

**Geology and Soil Mechanics**  
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**Lecture – 38**  
**Shear Strength of Soil - A**

Welcome back to the course Geology and Soil Mechanics. So, today we are going to start the new topic that is shear strength of soil. So, this is very important to understand the stability or the failure happening in the soil. Now any geotechnical structure if you think of like your earth retaining structure or your soil embankment, soil slope, and then foundations and any kind of say geotechnical structure if you consider so basically to find out the ultimate limit or ultimate strength of those structure that means the I mean till the failure how much it takes care of the load. So, if you want to find out that.

Say for example you might have seen this is quite common phenomena in the natural life that landslide right so you might have seen or you might have heard this phenomena that landslide. So, landslide is basically is caused by the instability happening in the soil. Now what kind of instability and how this instability is happening?

So, any material if you consider in the world every material will be having some limit okay to carry any kind of load right. So, if I give you or if I ask you to carry some 10 kg of load you can carry right. If I ask you 20 kg of load still you can carry. But if I ask you 100 kg of load you cannot carry. So, you have some inherent limit. So, that limit is nothing but the strength right. You have some inherent strength by which you can carry different kinds or different amounts of load right.

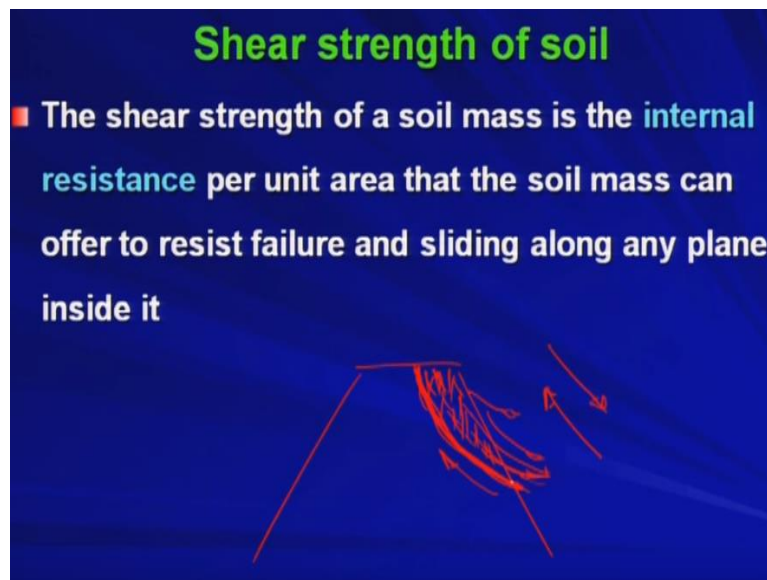
The same thing is applicable to the soil also. So, soil is having some inherent strength and that strength is nothing but your shear strength I mean we will see that. This strength is basically associated with the shear resistance or the shear I mean force I mean happening in the soil at a particular surface or particular plane. Now this resistance or this shear strength basically will try to resist whatever load is coming on top of the soil or whatever external forces are coming on top of the soil.

Suppose for example the foundation if you consider the building foundation so the foundation will be having some limit I mean because the soil will be having some limit so each foundation will be having some limit to carry the load. So, you cannot go on extending the load on a particular type of foundation right. You cannot I mean if your foundation can carry say 10 I

mean ton per meter square load you cannot apply or you cannot impose some say 25 kilo I mean ton per meter square load on top of the foundation.

So, your foundation will be undergoing failure right. Similarly, in case of your slope stability or the retaining structure or the even the landslide right so every soil will be having some resistance along any particular surface right. So, and if some instability is happening if that limit is reaching at that particular surface the soil will try to flow or try to slide over the surface and that is why the landslide happens right due to some instability.

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Now basically the shear strength of a soil mass is the internal resistance per unit area that the soil mass can offer to resist failure and sliding along any plane inside it. So, for example say I mean you have some embankment kind of thing okay or you have some slope I mean natural slope okay now the soil you might have seen the soil that landslide or the soil failure it is happening like that right. So, this part of the soil this shaded part of soil it will try to move in this direction flow in this direction right. Now why?

This if you consider this surface if you consider this curved surface right so along that surface some instability is happening that is why the shaded soil part is getting detached or getting separated out from the rest of the soil body right. So, this surface is becoming some critical surface and that is known as the failure surface and along that failure surface your resistance right so soil is I mean if you apply some load and that I mean due to that load basically the internal resistance will be getting developed right.

So, if I ask you to carry some load so your internal muscle or internal say strength will be coming out and that will try to resist that much of load external load. Similar kind of thing is happening in soil also. So, when this weight of the soil is causing that shaded part of the soil is causing some I mean extra load or the it is crossing the limit of the soil resistance then it will try to move in this direction or the move along this plane.

So, that is why it is known as the internal resistance though that is the internal or inherent property of the soil internal resistance per unit area that the soil must can offer to resist failure. So, when this wedge will try to flow in this direction so your resistance will be acting in the opposite direction right. So, the action so if this is the direction of action your direction of reaction or the resistance will be just in the opposite direction right. So, to I mean it will try to resist the failure on and sliding along any plane so along any plane as I showed you this is your this is the plane along this plane basically it will try to resist the sliding.

Now I mean unless I mean till you are within the limit of the soil strength basically this sliding or this failure will not be happening. Once you cross or once you reach that means on the verge of failure basically what will happen the wedge will try to I mean flow or try to slide over the surface right and that is why you get the I mean the landslide or the slope failure or the retaining structure failure some kind that kind of thing right. So, basically it tells you that if you have the instability so gradually you are applying load and I mean the gradually your strength is getting mobilized along that surface and once it reaches that strength I mean limit right it will collapse or it will fail right.

So, now we are going to I hope that you have understood what is the phenomena of shear strength. So, now we are going to analyse this shear strength and we will like to I mean establish the relations or the failure criteria for this kind of particular this kind of shear failure and also, we will see that what are the different types of laboratory experiments available to determine the shear strength of soil as well as some field experiment by which you can determine the shear strength of soil so those things we will see in this particular chapter.

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## Shear strength of soil

### Mohr-Coulomb Failure Criteria

- Mohr (1900) presented a theory for rupture in materials that contended that a material fails because of a critical combination of normal and shear stress, and not from either maximum normal or shear stress alone

i.e.  $\tau_f = f(\sigma)$  (5.1)

- The failure envelope defined by Eq. (5.1) is a curved line

So, first we should understand the Mohr-Coulomb Failure Criteria. I hope that you are quite familiar to the different failure criteria from your mechanics course like von Mises Failure Criteria, Tresca Failure Criteria, Drucker-Prager Failure Criteria similarly Mohr-Coulomb Failure Criteria. So, what is the Mohr-Coulomb Failure Criteria? Let us talk about that and once we establish the Mohr-Coulomb Failure Criteria and based on that we will try to establish the relation of shear strength and shear stress.

Now Mohr in 1900 presented a theory for rupture in materials that contended that a material fails because of a critical combination of normal and shear stress and not from either maximum normal or shear stress alone right. So, what it says that any I mean any material will under some failure when some critical combination of normal and shear stress is happening; critical combination means it is not I am I cannot get the failure if the normal stress is getting maximum or the shear stress is getting maximum.

So, this is the common I mean say common thinking right. Say if my normal stress is getting maximum then I may get the failure no, Mohr is saying no. You may not get the failure at the maximum normal stress or you may not get the failure at the maximum shear stress when the maximum shear stress is happening. That means you are applying some external load on top of the soil and due to that you are getting some shear stress developed in the soil as well as normal stress developed in the soil and those shear stress and normal stress when they are reaching maximum that may not be the point where you will be getting the failure rather you will be getting the failure at a critical combination of shear and normal stress right.

So, either maximum normal stress or maximum shear stress will not cause the failure rather you will be getting some combination critical combination of normal and shear stress where you will be getting or where the failure will be initiated in the soil or the material right. So, basically that is expressed in mathematically that is expressed  $\tau_f$  that is the shear strength is equal to function of normal and shear stress right.

So, this is the function and we will try to establish this function now. Now the failure envelope right so if this equation 5.1 is giving me the failure envelope, failure envelope means that is the limit or that is the surface okay on which the failure occurs right. If you draw in case of 2D situation you will be getting in case of 1D situation you will be getting one point right in stress strain in stress strain say space. In case of 2D you will be getting one plane okay along which once the critical once the combination of your loading is reaching that or touching that plane immediately you will be getting the failure that is known as failure plane. In case of 3D you will be getting the surface right.

So, along the surface suppose this is my surface this is creating the limit. So, if I am within this limit there is no failure, failure is not happening there but at some critical combination of shear and normal stress I am just touching this surface or in 2D it is plane so I am if I touch the plane immediately I will get the failure. So, that it says so the failure envelope defined by equation 5.1 is a curved line. Frankly speaking this is a curved line, this is a nonlinear failure envelope.

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**Shear strength of soil**  
**Mohr-Coulomb Failure Criteria**

- For most soil mechanics problems, it is sufficient to approximate the shear stress on the failure plane as a linear function of the normal stress (Coulomb, 1776)

i.e.  $\tau_f = c + \sigma \tan \phi$

Where,  $c =$  Cohesion  
 $\tau_f =$  Shear strength  
 $\sigma =$  normal stress on the failure plane

$\phi =$  Angle of internal friction (5.2)

However, for most soil mechanics problem it is sufficient to approximate the shear stress on the failure plane as a linear function of the normal stress okay. In most of the soil mechanics problem we generally we consider that the it is a linear function. This failure envelope whatever failure envelope whatever we have got from equation 5.1 so that was giving me the curved failure envelope right but Coulomb in 1776 I mean he proposed that basically you can consider this function as a linear function okay and it will not I mean cause any difficulty or cause any problem to understand or to predict the behaviour of soil okay.

So, this nonlinear failure envelope so earlier you were getting this kind of a failure envelope say suppose for example. Now you are approximating this failure envelope as nonlinear linear okay as per Coulomb okay. So, this nonlinear failure envelope is becoming linear failure envelope in by choosing the Coulomb's failure criteria.

So, now basically this I mean linear function or this linear failure envelope can be expressed by this expression  $\tau_f$  is equal to  $c$  plus  $\sigma \tan \phi$  where  $c$  is the cohesion,  $c$  is known as the cohesion okay in the soil so I mean this is grain-to-grain say cohesion okay,  $\tau_f$  is the shear strength that is the limit or the strength of the soil that is inherent limit or inherent strength and  $\sigma$  is the normal stress on the failure plane okay.

So, suppose if you have some failure plane and if you want to find out the shear strength along that failure plane because when I am talking about the failure then only you will be talking about the shear strength otherwise any surface any plane you consider along that plane some shear stress is acting but that shear stress when it will reach the maximum limit at that time failure will be happening and that plane will be identified as the failure plane and along that plane whatever shear stress is acting that is nothing but the shear strength that is the maximum limit of the soil okay. So, if I consider this is my failure plane along which whatever say normal stress is acting say  $\sigma$  okay and if I know the  $\phi$  what is  $\phi$  is nothing but angle of internal friction.

So, soil will be showing 2 different kinds of so basically the shear resistance will be dissipated through cohesion and through friction okay. So, this is this  $\phi$  is nothing but the angle of internal friction. If you recall 10 plus 2 standard friction chapter so basically if I mean that is nothing but the friction angle okay this  $\phi$  is nothing but the friction angle. So, if I consider  $\sigma$  is acting on this plane that is nothing but the normal stress and if I want to find out the shear strength  $\tau_f$  that  $\tau_f$  can be obtained by this expression where  $\tau_f$  is equal to  $c$  plus  $\sigma \tan \phi$  okay.

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**Shear strength of soil**  
**Mohr-Coulomb Failure Criteria**

- The preceding equation is called the “Mohr-Coulomb failure criterion”
- The M-C failure criterion expressed in terms of effective stress

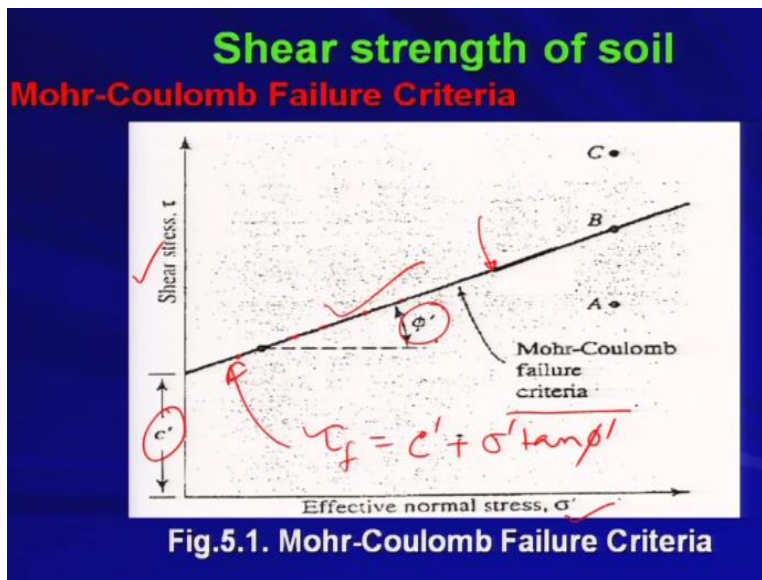
$$\tau_f = c' + \sigma' \tan \phi' \quad (5.3)$$

- The value of  $c'$  for sand and inorganic silt = 0

Now we can the preceding equation is called the Mohr-Coulomb failure criterion okay whatever equation we have seen in equation 5.2 that is basically known as Mohr-Coulomb failure criterion okay. Now if you try to obtain the Mohr-Coulomb failure criterion expressed in terms of effective stress of course  $\tau_f$  will be equal to  $c'$  that is the effective cohesion plus  $\sigma'$  that is the effective normal stress into  $\tan \phi'$  where  $\phi'$  is the effective angle of internal friction okay. In I mean everything is expressed in terms of effective parameter.

Now the value of  $c'$  that is the effective cohesion for sand and inorganic clay is 0 and this is quite obvious. If you take some dry sand or the inorganic silt you will see that the cohesion between the particle okay is negligible or simply zero. So, if you as you if you think of cohesion value is zero for sand and inorganic silt so that is quite good that is quite logical and that you can get it from the test also, laboratory experiment. So, for normally consolidated clay  $c'$  is, that is the effective cohesion is found to be zero. However, for over consolidated clay  $c'$  is greater than zero. So, these things generally happens okay in soil mechanics. Now this is your Mohr-Coulomb failure criteria.

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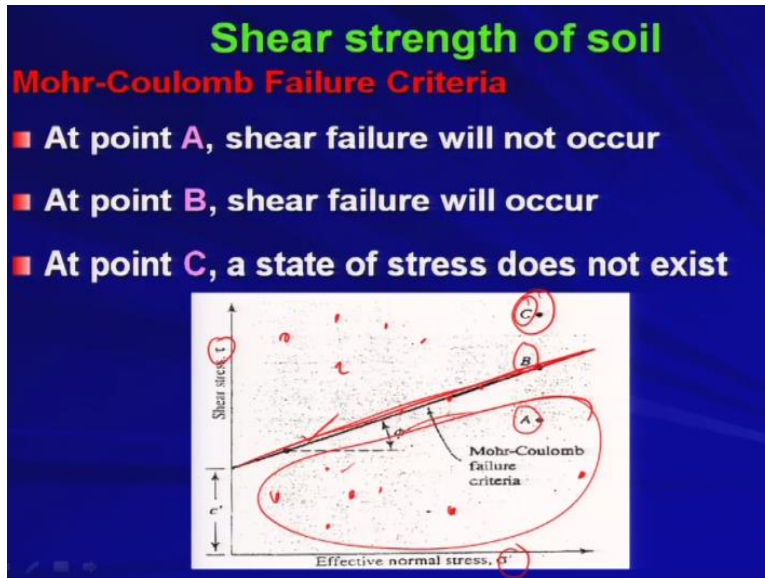


So, you are plotting the line between shear stress and normal stress okay so effective normal stress versus shear stress okay. Now basically if you look at this plot what you are getting you are getting some linear failure envelope. So, this is your linear failure envelope Mohr-Coulomb failure criteria that is given as  $\tau_f = c' + \sigma' \tan \phi'$  so where  $\phi'$  is the angle of inclination of this line and  $c'$  is the intercept on the shear stress axis.

So, now basically you are getting so this the equation of this line will give you the Mohr-Coulomb failure criteria that is nothing but so if you touch this line basically you are getting the failure. So, if you are on this line you are getting the failure. So,  $\tau_f$  that is the y axis is equal to  $c' + \sigma' \tan \phi'$  okay. So, this is basically the equation of this straight line and that is nothing but your Mohr-Coulomb failure criteria.

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Now if you look at this figure okay so now you have understood one thing you will be getting some critical combination of sigma and tau right shear stress and normal stress for which you are getting the failure. You are not getting the failure either at maximum normal stress or at maximum shear stress. Rather you are getting some critical combination of sigma prime and tau so that you are getting the failure along this line.

Now you are getting you are choosing 3 different points say point A, point B, and point C for example. You are choosing 3 different points at different locations in this plot. Point A is below the failure envelope, failure envelope means this straight line point B is lying on the failure envelope and point C is lying above the failure envelope and let us try to opine on these points I mean whether this what type of points and what these points are really mean for, so those things we are going to look at okay.

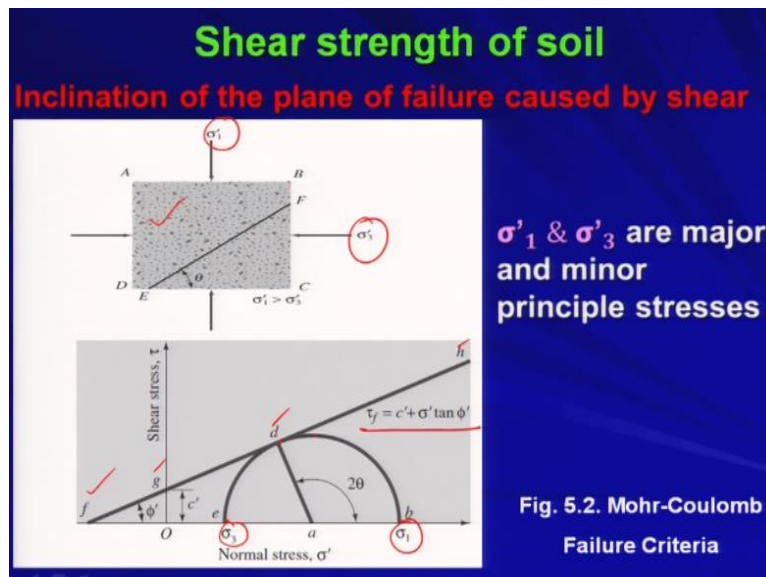
Now point A, at point A, shear failure will not occur agreed? So, any combination of sigma and tau whatever combination you choose if that combination is lying below the failure envelope like point A you can get another point here you can get another point here you can get another point you can get n number of points which are lying below the failure envelope, failure will not occur for that combination of sigma and tau agreed.

Now at point B shear failure will occur. What is point B? Point B is exactly lying on the failure envelope. So, any point which is touching or which is lying on this failure envelope therefore that combination of sigma and tau will be getting the failure right. Now what about point C? At point C, a state of stress does not exist because I mean you can move from any point to any point

by varying the sigma and tau and ultimately this is your boundary this is your limit that is the limiting condition that soil is saying beyond this limit I cannot take the load. So, you cannot have the access beyond this line.

So, you can roam around whatever points are here in this zone right. So, and maximum you can go up to this line. So, this is the maximum limit. Beyond that limit whatever point C is lying or whatever point are lying here. In this zone, basically you cannot have the access. You cannot go inside those region right. So, basically the region which is defined by this strength or by this failure envelope right within that region you can roam around and you can get different combination of shear strength and normal stress and for that you can say whether the failure is happening or the failure is not happening. You cannot go beyond the failure.

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Now basically we will look at this say Mohr-Coulomb failure criteria in different way not different way same way but in for different say combinations or say different concepts we are going to find out from this plot. Now what we are considering we are considering one soil element like this. So, ABCD is my soil element say. Along AB plane and along CD plane the sigma 1 that is the major principle stress is acting, everything is expressed in terms of effective stress, so I am putting prime so whenever I am putting prime so prime will indicate those parameters are effective parameters okay.

So, sigma 1 is acting on this plane so AB and CD okay. These 2 planes, horizontal planes, under I mean these 2 planes experiencing the major principle stress sigma 1 prime. So, obviously the

plane which is making 90-degree angle with the major principle plane because so therefore AB and CD become major principle plane am I right? So, on AB plane your major principle stress is acting so therefore this plane is I mean happening as major principle plane.

Similarly, BC and AD along this plane basically you are your minor principle stress is acting that is  $\sigma_3$ . So, therefore these planes will become minor principle planes right. So, now under this situation that means the soil element is experiencing major principle stress and minor principle stress on these planes and we would like to plot this thing in the Mohr-Coulomb failure criteria.

We would like to accommodate this thing in the Mohr-Coulomb failure criteria. Now so  $\sigma_1$  is your major principle stress. So, that is plotted here. So, that means along this plane why it is major principle stress? I mean this is the principle stress or principle plane because on this plane there is no shear stress. So, therefore this plane has to be the principle plane. So, therefore this  $\sigma_1$  that is point b is shown here right.

Similarly, this point, point say e ok so this point is basically the minor principle stress because again this plane is the minor principle plane and that plane basically will be considered as the minor principle plane and along on that plane basically your minor principle stress is acting so that is nothing but  $\sigma_3$ . So, from your basic say Mohr circle representation or the concept you are getting these 2 points  $\sigma_1$  and  $\sigma_3$  right and then you draw the Mohr circle and then you tell and then you get the Mohr-Coulomb failure criteria which will be plotted on the same plot so that is given by this line fgdh say.

So, this is my linear failure envelope and this is touching the Mohr circle. Now you see here what is your maximum normal stress this is nothing but the major principle stress that is your maximum normal stress. Are you getting the failure there? No right. Now what is your maximum shear stress. Your maximum shear stress is lying somewhere here that is the radius of the circle. Are you getting the failure there, no.

So, that means the maximum shear stress or the maximum normal stress they are not saying anything about the failure where you whereas you are getting the failure somewhere else. Where you are getting the failure? When the Mohr circle is touching the failure envelope at say point d okay. So, failure envelope is tangential at that point tangential to the Mohr circle at that point d and that point is basically telling about the failure right. So, well so I will stop here today. In the next lecture, we will continue this thing in with further details. Thank you very much.