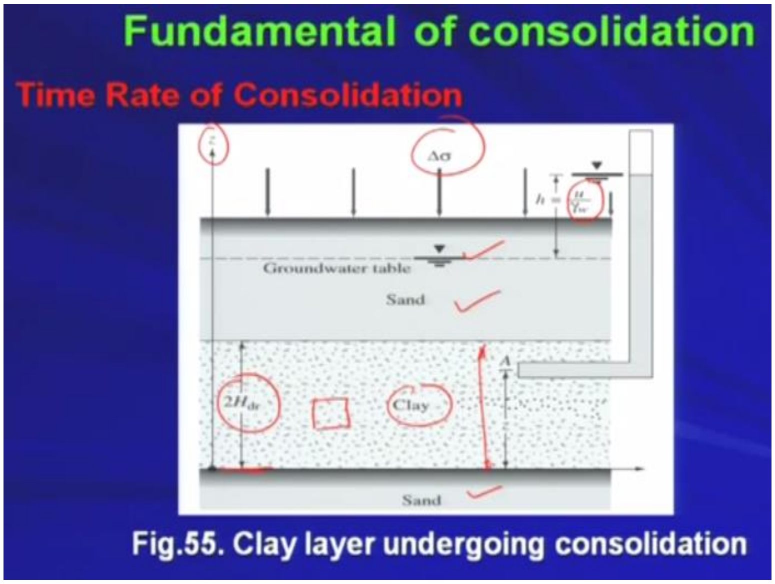
Geology and Soil Mechanics
Prof. P. Ghosh
Department of Civil Engineering
Indian Institute of Technology Kanpur
Lecture - 33
Consolidation - C

Welcome back. So, in the last lecture we just started the discussion on the 1D consolidation theory proposed by Terzaghi and there we have we discussed about the assumptions. Based on that this 1D consolidation theory was established. So, those assumptions already we have discussed.

(Refer Slide Time: 00:35)

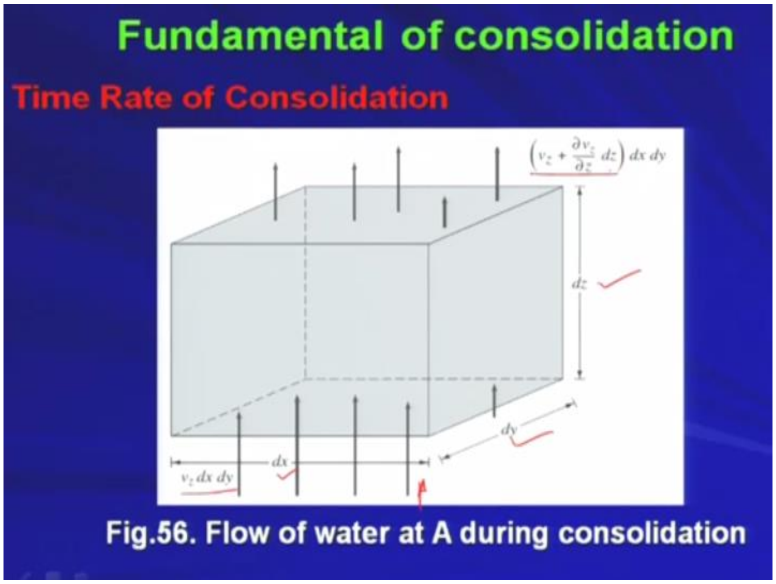


Now basically we are considering this kind of say soil deposit. So, you have sand at top and you have sand at bottom and in between that you have the clay layer which is getting sandwiched between these 2 sand layers. You have the ground water table is here so that the clay is or the clay deposit is completely saturated which is one of the assumptions right.

The depth of the clay layer is 2 times H dr so why 2 is coming to the picture we will be coming to that point later on because there is something else because I will use only H dr so 2 H dr will be is coming into the picture to take care of something. So, that I will come to that point later on okay. So, z that is the vertical direction so at this interphase z is 0 and z is continuously increasing in the vertically upward direction.

Now we are applying some stress increment delta sigma and you have at any point along this clay deposit you can find out the excess pore water pressure due to the application of this stress increment or the pressure increment as u by gamma w. So, that is nothing but the head of the that u is the excess pore water pressure and h is the head of water due to that. So, that h you can find out by putting the piezometer at any location in the clay deposit okay. So, this is the problem. So, this problem we are going to say analyse.

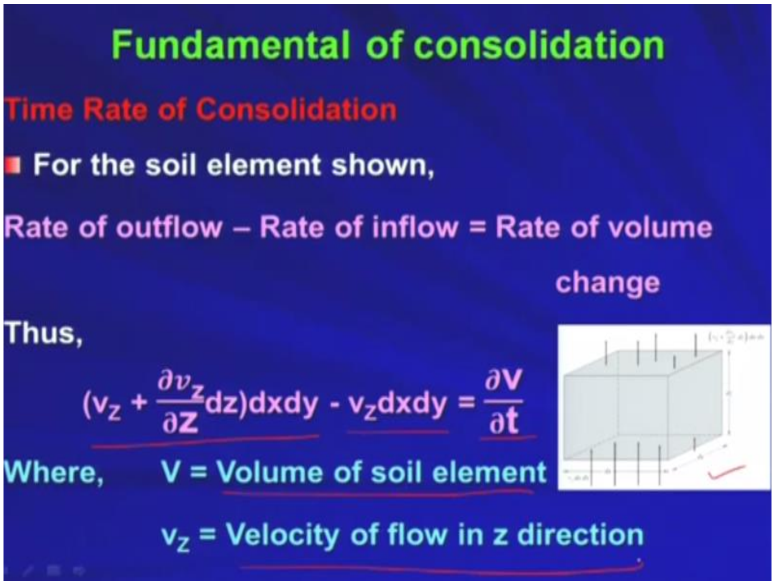
(Refer Slide Time: 02:05)



Now we are considering any say soil element okay some 3 dimensional soil element we are considering in the clay deposit so which is having the dimensions dz in the z direction dy in the y direction and dx in the x direction. So, dx, dy, dz is the total volume dx into dy into dz is the total volume of this soil element. Now we are considering 1D flow that means the flow will be only in the one direction as that is another assumption in the 1D consolidation theory establishment right.

So, the flow is happening so this is the inflow direction. The inflow is happening on this plane okay so on this plane so v x into dx into dy. So, v z is the velocity of flow or the water okay so on in the inflow when it is entering the soil element and when it is exiting the soil element at that time the velocity is v z plus del v z del z dz right. So, that is the incremental velocity.

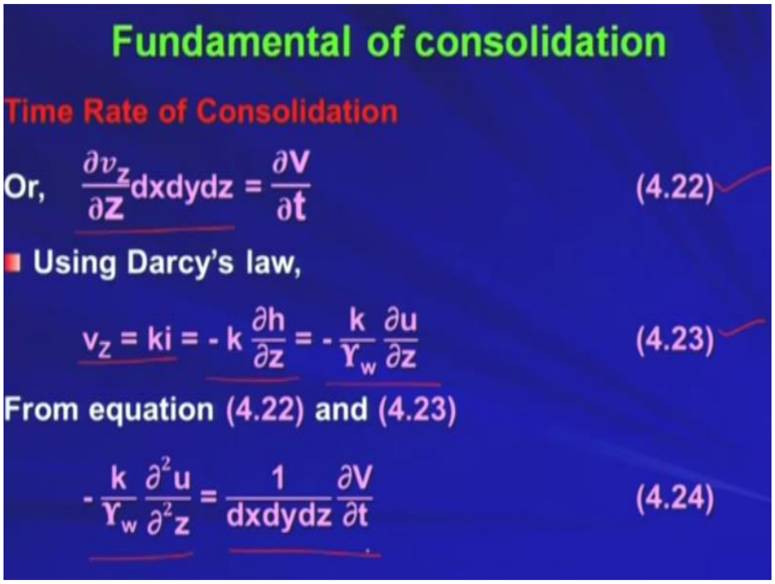
(Refer Slide Time: 03:08)



Now for the soil element now for the soil element shown okay in this figure okay rate of outflow please try to understand. So, this is completely mathematical. So, if you understand this thing then rest of the things will pretty I mean similar or simple right. So, rate of outflow minus rate of inflow that means whatever amount of water is going out and whatever amount of water is going in okay that difference between these 2 will be nothing but the rate of volume change am I right or not because the rate of inflow whatever is entering and whatever is going out the total quantity if they mismatch that mismatch is happening due to the volume change of the soil deposit right.

So, thus what is rate of outflow rate of outflow is nothing but v z into del v z del z dz into dx dy right on that area on that cross-sectional area dx dy the flow is happening so that is the quantity of outflow. What is the rate of inflow? Rate of inflow is nothing but v z into dx into dy that is nothing but your rate of inflow that is the amount of water or the I mean fluid is entering the soil element and that is nothing but rate of volume change del v dt okay where v is the volume of soil element and v z is the velocity of flow in z direction agreed. So, this is the expression for your fundamental 1D consolidation theory I mean first I mean this is the backbone equation or the expression for your 1D consolidation theory.

(Refer Slide Time: 04:48)

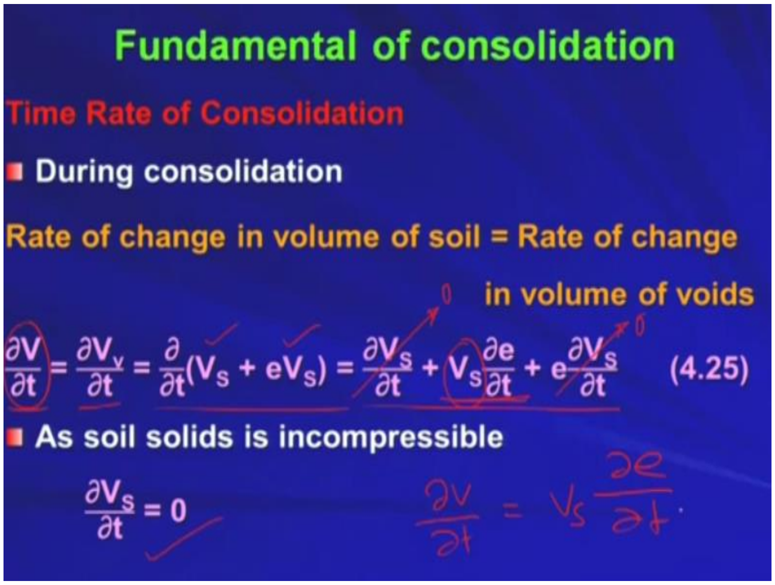


Now from that equation we can get del v z del z dxdydz is equal to del v dt where v is the volume of the soil. So, using Darcy's law I can write v z equal to k into i now k is the coefficient of permeability and i is the hydraulic gradient. Now that can be written as minus k into del h del z because minus sign is coming because your I mean you are getting basically the height or the pore fluid okay.

That is this del h del z is nothing but your hydraulic gradient right so minus sign is coming because of your pore I mean pore pressure right or pore fluid pressure is I mean coming down or it is decreasing with increase in the velocity. So, now that minus k into del h del z is equal to now in place of h you can write down u by gamma w right already we have seen u by gamma w we can write in place of h, h is the head of pore water pressure.

So, in place of h we can write down u by gamma w. So, if I put h equal to u by gamma w so I can write minus k gamma w del u del z right. So, that is inversely proportional so that is why this negative sign is coming into the picture. Now from equation 4.22 and 4.23 so from these 2 equations basically I can write in place of del v z del z I can write k by gamma w del 2 u del 2 z is equal to 1 by dxdydz into del v dt right very simple.

(Refer Slide Time: 06:55)

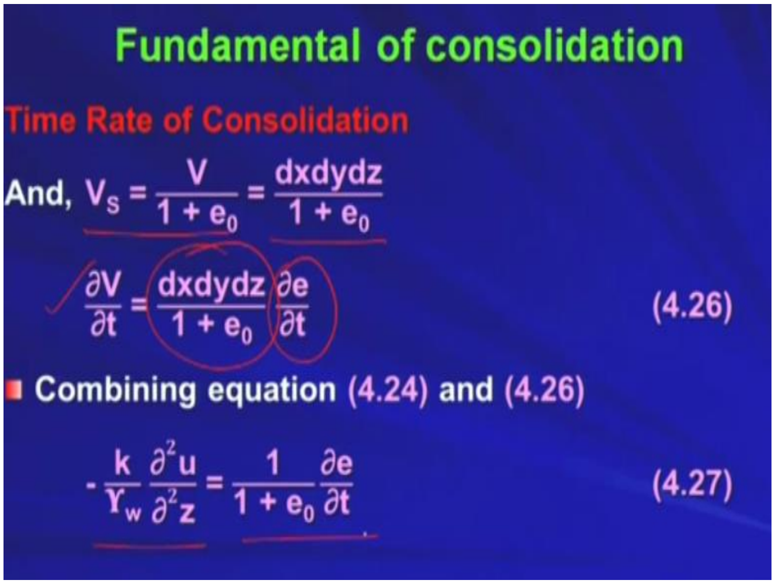


Now during consolidation, rate of change in volume of soil is equal to rate of change in volume of voids. Already several times we have discussed this thing. So, volume of soil whatever change is happening in the volume of soil that is nothing but equal to the volume of I mean change in volume of voids right. So, if that is true so del v dt that is the rate of change in volume of soil is equal to del V v by dt del t.

So, which is nothing but rate of change in volume of voids which can be written as in place of V v I can write V s plus e into V s agreed where V s is the volume of solid and e is the void ratio and we can write down V s plus V s is nothing but V v. So, I can write this. So, from this if you expand this term so I can write del V s del t plus V s into del e del t plus e into del V s del t.

Now as soil solids is incompressible so already this is one of the assumptions we are considering the soil solids are incompressible. As the soil solids is incompressible then basically del V s del t must be equal to 0. There will be no change in volume of solid with time. So, del V s del t must be equal to 0. If that is 0 so this term will be 0 and this term will be 0. So, this term will be left out.

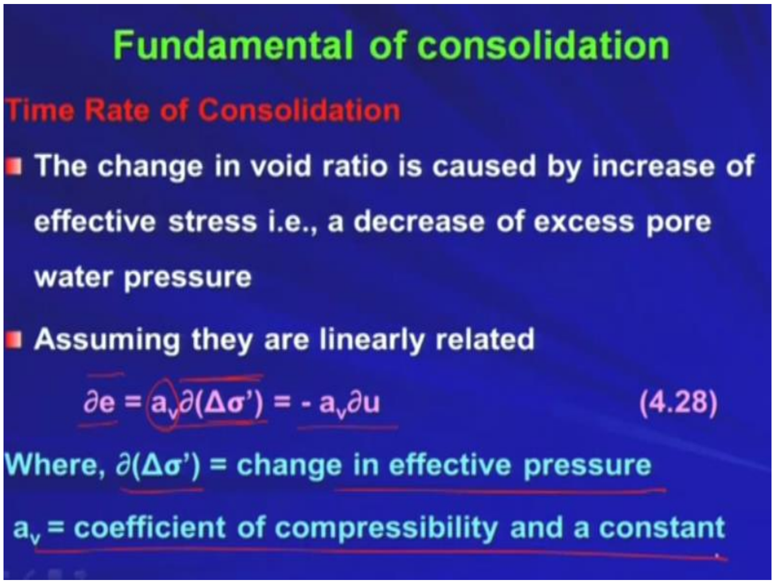
(Refer Slide Time: 08:32)



So, V s again we can write we can see or we can establish this thing from our previous background, previous knowledge that V s is equal to V by 1 plus e not. So, basically, we can establish this relation V s is equal to V by 1 plus e not from our previous knowledge whatever we have covered so which can be written as dx, what is V, Vis the volume of soil element that is nothing but dxdy and into dz. So, that divided by 1 plus e not.

So, del v dt is equal to so basically in place of V s so basically del v dt is equal to del v dt is equal to V s del e del t right. So, that is in place of V s del v dt is equal to V s into del e del t right. Now combining equation 4.24 and 4.26 okay we will be getting minus k by gamma w del 2 u del 2 z is equal to 1 plus 1 by 1 plus e not into del e del t right. If you combine these 2 equations you will be getting this expression.

(Refer Slide Time: 09:59)



Now the change in void ratio is caused by increase of effective stress that is a decrease of excess pore water pressure. That also we know from our earlier discussion right. The change in void ratio is caused by I mean which will cause this change in void ratio that will be basically your increase in effective stress. If you have the increment in the effective stress then only the volume will be changing right. Otherwise there will be no change in volume already we have seen.

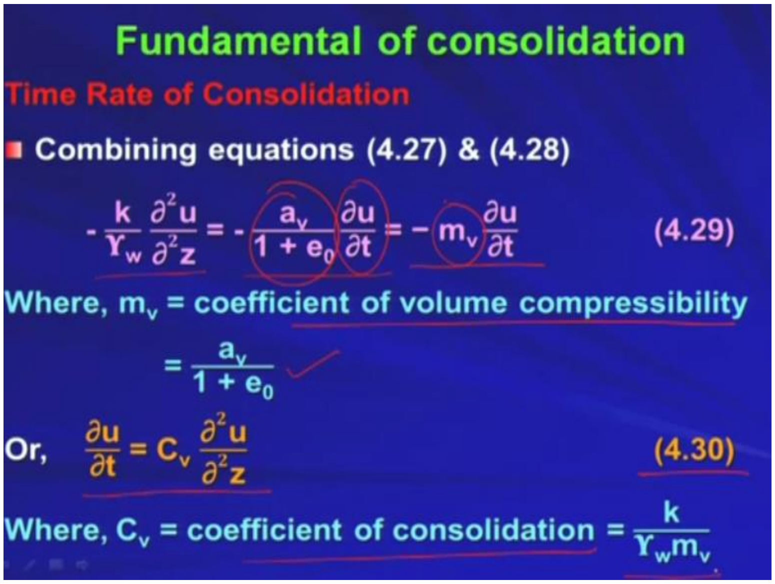
The volume change is completely associated with the increase or enhancement in the effective stress. So, and which will further tell you that there will be some excess pore water pressure will be going down right. So, decrease is decrease of excess pore water pressure. So, the change in void ratio is caused by increase of effective stress that is a decrease of excess of pore water pressure agreed. So, assuming they are linearly related.

That means this change that means change in void ratio okay caused by some increase in effective stress and decrease in pore water excess pore water pressure. So, these things are linearly related okay. So, the relation between these things are linear. If that is true then del e that is the change in void ratio is equal to some say I mean some constant a v into so if effective stress increases I mean change in void ratio will be happening.

So, a v some constant into multiplied by del into delta sigma prime right. So, that is nothing but this gives me the increase in or the change in effective stress. So, if it increases that also will be happening. So, this is some constant. We will come to that point what is this constant what is known as. Therefore, you have some that is equal to minus a v into del u. Why minus? Because width I mean enhancement in the or increase in the effective stress you will be getting decrease in the excess pore water pressure. So, that will be I mean inversely I mean not inversely that will be in the negative relation right.

So, minus a v into del u. Now where delta sigma prime is nothing but change in effective pressure and a v is coefficient of compressibility and a constant. So, a v is known as coefficient of compressibility. So, please remember this term like C c, C s all those things we remember. Similarly, a v you need to remember what is that is known as coefficient of compressibility. That is nothing but the change in void ratio divided by change in effective stress. So, that ratio because you have the linear relation so that ratio is nothing but the coefficient of compressibility.

(Refer Slide Time: 13:02)

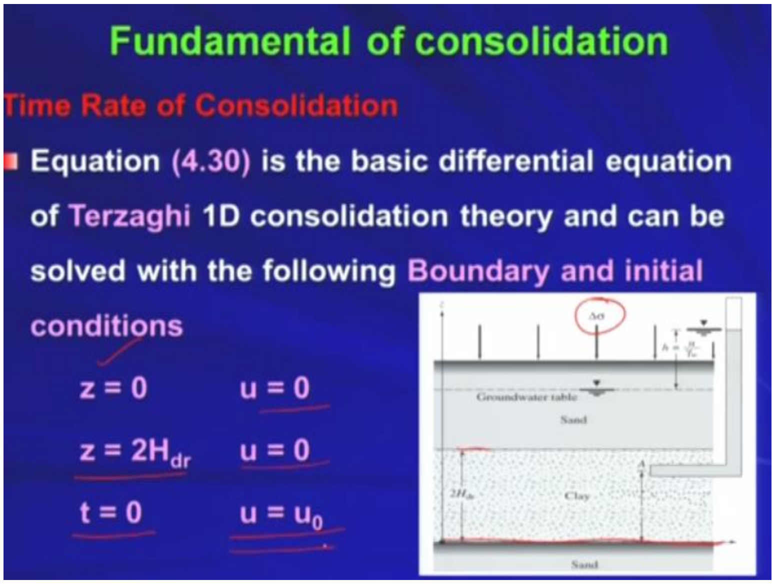


Now combining equations 4.27 and 4.28 now we can get so this was there earlier. Now we can write that is minus a v into 1 plus e not into del you del t. So, if you go back so del e del t in place of that we can write down this del u del t. In place of del e del t we can write down del u del t minus a v into del u del t right. That is nothing but equal to minus m v into del u del t where m v is the coefficient of volume compressibility and is equal to a v by 1 plus e not okay.

So, a v by 1 plus e not that is nothing but your m v and that is known as coefficient of volume compressibility okay. So, therefore we are getting one expression like this del u del t is equal to C v into del 2 u del 2 z okay which is nothing but equation 4.30. So, we are getting some partial differential equation which is nothing but the 1D consolidation equation okay so del u del t is equal to C v into del 2 you del 2 z where C v is the coefficient of consolidation and is given by k by gamma w into m v okay.

So, please now try to understand how you have got his 1D consolidation equation right. So, now if you try to solve this equation what are the things you need. You need boundary conditions as well as initial condition right. So, let us see what are the different boundary conditions and initial condition you have.

(Refer Slide Time: 14:39)

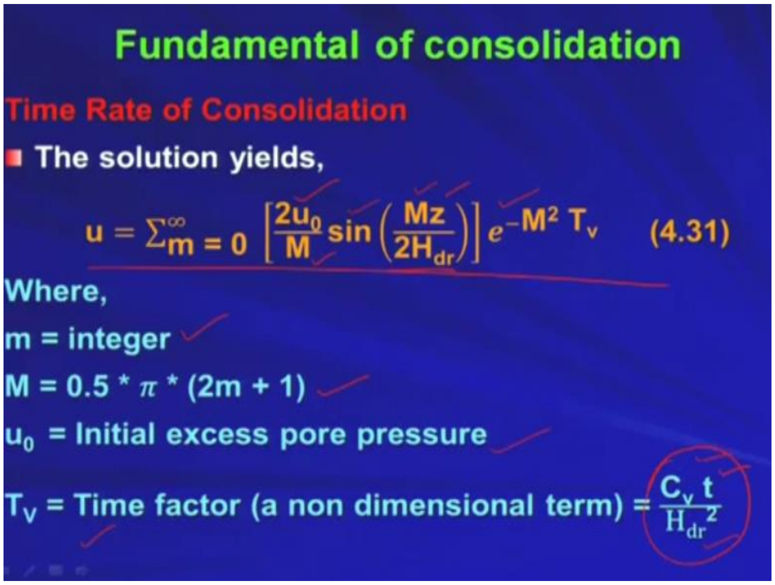


So, equation 4.30 is the basic differential equation of Terzaghi 1D consolidation theory and can be solved with the following boundary and initial conditions. So, what are the different boundary and initial conditions? At z equal to 0 so this is the first boundary condition at z equal to 0 that means you are here okay. What is your excess pore water pressure? That must be 0 because that is the interphase between the clay and sand layer.

So, the water will be draining out through the sand layer right. So, at that location you will be having the u equal to 0 that is the excess pore water pressure must be equal to 0. Then where will be the other location where excess pore water must be 0? That will be at 2 into H dr. So, z equal to 2H dr your excess pore water pressure must be 0 and at t equal to 0 your excess pore water pressure was u not u equal to e not right.

That means you are just applying the increment of effective I mean total stress that is delta sigma and immediately you will be observing that excess pore water pressure will be building up right and that is nothing but u equal to u not okay. So, these are 2 I mean different boundary conditions and initial condition.

(Refer Slide Time: 15:55)



Now if you apply those things and if you go back to your mathematics and if you solve this differential equation you will be getting the solution like this. So, u equal to summation m equal to 0 to infinity plus 2u not by M capital M into sin Mz by 2H d r into e to the power minus M square T v where m small m is an integer. Capital M is 0.5 into pi into 2m plus 1. u not is the initial excess pore water pressure as I told you and T v is the time factor that is a non dimensional term and this time factor is equal to C v into t what is C v coefficient of consolidation already we have seen C v into t, t is the time divided by H dr square okay.

So, H dr square means half of the soil deposit depth. So, that is why we have taken we had taken twice H dr that is why we will be we are getting this very simple expression like the twice H dr was the total depth of the clay deposit. Now we are getting H dr at the bottom. So, that means the I mean basically if you know T v that means for different values of t you can establish T v by knowing H dr right.

So, if you know T v if you know u not if you know all those things you can find out the u that is the what is you, u is the excess pore water pressure at different time right. So, at as time increases you can find out how your excess pore water pressure is going to build up. So, that is your main objective right in the and or how your excess pore water pressure is getting dissipated right. So, as you know at t equal to 0 your excess pore water pressure must be equal to u not.

At t equal to infinity your excess pore water pressure must be equal to 0 right? So, in between that from t equal to 0 to t equal to infinity how the pore water pressure is getting distributed so that you will be getting from this expression and once you know the excess pore water pressure dissipation or the excess pore water pressure I mean how that is getting distributed, based on that you can find out the effective stress distribution because if you know the total stress you can find out the effective stress by knowing the excess pore water pressure.

So, I will stop here today. So, in the next lecture we will be talking or we will be seeing or we will be observing the excess pore water pressure distribution and then other things we will discuss about consolidation theory in soil mechanics. Thank you very much.