

**Geology and Soil Mechanics**  
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**Lecture - 45**  
**Shear Strength of Soil - e**

Welcome back. So, in the last lecture we just discussed about the stress path for CD test, isotropically CD test.

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**Shear strength of soil**

**Stress Paths (SP)**

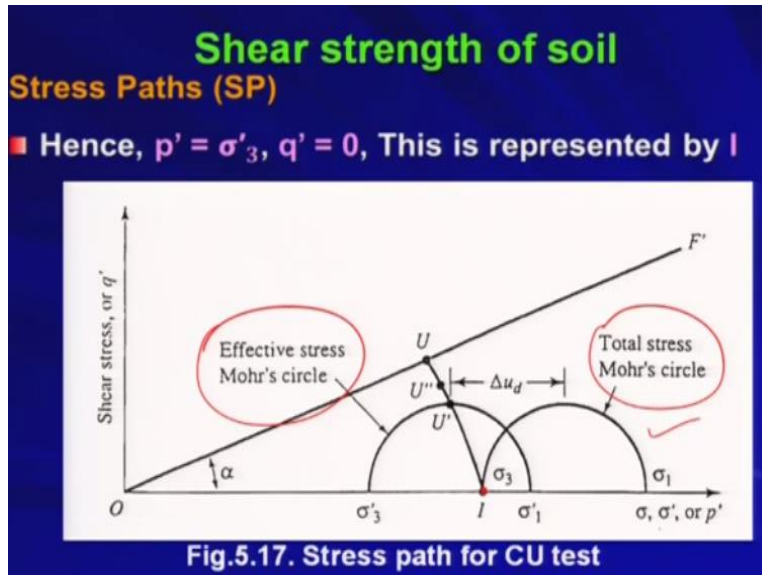
- From equations (5.33) & (5.34)  
$$\sin \phi' = \tan \alpha$$
$$\phi' = \sin^{-1}(\tan \alpha) \quad (5.35)$$
- Now let us consider a normally consolidated clay subjected to isotropically CU triaxial test
- At the beginning of the application of deviator stress  $\sigma'_1 = \sigma'_3 = \sigma_3$

And there we have seen the stress path is making an angle 45 degree with the horizontal and we have established one new kind of say failure envelope that is nothing but the modified failure envelope and that is known K f line and we have established the relation of for the inclination between the inclination of K f line and the original failure envelope like this. Now we are considering a new kind of test right normally consolidated clay we are considering and we are considering it is subjected to isotropically CU test and then would like to find out the stress path for this kind of test.

Now at the beginning of the application of deviatoric stress your sigma 1 prime is equal to sigma 3 prime is equal to sigma 3 because you are again considering the consolidated test so that consolidation is happening in the stage 1 and that consolidation is happening due to the cell pressure application that is sigma 3. So, at that time you are considering you are considering the drainage right. Otherwise consolidation will not be happening. So, you are considering the drainage. Therefore sigma 1 prime that is effective major principle stress is equal to effective

minor principle stress  $\sigma_3$  prime is equal to the total stress  $\sigma_3$  right because you are considering the complete consolidation.

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Now hence  $p$  prime is equal to  $\sigma_3$  prime right and  $q$  prime is equal to 0 for this kind of situation okay. So, this is represented by point I so the same point. So, in case of consolidated drained test and in case of consolidated undrained test this point is lying on your  $p$  prime axis because that is known as isotropically consolidation consolidated test right. So, all-round you are applying the  $\sigma_3$  and then you are consolidating the sample.

So, this I point will be represented by this state of stress okay. Now basically you in case now you are shearing it right. Now you are applying the deviatoric stress. So, that means you are applying the axial deviatoric stress and therefore you are increasing the total stress in the sample and eventually or effectively the total stress will be coming to the effective stress.

There will be 2 Mohr circles. One is total stress Mohr circle another one is effective stress Mohr circle for CU test because now you are considering undrained situation when you are shearing the soil sample right. So, now you have started the test from this point I okay. Now you are progressing, now you are applying the deviatoric stress. Now how the stress path will move. Let us see.

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## Shear strength of soil

### Stress Paths (SP)

■ At some other stage of the deviator stress application

So,

$$\sigma'_1 = \sigma_3 + \Delta\sigma_d - \Delta u_d \quad (5.36)$$

$$\sigma'_3 = \sigma_3 - \Delta u_d \quad (5.37)$$

$$\text{So, } p' = \frac{(\sigma'_1 + \sigma'_3)}{2} = \sigma_3 + \frac{\Delta\sigma_d}{2} - \Delta u_d \quad (5.38)$$

$$\& \quad q' = \frac{(\sigma'_1 - \sigma'_3)}{2} = \frac{\Delta\sigma_d}{2} \quad (5.39)$$

Now at some other stage of the deviatoric stress application that sigma 1 prime is equal to sigma 3 + delta sigma d minus the pore water pressure developed during the deviatoric stress application at any stage. I am not talking about the failure. That means you are you have started shearing and till the failure in between at any stage you consider this is the effective major principle stress that is sigma 1 prime that is equal to sigma 3 + delta sigma d. What is sigma 3 + delta sigma d? That is nothing but sigma 1 minus the pore water pressure delta u d will give u the sigma 1 prime.

Similarly, what will be the value of sigma 3 prime? That is nothing but sigma 3 that is the all-round cell pressure minus the pore water pressure sigma delta u d right. So, that will give you the sigma 3 prime that is effective minor principle stress. So, that means this point will be talking about sigma 3 and this point will be talking about sigma 3 prime. So, the difference between these 2 is delta u d. So, this is nothing but your delta u d okay.

Similarly, sigma 1 prime and sigma 1 so this is your sigma 1 this is your sigma 1 and this is your sigma 1 prime. So, this difference is also delta u d. So, the center of the Mohr circle will be also having the difference of delta u d. So, this is nothing but the center okay. The difference between the center of the total stress Mohr circle and the effective stress Mohr circle will be also delta u d okay so that is coming from your CU test.

So, if you recall your CU test concept okay, now in this situation in that stage whatever value of sigma 1 prime and sigma 3 prime have been obtained so based on that I can calculate p prime which is nothing but sigma 1 prime + sigma 3 prime by 2 which is nothing but sigma 3 + delta

$\sigma_3 + \frac{\Delta \sigma_d}{2} - \Delta u_d$  right. So, if you put these values you will be getting  $\sigma_3 + \frac{\Delta \sigma_d}{2} - \Delta u_d$ .

So, that is your  $p'$  okay. Now what is the value of  $q'$ ?  $q'$  is your  $\sigma_3 - \sigma_1$  by 2 is equal to  $\frac{\Delta \sigma_d}{2}$  okay. So, you have got  $q'$  and  $p'$ . Now you are going to plot this thing in the  $p'$   $q'$  plot or the space. Now which point will be denoted by this? The this point right.

If you see if you look at so  $p'$  is equal to  $\sigma_3 + \frac{\Delta \sigma_d}{2} - \Delta u_d$  here plus  $\frac{\Delta \sigma_d}{2}$ . So, you will be getting the center of the total stress Mohr circle minus  $\Delta u_d$  will be giving you the  $p'$  am I right. Similarly,  $q'$  is your  $\frac{\Delta \sigma_d}{2}$  that is nothing but the radius of the Mohr circle.

Now here you can see that when you are talking about  $p'$  and  $q'$  basically that is associated with the effective stress Mohr circle. So, we have got a point  $U'$ . So, this is the point which is nothing but  $U'$  right. So, similarly at any stage you draw the Mohr circle like this okay and you try to establish the topmost point on the Mohr circle you will be getting another point say  $U'$  and so on.

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**Shear strength of soil**

**Stress Paths (SP)**

- The conditions (5.38) & (5.39) will plot  $U'$
- Points such as  $U''$  represent values of  $p'$  &  $q'$  as the test progresses
- At failure

So, 
$$p' = \sigma_3 + \frac{(\Delta \sigma_d)_f}{2} - (\Delta u_d)_f \quad (5.40)$$

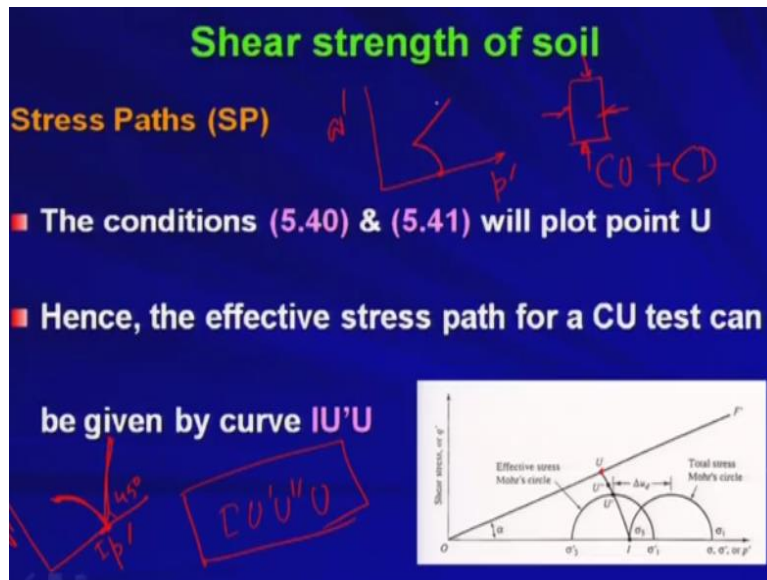
& 
$$q' = \frac{(\Delta \sigma_d)_f}{2} \quad (5.41)$$

And ultimately the conditions 5.38 and 5.39 will plot  $U'$ . So, points such as  $U''$  represent values of  $p'$   $q'$  as the test progresses so as I told you. So, initially you have got  $U'$  then as the test progresses you will be getting another point say  $U''$  and

another prime U triple prime and so on till the failure right. So, at failure so your p prime is nothing but  $\sigma_3 + \frac{\Delta \sigma_d}{2}$  and  $\frac{\Delta \sigma_d}{2} - u$ .

What is  $\Delta u$  that is the pore water pressure at failure and what is  $\Delta \sigma_d$  that is the deviatoric stress at failure. So, this is your p prime and this is your q prime which is defined by the point u okay. So, if you draw the Mohr circle you will get it. So, you draw the Mohr circle for total stress and effective stress you will be getting the point u which is on the failure envelope OF prime.

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So, the conditions 5.40 and 5.41 will plot point U okay. Hence the effective stress path for a CU test can be given by the curve IU prime U double prime U okay. So, IU prime U double prime U. So, this curve path is your stress path for the CU test agreed. So, how it is moving. So, it is not the straight line now. It is rather the curved line.

You can get it I mean for any say test CU test or CD test if you draw if you plot the stress path you will be getting one will be your straight line that is CD test will be giving you the straight stress path whereas the CU test will give you the curved stress path. So, basically what I am getting? I am getting say this is q prime this is p prime. So, I am starting from the isotropically consolidated test that is point I.

Now if I perform CD test on the sample I will be getting the stress path like that which is making an angle 45 degree. If I use or if I apply CU test or if I perform CU test on the same sample I will be getting a curve like that. Then that will be meeting the failure envelope okay. Is that clear? So,

I will be getting the failure envelope I mean by getting these 2 paths. Now the situation now the condition is that basically you have started with the same condition.

This is your condition initial condition before starting or before beginning of the test. Now you have moved through the separate path when you are talking about the CU test and when you are talking about the CD test. Now by seeing this stress path immediately you can identify that what type of test has been performed and how the test has progressed. So, this is the trace of the test right. So, if I say this is my stress path right so I can understand so the test is happened in CU test.

So, this is your CU test and CU test is happened along this path okay. Similarly, if I have the CD test CD test has happened along this path. I hope that you have understood this test because this concept is very fundamental and very important concept for understanding the critical stress soil mechanics. However, by knowing this you can plot for any test. It is not only CU test or CD test, any kind of test.

Suppose you are considering say CU test okay. So, suppose for example I mean you are doing first you have applied all-round say cell pressure. Now first you have performed CU test okay. Before going to the failure, you have opened the valve and you have done the CD test for the same sample. So, that means partly CU and then partly CD okay. This kind of situation may happen in the field right.

So, initially the consolidation has happened but initially there is no drainage and immediately after the application of stress some amount of drainage is happening. So, CU test combined with CD test so that kind of thing also you can perform by this. So, what will happen in that situation? It may so you this is your isotropically consolidation this is your  $q$  prime  $p$  prime space okay. So, initially you have started CU test okay and then you have done CD test okay. So, this kind of so by seeing the stress path basically you will be understanding that how the test has progressed, how the test you have performed right.

So, I will stop here today. In the next lecture, we will be talking about another type of shear test that is vane shear test and then we will be discussing about the Skempton pore water pressure parameter and that will conclude the chapter shear strength. Thank you very much.