Civil Engineering

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Lecture No. # 09

In the previous lecture, we have seen some aspects of flow through confined aquifers. We have seen the usage of mass balance along with the Darcy's law to derive an equation of motion which relates the variation of head for different times and distances.

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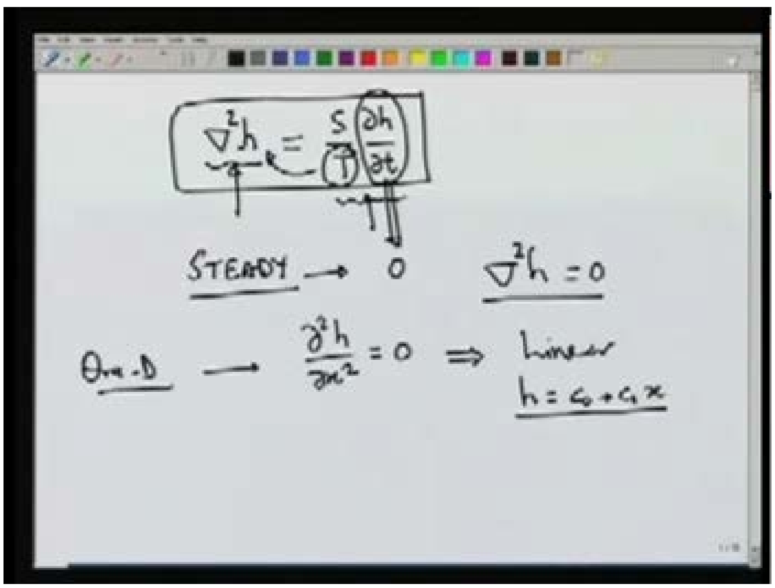

The equation which we have derived is del2h. So this equation tells us that the laplacian of h is equal to a storage coefficient divided by transmissivity times the time derivative of h. This term represents the change in water volume inside the aquifer and this term is represented with multiplication by t, the net inflow into the aquifer from all sides. In the confined aquifer case, we have seen that the thickness remains constant but for unconfined aquifers, the thickness varies and it is equal to h. So it is more difficult to derive equations for unconfined aquifers which is the reason why we have first started with the confined aquifer case. We have already seen how to solve this equation for a study state. If we have a steady state slope that means the parameters are not changing with time. The head remains constant with time on the changes with space and therefore del h/del t term can be ignored as 0. The equation which we get is the Laplace equation. If we assume that the flow is one dimensional, we have a very simple equation. So for one dimensional flow in the x direction, we can write but the second derivative of h with respect to x will be = 0 which gives us the linear head profile as h equal to $C_0 + C_1x$. The values of C_0 and C_1 would be obtained based on the boundary conditions.

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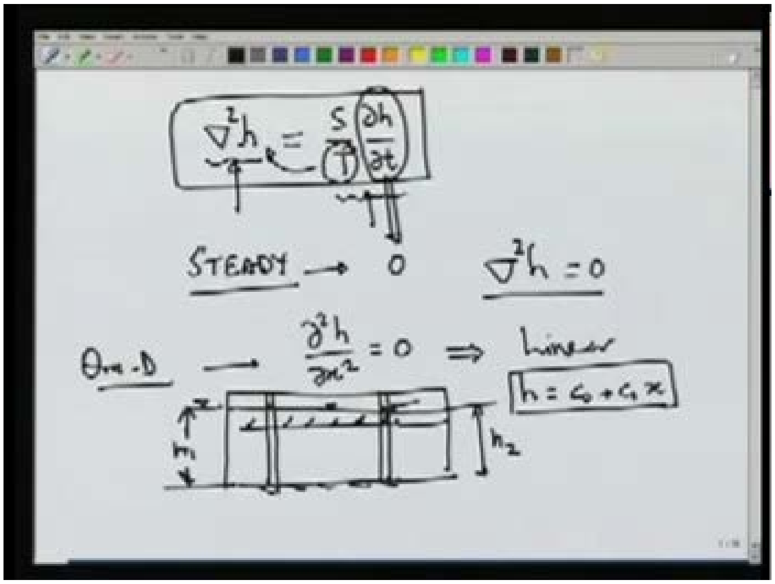

As we saw in the previous lecture, if the flow is occurring within two water bodies which have elevations of h_1 and h_2, the head variation within the aquifer is linear and therefore the pizometric level would be a linear profile joining h_1 and h_2. It means that if we install pizometer here, the water will rise up to this level in that pizometer. Similarly if you have a pizometer here, water will rise up to this level. Now this one dimensional flow is easy to solve but it is not very practical. In most cases we are into sudden flow through or towards a well. That is why in the next case we will discuss the flow of water in confined aquifer towards a well.

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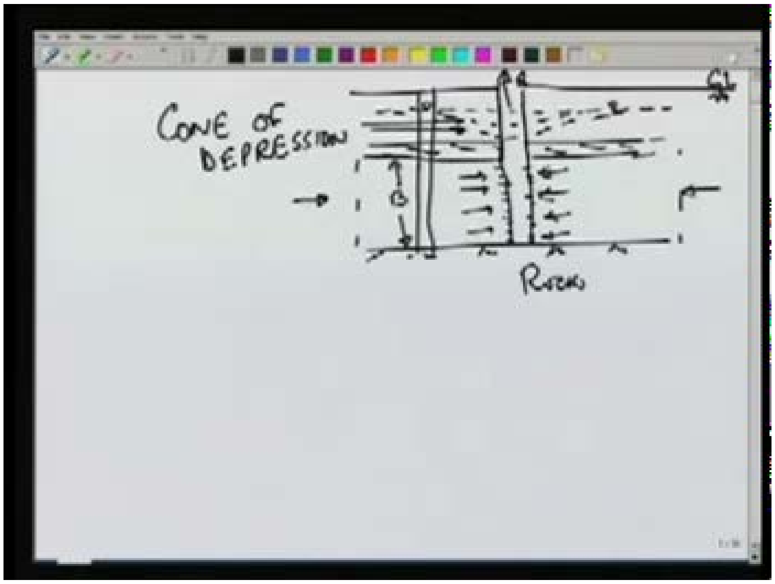

We have an impermeable bottom here which is rock. Then we have a confining layer which is another impervious layer, the ground level and this confined hook for hallow thickness of b. When we install a well, we will assume that the well is a screened throughout the aquifer thickness, so water can come in this well throughout this thickness and there is no water entry from above the aquifer. We have some pumping rate. Let us call it q and the conditions are, when we start the pumping, we can assume that the pizometric surface is horizontal. So before pumping begins, if we assume the the water level in any pizometer, and then start pumping then this pizometric head will go down and with time it will slow and after sometime it maybe like this (Refer Slide Time: 06:13), after some more time of pumping, it may go like this. With time this pizometer level is going down. If you look at this carefully it is in form of a cone. This is called a cone of depression.

This will go deep. As we keep on pumping, there is some recharge into the aquifer and ultimately this cone of depression will reach a stage where this q which is being pumped out will be balanced by the q which is coming in. We will say that it has reached a steady state. Now this cone of depression is important for us because we know that how much is the head at any location h. So let us draw it here again.

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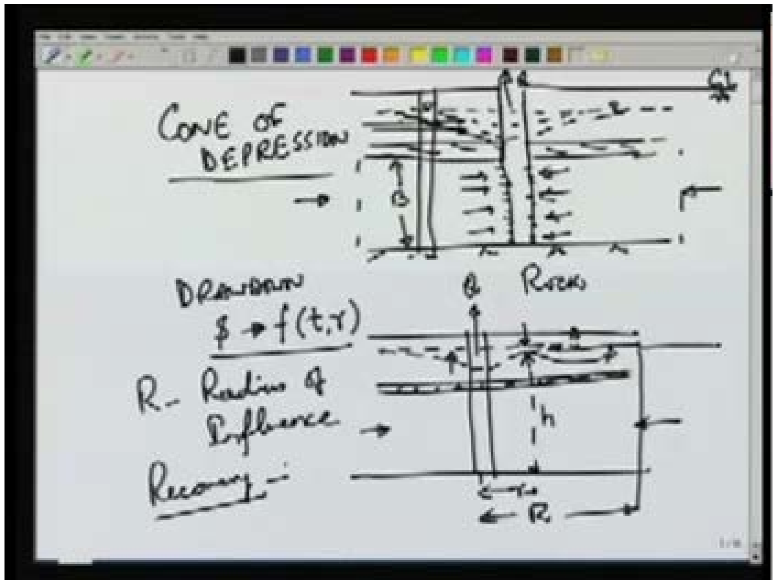

What we are interested in finding out is the location of the cone of depression at any distance r from the center of the well; let us say that the head is h. This is the initial pizometric level. The difference in the initial level and the cone of depression at any time is called the draw down, denoted by s. So as you can see with time s will increase. When we start, s is 0 because the pizometer level is horizontal and as we keep on pumping, s will increase and s will be a function of time and the distance from the well. Once it reaches steady state then that profile s belongs to a function of r. If we go very far from the well, we will reach a point beyond which the draw down is 0, this distance r may be thought of as radius of influence.

So this is the distance up to which the well is influencing the water level and therefore we call this the radius of influence. In other words there is no effect of the well felt beyond the distance of r from the well. If we stop pumping then slowly this cone of depression will be going up because, we stop pumping, the rest of the water starts coming in and that will cause the pizometric head to rise and that portion or this phenomenon is known as recovery. So we have the draw down in which by pumping we are lowering the pizometric head and then once we have stopped pumping the pizometric head we will try to go back to its initial level which is called the recovery. All these are important in the sense that if we measure draw down, it depends on the aquifer properties S and T. If we measure the draw down, we can estimate the parameter S and T from the observed values of small s which is the draw down. So we will see how h and therefore s because s is nothing but initial H – h.

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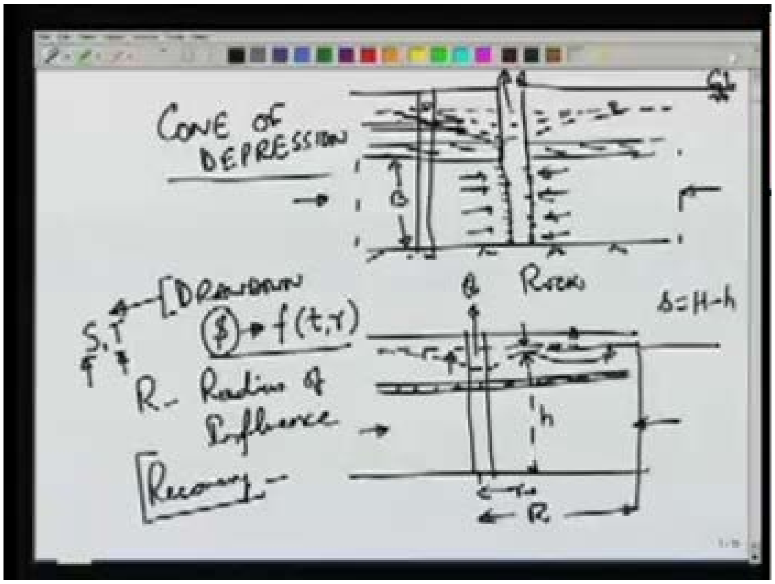

Capital h is height of the initial pizometer level so if we find out how h is changing with time we can derive an equation for the draw down and that will help us in finding out the storage coefficient and transmissivity of the aquifer. So we will first look at steady state condition because that is easier to derive. There is no time factor involved and s will only be a function of r so let us start with a steady state flow.

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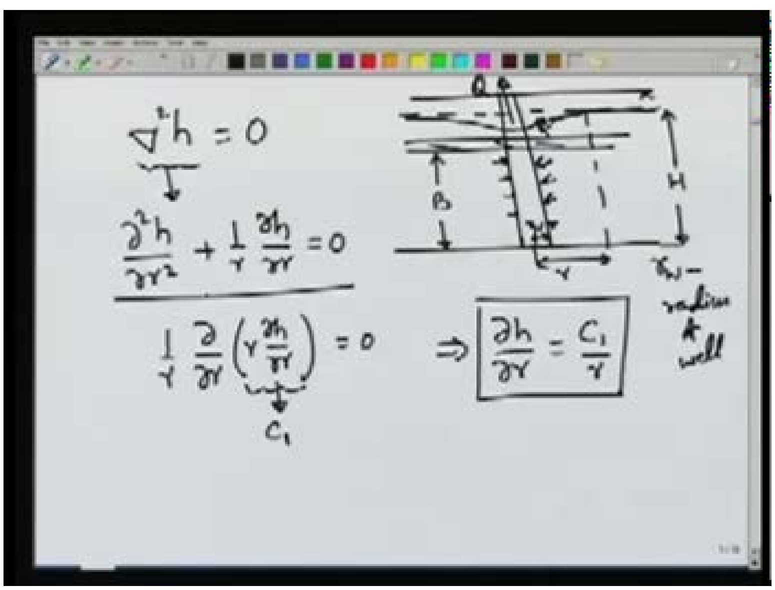

The assumptions we are making is that the thickness of the aquifer b is constant initially. We have a horizontal pizometric head level as capital h, the pumping rate q is assumed to be a constant. So we are pumping in the aquifer at constant rate and then we also assume that we have reached a steady state condition. The draw down is not changing with time, i.e., the cone of depression remains constant with time. If we use the equation, the Laplace equation del 2 h is = 0. Since this is a readily symmetric flow condition, we would like to use radial coordinates here and therefore the equation becomes this (Refer Slide Time: 13:08). The laplacian operator in radial coordinates also involves partial derivative with respect to theta, but here because of the radial symmetry, h will not be a function of theta, in other words, at any distance r from the well, the value of h will be same irrespective of what the angular direction is in which we are measuring. Due to radial symmetry del 2 h/del theta square term is ignored form here. So this is the equation of motion for steady state flow in a confined aquifer towards a well which is a screened throughout aquifer thickness and is pumping at a constant rate of q. To solve this equation, we can write this as 1 over r del/del r of r del h/del r and this implies that partial of h with respect of r will be equal to some constant over r and from here we can say that this will be a constant because its derivative with respect to r is 0. So if we assume the constant to C_1, then we can write partial of h with respect to r will be equal to C_1 over r. So this gives us the derivative for the gradient of head at any location r and you can see that the gradient is inversely proportional to r.

As you go away from the well the gradient will decrease which is also clear from the cone of depression because the slope will be high in the early portion and then as you move away it becomes flat until the slope becomes 0, very far away from the well. The radius of the well is typically very small and denoted by rw, so rw is the radius of well and capital R will be the radius of influence just we have already discussed. So to solve this equation, we would need some boundary conditions and for example in this case we can use the conditions that r = rw and s = to sw where sw is the draw down in the well.

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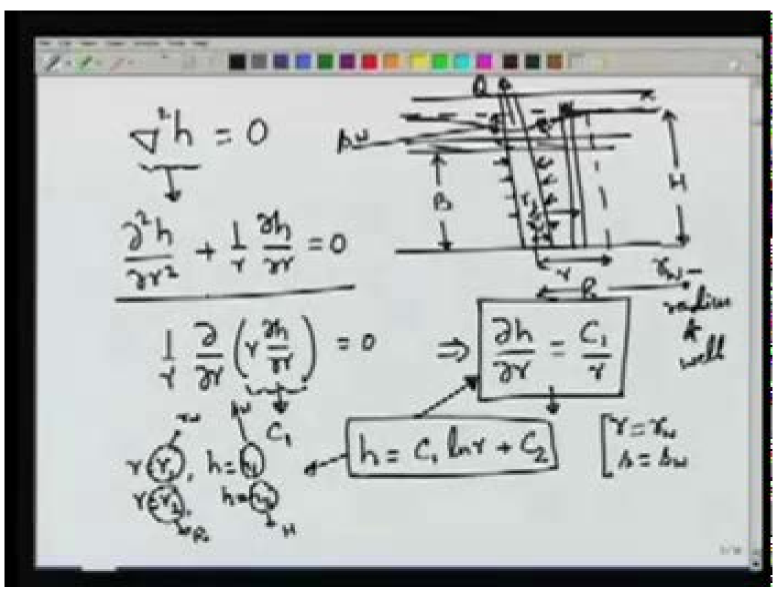

We can show it here, s w is the draw down in the well. So one condition may be that r = rw s equal to sw but typically the draw down in the well is affected by some losses in the well. The flow is also not laminar. It is turbulent therefore sw is not used and we take some other location. Let us say $r = r_1$ and find out the draw down there as s_1. This boundary conditions need to be used in order to be able to solve this equation. But this equation tells us that this is the governing equation for steady state flow in a confined aquifer. If you solve this equation you can write h + C_2 and as you can see there are two constants here, C_1 and C_2 which need to be evaluated. This means that we need to have boundary conditions at 2 different distances for example r_1 and r_2. So if you assume that at $r = r_1$, h is h_1, h is h_2. Given r_1 and r_2 may be any two radiuses. For example, r_1 maybe rw and r_2 may be r capital R. If we take this as rw then, this would be sw and if we take this as capital R then this would be h because of that the radius of influence, the head remains constant at the initial value. We will not assign these values. We will just assume that they are two different r values r_1 and r_2 and 2 different h values, h_1 and h_2 which allow us to obtain the value of C_1 as $h_2 - h_1$ over natural log of r_2 over r_1.

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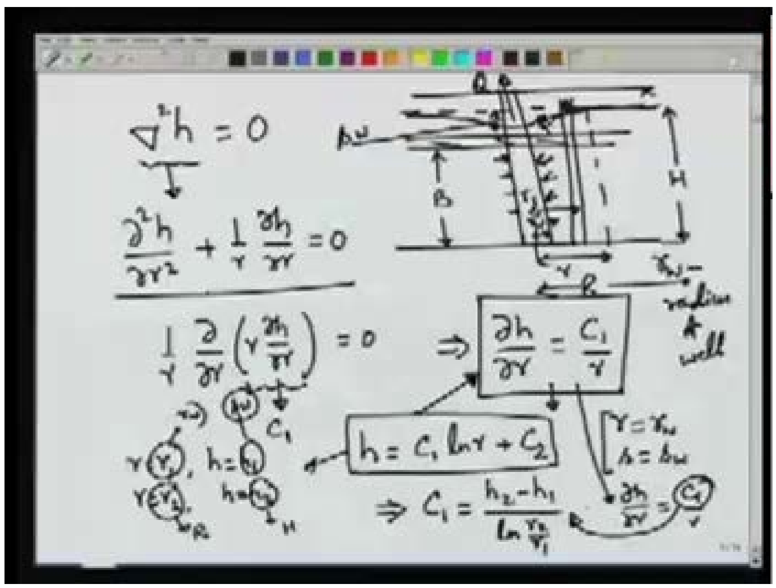

If you look at this equation del h/del r is C_1 over r and C_1 is obtained from here.

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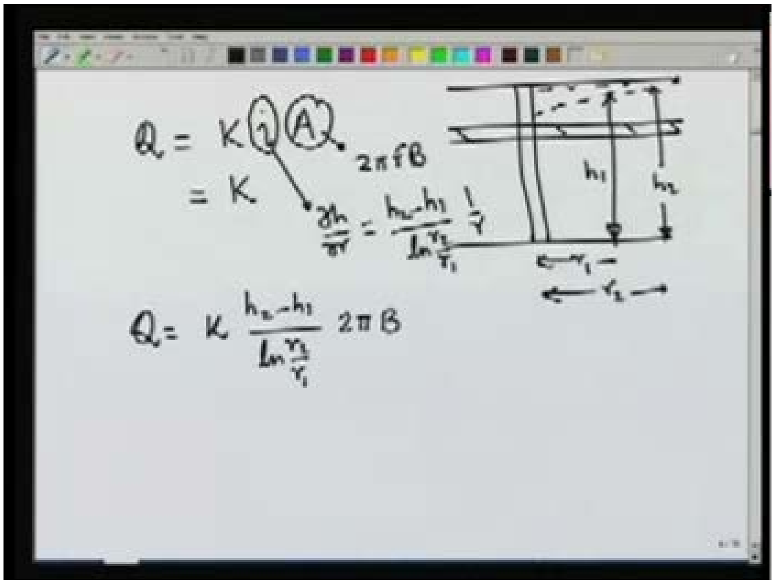

What it tells us is that by measuring the draw down at two different locations, r_1 and r_2 and therefore h_1 and h_2. We can get the gradient del h/del r and we can also write the discharge as k i A. At any distance r, the area would be equal to 2 pi r into the thickness of the aquifer B and i is nothing but del h/del r which we have already seen from here is C_1/r and C_1 is given by this, h_2 – h_1/r. If we put the values of i and a in this equation, then q can be written as kh_2 – $h_1$then r_2 over r_1 2 pi B because this r and 1 over r will cancel out. kB as we know k and b can be combined as a transmissivity.

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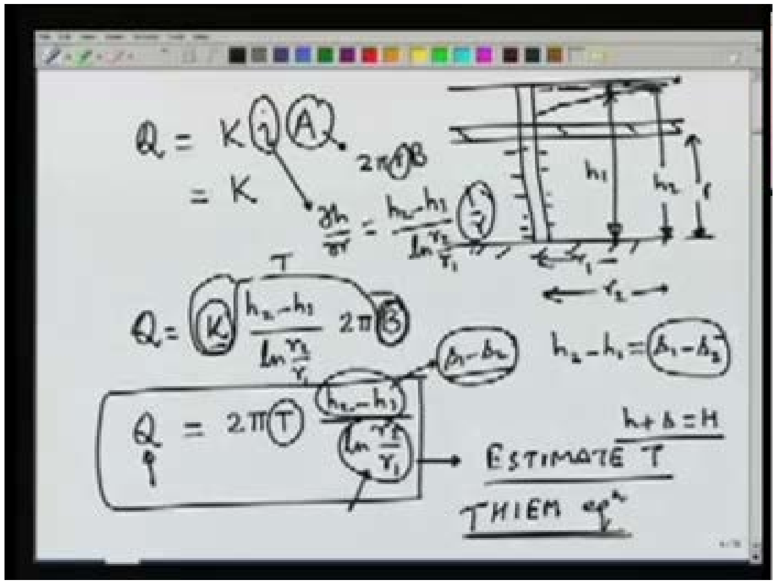

We get a simple expression for q. In practice it is difficult to measure h because impermeable layer here would be quite deep. We may not even know how deep it is but if we measure s what is draw down, we can see that $h_2 - h_1$ would be equal to $s_1 - s_2$ because h + is constant which is equal to h, so $h_2 + s_2$ and h_1+s_1 will be same and therefore $h_2 - h_1$ will be equal to $s_1 - s_2$ and therefore typically we replace this term by $s_1 - s_2$. s_1 and s_2 were easy to measure because we know the initial water level or the pizometric head level in this case and the final pizometric head level the difference between those two will be the draw down s. This gives us the method of estimating t. If we measure q, the draw down and the tensile of course; we know where we have put the pizometer. So knowing all these three terms we can obtain the value of transmissivity. This test is very useful in estimating the transmissivity or hydraulic conductivity because the thickness B will be known and therefore we can obtain hydraulic conductivity.

Also, this equation is commonly known as the THIEM equation based on the first derivation. The THIEM equation is valid for a steady state flow towards a well in a confined aquifer assuming that the well is fully screened. It is drawing water throughout the aquifer width and all the assumptions which we have earlier made in deriving the equation still holds good. For example we had assumed that the aquifer is isotropic, so kx, ky, kz are all same. We have also assumed that the aquifer is homogeneous. That means k is not changing from place to place. We have assumed that the thickness of an aquifer remains constant. So this b is a constant value. So if all these assumptions are satisfied then for steady state flow towards radial, THIEMS equation can be used. The steady state equation although useful but will take a very long time to reach the steady state and therefore it is better to derive some equation which accounts for the transient behavior and tells us how the cone of depression is decreasing.

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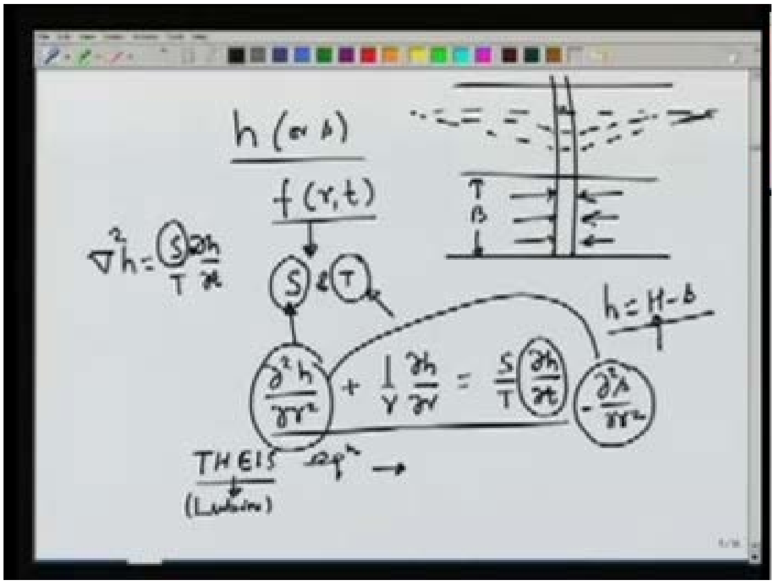

Initially we have some pizometric level then we start pumping with time. We have this level going down the radius of influence would be a distance beyond which there is no influence and at different times, the radius of influence will also be increasing. So as we go on pumping, the cone of depression will expand and the radius of influence will be increasing. If you are able to derive an equation which tells us how the h or s varies with radius as well as time, it will be very useful to know how the cone of depression expands, how the radius of influence is increasing and it will also help us in deriving a technique to estimate s and t. As we have seen in steady state case we would be able to estimate the transmissivity but not the storage coefficient because of the nature of the equation, the term s affects the transient behavior and if we assume a steady state then there is really no effect of the storage coefficient. Things are not changing but time is, so the storage coefficient does not enter into the equation but if we are using a transient flow condition then, we get an equation where s will also be used in the equation and therefore we can derive the values of both s and t from that equation. We will now look at the transient flow radial again towards a well.

All the other assumptions will still be valid so we have a fully screened well with a constant thickness of B. The only assumption which we have not alleged is that instead of being steady state flow, it is now transient. Therefore the draw down and the h will both depend on time and therefore the equation governing (Refer Slide Time: 28:02) will include the transient term, del h over del t. Now partial of h with respect to t will not be 0 in this case while in the steady state case, we have taken it as 0. The solution of this equation is a little complicated. That is not as easy as it was for either one dimensional flow or for a steady state flow and therefore the person who derived this solution first was Thies. Infact, Theis and Luban both contributed to the solution but it is commonly known as the THEIS equation which relates s because as we know h is h – s, h is a constant. So the same equation can be written in terms of the draw down, so for example if we take del it can be written as – because h is capital H – s and capital H is a constant. It is in determinant of r. Therefore the second derivative of h with respect to r would be equal to negative of the second derivative of s with respect to r.

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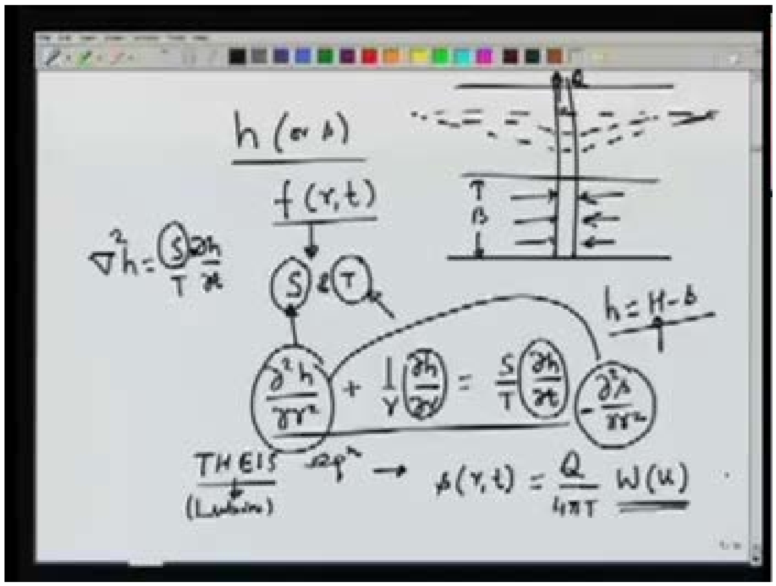

Similarly we can put these terms also in terms of s and the Theis solution gives the draw down s at any radius r and any time t, Q the pumping rate. We assume that q is a constant and Theis introduced a function which is called the well function. We will now look at what is u and what is Wu. This is the THEIS equation.

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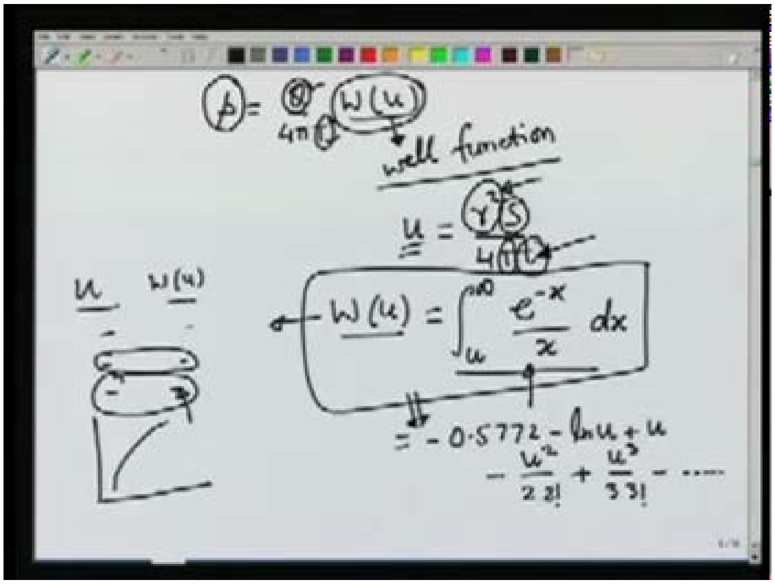

W is called the well function; u is a function of time and distance. You can see that u increases as t decreases and u increases as r increases, wu is an integral. This integral is not very easy to obtain and therefore W values are given in some books and you can refer to any ground water hydrology book. It will have a table of u versus W u values. Knowing value of u, r square is known to us, suppose s and t are known too, it is not an estimation of parameter problem. Suppose it is a problem in which s and t are given and we want to estimate the draw down at any observation, at any given time, then in that case, we will be able to obtain the value of u for any time. Then from this table, for a given value of u we mean it to interpolate the values here or you can directly get the value from this table. Sometimes we prepare a curve also and from that curve we can obtain the value of Wu.

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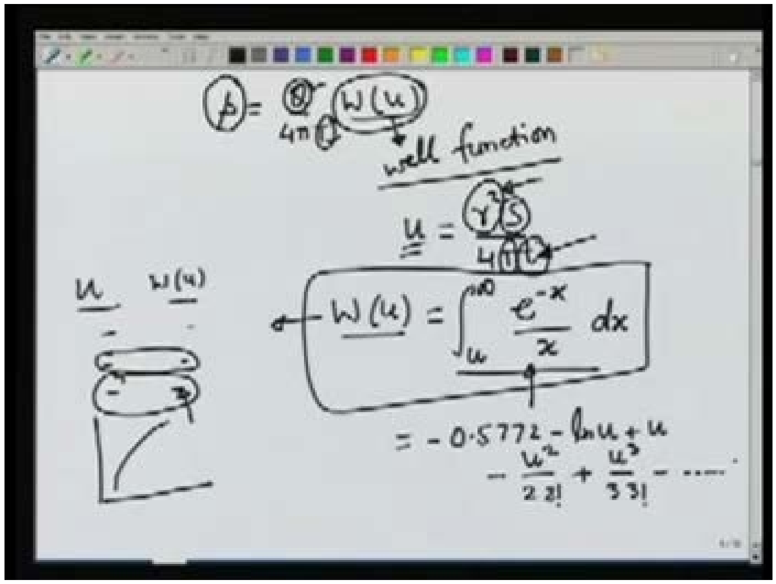

Once we obtain the value of Wu getting the draw down is straight forward because q and t are assumed to be known. If you look at this expansion of exponential function, we can expand Wu in terms of a series of infinite series and that series is written as 0.5772. If we do term by term, – u square over 2 factorial 2 + u cube over 3 at rate 32 and so on. If u is small, we can ignore the higher order terms and there is an approximation which says that if u is less than 0.01 only the first two terms can be used. We shall look into that approximation a little later. But Wu can be obtained using this series and once either from table or from figure or from this equation W is known as the draw down and can easily be obtained by the approximation which is known as Jacobs approximation.

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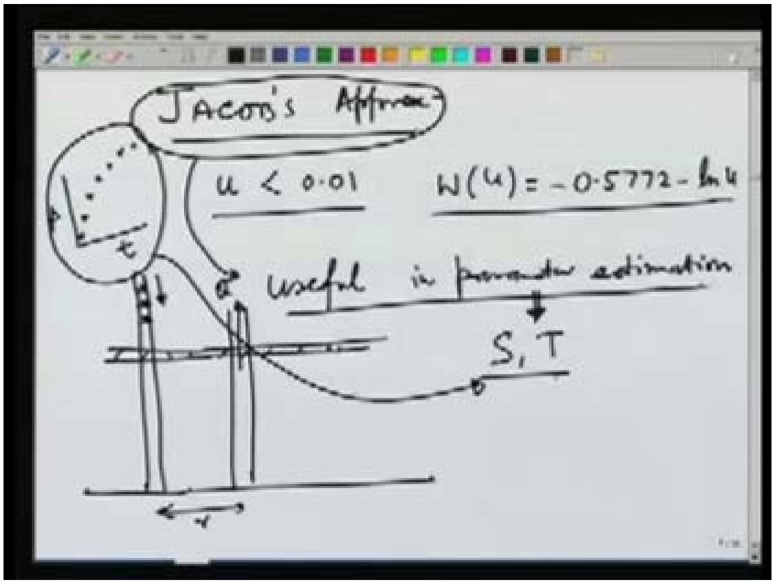

It says that if u is less than about 0.01 then only the first two terms of the series can be used and this approximation is very useful in deriving or estimating the parameters. Parameter estimation means estimating the values of a storage coefficient s and transmissivity t. Since the transient method involves betterment for smaller times, it is preferred over the steady state method. In transient conditions, what can be done is, an observation well can be selected, so the pumping well is here. We can select an observation well at distance of r and note down how the water level in this observation is decreasing. So with time the draw down s will increase and using this t and s value we want to estimate the parameters s and t. That is what we will look at next, how to estimate or compute the values of a storage coefficient and transmissivity from a given observed state of data. For draw down at a well which is located at a distance of r, sometimes we may choose wells at different radial distances, so we may have an another pizometer and you can measure the water level in this with time, may be at a distance r_2 and r_1.

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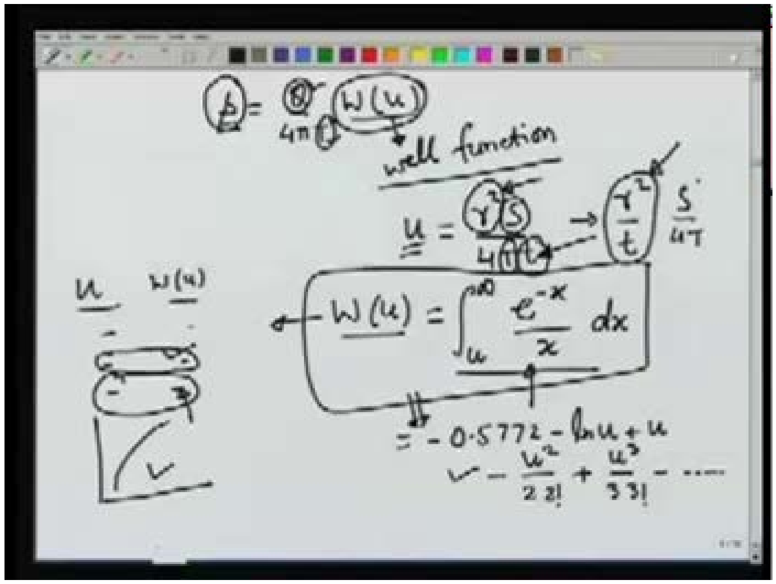

If you have two different wells, we can use that data to estimate the value of s and t using both of the wells and if you look at the definition of u it is r square over t, s 4over t so individual values of r and t are not important this r is square over t factor is what will affect the draw down. If you have two different wells at distance of r_1 and r_2 and plot t versus s, they would be on different lines, but if you plot r square over t, or t over r s square versus draw down, both of them will fall on the same curve and that curve can be used to estimate the value of s and t.

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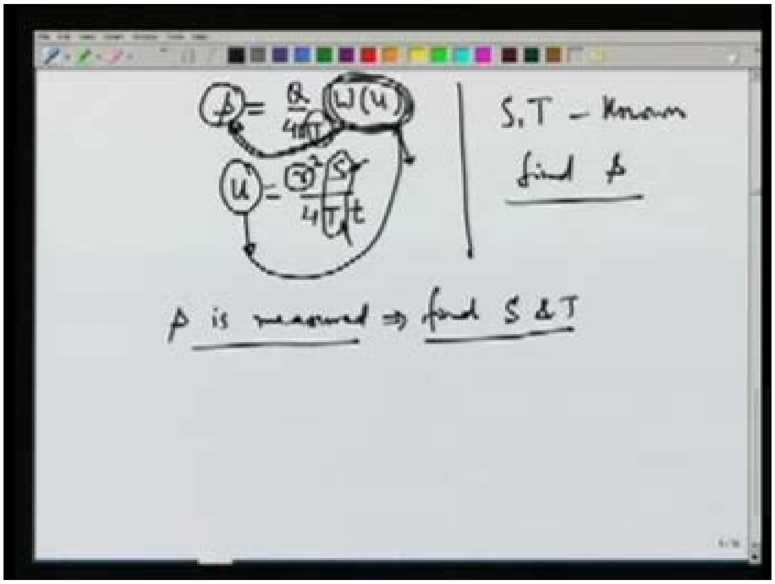

If you look at the equation for s and u, one thing which is clear is that if s and t are known and we want to find out draw down, there is no problem. We for any given s and t and knowing the distance of the observation u, can be computed. Once u is computed Wu can be easily obtained from the table's graphs or some equation. Therefore s will be known, so knowing u we get Wu, knowing wu we get s. But in the other problem where s is known, or measured we find s and t. This is an inverse problem in which we are estimating the parameters and this is not directly solvable because if s and t are not known then u cannot be computed and if u cannot be computed then we cannot obtain Wu because this also involves the unknown t. So we will look at some of the methods from which try to estimate values of s and t from the measured data of s and there are some methods which are graphical, some which are numerical and we shall look at all these methods. The simplest method to estimate the values of s and t is based on the Jacob's approximation.

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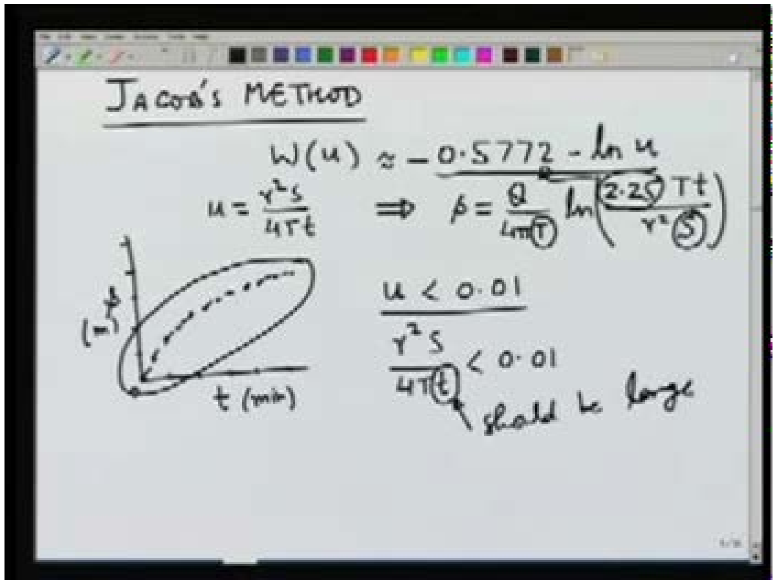

In the Jacobs method as we have discussed we used approximation that Wu and since u is r square s over 4 Tt, it gives us the value of the draw down s as Q over 4 piT W u which is approximated by this and it can be written as natural log of 2.25 where this2.25 comes from this factor of .5772. Using this approximation and from the data of s versus t, what we have in the field is measurement of draw down at different times. The t value is taken in minutes but any other unit and can also be used. Generally it will be in meters and the plot would look like this. The rate of increase will go down as the continued of the pumping; initially the draw down will be increasing at a fast rate. It then becomes almost flat, as we move away. So this one, the data available to ourselves this and what we want is estimate t and s. We assume that Jacob's approximation is valid but we know that it is valid only for u less than 0.01, so that is an implicit assumption. When we use Jacob assumption whatever data we are using has values of u less than 0.01 and if you look at the definition of u, you will notice that t has to be large which means that some of the ugly data will typically not follow the Jacobs approximation. What we do using this approximate equation is that we see that s and log of t natural log of t should fall on a straight line. So that is what we would plot.

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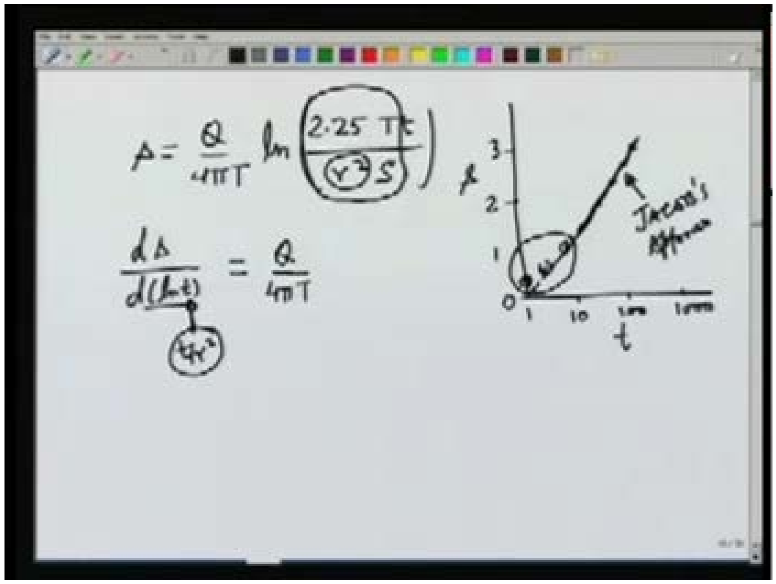

We prepare the curve in which t is 1. The log scale may start with 1, 10, 100, 1000 and s is on simple arithmetic scale. So s may be 0, 1, 2, and 3. The data once is plotted on this; let us say the data points lie like this in which as you can see, there is a straight line which fits the data. Some of the data which are circled here will not fall on the straight line. This is because the time is small and therefore u value will large than 0.01 but once we see the data point which is falling on the straight line, we know that this data follows Jacobs approximation and therefore we can use this straight line to obtain the parameters values s and t. Let us write it again. The value of s in terms of (Refer Slide Time: 45:20). If we take the slope of this straight line, the slope of the straight line which is ds over d of l and t, it would give us q over 4 piT. We are assuming that r is constant. So we are using data from a single well otherwise instead of t we have to use t over r square. For this analysis, let us assume that r is a constant. So we have a single observation, at which our measurement draw down, if we assume that r is a constant then, we can use here l and t and keep this r s square within this constant value here. The slope of the draw down versus lock time curve will give us q over 4 piT and from that slope, t is unknown. Once we find the slope, we can obtain the value of t. The slope ds by d_1 and t can be easily obtained from this plot.

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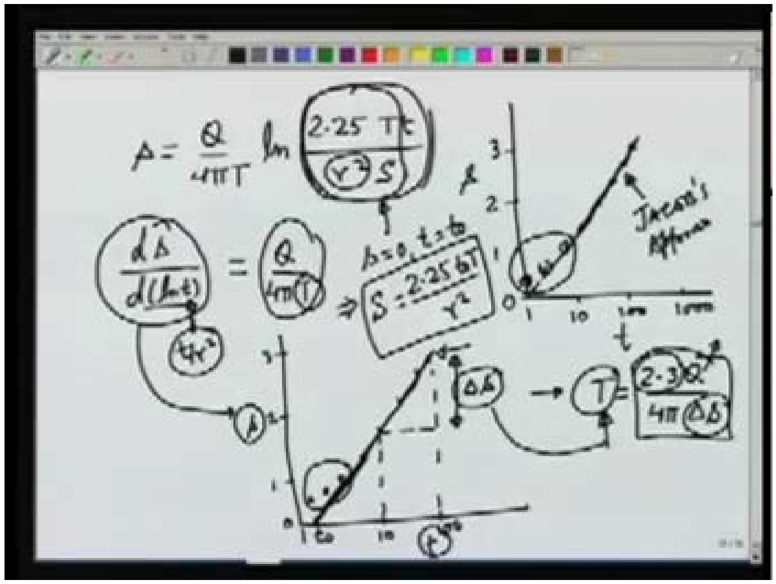

Let me draw the plot here, a little more detailed. So these are the data points which follow a straight line and then there are some data points which are away from the line. This is the line which represents the Jacobs approximation. The t value is let us say 1, 10, 100 and s value may be 1, 2, 3 in order to find out the slope, what we can do if we take one log cycle of t, and we can find out what is the change in draw down in log cycle of t, that will give us the slope of s versus log t on base 10 and then we can convert it into measure log and we get the equation for estimating T = 2.3q over 4 pi delta s, where delta s represents the change in s in one log cycle of t and 2.3 comes because of conversion of a base from 10 to natural log t. We can see that the slope of this line gives us transmissivity directly. This line intersects the s = 0 line at some time which we will call as t_0. This means that t = t_0, s is = 0 and if we use that condition, we can see from this expression s = 0, t = t_0 which means that this term should be equal to 1 because natural log of 1 will be = 0 and from there we can obtain the value of s. t is already known from here and 25 t_0 t over r square.

The procedure is first we have to plot the data on a semi log plot, where t will be plotted on log axis; s will be plotted on arithmetic simple axis. Once we plot the data, we look at data and see what portion can be fitted by a straight line. Early portion will not be on the straight line but the late portion will be on the straight line. We extend the straight line to intersect the x axis at time t = t_0. We also note down the change in draw down in one log cycle of t and we call that delta s. Using the value of delta s, we can estimate t, using the equation 2.3 q 4 pi delta s, q of course is known to us, the pumping rate constant value. Delta s can be obtained from this line. Once we get the value of t, we use this intercept t_0 we say that s will be 0 at t = t_0 from the Jacobs approximation. For s to be 0, the time here for the argument of natural log should be = 1.

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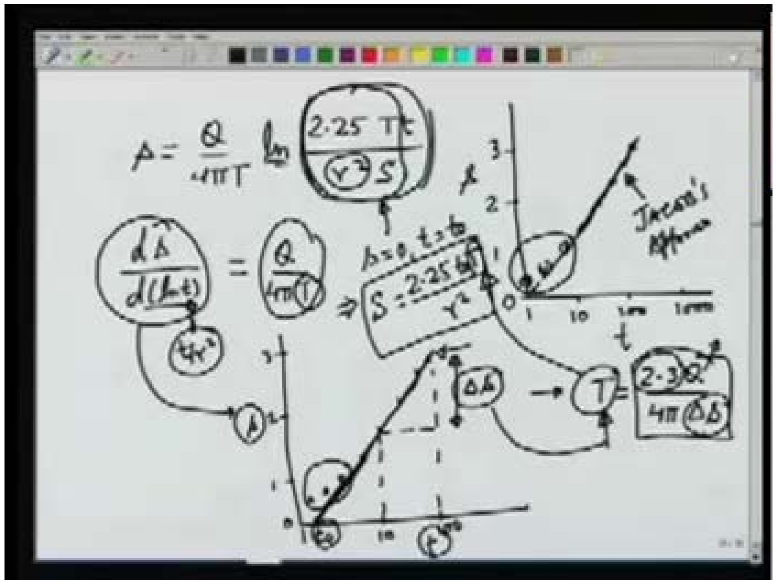

Therefore s will be obtained as 2.25 t_0 t over r square where t has already been obtained from here. In this way both t and s can be estimated from the data from an observation well draw down data from an observation well. If we have more than one well, then instead of t, we will be using t over r square. The only problem with this method is that we need data for a very long time.

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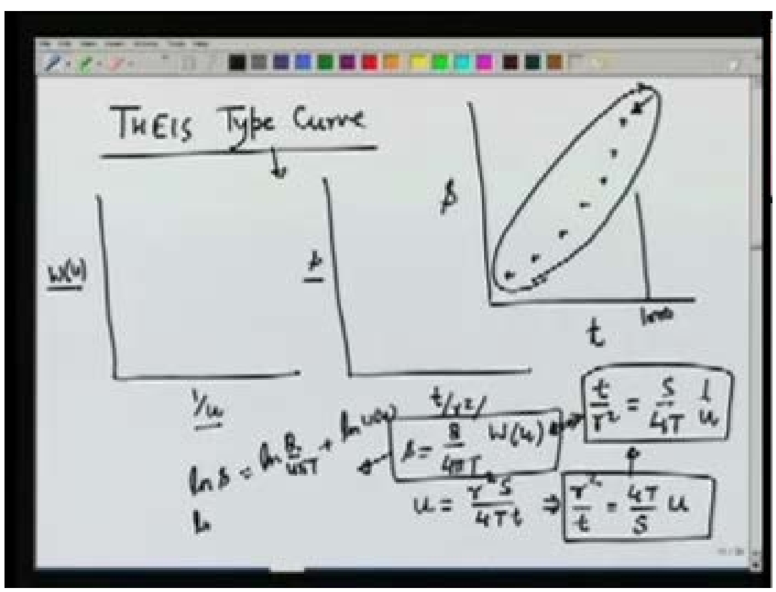

Sometimes when we plot t versus draw down, the data may not show a straight line behavior even for very long time. So this may be a 1000 min or sometimes even more than that. It is possible that depending on the storage, coefficient and transitivity values the value of u corresponding to this time is the largest time may be more than 0.01. So in that case we need to continue the test for a very long time which may not be economic. Therefore we should look at some method that uses the entire data set rather than depending on Jacob's approximation for fitting this straight line. This is called the THEIS type curve method. In this method we use the entire curve. The type curve tries to compare two different curves. In one of them we plot the well function variation with respect to u or one over u and in the other curve; we plot t over r square versus draw down s. The idea is that these two curves should be similar because if you compare these two equations, the t over r s square. Let me write it here in the form of t over r square. Sometimes we use W u and u here and then s versus r square over t but most of the times it is preferred a Wu versus 1 over u, and s over t over r square. If you look at these two equations, we can see that ratio between s and t over r square and the ratio between Wu and 1 over u would be similar. Only a shift of the axis would be there.

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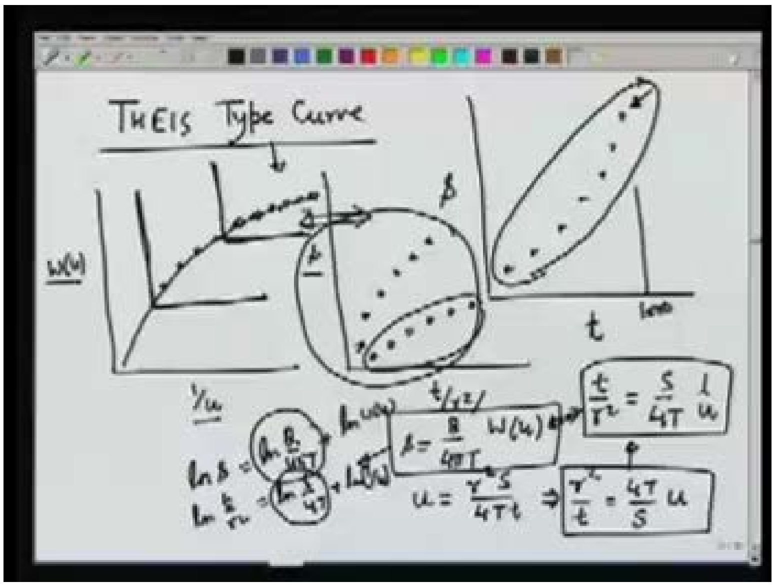

For example if you take the log of this and similarly if you take the log of the second equation, this tells us there is only a shift which is given by these two parameters. In the curves s versus t over r square on log scale and Wu is to 1 over u again on log scale. Wu versus 1 over u does not depend on the data points. This is a fixed function W of 1 over u and the plot would look like this. This is the type curve with which we have to match our plotted data, s versus t over r square and that data may be like this or it may be like this (Refer Slide Time: 56:42). Depending on the values of s and t, you may have a curve like this. The idea is that we match these 2 curves. So we plot the values in these curve, move this curve here in such a way, that the data lies exactly on the type curve or in some cases, if the data is like this, the curve may have to be plotted some where here or the excess has to be some where here to match with the type curve. Depending on the s versus t over r square data, the shifting of the axis will be different. This is known as the type curve matching and this gives us the method of estimating the values of s and t.

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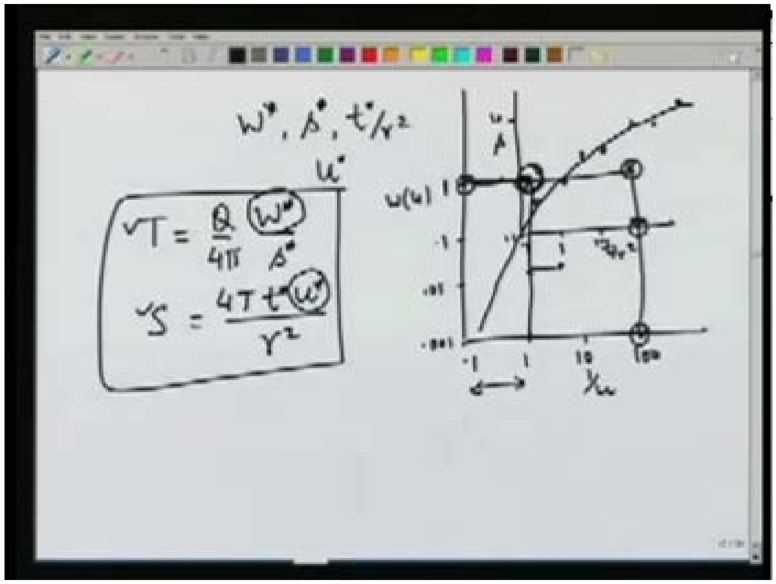

Let us say that we have matched the type curve this is$_1$ over u versus Wu which is a fixed curve and let us also say that we have matched the data by shifting the axis here. Now we want to find out the values of s and t. We can choose any point on this graph. Suppose 1 over u values are points 1, 10, 100 W u values may be .001, .01, .1, 1. Similarly this t over r square values May be .1, 1, 10 and s value again may be also be .1, 1, 10 and so on. The scale of course has to be same. The log scale here and log scale here would be the same. Now we can choose any point on this. Let us choose points which give us easier computations. But we can choose any point on this curve. After matching, we can choose any point on this graph and we note down the co ordinates for 1 over u for W u for t over r square and for s. Let us call these values as W star, s star, t star over r square and u star. We are measuring 1over u. Let us call that value as 1 over u star. So this will be 1 over u star. This will be W star. This value on t over r square scale will be t star over r square and this value will be on s scale s star and using these we can easily obtain the value of transmissivity as t over q over 4 pi and then the storage coefficients s. This is using the 2 equations which we had derived earlier.

Once W star, s star, t star, u star are known then t and s can be estimated. The only thing is that it is a graphical method. So there is some subjectivity involved in choosing or matching the type curve. But once it is matched the computations are very straight forward and as you can see if you take W star as 1 and u star as 1, the computations becomes little simpler. That is why we choose the point which gives us (Refer Slide Time: 1:02:29), so this point will give us W star of 1 and u star also of 1. For that we can find out s star and t star and use this to get t and s. We have seen some methods of estimating the parameters s and t. For example Jacob’s method which required data for very long time. Then we have these type curves which can use all the data set and compute the value of s and t. We have also discussed all these methods only for confined aquifers. Unconfined aquifers are a little more complicated because the thickness is not constant. It depends on the head h and we will have to make some simplifying assumptions to obtain the equation of motion that we will do in the next lecture.