Water Resources Engineering

Prof. R. Srivastava

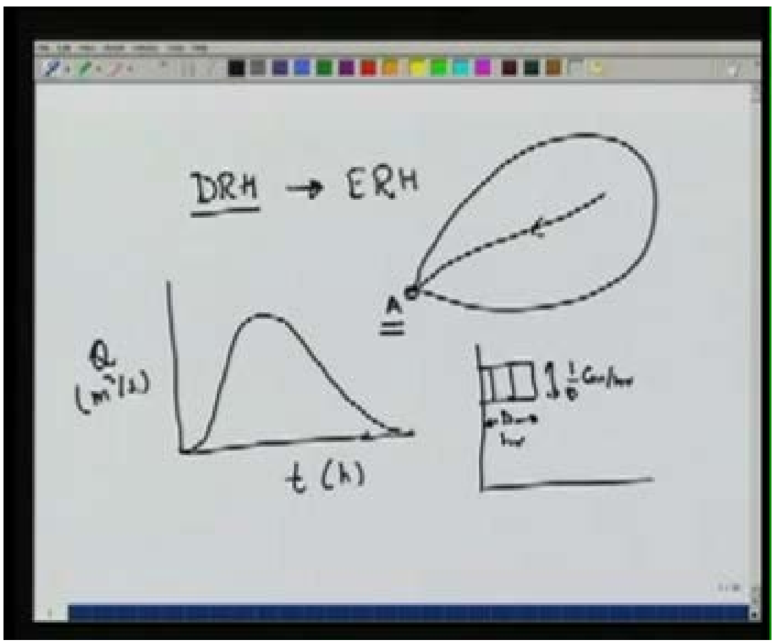
Department of Civil Engineering

Indian Institute of Technology, Kanpur

Lecture No. # 03

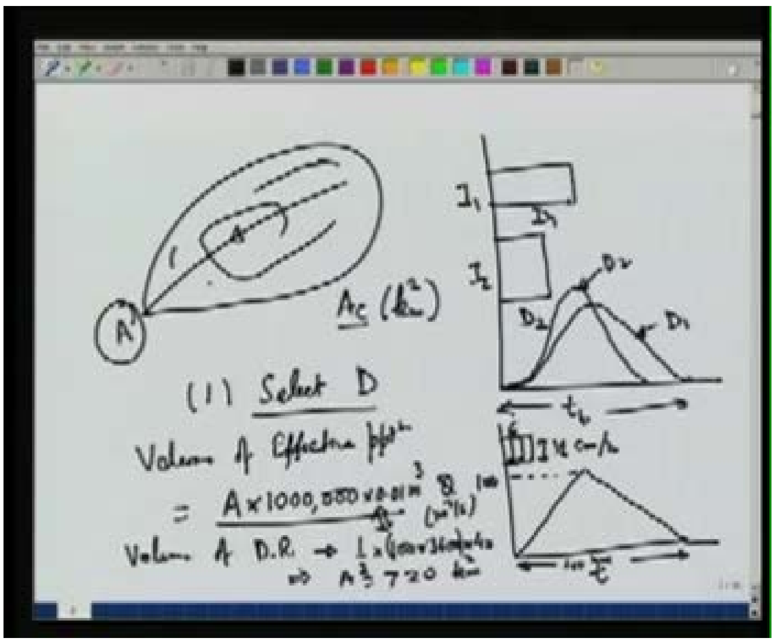
In today's lecture we are going to look at some aspects of the unit hydrograph. We will see how to obtain unit hydrograph for a basin and how to use it to predict the runoff for any given rainfall. As we have seen, the unit hydrograph describes the direct runoff due to a unit depth of rainfall, uniformly distributed over the whole catchment area. So let us look at the catchment area.

(Refer Slide Time: 00:49)



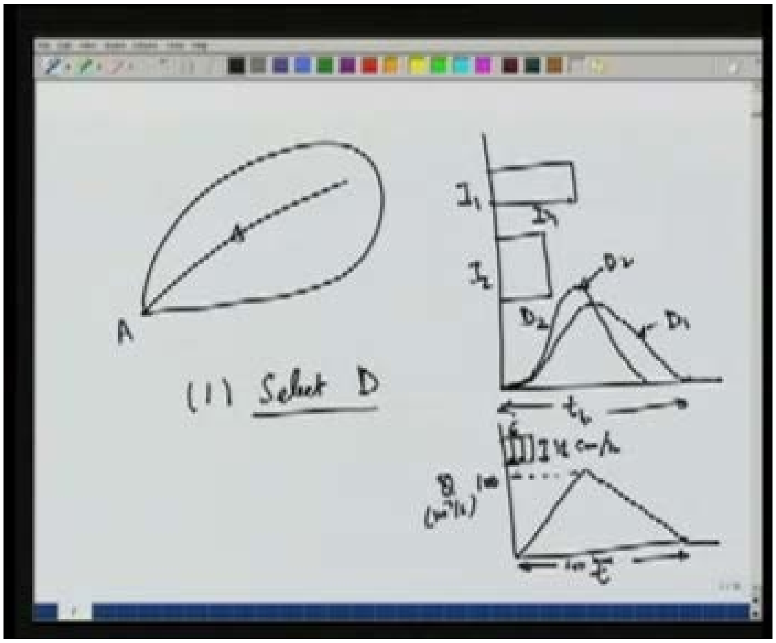
We are measuring the runoff or the stream flow at point A. Suppose the rainfall occurs over the basin or the catchment, the requirement is that it should be uniform all over the catchment and for the unit hydrograph the depth of rainfall should be unit which we take as 1 cm. At any given duration, D hour, then the intensity of the rain would be 1 hour D cm per hour, so as to get total rainfall depth of 1 cm. Now if this effective rain, when we say effective rain, it means rainfall minus whatever abstractions have taken place, so this is the part which directly contributes to direct runoff. We have already seen that the runoff consist of mainly two components, base flow and direct runoff. The direct runoff is the part which occurs immediately or with a very little land after the rainfall. What we are interested in is correlating the direct runoff DRH, the direct runoff hydrograph with the effective rainfall hydrograph ERH. If a rainfall of duration D hour occurs with the intensity of 1 hour D cm per hour indicating the total rain effective rain is 1 cm and if we measure the direct runoff at point A, that would give us the unit hydrograph of D hour duration and it would look like this (Refer Slide Time: 03:10). Where the time may be in hours, the stream flow is generally in meter cube per second. The use of the unit hydrograph is to estimate the runoff for any rainfall. For example we can easily estimate the amount of rainfall which is likely to occur once in 100 years and based on that pattern of the rainfall, we can apply the unit hydrograph and estimate the runoff for that once in 100 year rainfall. We can use that to design flood control structures. So let us look at the methodology of deriving the unit hydrograph for a basin.

(Refer Slide Time: 03:58)



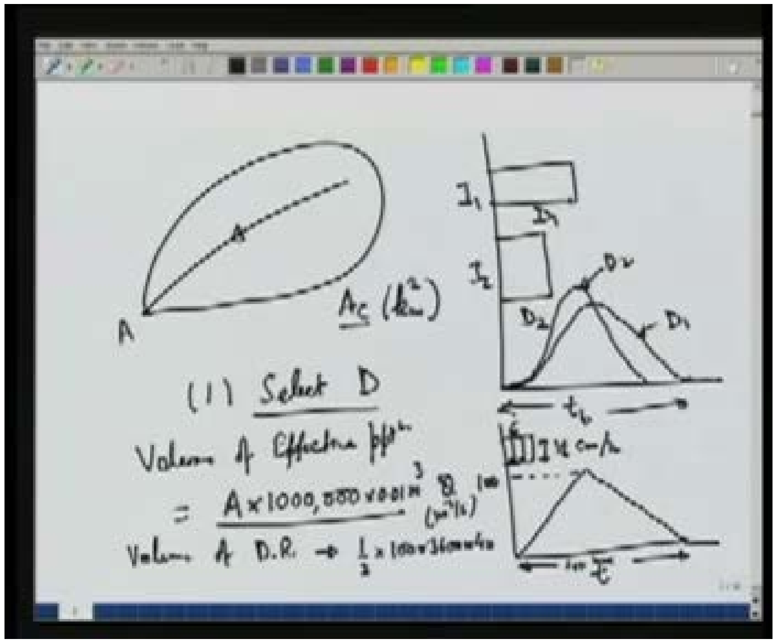
The first step is to select a duration D because the unit hydrograph changes with duration. If we have a rainfall of longer duration the intensity would be smaller to maintain the unit area of 1 cm. If we have a rainfall of shorter duration, the intensity would be higher in such a way that I_1 D_1 should be equal to 1 and I_2D_2 should also be equal to 1 and naturally these two different rainfalls will give us different hydrographs. If we have a shorter duration, the time base will be smaller and the peak would be larger to maintain the same area. So for duration D_1 we would have longer time base of the hydrograph and for the rainfall of duration D_2, we will have a smaller base but a higher peak because the area under the hydrograph should be equal to the rain falling over the catchment. In fact if we look at an ideal or theoretical unit hydrograph and assume it to be rectangular then we can see that the area of the rain or let us say the volume of the rain falling over the catchment would be equal to volume of the direct runoff at point A. Now let us just for example take this Q peak flow as 100 meter cube per second and let us say the time base is 40 hours. Let us say that the rainfall which occurs is of 6 hour duration with intensity of 1/6 cm per hour.

(Refer Slide Time: 06:42)



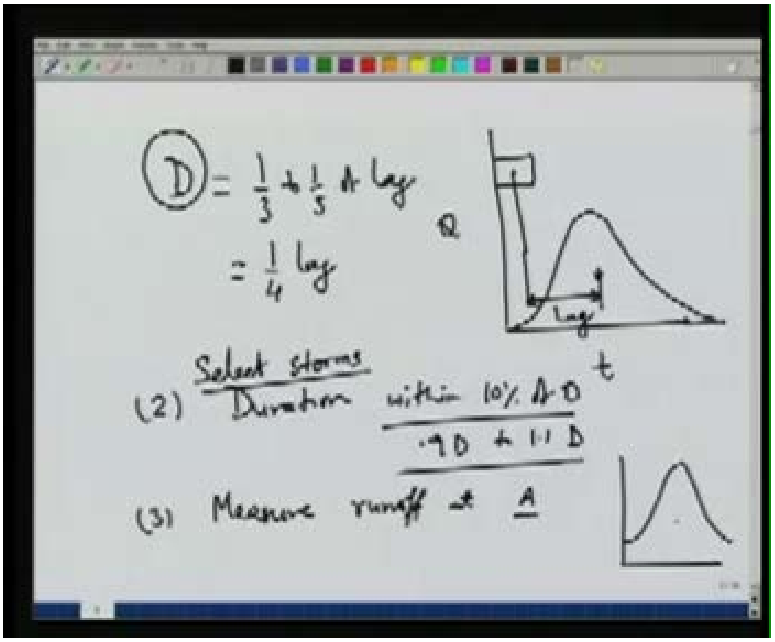
So the area of the rain is 1 cm. The hydrograph is 1 cm. This is the effective depth of rainfall which is falling over the whole catchment. Let us say that area of the catchment is AC and typically these areas are measured in km square.

(Refer Slide Time: 8:21)



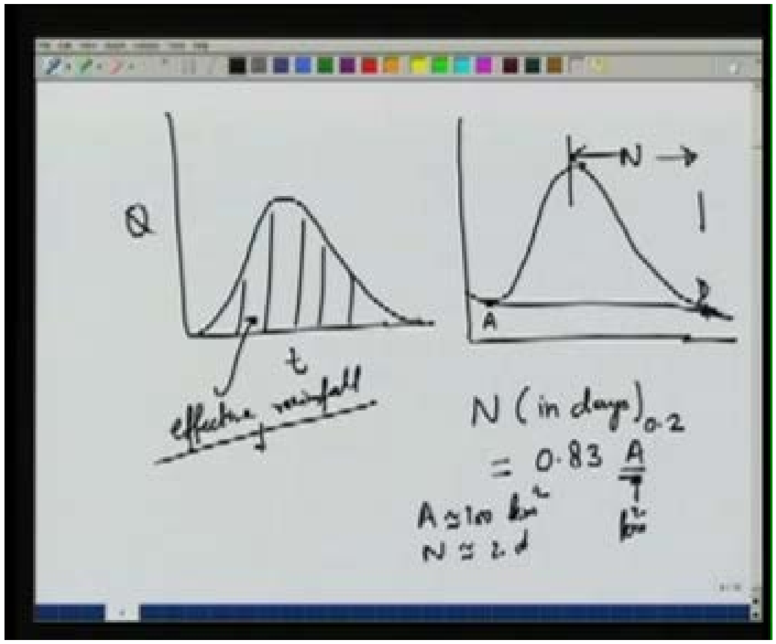
Therefore the volume of effective precipitation would be A in km square. So we will multiply it with 1000, 1000 to get in meter square the depth of rainfall which is 1 cm. It would be 0.01m. This is the volume of the effective precipitation because we are saying that the 1 cm of depth falls uniformly over the entire catchment area. Now this should be equal to the volume of direct runoff which we can obtain from this direct runoff hydrograph and if you look at this figure, the area would be 1/2 in to the peak flow. This peak flow typically is in meter cube per second. So we have to multiply it by 3600 to get in m cube per hour and then multiply by the time base assuming this to be a triangle. The area will be given by ½ in to peak flow in to the base and these should be equal. If we equate these two, then the area of the catchment comes out to be 720 km square. So this tells us that the area under the direct runoff hydrograph and the volume of rain falling over the catchment should be equal, so that gives us an idea of the sketching the hydrograph in such a way that the area matches with the catchment area to select the duration. We have to define or we have to select what is the best or most likely duration of rain fall over the catchment. There are some empirical relations.

(Refer Slide Time: 09:16)



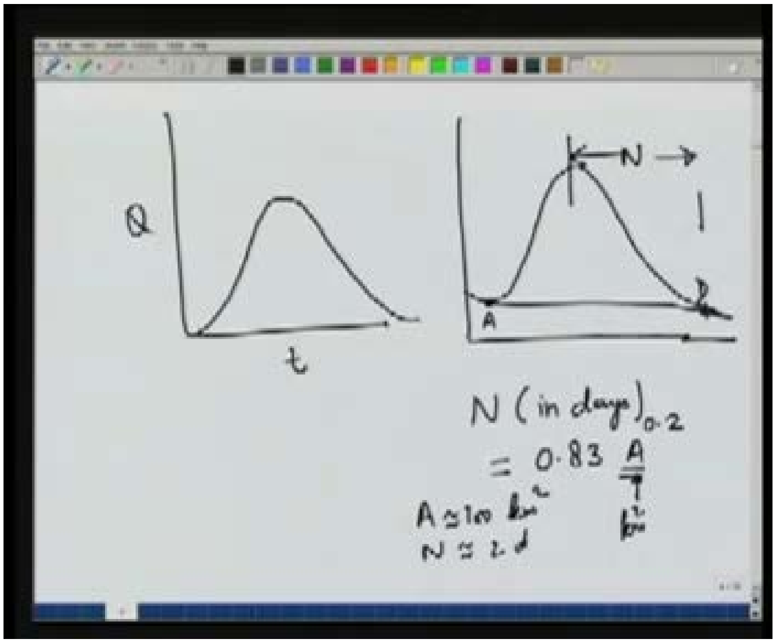
For example one of the relations is that if you find out the basin lag, which is defined as the time lag between the centre of the precipitation, this point and the centroid of the runoff, the time lag between these two points is known as the basin lag. A good duration for the unit hydrograph should be about 1/3 to 1/5 of the lag. After study of various lessons, it has been found that this would be an ideal location and typically 1/4 of the lag is taken as a good unit hydrograph duration D. Now once we select the duration D, the second step would be to select a few storms which occur over the catchment area with duration D. In nature typically the storms will not occur with exactly the same duration. So we can say that if the duration of rainfall is within 10 percent of D. This indicates that it should be let us say 0.9 D to 1.1 D, so we select a few storms in which we have uniform flow occurring over the entire catchment and the duration of rainfall is 0.9D to 1.1D. We may get 3 or 4 such storms and for each storm, we select storms with duration within this 0.9 to 0.1 D. For each storm, we measure the runoff so at the outlet of the catchment area which is this point (Refer Slide Time: 11:58), due to a uniform rain over this entire catchment area falling in duration 0.9 to 1.1 D. We measure the stream flow and once we get the stream flow we have already seen various methods of separating the base flow and get the direct runoff hydrograph. We have already discussed 3 different methods of separating the base flow. There is another empirical method which is sometimes used to locate the end of the direct runoff.

(Refer Slide Time: 12:34)



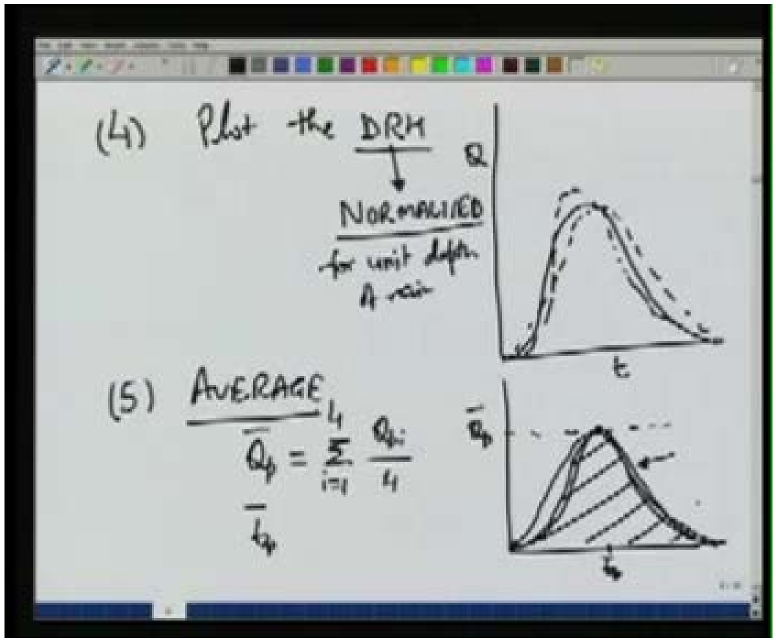
If this is the total runoff, the starting point A is easy to see because this is the point at which the hydrograph starts raising the ending point D. This is estimated by I to see where the base flow has started. Direct runoff has entered but there are some empirical relations which give us the location of the point D and one of the commonly used equation is that from the peak, the point D has a distance of N in days which is related to the catchment area where A is in Km square. If we know the catchment area which of course we know all the time, we can estimate the distance between the end of direct runoff from the peak of the hydrograph and for example if area is about 100 km square then N would come out to be about 2 days. If you have this hydrograph measured, you can estimate the end of the direct runoff. By using this equation, separate the base flow by using any of the methods.

(Refer Slide Time: 14:31)



For example the straight line method and the rest of the curve would be the direct runoff hydrograph which would look like this. So we measure runoff at A, we then estimate the direct runoff hydrograph using the techniques which we described now. We also should be careful that the area under this curve which represents the effective rainfall should be close to 1 cm because that is what our aim is. We need to derive hydrographs for 1 cm of rain. This should be 1 cm roughly but even up to 4 cm, storms can be used and once we find the area, it will divided by that ordinate. So let us look at the next step.

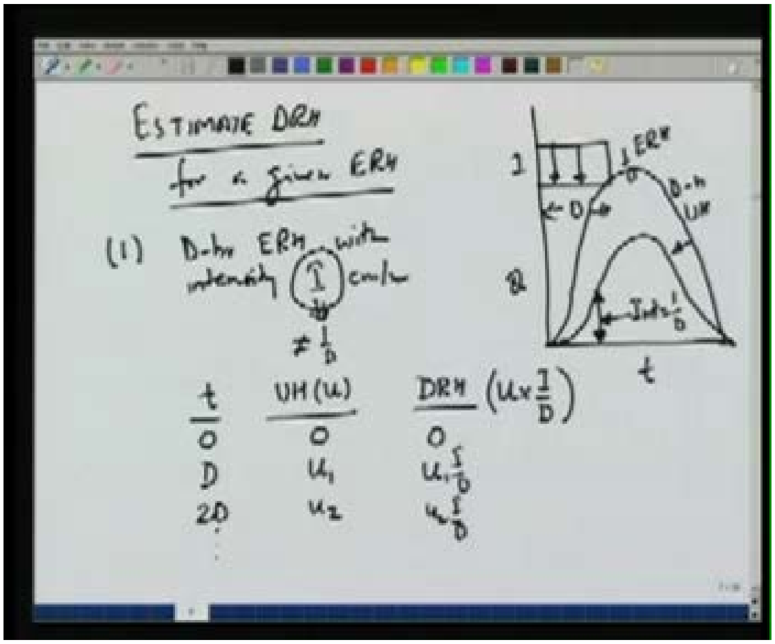
(Refer Slide Time: 15:30)



The 4th step would be to plot the direct runoff hydrographs. Suppose we have a 3 or 4 different storms which we have selected and obtained the direct runoff hydrograph for those 3, then under ideal conditions all the DRH should be same because we have assumed that the runoff is time invariant and linearly related. So we plot the DRH and when we say DRH, all these are normalized, which means divided by the depth of rain or the area under the curve. So all these DRH, (for unit depth of rain) should be same. But in nature we have a lot of variations. All the assumptions which we have made in the unit hydrograph theory will not be true. For example storm may not be uniformly distributed over the area. It may be moving or it may not be covering the whole area at a time. All the 3 - 4 storms we have selected, will have different DRH and our aim now would be to average these and the best way to take the average is to take average of all the peaks.

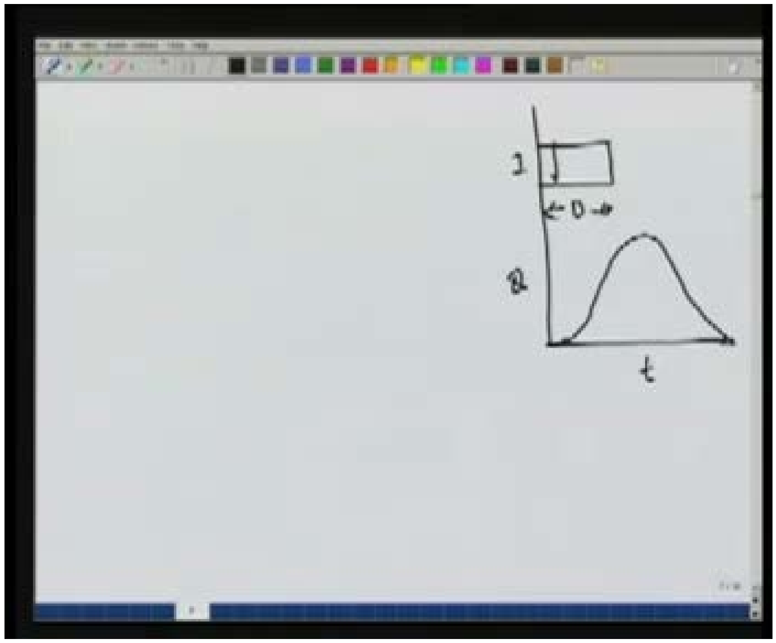
So suppose we have 4 storms, we add all the peaks and divide by 4. So this average peak will be marked on the curve and we also find out what is the average time to peak. So similarly we find out t_P bar and this way we will locate the peak of the hydrograph. Now we draw an average line looking at the individual hydrograph. We try to draw an average line for the rising limb and then similarly an average for the falling limb. Once we average out all the variations in these 3 - 4 hydrographs, this is an averaged unit hydrograph and the next step would be to find out the area under the curve which should be equal to 1 cm rainfall depth. Since we have drawn it approximately, the area will not be exactly 1 cm and if the area is more than 1 cm, we can reduce the area by moving our averaged curve a little so that the area becomes 1. If it is less than 1, then we would increase this and try to get a different curve. So this will be an iterative process in which we have to draw a number of curves and fix the one which has an area of 1 cm of rainfall depth, over the catchment area as the unit hydrograph. This way we can obtain the unit hydrograph for any basin, provided we have storms of that duration. They are uniform, over the area and all the assumptions are satisfied. Now once we get the unit hydrograph, the next step would be to use the unit hydrograph to estimate runoff for any given rainfall.

(Refer Slide Time: 20:28)



The simplest case would be when a storm, of the same duration as the one for which we have developed the unit hydrograph occurs,

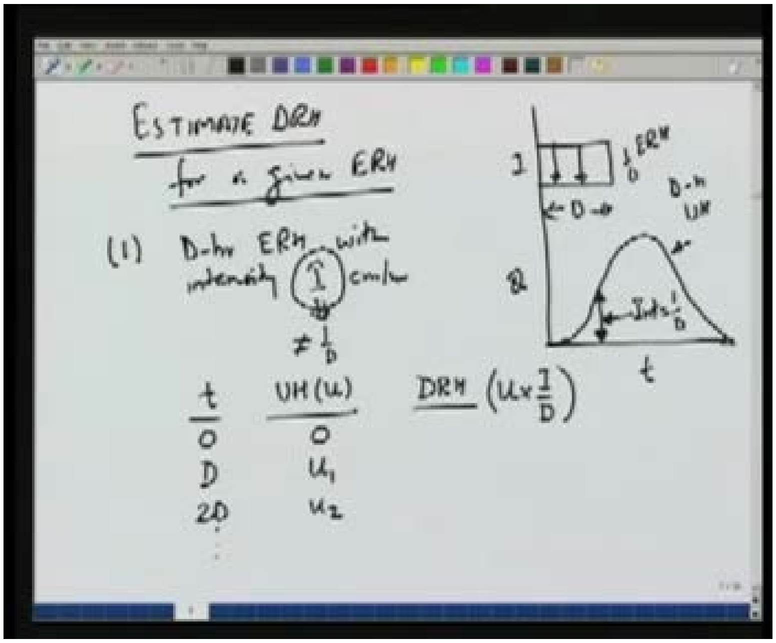
(Refer Slide Time: 21:00)



suppose this is the ERH having intensity 1/D and duration D and this is the unit hydrograph corresponding to duration D, we will call it D hour unit hydrograph. How to estimate DRH for any given rainfall pattern? For a given ERH the main purpose of developing the unit hydrograph is to estimate DRH. So let us look at a few cases, the simplest case which we are looking at now is a rainfall which occurs for the same duration but with a different intensity. So let us say case one would be D hour ERH with let us say intensity is I cm per hour and I is not equal to 1 hour D because if I is equal to 1 hour D then whatever we have derived would be giving us the DRH. Now we have to invoke some of the principles which we have discussed earlier. In one was the principles of linearity, we said that if a rainfall of some intensity occurs and we have a DRH for that rainfall and then if a rainfall of let us say twice the intensity occurs then all the DRH ordinates will be doubled.

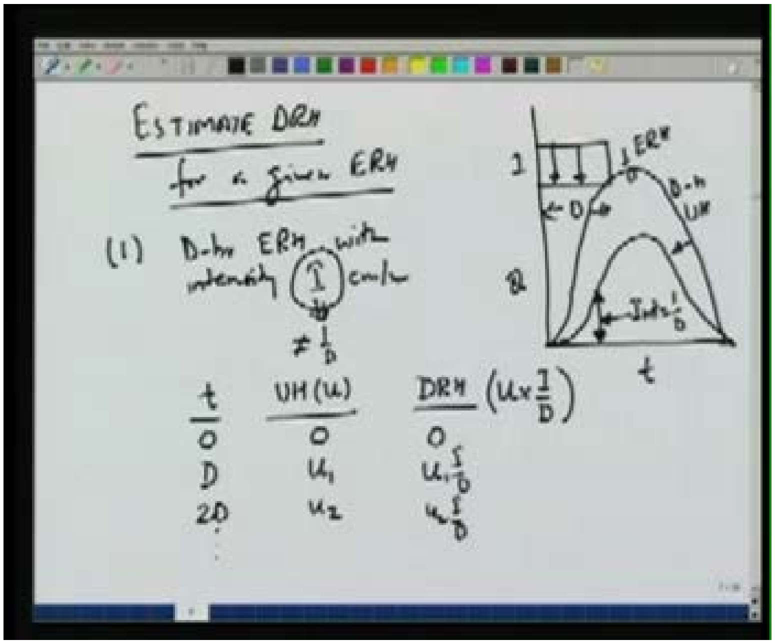
So we will use the same principle here. We know that this is ordinate DRH. Since this is ordinate corresponds to I equal to intensity, equal to 1 hour D. Now if we have a different intensity with I, then this ordinate will be multiplied accordingly with that intensity. Generally before computers were so popular, it was all done in tabular forms, we used to make a table of time unit hydrograph ordinate. Generally unit by u, the direct runoff hydrograph ordinate, so at time t = 0. Naturally u will be 0. Let us say at some time, we can have the time interval same as D or we can have 1/2 D or 2D interval. Let us take the interval as D, then at time t = D, the ordinate of the unit hydrograph is u_1 and at 2D, it is u_2 and so on.

(Refer Slide Time: 24:43)



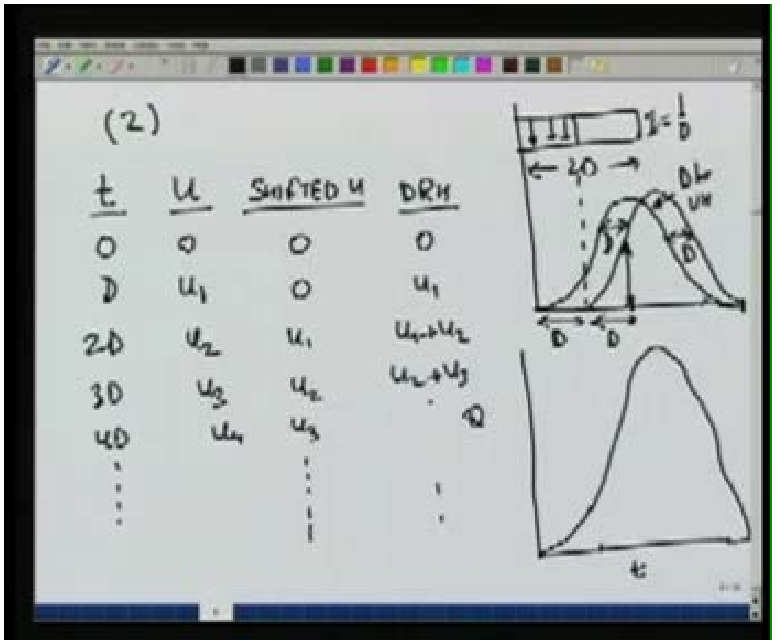
Now if a rainfall of intensity I occur then the DRH ordinate will simply be u into I_1 over D because 1 hour D is the intensity for which we have unit hydrograph and I is the actual intensity of rainfall. So DRH ordinate will be $0u_1I$ over D, u_2I over D which is basically the same as this whole curve will shift up or down depending on whether I is more than 1 hour D or less than 1 hour D.

(Refer Slide Time: 25:20)



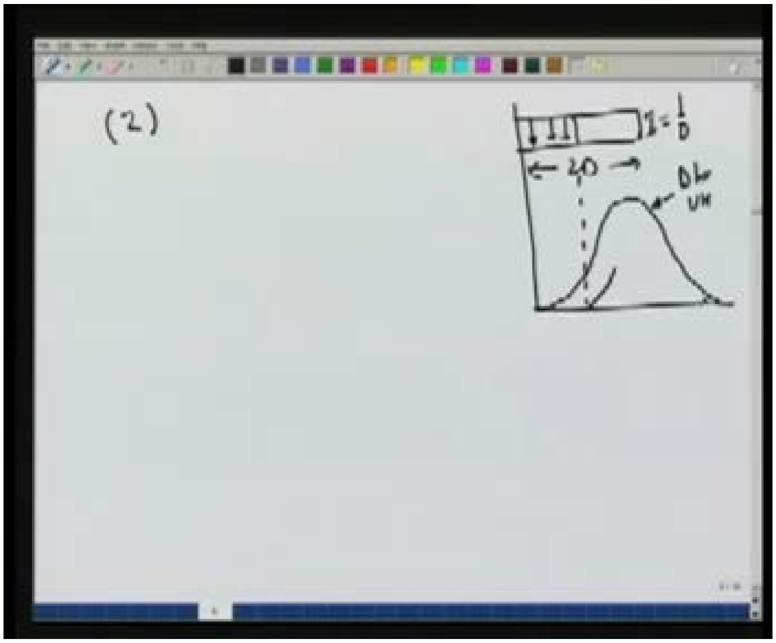
If we look at a case where I is 1.5 D, 1.5 over D, the hydrograph will look like this. All the ordinates will be multiplied by 1 and a ½. Now this was the simplest case where the duration of the rainfall was same as the duration for which we have derived the unit hydrograph.

(Refer Slide Time: 25:32)



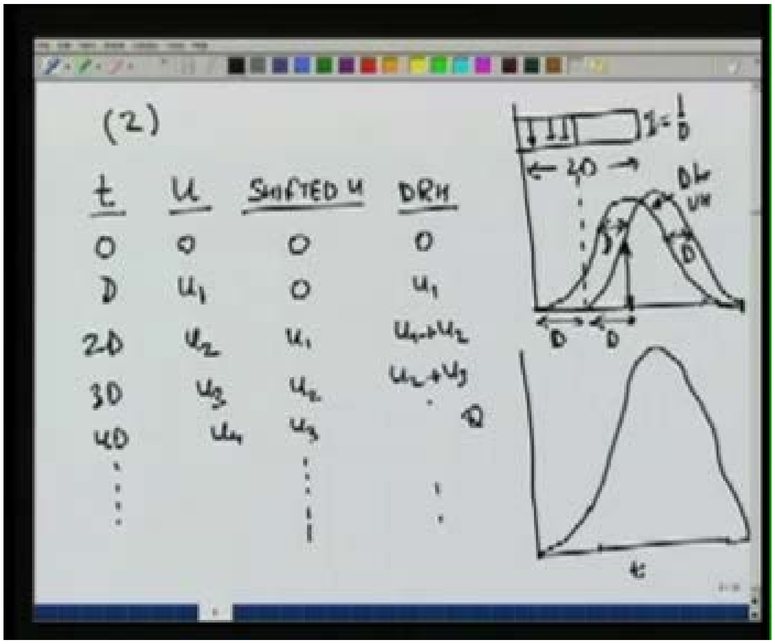
There may be cases when duration is more than D or less than D. The next case which we will discuss is, suppose we have rain, our ERH is now of duration 2D. We have a D hour unit hydrograph and we want to estimate the DRH for a rainfall of duration 2D and intensity. Let us assume that there is a constant intensity I to make the calculations easier we would assume that this I is 1 hour D, but this I can be anything here. We would use again the concept of time invariance and linearity. So if we use the concept of time invariance and linearity it leads us to method of support position. So we say that if there are two different storm events occurring then we can add the runoff due to each single event and get the total runoff.

(Refer Slide Time: 27:26)



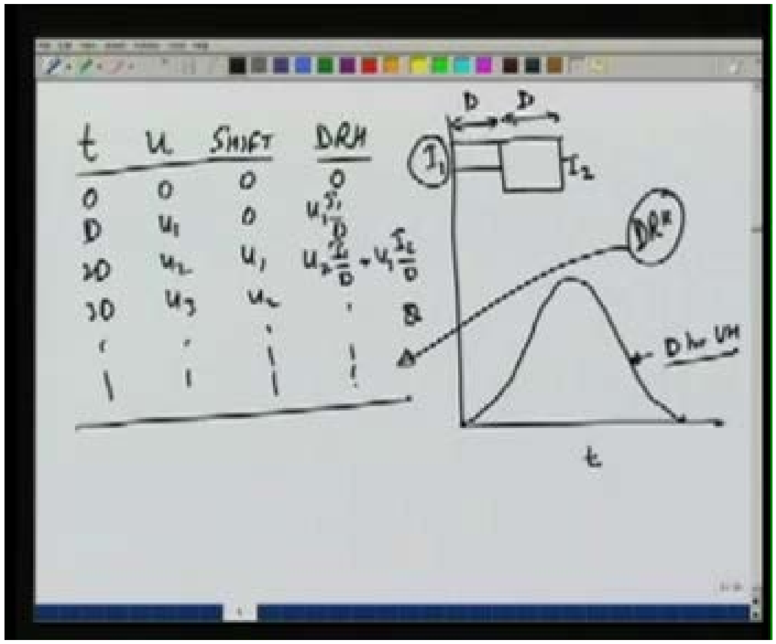
If this is the D hour for the first storm it would work fine but for the second storm we can think of the second storm as the storm starting at time T = D and then the direct runoff hydrograph for this would be identical but shifted by duration D. So all the ordinates of this curve would be shifted by D and then to get the total runoff, we just add these 2 curves and get the total runoff. For example in this case up to D the curve will look exactly the same, because there is no second curve. Beyond that we need to add these two and get a different curve and the time base will also extend up to the end of the second UH. In this way once again we can do it in the computer very easily. Earlier we use to do it using tabular forms. If we do the tabular form calculation in this case, we have this known to us. I am showing here 0D, 2D, 3D these do not have to be at D. They can be 1/2D, they can be 2D intervals but let us assume that the ordinates are tabulated at every D.

(Refer Slide Time: 30:45)



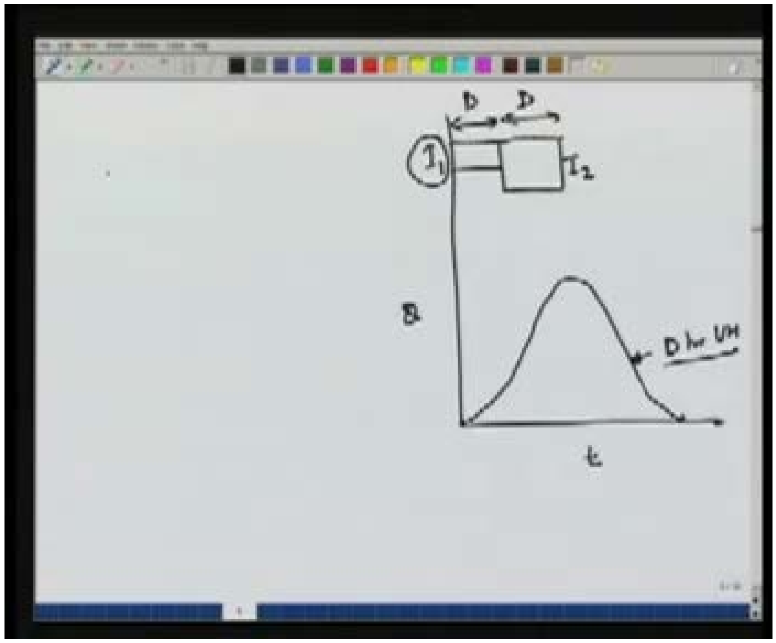
If you look at the procedure here, in addition what we are doing is we are shifting the second curve by an amount D, so we would call shift and write shifted u for the first and second time it would be 0 because we are shifting all the ordinates by an amount D and then for 2D which is the time here the ordinate of the second would be u_1 which was the ordinate of the first curve at time D. So all the ordinates are now shifted and then we have to add these 2 to get our DRH and so on. This was for the case when the intensity of rain was constant and was equal to 1 over D. If we have a different intensity then we can accordingly multiply the ordinates by the intensity.

(Refer Slide Time: 30:58)



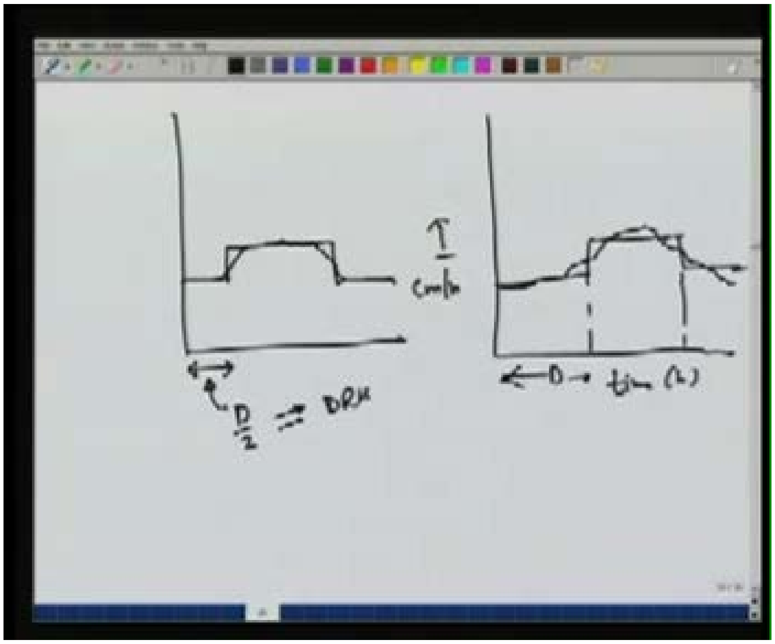
For example let us say that the ERH looks like this where this is intensity I_1 this is intensity I_2, we still assume it to be same and a multiple of D of course.

(Refer Slide Time: 32:09)



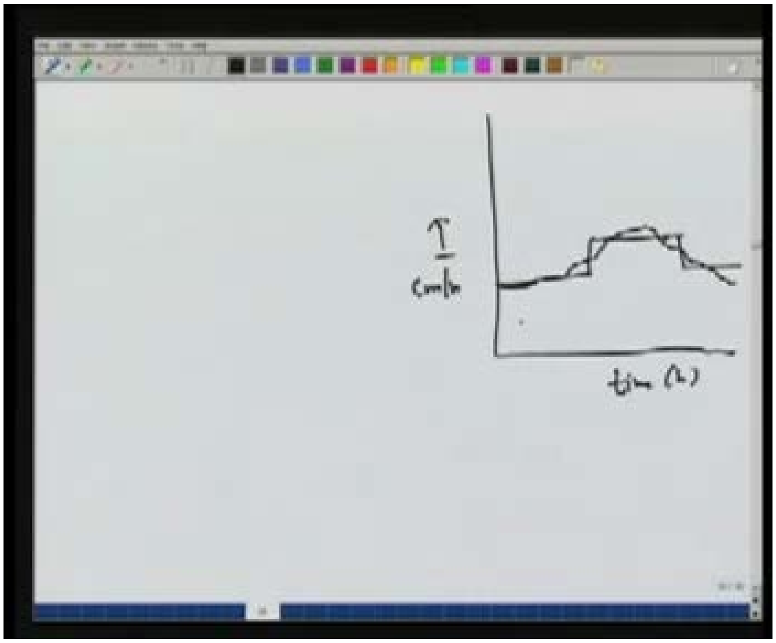
We will have all these terms we considered here, occurring over a period of D so that we can use the D hour unit hydrograph. This is the D hour unit hydrograph. In order to find out the runoff due to this combination of rainfall we would again use the same tabular form, but now we would multiply the ordinates by I_1 over D. In this case, table would look like this. You have t u, we can shift it and then the DRH would be the first ordinate multiplied by I_1 over D plus the second ordinate multiplied by I_2 over D, so $0u_1I_1$ over Du_2I_1 over D plus u_1I_2 over D and so on. In this way even if the intensities are varying we can find out the DRH in tabular form and these days with the advent of computers, it is very easier to do these computations in spread sheet and we will not discuss it here. But the procedure remains the same that you will shift the UH multiplied by the intensity and add them to get the DRH. These are simple cases when the duration of the rainfall was same as the duration of the unit hydrograph.

(Refer Slide Time: 33:50)



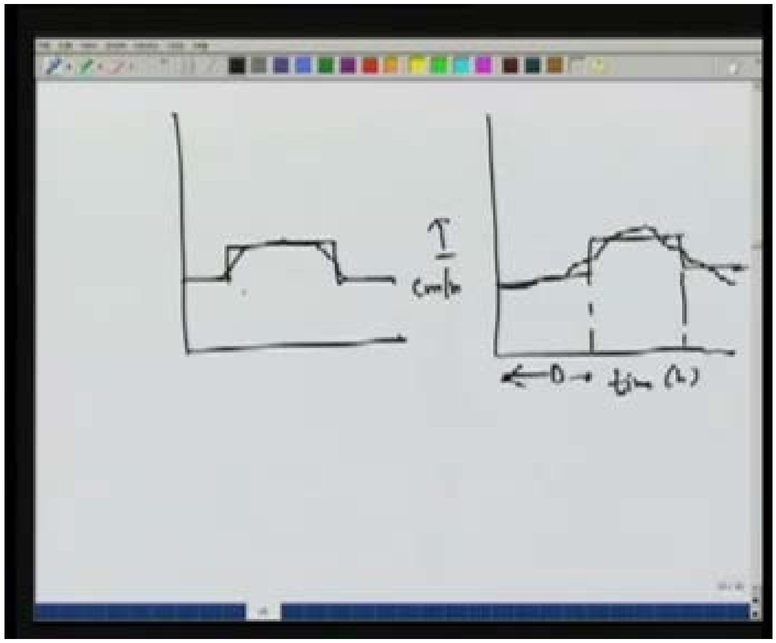
There may be cases when the rainfall occurs for a different duration and the intensity of the rainfall may not be constant for a duration which is very long.

(Refer Slide Time: 34:46)



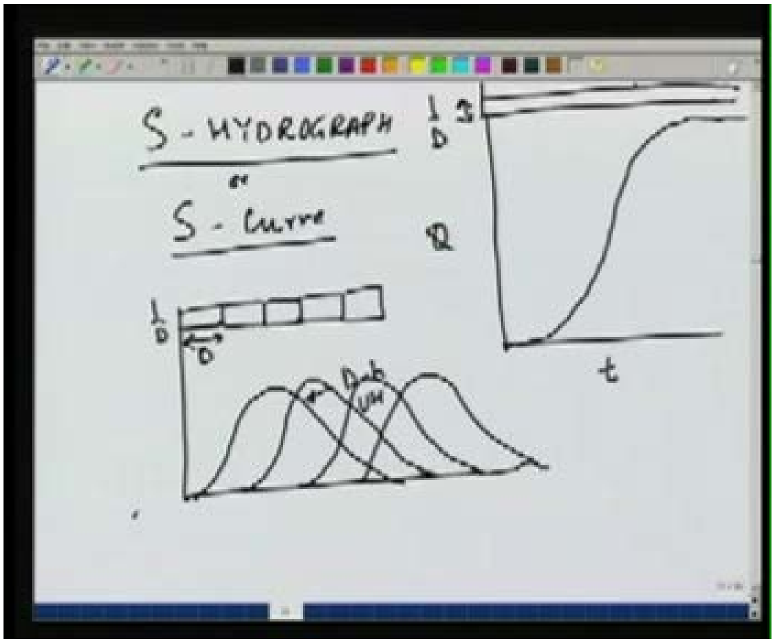
What we would do most of the times is we assume that it remains constant for some time. For example if the rainfall hydrograph is given like this, we may assume it to be constant for some time. So we may idealize that like this and then based on that we can analyze and find out the DRH.

(Refer Slide Time: 35:21)



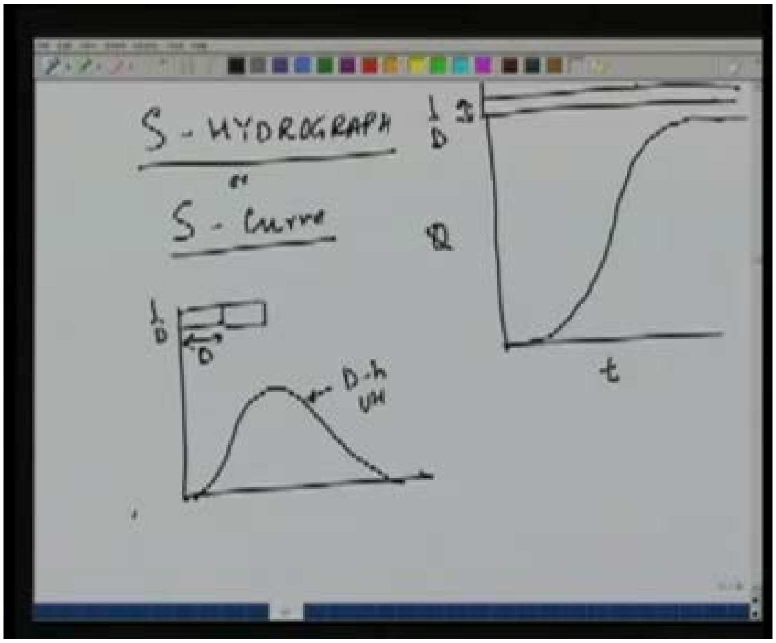
If suppose we idealize this as D hours rainfall, then we can use the D hour but if the rainfall cannot be idealized like this, suppose the rainfall looks like this then we may be able to idealize it like this, in which case, the intensity of rainfall is constant for a duration. This is smaller than D or not a multiple of D. So let us make it D/2. Now the question is how would we obtain the DRH for this rain? This because the DRH which is given to us has a duration of D and this rainfall which is given ERH has a duration which is smaller than D. Now to solve this problem we introduce a concept which is known as S curve or S hydrograph.

(Refer Slide Time: 36:11)



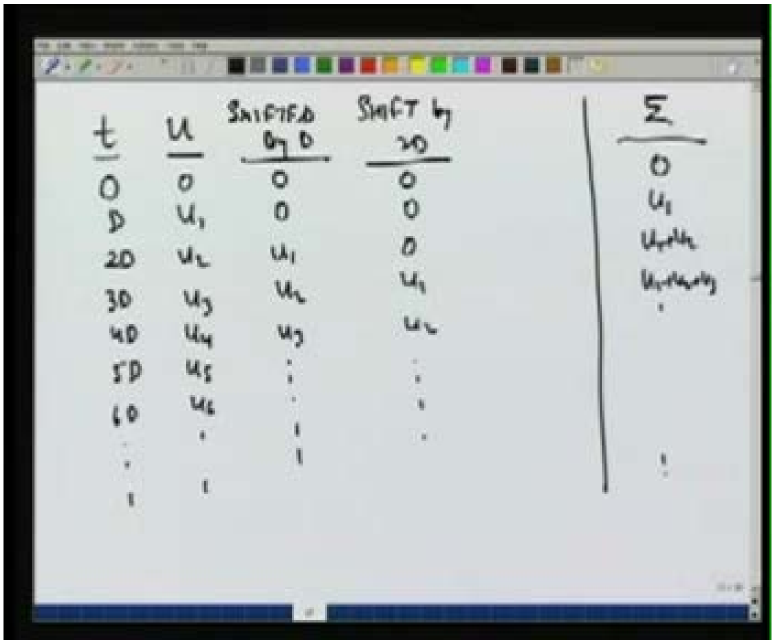
What this hydrograph is is the direct runoff hydrograph for rainfall of a certain intensity occurring for infinite time. If the rainfall occurs for a very long time uniformly over the entire catchment, then the relating DRO DRH would be called S hydrograph. It looks like this and since it looks like an S that is why the name S hydrograph or S curve. The ERH for this case is a rainfall of infinite duration with certain intensity. Let us say 1 over D, if we obtain the S curve for a given rainfall, let us look at how to obtain S curve for the given duration D, if we know the D hour unit hydrograph. D hour unit hydrograph is given to us and the rain which is causing this is of duration D and intensity 1 over D.

(Refer Slide Time: 38:22)



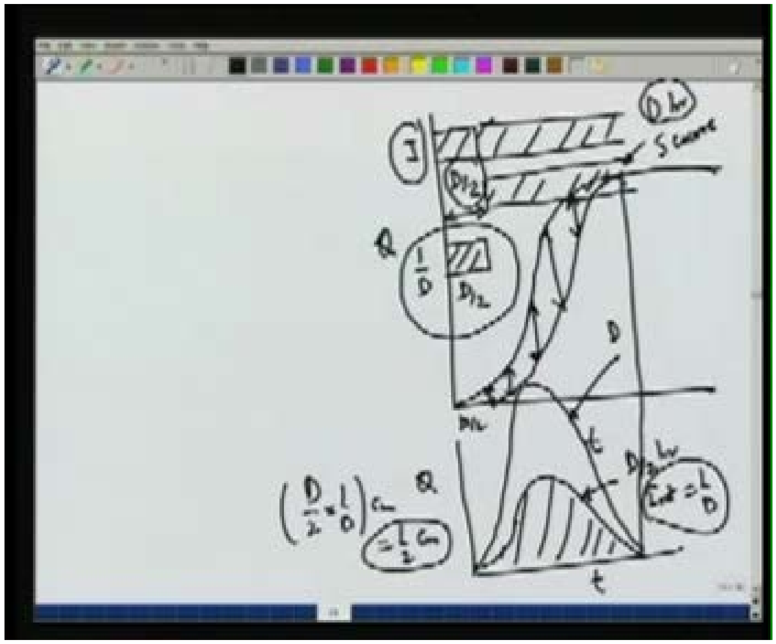
Suppose consider another rainfall occurs of the same duration and intensity 1 over D. If we have a D hour unit hydrograph available to us and we want to derive the S curve for this, we can say that the rainfall of duration D repeats itself a number of times, until infinity and we have already seen that using the principle of superposition we can draw all these unit hydrograph shifted by the interval D and a large number of them too. What it means is that if we add all these ordinates, we would get the S curve. So in the tabular form we can perform the S curve calculations in the same way as we have done earlier.

(Refer Slide Time: 39:07)



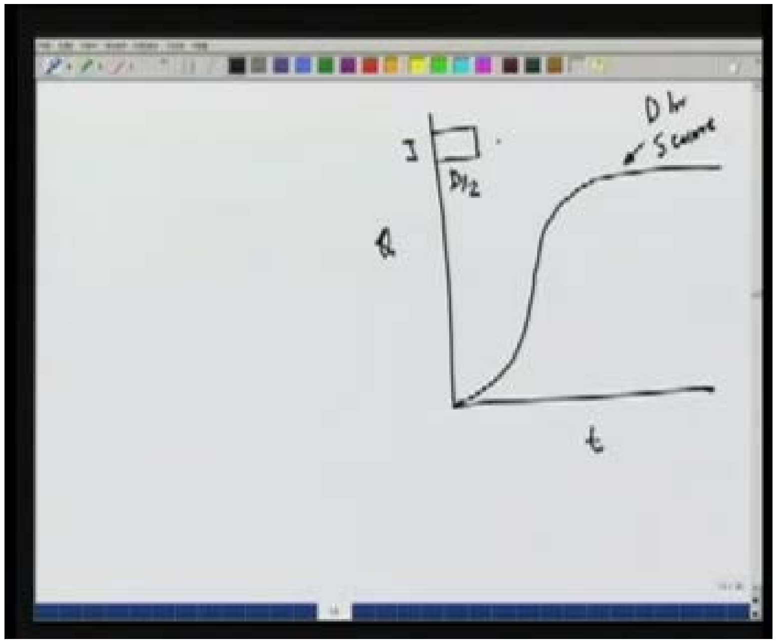
For example if we have the ordinates of unit hydrograph at every D hours, we can shift it but now we have to add a lot of these ordinates. Therefore shift by D 0 0, then u_1, u_2, u_3 and so on. Then we shift it by another D or shift it by 2D and will have 0 0 0 u_1, u_2 and so on and similarly a lot of them. We then add them all up, we would get here 0, u_1, u_1 + u_2 and so on and ultimately we would get this kind of curve. This is known as the S curve and it represents the direct runoff due to intensity 1/D cm of rainfall occurring over the whole catchment for infinite time. Now once we have this S curve, we can derive the runoff or any duration which is not a multiple of D.

(Refer Slide Time: 40:48)



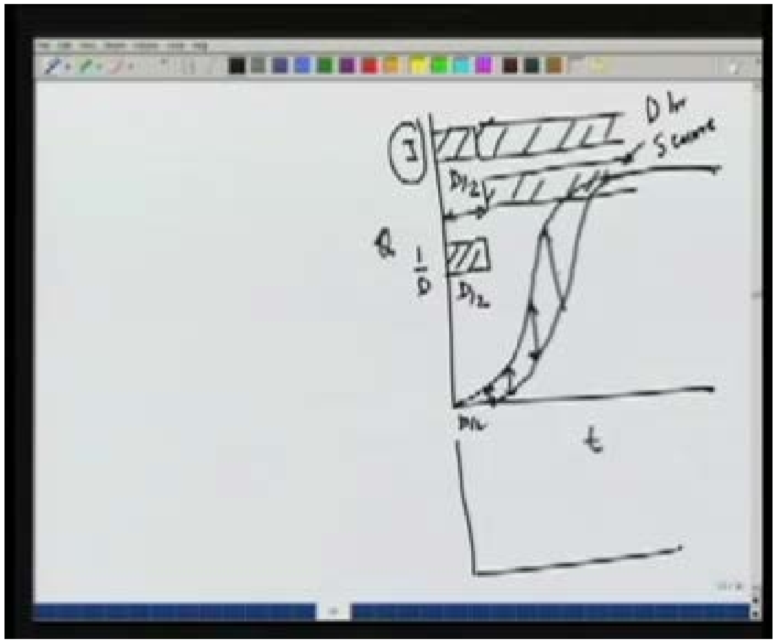
Again if you look at the effective rainfall, suppose this is the effective rainfall for the duration which is D/2, the intensity is high, we know the S curve for duration D or intensity 1 over D.

(Refer Slide Time: 41:40)



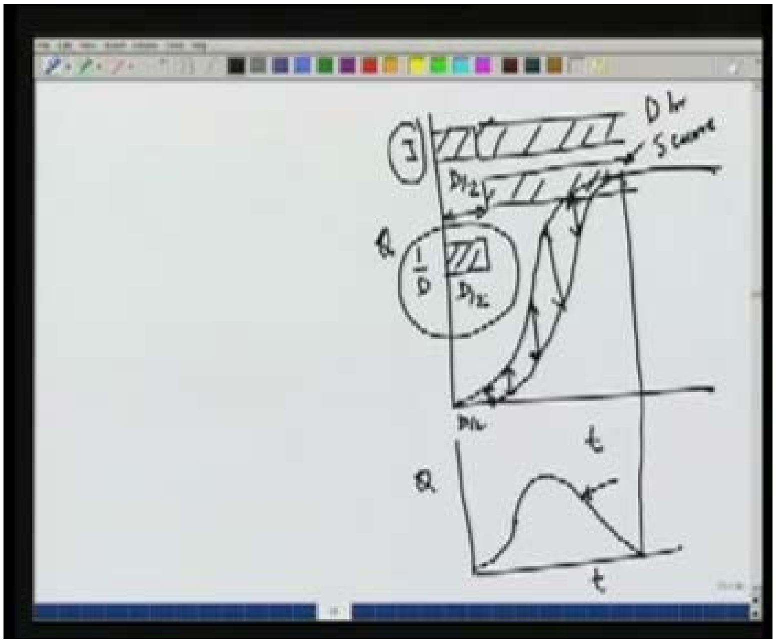
Let us assume that S curve is given by this which represents infinite duration of DRH. What we say is if we lag the ERH by this time D/2, in the first case our ERH starts from 0. In the second case the ERH starts from D/2, then we would get S curve which would be shifted by the amount D/2 and would be parallel to the first S curve.

(Refer Slide Time: 42:32)



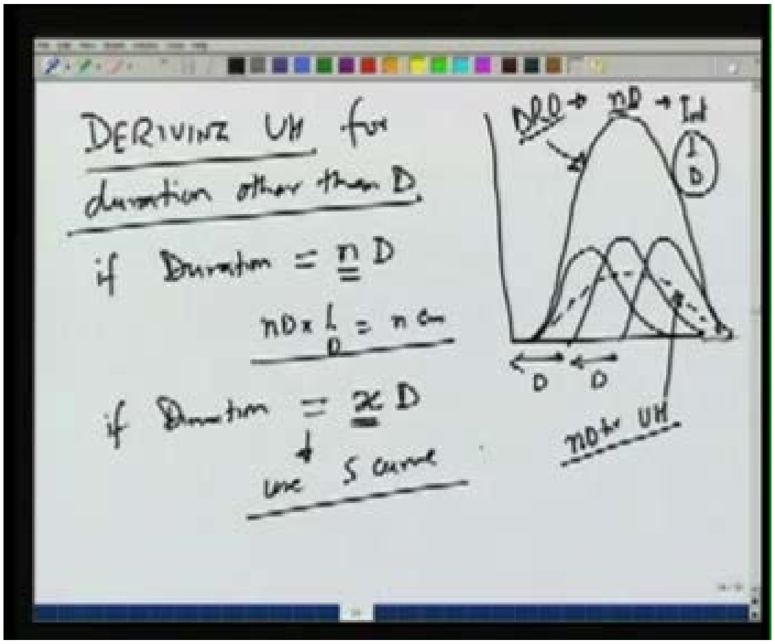
The difference between these 2 S curves at any time is because of the difference of rainfall between the first and the second case which is nothing but a rainfall of duration D/2 and intensity 1 over D. To summarize, if we have a D hour, S curve we shift it by D/2, find out the difference and ordinates where the difference will give us the hydrograph due to rainfall of intensity 1 over D over a duration D/2. Once we have this, we can find out, for any other intensity I, what the ordinate would be. If we take the difference of these 2 S curves you will see that the difference is first increasing and then it is decreasing and becoming 0 at certain point.

(Refer Slide Time: 44:08)



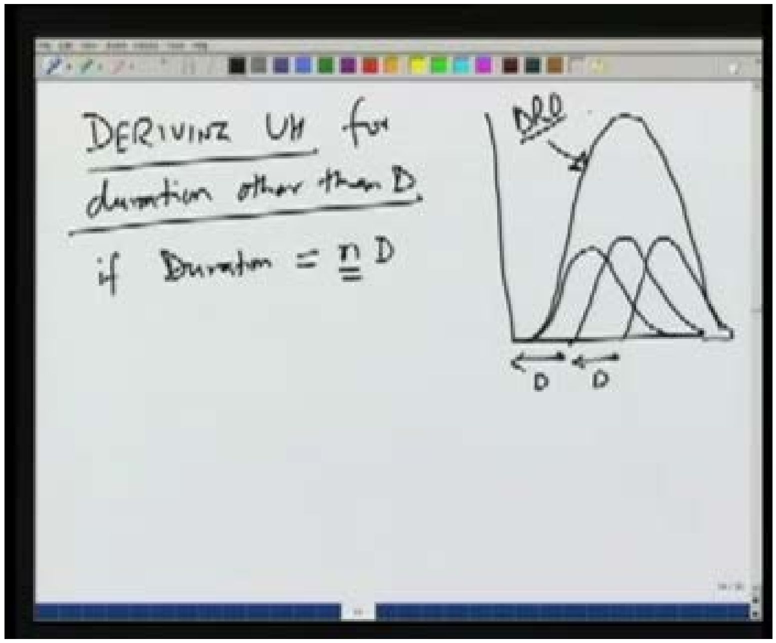
If we plot it, it would look like this and this direct runoff hydrograph is because of the rain of intensity ½, 1/D over duration D/2. So we multiply each ordinate by I over D and we will again get the required direct runoff hydrograph because of rainfall of intensity I and duration D/2. So this again we can do it in a number of ways. We can first derive the D/2 hour unit hydrograph. This is due to rain of D/2 hour and intensity of the rain which is 1/D and because this, the difference of the S curves which is for a D hour hydrograph. D hour unit hydrograph has rainfall of intensity of 1/D cm per hour. Therefore this hydrograph represents a D/2 hour hydrograph with intensity of 1/D. Therefore the volume of this will not be equal to 1 cm of depth. It will be D/2 into 1 by D which is 1/2 cm. So this direct runoff represents 1/2 cm of rain over the entire catchment. If you want to find out the unit hydrograph for D/2 hours, we will have to multiply all the ordinates by 2 and get D/2 hour unit hydrographs. So in this way for a given D hour unit hydrograph, we can derive for any other duration. If that duration is a multiple of D then it is very easy. We do not have to go through S curve.

(Refer Slide Time: 46:18)



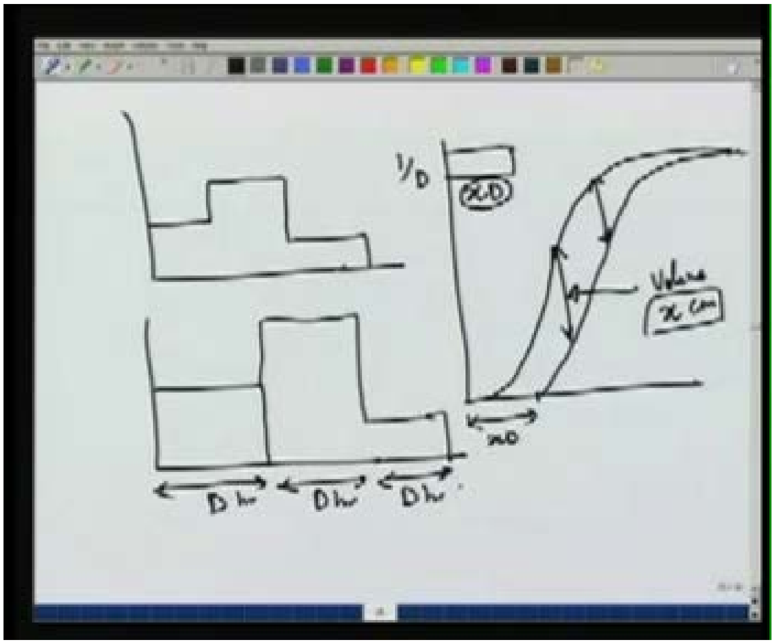
So let us look at deriving UH for any other duration. In a way we have already discussed this, but let us summarize this.

(Refer Slide Time: 47:42)

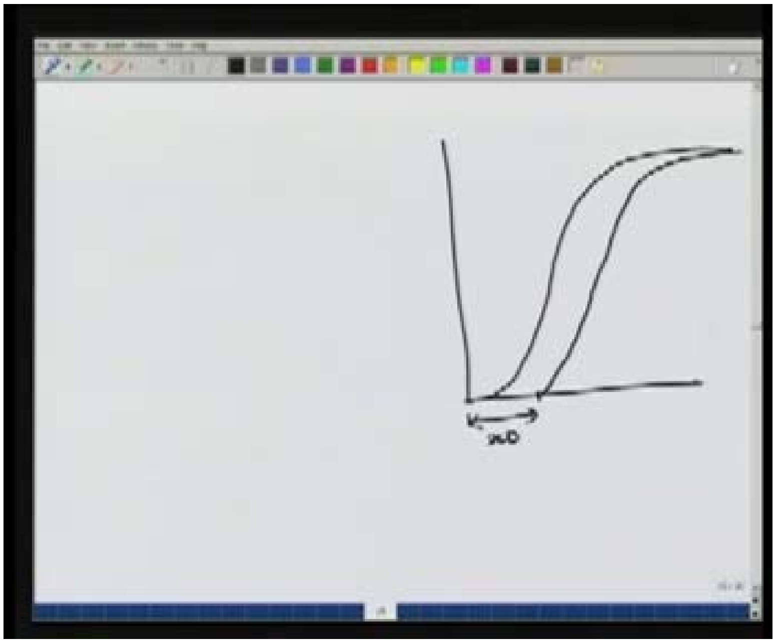


When we derive for any other duration, if the duration is nD, where n is some integer value, then we just add up a lot of unit hydrograph. Suppose n = 3, then we draw 3 unit hydrograph shifted by D. Add them up and get a direct runoff hydrograph. This would be DRO direct runoff ordinate, which would correspond to rainfall of duration n D. In this case, 3D and intensity 1/D therefore the volume of rain which causes this direct runoff hydrograph will be nD = to n cm and therefore once we get this DRH, we have to multiply by 1/n or we divide each ordinate by n. In this case since n = 3, all the ordinates have to be divided by 3 and we would get the one that corresponds to nD hours. The second case is if duration is equal to some fraction, X into D where X is not an integer then we have to use S curve.

(Refer Slide Time: 49:13)

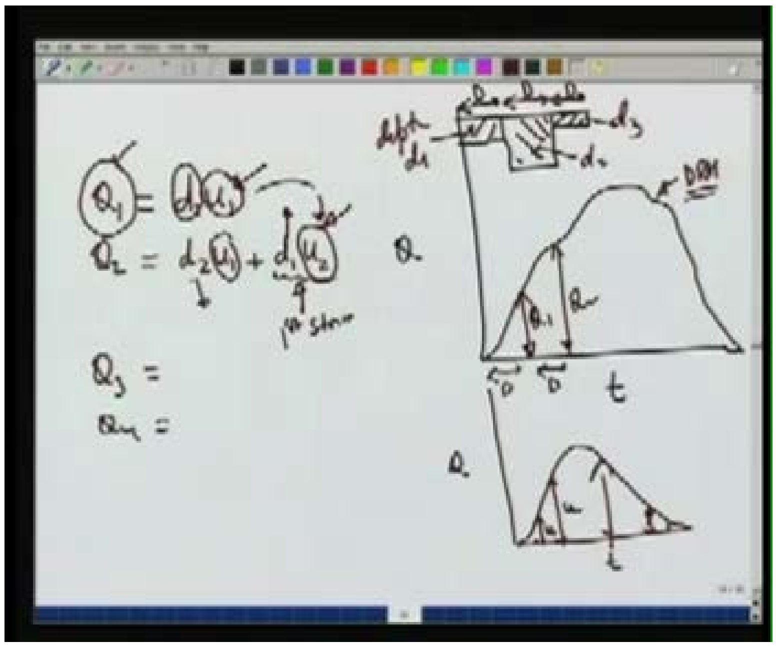


(Refer Slide Time: 49:29)



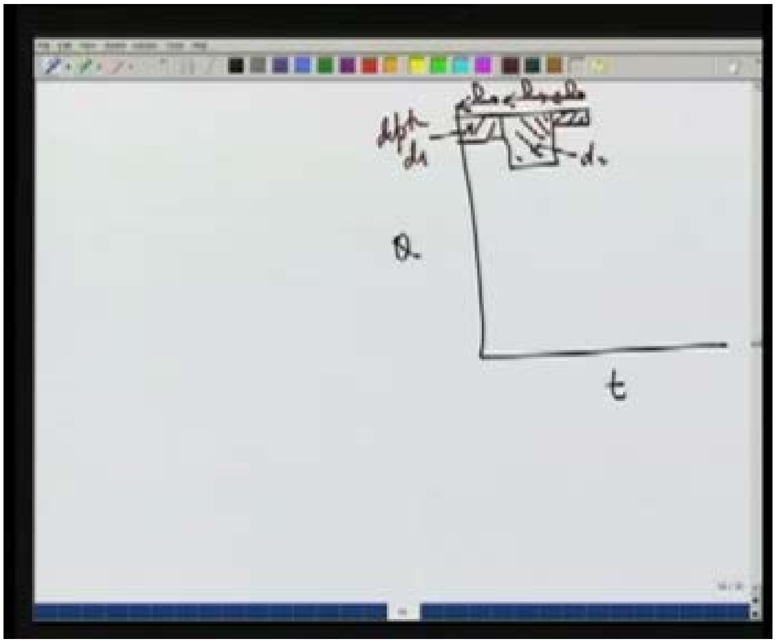
As we have discussed, you can get hydrograph for any other duration by drawing the S curve and shifting it by XD. Now these direct runoff ordinates they are due to storm of intensity 1 over D and period 1XD therefore, the rain, volume or depth of rainfall would be X and therefore in order to compute the unit hydrograph for XD, we will have to divide all these ordinates by X. Now this works well when we have rainfall hydrograph which can be idealized as rainfall of constant intensity over certain duration. We have also seen that for derivation of the UH, we need rainfall of similar kind where we have a constant intensity D hour rainfall. In some cases it is not possible to get these very ideal kinds of storms and sometimes it may happen that we have rain occurring over D hour periods but with varying intensities. So if we have the runoff corresponding to these complex storms, we can still derive the unit hydrograph but then the computations become a little bit complicated.

(Refer Slide Time: 51:37)



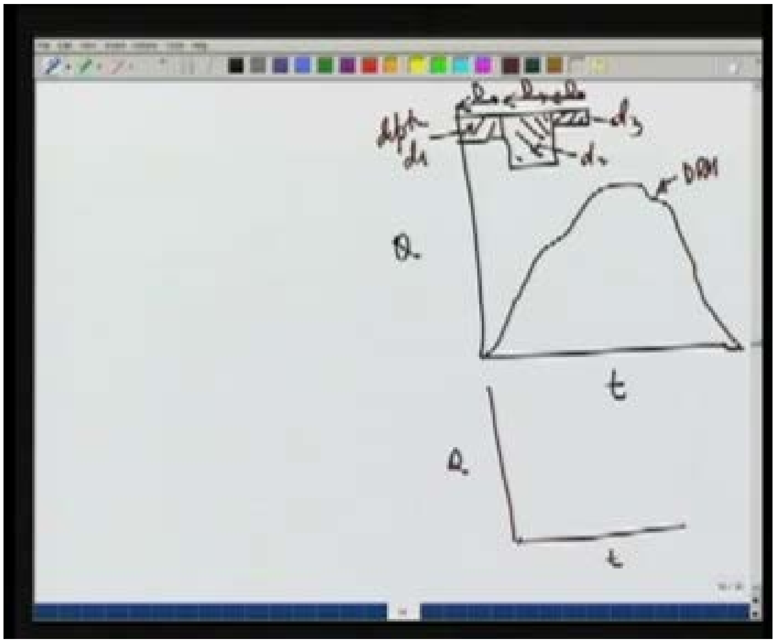
Let us look at a complex storm in which the ERH is given by all these. There are D hour duration but the intensities are different.

(Refer Slide Time: 52:24)



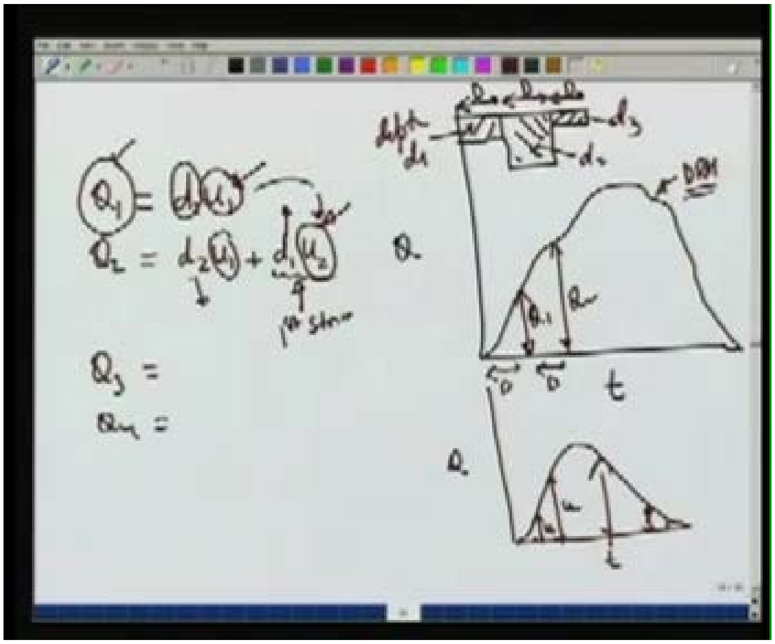
So let us say that this ERH has depth d_1. This has d_2 and this has depth of d_3 and what we have is we have the measurement of the runoff at the basin station A which looks like this. Now from this measurement of direct runoff hydrograph, what we want to estimate is what will be the unit hydrograph for a D hour duration rainfall and in order to do that we again use the principle of super position.

(Refer Slide Time: 53:15)



Let us assume that there is a unit hydrograph corresponding to duration D which looks like this and it has ordinates u_1, u_2 and so on which we do not know. This is inverse to what we were discussing till now and we want to estimate the values of u_1 u_2 e.tc. Given the direct runoff values let us say Q_1, Q_2 at intervals of D like this.

(Refer Slide Time: 54:59)



See using the principle of super position we can write equations like Q_1 and since the depth of rainfall for the first storm is small d_1, we can write the ordinate of DRH after time D as d_1u_1 because the second storm has not started yet. Once the second storm starts, we will have to account for the second storm also and now this is the first storm contribution because now it has already lasted for 2D hours. The unit hydrograph ordinates at 2D into the depth of the first storm plus the second storm have only lasted for D hours. So it is ordinate u_1 multiplied by the depth of the second storm. Similarly we can write Q_3, $Q_{4.}$ These are known to us from the direct runoff hydrograph which we have measured at A. So now the problem is knowing Q_1, d_1, Q_2, d_2 and so on, we have to estimate the values of u_1, u_2 and u_3 and so on. So the procedure is very simple that first equation will give us the value of u_1 because q one and d one is known once we know u_1 in the second equation we can find out u_2 because then u_1, d_2, d_1 and q_2 all are known. So the procedure is simple but sometimes it leads to oscillations and the computed ordinates u_1, u_2, u_3, u_4 as we go towards the tail end, may become unrealistic and some of them may become negative because we are going in a sequential order and any errors in computations will add up and will cause non physical behavior of the curve.

So at times, we just fit even if we get negative values here. We just fit and make it a smooth curve which has a unit area under the curve. So let us summarize what we have discussed today. We have looked at unit hydrograph which is nothing but the direct runoff produced by unit depth of rainfall occurring uniformly over the catchment area in a certain given durations. This duration is important for the hydrograph therefore all the hydrograph denotes the duration of ERH. We say that it is a 6 hour unit hydrograph or 8 hour unit hydrograph or 4 hour unit hydrograph. We have looked at the ways to derive the unit hydrograph for a given catchment from a simple storm as well as from a complex storm. After having derived it, we have also seen how to use it to predict direct runoff for various other storms of different durations and intensities.