

Geology and Soil Mechanics
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Lecture - 32
Consolidation - B

Welcome back. So, in the last lecture we discussed few issues, few important issues related to the fundamental of consolidation and we have seen 2 different types of soil one is normally consolidated soil another one is over consolidated soil and at this moment we are quite familiar to these 2 terms right. So, what is normally consolidated what is over consolidated and how to find out this preconsolidation pressure which will govern or which will tell about the state of the soil whether it is over consolidated or whether it is normally consolidated right. So, these are the few things we have discussed in the last lecture.

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Fundamental of consolidation

Calculation of settlement from 1D primary consolidation

- Let us consider a saturated clay layer of thickness H and cross sectional area A under an existing average effective overburden pressure σ'_0
- Because of an increase of pressure, $\Delta\sigma'$ let the primary settlement be S_c

$$\Delta V = V_0 - V_1 = HA - (H - S_c)A = S_c A \quad (4.10)$$

Where, V_0 and V_1 = initial and final volume

So, today we will be obtaining the calculation of settlement from 1D primary consolidation. So, let us consider a saturated clay layer of thickness capital H and cross-sectional area capital A under an existing average effective overburden pressure σ'_0 okay. So, we are considering one clay deposit which is completely saturated whose depth or the thickness is H and cross-sectional area is A and the it is under existing average effective overburden pressure σ'_0 .

Now because of an increase of pressure say $\Delta\sigma'$, so this σ'_0 was there okay that was the initial starting point of the calculation. Now because of an increase of pressure

delta sigma prime let the primary settlement be S_c okay. So, if that is true then ΔV that is the change in volume is nothing but $V_{v0} - V_{v1}$ where V_{v0} and V_{v1} are the initial and final volume of the soil deposit is equal to HA .

HA is nothing but the height thickness into cross sectional area that will give me the initial volume minus $H - S_c$ so that is the changed or the reduced thickness of the soil deposit because once you are applying some delta sigma prime on top of the soil deposit it will undergo some volume change and that volume change is proportional to the change in thickness as well already we have discussed this thing in the last lecture because you are providing some confinement in the lateral direction.

So, $H - S_c$ is now your new I mean thickness of the soil deposit where S_c is the settlement that means the deformation or the change in your thickness okay. So, $H - S_c$ into A . So, that will give me the final volume V_{v1} so which will give me final expression as S_c into A . So, S_c into A is nothing but ΔV which is nothing but the volume change okay or change in volume.

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Fundamental of consolidation

Calculation of settlement from 1D primary consolidation

- The change in total volume

$$\Delta V = S_c A = V_{v0} - V_{v1} = \Delta V_v \quad (4.11)$$
- Where, V_{v0} and V_{v1} = initial and final void volume
- We know $\Delta V_v = \Delta e V_s$ (4.12)
- And
$$V_s = \frac{V_0}{1 + e_0} = \frac{AH}{1 + e_0} \quad (4.13)$$
- Where, e_0 = initial void ratio at V_0

Now the change in total volume ΔV already we have seen that is equal to S_c into A and which is equal to $V_{v0} - V_{v1}$. What is V_{v0} ? V_{v0} is the initial void volume and V_{v1} is the final void volume because whatever volume change you are going to observe so that is simply the volume change in the void space right. So, the change in volume of void is nothing

but the change in actual volume of the soil deposit because soil grains or the soil solids are not going to exhibit any volume change right. Already we have discussed this thing.

So, V_v not minus V_v 1 is nothing but the volume change that is the initial void volume minus final void volume which is equal to ΔV_v so that is nothing but the change in void volume okay. So, change in volume is nothing but equal to change in void volume. So, now we know from our fundamental definition of void ratio that ΔV_v that is the change in void volume is nothing but equal to Δe into V_s where Δe is the change in void ratio right. So, we know that e is equal to V_v by V_s if you recall. So, basically Δe there is a change in void ratio is nothing but equal to change in volume divided by the volume of solid. So, ΔV_v is nothing but equal to Δe into V_s right.

So, from this I can write V_s is equal to V not by $1 + e$ not that we can establish from our previous say definition whatever we have covered for our void ratio and other things. So, V_s is nothing but the volume of solid is equal to V not okay divided by $1 + e$ not right. V not is the volume I mean initial volume is equal to H that is the initial volume already we have seen that is A is the cross-sectional area and H is the thickness and $1 + e$ not right where e not is the initial void ratio at V not.

So, when you have the volume of soil is V not at that time your void ratio is e not. So, we can establish this equation 4.13. So, equation 4.12 and 4.13 basically these equations are not new to you. These equations already we have covered. We already we have discussed and established earlier.

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Fundamental of consolidation

Calculation of settlement from 1D primary consolidation

From equation (4.10) - (4.13)

$$\Delta V = S_c A = \Delta e V_s = \frac{AH}{1 + e_0} \Delta e$$

Or $S_c = H \frac{\Delta e}{1 + e_0}$ (4.14)

■ For normally consolidated clays that exhibit a linear $e - \log \sigma'$ relationship

$$\Delta e = C_c [\log (\sigma'_0 + \Delta \sigma') - \log \sigma'_0]$$

So, from equation 4.10 that means 4.10 to 4.13 basically what we are getting delta V is equal to S c into A okay is equal to delta e into V s right is equal to AH divided by 1 plus e not into delta e right. So, from this I can find out or I can write S c that is the primary consolidation settlement S c is equal H into delta e by 1 plus e not. So, I mean this is the expression by which I can find out the primary consolidation settlement in any soil deposit.

So, you need to know the initial void ratio e not, you need to know the volume change that is nothing but the change in void ratio that is delta e, and you need to find out or need to observe the thickness or the initial thickness of the soil deposit. If you know these 3 parameters you can find out the consolidation settlement S c. Now based on different conditions whatever we have covered like normally consolidated or over consolidated you will be getting the different expression for S c. We will see that. So, but the backbone expression of S c is 4.14.

Now for normally consolidated clay that exhibit a linear e-log sigma prime relationship already we have discussed this thing that you will be getting some steep slope with linear relationship when the soil will be behaving as normally consolidated soil. When the soil will be behaving as over consolidated soil at that time you will be getting later rather flatter slope and some nonlinear portion. Well so delta e is equal to in case of in case of normally consolidated soil what is your delta e? Delta e is equal to C c into log sigma not prime plus delta sigma prime minus log sigma not prime.

So, this is basically if you recall your e-log sigma prime curve so basically this is this was the portion for normally consolidated soil. So, basically from this you will be getting so your delta C

e is nothing but so this is your say sigma not prime and this is your sigma not prime plus del sigma prime. So, from there I can establish so delta e is equal to some constant C c into log sigma not prime plus delta sigma prime minus log sigma not prime right. So, that we can establish from this plot right. So, that we are getting here okay.

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Fundamental of consolidation

Calculation of settlement from 1D primary consolidation

Or
$$S_c = \frac{C_c H}{1 + e_0} \log \left(\frac{\sigma'_0 + \Delta \sigma'}{\sigma'_0} \right) \quad (4.15)$$

Where, C_c = Slope of the e - $\log \sigma'$ plot and is defined as compression index

■ For Over consolidated clays for $\sigma'_0 + \Delta \sigma' \leq \sigma'_c$

$$\Delta e = C_s [\log (\sigma'_0 + \Delta \sigma') - \log \sigma'_0]$$


Now what is C_c ? C_c is a constant right C_c is a constant that is nothing but the slope of the e - $\log \sigma'$ plot and is defined as the compression index so that is known as compression index and you please remember these terms because frequently we will be using these terms later on when we will be talking about different issues related to the consolidation. So, compression index is nothing but the slope of e - $\log \sigma'$ plot under normally consolidated soil condition. Therefore, S_c can be written as C_c into H by $1 + e_0$ not, H by $1 + e_0$ not is nothing but delta. So, if you go back to this expression so your delta e S_c is equal to H into delta e by $1 + e_0$ not right

So, instead of in place of delta e we are putting this expression and we will be getting S_c equal to C_c into H by $1 + e_0$ not into $\log \sigma'_0 + \Delta \sigma' - \log \sigma'_0$ right. So, basically in case of normally consolidated soil if you want to find out the 1D primary consolidation settlement basically you need to know what was the initial effective stress and then after incremental load because you are going to find out the settlement for some increment of load or the increment of pressure right.

So, that increment of pressure if you know that $\Delta \sigma'_{\text{okay}}$ if you know the initial thickness of the soil if you know the initial void ratio e not okay these are the things can be obtained right or can be observed or can be determined and C_c is the constant some constant we will find out or we will see that how to find out this constant. So, if you know these parameters you can find out the primary consolidation settlement in the normally consolidated soil okay. Now for over consolidated clay what will happen?

So, for over consolidated clay you have to desiccate 2 different regions. One region is that when $\sigma'_{\text{not prime}} + \Delta \sigma'_{\text{prime}}$ that is the increment of the total pressure is less than equal to the preconsolidation pressure $\sigma'_{c \text{ prime}}$ at that time soil will be behaving as complete over consolidated soil am I right. So, basically if you recall your e log σ'_{prime} plot so basically this is the something like this.

So, you are basically if you have this condition so this is basically your say $\sigma'_{c \text{ prime}}$ okay. So, you are within this region. So, if you are within this region then that means whatever I mean increment of load or whatever total enhancement or increment is happening in the application of the load that is well below the preconsolidation pressure so will be behaving as over consolidated soil by definition right.

So, in the over consolidated clay Δe is equal to C_s some constant okay. We will see that what is C_s , C_s into $\log \sigma'_{\text{not prime}} + \Delta \sigma'_{\text{prime}} - \log \sigma'_{\text{not prime}}$. So, it is as simple as that I mean because this is the linear regression again. You have the linear portion. This portion is linear portion but that is under over consolidated part so this linear portion can be defined by this expression.

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Fundamental of consolidation

Calculation of settlement from 1D primary consolidation

Or
$$S_c = \frac{C_s H}{1 + e_0} \log \left(\frac{\sigma'_0 + \Delta\sigma'}{\sigma'_0} \right) \quad (4.16)$$

Where, C_s = Slope of rebound curve and is defined as "swell index"

And for $\sigma'_0 + \Delta\sigma' > \sigma'_c$

$$S_c = \frac{C_s H}{1 + e_0} \log \left(\frac{\sigma'_c}{\sigma'_0} \right) + \frac{C_c H}{1 + e_0} \log \left(\frac{\sigma'_0 + \Delta\sigma'}{\sigma'_c} \right) \quad (4.17)$$

So, now if I put the expression or the I mean equation for delta e then we will be getting S c is equal to C s into H by 1 plus e not into log sigma not prime plus delta sigma by sigma not prime. Now what is C s? C s is nothing but slope of rebound curve that is the unloading curve right. Basically, that part whatever I have shown you that is the unloading part right, slope of the rebound curve and is defined as swell index that means you are allowing the soil to swell.

That means whatever load you have applied now you are coming back right so C s is known as swell index. Now in case of if you have the second region so as I told you that in case of over consolidated soil you will be having 2 different region so one region is that when you are below the preconsolidation pressure and when you are above the preconsolidation pressure. So, if you are I mean if you are I mean within the or above the preconsolidation pressure that means sigma not prime plus del sigma prime is greater than sigma c prime.

At that time S c is given by this part so C s into H by 1 plus e not log sigma c prime by sigma not prime. So, this part will give you the settlement due to your rebound curve or the over consolidated part and this part will give you normally consolidated path okay. So, this 4.17 will give you the total settlement for over consolidated soil when your stress increment or the load increment is greater than the preconsolidation load.

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Fundamental of consolidation

Compression index (C_c) & swell index (C_s)

- Skempton (1944) suggested for compression index for undisturbed clays

$$C_c = 0.009(w_L - 10) \quad (4.18)$$

- The swell index is appreciably smaller in magnitude and is given by

$$C_s \approx \frac{1}{5} \text{ to } \frac{1}{10} C_c \quad (4.19)$$

So, Skempton 1944 suggested now we will find out this compression index C_c because these are 2 constants we have just now observed that compression index C_c and swell index C_s . Now we will find out how to find out this this index or these constants right. So, Skempton 1944 suggested for compression index for undisturbed clays C_c is equal to 0.009 into W_L minus 10. What is W_L ? W_L is nothing but the liquid limit already you know right.

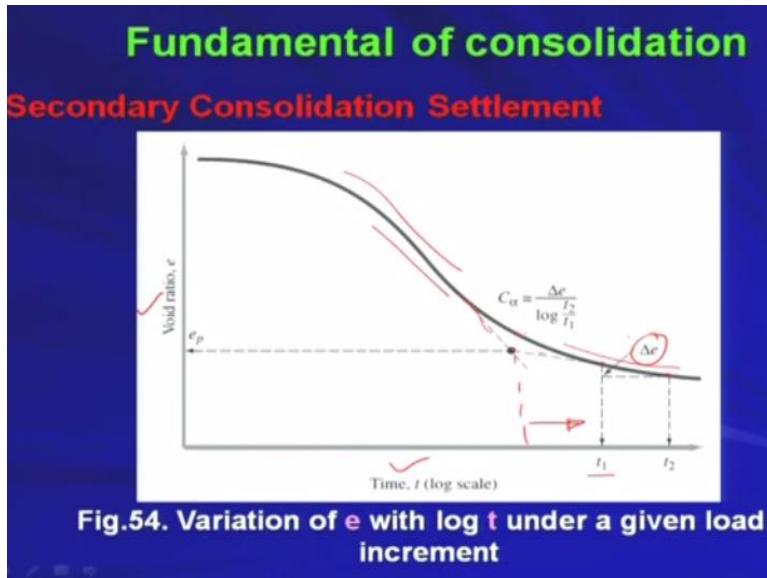
So, if you know the liquid limit of that clay you can find out C_c which will be constant for that particular type of soil. Similarly, the soil index is appreciably smaller soil index as I told you because you will be getting some flatter slope. C_c will be much more higher than C_s because in case of normally consolidated soil you will be getting some steeper slope whereas in case of over consolidated soil you will be getting some flatter slope.

So, C_s is associated with the over consolidated soil. So, the soil index is appreciably smaller in magnitude and is given by C_s approximately equal to one fifth to one tenth of C_c okay. So, now you can see I mean what will be the order of C_s . So, C_s is much more lower than C_c .

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Now coming to the secondary consolidation. As I told you will be having 2 different type of consolidation settlement one is the primary consolidation settlement another one is the secondary consolidation settlement. So, in case of secondary consolidation settlement let us find out when you will be having or when you will be exactly talking about secondary consolidation settlement.

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So, basically you have the curve like that okay. So, initial flatter part, then the steeper part right. In this part, you will be having some steeper slope and then basically after this point basically you will be getting some flatter I mean curvilinear part. So, now basically if you extend this straight-line part and this straight line parts you will be getting some point here. So, from this point onward you will be getting some secondary consolidation okay.

So, now let us find out the secondary consolidation. So, here actually if you see this is with respect to time so void ratio versus time plot in log scale. So, this is t_1 . At t_1 whatever void ratio was there at t_2 this is the void ratio so your difference in void ratio is Δe . So, you can establish right the equation for this part or this secondary consolidation part.

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Fundamental of consolidation

Secondary Consolidation Settlement

- The secondary compression index can be defined

$$C_{\alpha} = \frac{\Delta e}{\log t_2 - \log t_1} = \frac{\Delta e}{\log(t_2/t_1)} \quad (4.20)$$

- The magnitude of secondary consolidation can be calculated as,

$$S_s = C'_{\alpha} \cdot H \cdot \log(t_2/t_1) \quad (4.21)$$

Where, $C'_{\alpha} = \frac{C_{\alpha}}{1 + e_p}$

So, let us see what is the equation. C_α that is the constant already we have seen in the last figure that is C_α okay. So, that is nothing but the slope of this line in the secondary consolidation part. So, C_α is equal to Δe by $\log t_2$ minus $\log t_1$ agreed? Just now we have seen. So, that is nothing but Δe by $\log t_2$ by t_1 . So, the magnitude of secondary consolidation can be calculated as so this is the secondary consolidation settlement S_s is equal to C_α prime into H into $\log t_2$ by t_1 . So, where C_α prime is nothing but C_α divided by $1 + e_p$ okay.

So, it is I mean the expression for the settlement will be remaining same whatever we have discussed right H into Δe by $1 + e_{not}$, e_{not} is the initial void ratio. In case of secondary consolidation what is the initial void ratio that is the final void ratio of the primary consolidation because once you cross the primary consolidation then only you will be entering the secondary consolidation part. The final void ratio of the primary consolidation will be the initial void ratio of the secondary consolidation.

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Fundamental of consolidation
Secondary Consolidation Settlement
 e_p = void ratio at the end of primary consolidation
 H = Thickness of clay layer

- Secondary consolidation settlement is more important than primary consolidation in organic and highly compressible inorganic soils
- In over consolidated inorganic clay the secondary compression index is very small and of less practical significance

So, here in this expression e_p is void ratio at the end of primary consolidation and H capital H is the thickness of clay layer. So, if you know these things so you can find out so at you observe some time t_2 and t_1 and you find out C_α prime. You know H so you can find out secondary consolidation settlement right. So, secondary consolidation settlement is dependent on time. So, as you because that will be happening throughout the life of the structure so it may

happen. So, immediate settlement or the primary settlement will be happening based on the primary consolidation.

After that the secondary consolidation will be always there and secondary consolidation settlement will be negligible I mean we will come to that point. So, secondary consolidation settlement is more important than primary consolidation in organic and highly compressible inorganic soils. So, please try to remember secondary consolidation settlement is more important okay. It will be giving you some significant value in case of organic as well as highly compressible inorganic soil.

So, the soil which is highly compressible that means even after the primary consolidation you will be observing some significant amount of secondary consolidation. However in most of the cases in over consolidated inorganic clay the secondary compression index is very small and of less practical significance. So, that is why people generally for most of the general cases people only calculate the primary consolidation settlement and based on that they design the whole structure right.

Say I mean the secondary consolidation will be much more lower than the primary consolidation settlement in case of normally I mean normal soil over consolidated normal soil. So, in case of that type of soil your primary consolidation has to be given more emphasis than the secondary consolidation. So, that is why people are not that much interested to find out the secondary consolidation settlement because it depends on time so as the time goes on you will be observing some amount of settlement.

However, if you are dealing with organic soil or highly compressible inorganic soil then you must I mean think about the secondary consolidation and you must calculate the secondary compression or the secondary I mean consolidation settlement and then based on that you need to design the structure.

So, if you design the foundation in organic soil or highly compressible inorganic soil you need to take care of the settlement even under secondary consolidation. So, that I mean to say okay so that basically you need to think about so first you do some experiment and then you find out what type of soil you are dealing with then you go for whether you will consider the secondary consolidation settlement or whether you ignore the secondary consolidation settlement.

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Fundamental of consolidation

Time Rate of Consolidation

- Terzaghi (1925) proposed the 1st theory to consider the rate of 1D consolidation for saturated clay soils

Assumptions

1. The clay water is homogeneous
2. Saturation is complete
3. Compressibility of water is negligible

Now time rate of consolidation. Now Terzaghi in 1925 proposed that first theory proposed the first theory to consider the rate of 1D consolidation for saturated clay soils okay. Now we will try to develop the 1D consolidation theory okay. Based on that we can calculate the consolidation for a particular deposit okay under completely saturated situation. Now what are the different assumptions. Now the clay water is homogeneous.

So, the clay water is homogeneous means the clay is completely saturated. That means you do not have any kind of different types of fluid present in the soil so whatever water is there that is completely homogeneous and clay is completely saturated. Saturation is complete as I told you. The complete saturation is required. So, you do not have any partially saturated soil. So, if you deal with partially saturated soil what will happen because of your air will be taking care of or air will be occupying some amount of voids.

So, that voids will be behaving in a different way than the voids which will be filled with complete water right. So, that is why the consolidation theory is based on completely saturated soil. If you have partially saturated soil you will be having something else. So, that is not the scope of this particular course. However, you need to I mean most of the times you will be finding out the settlement or the consolidation settlement which will be the under that situation the soil will be completely saturated okay.

Then compressibility of water is negligible. So, that means water is completely incompressible material that you are considering and that is I mean that, that will be a good say assumption I mean there is no problem in that. So, we know from our mechanics point of view that water is

incompressible material and therefore the compressibility of water if we neglect in the determination of consolidation theory then there will be no harm.

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Then fourth one is compressibility of soil grains is negligible. So, frankly speaking you will be getting some if you go for some higher load application basically you will be observing that the some compressibility is happening in the soil I mean soil fabrics or the soil particles but that compressibility is very say negligible as compared to the compressibility of the soil matrix as a whole.

So, that compressibility of soil grains each individual soil grain is negligible and hence we are not considering that thing in the determination or in the analysis. Then fifth one is the flow of water is in one direction only. So, when we are talking about the 1D consolidation theory when we are trying to develop or establish the 1D consolidation theory basically the flow of water is in one direction.

So, you can have the flow in different directions right. All 3 directions you can have the flow but that we are not considering for understanding the 1D consolidation theory. So, that can I mean that could come in case of when we are talking about the 3D consolidation theory. At that time 3 direction flow you are considering but however that is not the scope or that is beyond the scope of this particular course.

Then the flow is I mean not turbulent okay the flow is laminar therefore the Darcy's law is valid. So, whatever law you have observed or you have established in case of permeability of soil so

that Darcy's law is valid. That means V that is the velocity is equal to K that is the coefficient of permeability into i that is the hydraulic gradient. So, that equation is valid and based on that we can establish the 1D consolidation theory.

So, thank you very much. So, I will stop here today. So, in the next lecture we will talk about or we will see or we will establish the 1D consolidation theory. Thank you.