by a rain of 2 hour duration, 1 by 4 centimetre per hour intensity. If we take two S curves shifted by this 2 hour duration, take the difference of the ordinates that difference will be because of this effective rain and if we want 2 hour unit hydrograph then we will have to multiply it by 2 because this represents 1 by 4 into 2 centimetre equal to 1/2 centimetre of rain.

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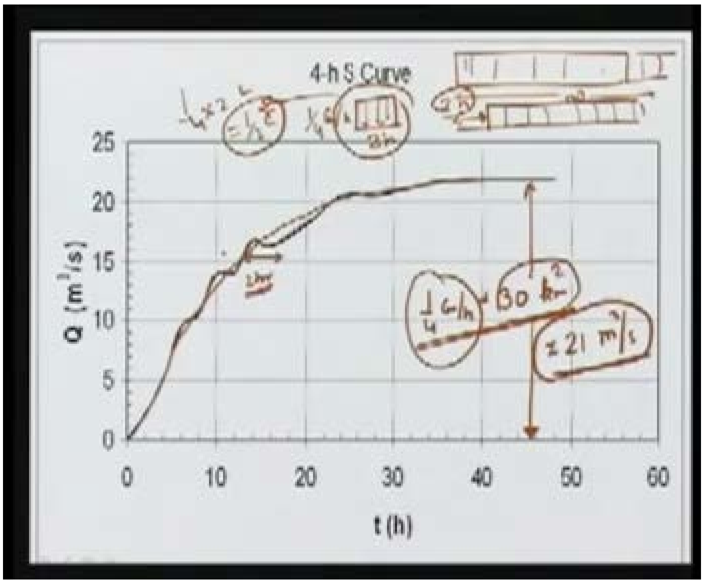

That is what we have done here in the next table. This is time. This is the smooth curve which we have fitted here. The ordinates of that smooth curve are given here and then we lag it by 2 hours. This zero corresponds to this zero with the lag of 2 hours. Similarly this 50 at 10 corresponds to 50 at 8 and so on.

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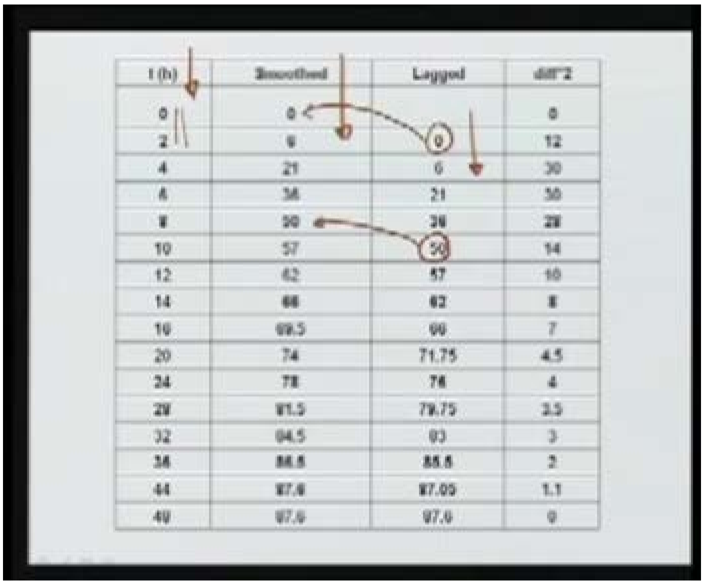

t (h)	Smoothed	Lagged	diff*2
0	0		0
2	6	0	12
4	21	6	30
6	36	21	30
8	50	36	28
10	57	50	14
12	62	57	10
14	66	62	8
16	69.5	66	7
20	74	71.75	4.5
24	78	76	4
28	81.5	79.75	3.5
32	84.5	83	3
36	86.5	85.5	2
44	87.6	87.05	1.1
48	87.6	87.6	0

This difference is multiplied by 2 so that we get unit hydrograph for 2 hour duration and this is plotted in this figure. You can see that a 2 hour unit hydrograph is obtained using the S curve; two different S curves one starting from zero, the other lagged and then we take the difference, multiply it by 2 to get the 2 hour unit hydrograph.

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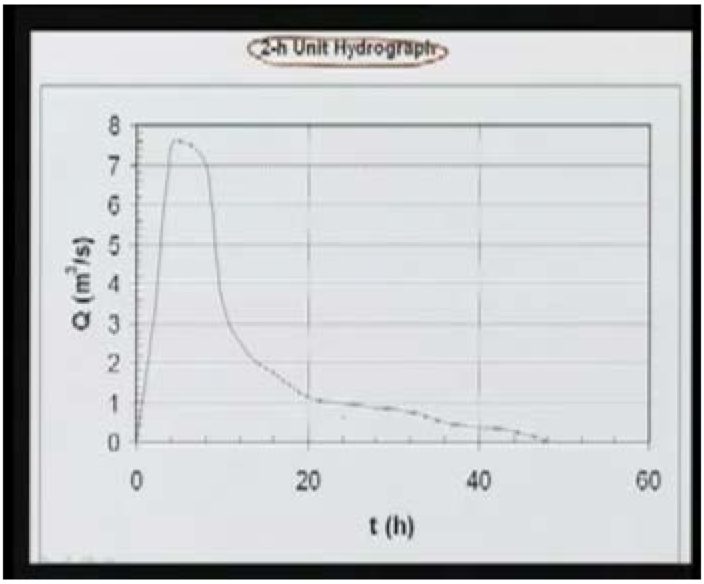

In this way we can obtain the unit hydrograph for any duration which is not a multiple of the given duration. It may be less than 4 hours it may be more than 4 hours; for example 5 hours also, 6 hours also. We can do the same thing; we plot the S curve lag it for the given time period. Suppose we want to derive a 6 hour unit hydrograph; we can make the 4 hour S curve lag by 6 hours. The resultant difference is because of a rain of intensity 1 by 4 over a period of 6 hours. That means a total rainfall of 1.5 centimetres. Whatever ordinate we get as the difference we have to divide them by 1.5 and we will get the 6 hour unit hydrograph.

In today's lecture we have seen how to obtain the flow duration curve from the stage measurement at the given location; how to obtain reliability like what is the reliability of a discharge 85% or 90%? How much time it will be available in the river. In other words we can say what is the 85% or 90% dependable discharge which will be equalled or exceeded for a certain percentage of time? Then we have seen how to obtain a direct runoff hydrograph from the total runoff measured at a point by separating the base flow. There are lot of techniques of base flow separation but we looked at one particular technique which works very well in this case. After separating the direct runoff hydrograph we can find out the volume of direct runoff and therefore we can find out the effective rainfall and from the total rainfall and effective rainfall we can obtain a phi index which will then give us effective rainfall hydrograph or ERH.

From an ERH and DRH we can obtain unit hydrograph which we will see little later. But suppose we know the unit hydrograph for a particular duration and we have rainfalls of the same duration occurring we can obtain the direct runoff because of these rainfalls of same duration by using the unit hydrograph for the duration of 4 hours. We have seen how to obtain DRH given the unit hydrograph and we have also seen how to obtain unit hydrograph for different durations which may or may not be a multiple of the duration of the given unit hydrograph.