

Geology and Soil Mechanics
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Lecture - 23
Seepage and In-situ Stress

Welcome. So, in the last lecture we have seen that how we can find out the seepage through an earthen dam and there we have established this equation.

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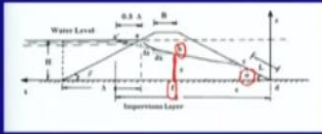
Seepage

Seepage through earthen dam on an impervious base

$$i = \frac{dz}{dx} = \tan\alpha$$

$$A = (ce)(1) = L\sin\alpha$$

$$q = k(\tan\alpha).(L\sin\alpha) = kL.\tan\alpha.\sin\alpha \quad (2.25)$$



Again, the rate of seepage (per unit length of dam) through the section bf is,

$$q = kiA = k\left(\frac{dz}{dx}\right).(z).(1) = kz\left(\frac{dz}{dx}\right) \quad (2.26)$$

That is rate of seepage is given by k into L into $\tan\alpha \sin\alpha$ where α is nothing but the slope of the downstream phase that is given by this figure and k is the coefficient of permeability whereas L is completely unknown to me as I told you in the last lecture that L is not known to me so I need to find out L . Now again so the rate of seepage per unit length of dam through the section bf any section say any section in between say that is bf okay.

Along that section if you say if I want to find out the rate of seepage that is given by the same q equal to k into i into A . So, what is i ? i is nothing but $\frac{dz}{dx}$. Now what is the area, cross sectional area at section bf is nothing but z into 1 , that 1 is nothing but the unit length normal to the length right. So, z is the depth of section bf . So, I can get the expression kz into $\frac{dz}{dx}$. So, that is the rate of seepage at any section bf okay which is say some distance away from point d okay.

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Seepage

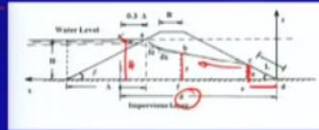
Seepage through earthen dam on an impervious base

For continuous flow,

$$kz \left(\frac{dz}{dx} \right) = kL \cdot \tan \alpha \cdot \sin \alpha$$

$$\text{Or, } \int_{L \sin \alpha}^H kz dz = \int_{L \cos \alpha}^d kL \cdot \tan \alpha \cdot \sin \alpha \cdot dx$$

$$\text{Or, } L = \frac{d}{\cos \alpha} - \sqrt{\frac{d^2}{\cos^2 \alpha} - \frac{H^2}{\sin^2 \alpha}} \quad (2.27)$$



So, now this for continuous flow I mean if you want to satisfy the equation of continuity and for the continuous flow whatever flow is happening through this section bf the same amount of flow will be happening in the section through the section ce, there is no issues because whatever flow is entering the section bf the same amount of flow or same amount of seepage will be going out from through section ce.

So, I know the rate of seepage through ce that is nothing but $kL \tan \alpha \sin \alpha$. Do you remember that? In the last lecture as well as today already we have seen that. So, $kL \tan \alpha \sin \alpha$ is the rate of seepage through section ce whereas along or the across the section bf the rate of seepage is kz into $dz dx$. So, they must be equal for the continuous flow.

Now we are taking the integration we were doing the integration and the limit is $L \sin \alpha$ that is here from this point because x is varying from section ce to bf x is varying from $L \sin \alpha$ to sorry z is varying from $L \sin \alpha$ to h because you are considering the whole thing say now you are considering from a prime to c okay. I hope that you are understanding this thing right. So, this is the my starting point right. I can go, by following this, I can go up to point a prime right.

So, the limit should be the integration limit should be $L \sin \alpha$ to h this is nothing but h okay $kz dz$ equal to $L \cos \alpha$ that is the x , x is nothing but $L \cos \alpha$. So, this is your $L \cos \alpha$ to d , d is given here. You see this is the distance from the point d , small d to the point a prime okay along the x axis. So, if you do the integration, if you perform the integration you will be getting the expression for L , L is given by this okay. So, once you know L you can find out the rate of seepage through the dam body okay.

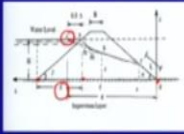
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Seepage

Seepage through earthen dam on an impervious base

Following are the steps to calculate q

- 1) Obtain α
- 2) Calculate Δ and 0.3Δ to get point a'
- 3) Calculate d
- 4) From α and d , calculate L from equation (2.27)
- 5) From L , Calculate q from equation (2.25)



Now following are the steps to calculate q . So, whatever we have discussed so far now we will be talking about those steps in brief okay. So, first you obtain alpha. What is alpha? Alpha is nothing but the slope of the downstream surface or the downstream slope okay, the inclination of the downstream slope. First you obtain alpha okay then you calculate capital delta. What is delta? So, if you see the figure, delta is nothing but the distance between this point and actual entry point of water that is a okay that is my delta okay.

So, and you also calculate 0.3Δ to get a to get a point a prime okay as I told u. So, that is located at 0.3Δ distance from the actual entry point a. Then you calculate d. How you can calculate d? So, you know this point because the dam geometry is known to you. You know this point. Now this a point is fixed so you can find out the distance between these 2 points. So, small d is known to you now.

Now from alpha and small d calculate L from equation 2.27 whatever I just discussed okay so you find out L. Once you know the magnitude of L then from L basically you calculate small q from equation 2.25 that is $kL \tan \alpha \sin \alpha$ if you recall $kL \tan \alpha \sin \alpha$. So, these are the steps involved to obtain the seepage through earthen dam.

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Seepage

Casagrande's solution for seepage through earth dam

- Equation (2.27) is derived on the basis of Dupuit's assumption

$$\text{i.e. } i \approx \frac{dz}{dx}$$

- It was shown by Casagrande that, when the downstream slope angle α becomes greater than 30° , deviation from Dupuit's assumption become more noticeable

Now basically as I told you that we assume that i is equal to $\frac{dz}{dx}$ right. So, sometimes this assumption basically is not serving well for the steeper slope. So, basically Casagrande has proposed some modification in this assumption. So, equation 2.27 is derived on the basis of Dupuit's assumption that is i approximately equal to $\frac{dz}{dx}$ right. Already we have seen that, already we have derived based on that.

Now it was shown by Casagrande that when the downstream slope angle that is α becomes greater than 30 degree that means when the downstream slope is becoming little bit steeper then the deviation from Dupuit's assumption become more noticeable. That means as I told you that if your slope is flatter that is the downstream slope, the downstream slope is flatter then this assumption i is approximately equal to $\frac{dz}{dx}$ will hold good.

But if you go for the steeper slope then this assumption will not be predicting the things properly. Then at that time you will be getting more deviation from the actual solution and the actual and the theoretical solution right. So, then Casagrande proposed that you have to modify this assumption little bit. How you can modify?

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Seepage

Casagrande's solution for seepage through earth dam

■ Thus, Casagrande suggested that,

$$i = \frac{dz}{ds} = \sin\alpha$$

Where, $ds = \sqrt{dx^2 + dz^2}$

Equation (2.25) becomes,

$$q = kiA = k(\sin\alpha)(L\sin\alpha) = kL\sin^2\alpha$$

Again, $q = kiA = k\left(\frac{dz}{ds}\right)(z)(1)$

So, the modification is suggested by Casagrande and he suggested that i should be equal to $\frac{dz}{ds}$ okay. So, now instead of taking $\frac{dz}{dx}$ you are taking $\frac{dz}{ds}$. What is s ? s is nothing but the surface or the parabolic or the Phreatic line surface okay. So, that $\frac{dz}{ds}$ by $\frac{dz}{ds}$ is nothing but equal to $\sin\alpha$ if you go back to the previous figure you will see that $\frac{dz}{ds}$ is nothing but $\sin\alpha$.

So, where ds is nothing but $\sqrt{dx^2 + dz^2}$ to the power half that is root over $dx^2 + dz^2$ okay. This is now what I mean to say Dupuit proposed that hydraulic gradient okay should be equal to $\frac{dz}{dx}$ where dz is the vertical length and the dx was the horizontal length. Now instead of taking $\frac{dz}{dx}$ you are considering $\frac{dz}{ds}$ where ds is the actual curve length okay the parabolic curve or the Phreatic line curve.

Now equation 2.25 then becomes then k that will be remaining same kiA q equal to kiA is equal to $k\sin\alpha L\sin\alpha$ is equal to $kL\sin^2\alpha$. So, if you go back to equation 2.25 you will see that now your basically i is $\sin\alpha$ right. So, you can multiply k with so i that is nothing but $\sin\alpha$ in this case. Earlier it was $\tan\alpha$ right, do you remember that? Earlier it was $\tan\alpha$ now it is $\sin\alpha$ and of course $L\sin\alpha$ is the cross-sectional area at location ce or at the location S so along this, this is your $L\sin\alpha$ okay.

So, $kL\sin^2\alpha$ is your rate of seepage. Now again q can be written as kiA which is nothing but k at any section say bf whatever we consider in the last time. So, k into $\frac{dz}{ds}$ that is your i into z that is the depth of section bf and into 1 that is the unit length normal to the board.

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Seepage

Casagrande's solution for seepage through earth dam

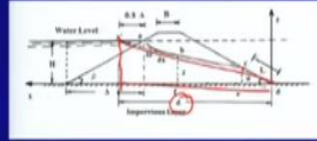
$$kz\left(\frac{dz}{ds}\right) = kL\sin^2\alpha$$

$$\text{Or, } \int_{L\sin\alpha}^H z dz = \int_L^S L\sin^2\alpha dS$$

Where, S = length of curve a'bc

$$\text{Or, } L = S - \sqrt{S^2 - \frac{H^2}{\sin^2\alpha}} \quad (2.28)$$

Where, $S \approx \sqrt{d^2 + H^2}$

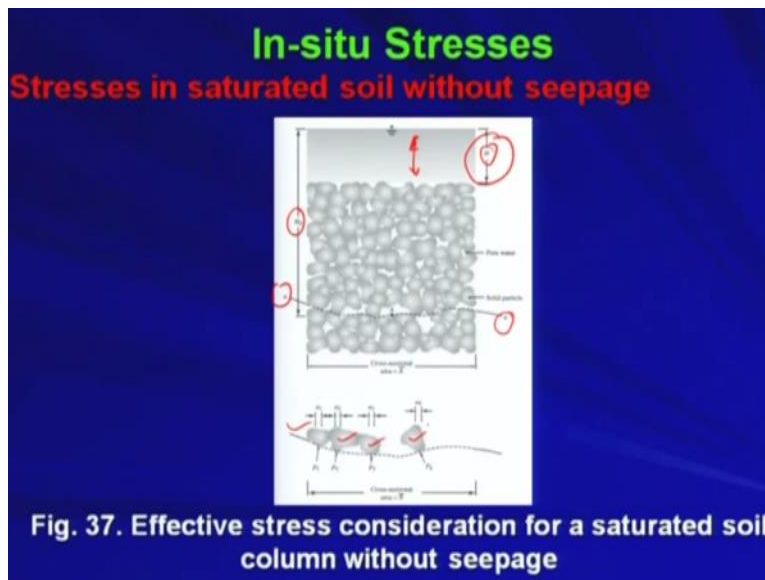


So, then this must be same as before and the integration limits will be $L\sin\alpha$ to H and L to S that is the total length of the curve. So, this if you perform that S is length of curve a prime bc that means the length of the parabolic curve that is the Phreatic line okay. So, from this you can find out L where S is given by root over approximately. This is if you look at the figure, if you look at the figure basically now basically you are starting from this point you are going up to this point.

So, you are moving along the curve itself not along the x direction okay and S , capital S , that is the length of the curve a prime bc is equal to approximately equal to d square small d square plus H square okay basically this is the approximation is done okay. So, if you now the steps will be remaining same right. So, you are first you find out L . Once you know the magnitude of L then you can find out the rate of seepage.

Well so we have already seen the seepage and all now we will start the new topic that is in-situ stresses. Now once we complete these 2 things then we will be solving some numerical problems because this in-situ stresses also will involve the concept of seepage so I thought that I will consider I will cover this part and then we will go for some numerical problems okay. So, what do you mean by in-situ stresses?

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So, basically if you consider a saturated soil without any seepage, suppose the soil deposit is there and you have some stagnant water level on top of the soil. So, basically if you have this kind of situation then if you see this is your water level on top of the soil deposit and you have the soil grains as shown in this figure and you are considering one say one section, section a okay small a so section a you are considering at some depth h a from the top surface and where capital H is the depth of water level okay on top of the soil grains okay.

So, you will be having the pore water as well as some extra water deposited on top of the soil layer. Now if you take out this section a away from this matrix, soil matrix, you will be seeing that how the soil grains are I mean as you see the soil grains are lying on that particular section. So, each soil grain will be having some contact okay with the next or the subsequent soil grain right. So, based on that you will be getting 2 different kind of stresses. So, we will come to that point later on.

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In-situ Stresses

Stresses in saturated soil without seepage

- The total stress at the elevation A can be obtained as,

$$\sigma = H\gamma_w + (H_A - H)\gamma_{sat} \quad (3.1)$$

- The total stress, σ can be divided into two parts,
 1. A portion is carried by water in the continuous void spaces
 2. The rest of the total stress is carried by the soil solids at their point of contact

So, the total stress at the elevation A can be obtained as, so the total stress means the whatever stress at elevation A, what is A, if you look at so at this point basically we are trying to find out the total stress. So, the total stress at elevation A can be obtained as sigma that is the total stress is equal to H, H into gamma w. What is H? H is nothing but the depth of water on top of the soil top surface right. So, that is the depth of free water basically okay.

So, H into gamma w plus H A, H A minus H. So, what is the H A? HA is nothing but the total depth from the top surface to the point A minus H that is the free water depth is equal to the depth of soil layer or the soil deposit multiplied by gamma, saturated gamma, saturated is the property of the soil itself right. So, that will give me the total stress at point A. I hope that you have understood right.

That is the free water, if you have the free water, that is the I mean pressure due to the free water plus the pressure due to the saturated soil. Now the total stress sigma can be divided into 2 parts okay. This total stress whatever stress you are getting or it is getting generated at point A okay so that can be divided into 2 parts.

First one is a portion is carried by water in the continuous void spaces that means basically you are applying the applying some load or whatever on the on the soil matrix. Now the total load will be taken care of by some amount, will be taken care of by the water and some amount will be taken care of by the soil solids right. In that way, you can decouple or you can differentiate or you can separate the total stress in 2 components right.

The first one is the a portion is carried by the water in the continuous void spaces okay is that clear and the second one the rest of the total stress whatever is remaining the rest of the total stress is carried by the soil solids at their point of contact. So, that means the rest of the total stress that means whatever total stress is remaining after deduction of the water contribution or the contribution from the water so that will be carried by the soil solids. Now how the soil solids will be taking care of that, at the contact point. So, it will be transferring the stresses through its contacts.

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In-situ Stresses

Stresses in saturated soil without seepage

The sum of the vertical components of the forces developed at the points of contact of the solid particles per unit cross sectional area of the soil mass is called as “effective stress”

The effective stress,

$$\sigma' = \frac{P_{1(V)} + P_{2(V)} + P_{3(V)} + \dots + P_{n(V)}}{A} \quad (3.2)$$

Where, $P_{1(V)}, P_{2(V)}, P_{3(V)}, \dots, P_{n(V)}$ = the vertical components of $P_1, P_2, P_3 \dots, P_n$

Now the sum of the vertical components of the forces developed at the points of contact of the solid particles per unit cross sectional area of the soil mass is called as effective stress. So, that means the effective stress is purely the property of the soil grain. Now the soil grain basically soil particles they are transferring the stresses through its contact point okay and these stresses will be known as the effective stress.

Now so the effective stress that is nothing but the sigma prime can be expressed in terms of $P_{1v} + P_{2v} + P_{3v}$ up to P_{nv} that means the say suppose n number of particles are lying over that section over that cross section divided by the cross-sectional area A bar. Now where P_{1v}, P_{2v}, P_{3v} up to P_{nv} are the vertical components of P_1, P_2, P_3 up to P_n . Now what are these P_1, P_2, P_3, P_n ? So, already we have seen in the figure they are nothing but the contact force.

So, whatever contact force you are getting the vertical component of that, that is nothing but P_{1v}, P_{2v}, P_{3v} up to P_{nv} summation of that divided by the total cross-sectional area will be

giving you the effective stress. That means this stress has nothing to do with the pore water or void space or something else. This is the sole property of the soil grains.

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In-situ Stresses

Stresses in saturated soil without seepage

■ Again, if a_s is the cross sectional area occupied by solid-to-solid contacts, then the space occupied by water = $(\bar{A} - a_s)$

$$\sigma \bar{A} = \sigma' \bar{A} + u(\bar{A} - a_s)$$

Or, $\sigma = \sigma' + \frac{u(\bar{A} - a_s)}{\bar{A}}$

$$\sigma = \sigma' + u(1 - a'_s) \quad \sigma = \sigma' + u \quad (3.3)$$

Now again if small a_s is the cross-sectional area occupied by solid-to-solid contacts then the space occupied by water is total cross-sectional area is \bar{A} minus the solid-to-solid contact area is say small s so \bar{A} minus a_s is nothing but the area or the space occupied by the water as simple as that.

So, σ that is the total stress multiplied by the total cross-sectional area \bar{A} is equal to σ' into \bar{A} plus u into $(\bar{A} - a_s)$. What is σ' ? σ' is effective stress into the cross-sectional area plus u , u is the water pressure or the fluid pressure right. So, u into the total area occupied by the water or the fluid. So, u into \bar{A} minus a_s . So, from this we can get σ equal to $\sigma' + u(1 - a'_s)$. Now what is a'_s ?

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In-situ Stresses

Stresses in saturated soil without seepage

Where, $a'_s = \frac{a_s}{A}$ = fraction of unit cross sectional area of the soil mass occupied by solid to solid contacts

$u = H_A \gamma_w$ = pore water pressure i.e., hydrostatic pressure at A

■ The value of a'_s is extremely small and can be neglected, thus

So, where a'_s is a_s by A bar that is the fraction of unit cross sectional area of the soil mass occupied by solid-to-solid contacts. That means whatever you have or total say total cross-sectional area is say A bar. Now the fraction of that which will be occupied by the solid-to-solid contact is nothing but a'_s that is a_s by A capital A bar and u is nothing but the pore water pressure which is given by H_A into γ_w okay that is the total depth that is the H_A into γ_w that is pore water pressure that is the hydrostatic pressure at A okay.

I hope that you have understood right. So, 2 components you are getting in the total stress. One is coming from the soil grain contacts another is coming from the simple pore fluid or the water. So, that thing that which is which the pressure which is coming through the pore water or the pore fluid that is known as pore water pressure in the soil mechanics and it is very very important.

So, most of the theories most of the important say mechanism have been developed based on this concept effective stress concept okay. So, you need to understand that how we can find out effective stress and how we can find out total stress and how we can find out pore water pressure and how the pore water pressure is affecting the effective stress okay. This is very important phenomenon okay. The value of a'_s is extremely small as you can imagine right.

So, if you consider the whole domain or the whole cross-sectional area and now on top of the cross-sectional area basically the particles are lying individually and the contact area of the particles as compared to the total cross-sectional area is very negligible. So, if it is very

negligible then we can ignore that. If it is ignored then if a s is ignored then we can write sigma is equal to sigma prime plus u okay.

So, I hope that you have got it right. So, total stress very simple so in soil mechanics it is very important equation important expression to understand. So, total stress is equal to effective stress that is coming from the soil grain and pore water pressure that is coming from the pore fluids. So, sigma is equal to sigma prime plus u.

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In-situ Stresses

Stresses in saturated soil without seepage

Putting this in equation (3.1)

$$\sigma' = [HY_w + (H_A - H)Y_{sat}] - H_A Y_w$$

$$\sigma' = (H_A - H)(Y_{sat} - Y_w)$$

$$\sigma' = \text{(height of soil column)} \times Y' \quad (3.5)$$

i.e., the effective stress at any point, is independent of the depth of water H, above the submerged soil

Now putting this in equation 3.1 so sigma is equal to sigma prime plus u. If you put that thing in equation 1 so basically you will be getting sigma prime in this fashion. So, sigma prime is equal to H into gamma w plus H A minus H into gamma saturated minus H A into gamma w so from that you can get sigma prime equal to H A minus H into gamma sat minus gamma w. Now what is H A - H so that is nothing but height of soil column right.

So, capital H is the free water depth. It could be anything right. So, H A - H is nothing but the height of soil column into what is gamma sat minus gamma w if you recall your previous knowledge that is nothing but gamma submerged. So, sigma prime is equal to the height of soil column into gamma submerged that is all. So, wherever you go at whatever depth you want okay you can find out the sigma prime that is the effective stress is equal to height of soil column on top of this on top of that particular point into gamma submerged.

So, that is the effective stress at any point is independent of the depth of water that is capital H depth of free water right above the submerged soil. So, this is very important. So, whatever

magnitude of capital H you consider that is the depth of water you consider depth of free water rather you consider on top of the soil matrix or the soil body so that will not affect the effective stress. So, effective stress is independent of that particular depth capital H and so only the submerged unit weight will be coming into the picture and depth of soil layer.

So, I will stop here today. In the next class, we will be continuing this thing and we need to understand this stress in-situ stress concept in very effective way so that later on when we will be talking about the consolidation, shear strength, those things basically are based on this concept okay. Thank you very much.