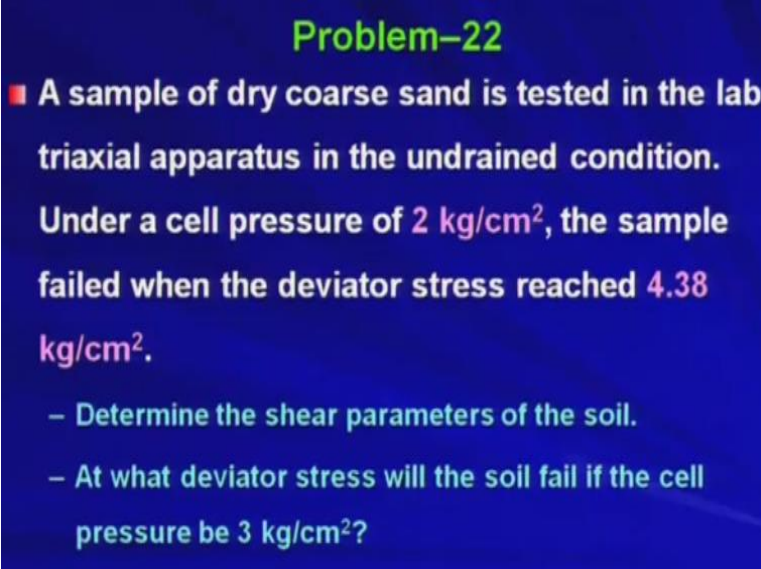


Geology and Soil Mechanics
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Lecture - 49
Problem on Shear Strength of Soil _a

Welcome back. So, in the last lecture we solved couple of problems on shear strength of soil. Today we will be taking another few problems.

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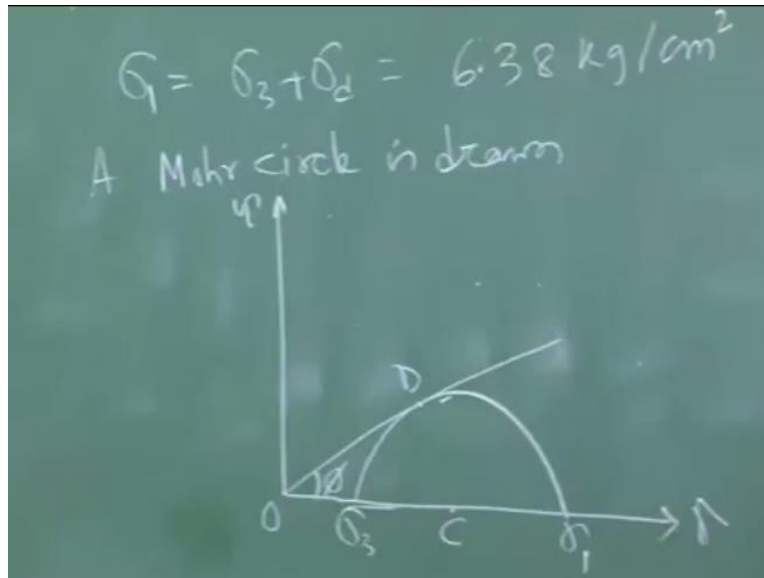
Problem-22

■ A sample of dry coarse sand is tested in the lab triaxial apparatus in the undrained condition. Under a cell pressure of 2 kg/cm^2 , the sample failed when the deviator stress reached 4.38 kg/cm^2 .

- Determine the shear parameters of the soil.
- At what deviator stress will the soil fail if the cell pressure be 3 kg/cm^2 ?

Today's first problem is a sample of dry coarse sand is tested in the lab triaxial apparatus in the undrained condition under a cell pressure of 2 kg/cm^2 . The sample failed when the deviator stress reached 4.38 kg/cm^2 . Now determine the shear parameters of the soil. As I told you shear parameters means c and ϕ . At what deviator stress will the soil fail if the cell pressure be 3 kg/cm^2 okay. So, under the cell pressure of 2 kg/cm^2 the soil failed okay when the deviator stress reached at 4.38 kg/cm^2 and now we are going to solve this problem.

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The first so problem 22 so here σ_3 is equal to 2 kg/cm^2 . σ_d that is deviator stress is 4.38 kg/cm^2 . Therefore σ_1 that is the major principle stress will be $\sigma_3 + \sigma_d$ as we know that gives me 6.38 kg/cm^2 okay. Now we are going to draw a Mohr circle. Now a Mohr circle is drawn considering this situation so τ σ space okay.

So, this is the Mohr circle okay. Now since the sample is made of coarse sand as it is given in the problem dry coarse sand okay you are using. Since the sample is made of dry coarse sand and since it is in the dry state right so no apparent cohesion will be coming into the picture. So, you will be getting cohesion is 0. So, therefore the Coulomb failure envelope will pass through the origin right. If c is 0 so there will be no intercept on the τ axis so your Coulomb failure envelope will pass through the origin right.

So, now this is the Mohr circle. This is your σ_1 . This is your σ_3 . Now one thing we have decided because you are using the dry coarse sand therefore c is 0 so the Mohr I mean Mohr-Coulomb failure envelope will pass through the origin, origin means this point okay and it will be tangent to this Mohr circle right. Otherwise it will not be I mean the Mohr circle should touch the failure envelope.

Then only it will be the Mohr circle corresponding to the failure of the sample. So, it will be tangent to the Mohr circle at failure. So, this is say center of the Mohr circle say c . This is the point where the Mohr circle is touching the failure envelope is D and this is the origin O okay and so therefore this angle is ϕ . So, you need to find out the magnitude of ϕ because c is 0 okay.

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$$\begin{aligned}\bar{OC} &= \frac{\sigma_1 + \sigma_3}{2} = \frac{6.38 + 2}{2} = 4.19 \text{ kg/cm}^2 \\ \bar{CD} &= \frac{\sigma_1 - \sigma_3}{2} = \frac{6.38 - 2}{2} = 2.19 \text{ " } \\ \therefore \sin \phi &= \frac{\bar{CD}}{\bar{OC}} = \frac{2.19}{4.19} \\ \Rightarrow \phi &= 31^\circ \\ \boxed{c = 0 \text{ \& } \phi = 31^\circ}\end{aligned}$$

So, we can if you look at this figure we can write down OC is equal to sigma 1 + sigma 3 by 2 right. What is OC? OC is the distance between the origin and the center of the Mohr circle. So, that is nothing but sigma 1 + sigma 3 by 2 which is 6.38 + 2 by 2 is equal to 4.19. Similarly, CD, what is CD? CD is nothing but your radius of the Mohr circle right. CD is the radius of the Mohr circle which will be making an angle 90 degree at the tangent point okay so CD is equal to sigma 1 - sigma 3 by 2 which is equal to 6.38 - 2 by 2 is equal to 2.1 okay. Therefore, from the figure I can write sin phi is equal to CD bar by OC right CD by OC is your sin phi. So, which is nothing but 2.19 by 4.19.

So, from this I get phi is equal to 31-degree okay. So, therefore your c is 0. What are the shear strength parameters? C is 0 and phi is 31-degree okay. So, now we have seen that how I can find out the shear strength parameters for this arrangement like when your cell pressure is 2 kg/cm square and the deviator stress at failure at is 4.38 kg/cm square. Now we will go to the second part. Now in the second part it is asked that at what deviator stress will the soil fail if the cell pressure is 3 kg/cm square.

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P-22

$$\sigma_1 = \sigma_3 \tan^2 \alpha + 2c \tan \alpha$$

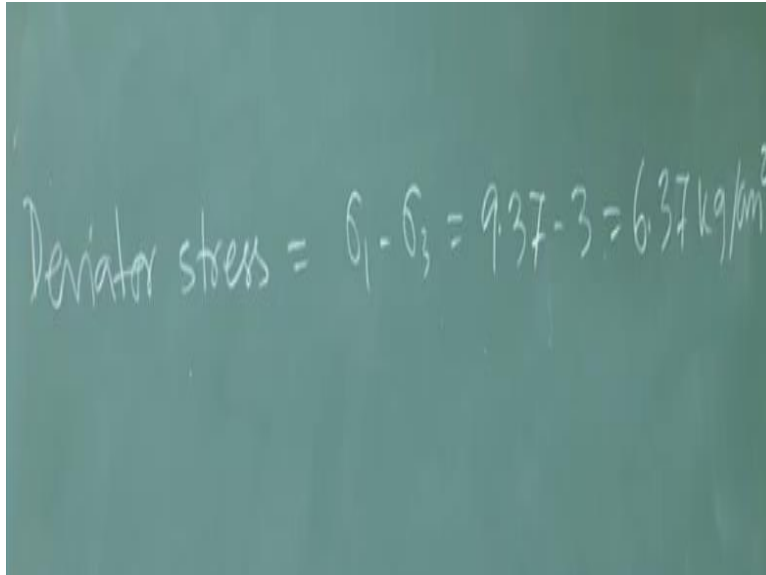
$$\sigma_1 = \sigma_3 \tan^2 \alpha = \sigma_3 \tan^2 \left(45^\circ + \frac{\phi}{2} \right)$$

Here $\sigma_3 = 3 \text{ kg/cm}^2$, $\phi = 31^\circ$

$$\sigma_1 = (3) \tan^2 \left(45^\circ + \frac{31^\circ}{2} \right) = 9.37 \text{ kg/cm}^2$$

Now the second part we know that sigma 1 is equal to sigma 3 tan square alpha + 2 c tan alpha. So, as your c 0 so this term will be coming 0. So, therefore I can write sigma 1 equal to sigma 3 tan square alpha okay. So, which is nothing but sigma 3 tan square 45 degree + phi by 2 okay. Now in the second part it is given sigma 3 is equal to that is the cell pressure is becoming 3 kg/cm square and already we have calculated phi equal to 31 degree because you are dealing with the same sample. Just you are varying the cell pressure. So, phi will not be changing right. Phi cannot be changed, cannot be varied with the variation of the cell pressure right. So, phi will be remaining same whatever you have got with some other cell pressure that is 2 kg/cm square earlier okay. So, from this I can find out sigma 1 is equal to 3 tan square 45 degree plus so which gives me 9.37 kg/cm square. So, you have got the major principle stress corresponding to the cell pressure 3 kg/cm square okay at failure.

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Deviator stress = $\sigma_1 - \sigma_3 = 9.37 - 3 = 6.37 \text{ kg/cm}^2$

So, if this is the major principle stress at failure then you can find out the deviator stress is nothing but sigma 1 - sigma 3 so sigma 1 - 3 which is nothing but 9.37 - 3 is equal to 6.37 kg/cm square. So, if you go so if you increase the cell pressure as 3 kg/cm square then you have to you have to apply 6.37 as deviator stress to get the failure okay for the same soil. So, we have solved this problem. So, I hope that you have understood the concept and how we are using the parameters and how we are playing with the parameters I hope that you have understood.

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Problem-23

■ The shear strength parameters of a given soil are $c = 0.26 \text{ kg/cm}^2$ and $\phi = 21^\circ$. Undrained triaxial tests are to be carried out on specimens of this soil. Determine

- Deviator stress at which failure will occur if the cell pressure be 2.5 kg/cm^2 .
- The cell pressure during the test, if the sample fails when the deviator stress reaches 1.68 kg/cm^2 .

Now we will take the second problem. The problem says the shear strength parameters of a given soil are c equal to 0.26 kg/cm square and phi equal to 21 degree. Undrained triaxial tests are to be carried out on specimens of this soil. Determine first, deviator stress at which failure will

occur if the cell pressure be 2.5 kg/cm square and the second part the cell pressure during the test determine the cell pressure during the test if the sample fails when the deviator stress reaches 1.68 kg/cm square okay.

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P.23
We know $\sigma_1 = \sigma_3 \tan^2 \alpha + 2c \tan \alpha$
For the given soil $c = 0.26 \text{ kg/cm}^2$, $\phi = 21^\circ$
 $\therefore \tan^2 \alpha = 2.117$, $\tan \alpha = 1.455$
Hence, $\sigma_1 = 2.117 \sigma_3 + 0.757$ (i)

So, now again this is pretty simple problem already we have solved so we know sigma 1 equal to sigma 3 tan square alpha + 2 c tan alpha right. For the given soil c equal to 0.26 kg/cm square and phi equal to 21 degree. Therefore, tan square alpha will be 2.117 and tan alpha will be 1.455 okay. Now hence sigma 1 is equal to 2.117 sigma 3 + 2 into c, c is 0.26 so and tan alpha is 1.455 so that gives me sigma 1 is equal to 2.117 sigma 3 + 0.757. So, I am calling this as equation 1.

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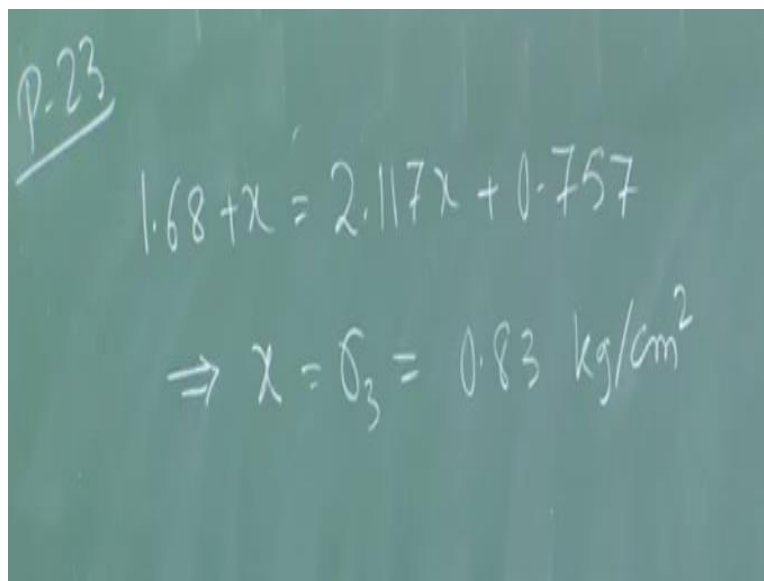
When $\sigma_3 = 2.5 \text{ kg/cm}^2$
 $\sigma_1 = 6.05 \text{ kg/cm}^2$
Now $\sigma_d = \sigma_1 - \sigma_3 = 3.55 \text{ kg/cm}^2$
ii) Let the reqd. cell press. be $x \text{ kg/cm}^2$
 $\sigma_1 = \sigma_3 + \sigma_d = 1.68 + x \text{ kg/cm}^2$

Now when your σ_3 is 2.5 kg/cm square so that is the first part that determine the deviator stress at which failure will occur if the cell pressure be 2.5 kg/cm square. So, if the cell pressure becomes 2.5 kg/cm square then I can find out σ_1 from equation 1 as 6.05 right. You place σ_3 equal to 2.5 and you get the magnitude of σ_1 from equation 1.

So, once I know σ_1 then basically deviator stress σ_d is nothing but $\sigma_1 - \sigma_3$ which gives me 3.55 kg/cm square okay. So, hence the required deviator stress is 3.55 kg/cm square to get the failure with cell pressure as 2.5 kg/cm square. Now in the second part let the required cell pressure be x kg/cm square.

So, therefore σ_1 is equal to $\sigma_3 + \sigma_d$ is equal to $1.68 + x$. What is 1.68? 1.68 is the deviator stress okay. So, if you apply if you get the failure at deviator stress 1.68 then what will be the cell pressure? That is the question right. That is the second part of the problem okay. So, I have got σ_1 is equal to $1.68 + x$. Now I put that thing in equation 1 so I have got σ_3 which is nothing but x kg/cm square and σ_1 which is nothing but $1.68 + x$ kg/cm square okay. So, I will put that thing in the equation 1.

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The image shows a chalkboard with handwritten equations. In the top left corner, 'P-23' is written and underlined. The main equation is $1.68 + x = 2.117x + 0.757$. Below it, the solution is given as $\Rightarrow x = \sigma_3 = 0.83 \text{ kg/cm}^2$.

So, $1.68 + x$ is equal to $2.117x + 0.757$ so from this I will get x which is nothing but your required σ_3 which is nothing but 0.83 kg/cm square okay. So, I hope that you have understood the problem, very straightforward problem right. So, you have got the shear strength parameters and you are finding out that how much pressure you need to apply to get the failure. Now we will go for the next problem.

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Problem-24

■ The following are the results of a set of drained triaxial tests performed on three identical specimens of 38 mm diameter and 76 mm height. Determine the shear parameters of the soil.

Sample No.	Cell pressure (kg/cm ²)	Deviator load at failure (kN)	Change in volume (cc)	Axial deformation (mm)
1	50	0.0711	-0.9	5.1
2	100	0.0859	-1.3	7.0
3	150	0.0956	-1.6	9.1

So, next problem says the following are the results of a set of drained triaxial tests performed on 3 identical specimens of 38 mm diameter and 76 mm height. Determine the shear parameters of the soil. So, you have done some drained test on 3 identical specimens of this size 38 mm diameter and 76 mm in height and you need to find out the shear strength parameters.

So, sample, there are 3 samples, sample 1, 2, and 3. So, you have got the cell pressure, different cell pressure 50, 100, and 150 kg/cm square. You have got the deviator load okay so basically you are calculating or you are reporting the deviator load at failure is 0.0711 kN and so on. Change in volume so when you are applying because you will be you are doing the drained test so you will be observing some volume change because consolidation is happening right.

I mean you will be getting the water will be draining out so you will be getting the volume change during the shear so you have got 3, 2 different say stages right. One is the in the first stage you do the consolidation and second stage you do the shearing okay. So, you have got the change in volume and you have got the axial deformation also so that is 5.1 mm for sample 1, 7 mm for sample 2 and 9.1 mm for sample 3. Now let us do this problem.

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P-24
The deviator loads at failure corresponding to each cell press. are given. In order to determine the corresponding deviator stresses, these loads are to be divided by the corrected area of the sample, which can be obtained from

So, problem 24. The deviator loads at failure corresponding to each cell pressure are given okay. In order to determine the corresponding deviator, stress these loads are to be divided by the corrected area. Why corrected area? Because when you are considering the shearing that means the volume change you are considering so the I mean you may get the decrease in the volume or increase in the volume right whatever may be the situation you will be getting the volume change.

So, when the volume change is happening so you will be getting the area whether it is bulging or contracting depending on that you will be getting the different areas at different deviator stress. So, you need to correct the I mean you need to find out the actual area on which the deviatoric load is acting and based on that you can find out the deviator stress okay. So, by the corrected area of the sample which can be obtained from this equation.

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$$A_c = \frac{V_1 \pm \Delta V}{L_1 - \Delta L}$$
$$V_1 = \left(\frac{\pi}{4}\right)(3.8)^2(7.6) \text{ cc} = 86.19 \text{ cc}$$
$$L_1 = 7.6 \text{ cm}$$

A_c is equal to V_1 plus minus ΔV divided by $L_1 - \Delta L$ okay so where V_1 is the initial volume of the specimen. ΔV is the volume change. Now it could be additive it could be subtractive depending on whether you are getting the increase in the volume or decrease in the volume. So, already you have seen that if you consider the dense sand you generally get the increase in the volume right. At that time ΔV will be additive.

So, if you get the decrease in the most of the times you will be getting the decrease in the volume so then your Δv will be subtractive. Well L_1 is the initial length of the specimen and ΔL is the change in length or the axial deformation okay. So, now your V_1 that is the initial volume of the specimen is π by 4, 3.8 square now 3.8 cm is the diameter and 7.6 cm is the length. So, cc which is coming as 86.19 cc. So, L_1 is 7.6 cm as given in the problem.

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P-24

For the 1st sample $\Delta V = -0.9 \text{ cc}$, $\Delta L = 5.1 \text{ mm}$

$$A_c = \frac{86.19 - 0.9}{7.6 - 0.51} = 12.03 \text{ cm}^2 = 12.03 \times 10^{-4} \text{ m}^2$$

$$\sigma_d = \frac{0.0711}{12.03 \times 10^{-4}} = 59.10 \text{ kN/m}^2$$

So, for the first sample your delta V is given as - 0.9 cc okay and your delta L is given as 5.1 mm okay that is given in the problem. You see the table you will see that. Therefore, A c corrected area will be 86.19 - 0.9 divided by 7.6 - 0.51 okay which gives me 12.03 cm square which is nothing but 12.03 into 10 to the power - 4-meter square okay. Therefore, your sigma d that is deviator stress is now the deviator load by the corrected area. So, deviator load is given as 0.0711 kN by 12.03 into 10 to the power - 4 m square that is the corrected area which gives me 59.10 kN/m square.

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$\therefore \sigma_1 = \sigma_3 + \sigma_d = 50 + 59.10 = 109.10 \text{ kN/m}^2$

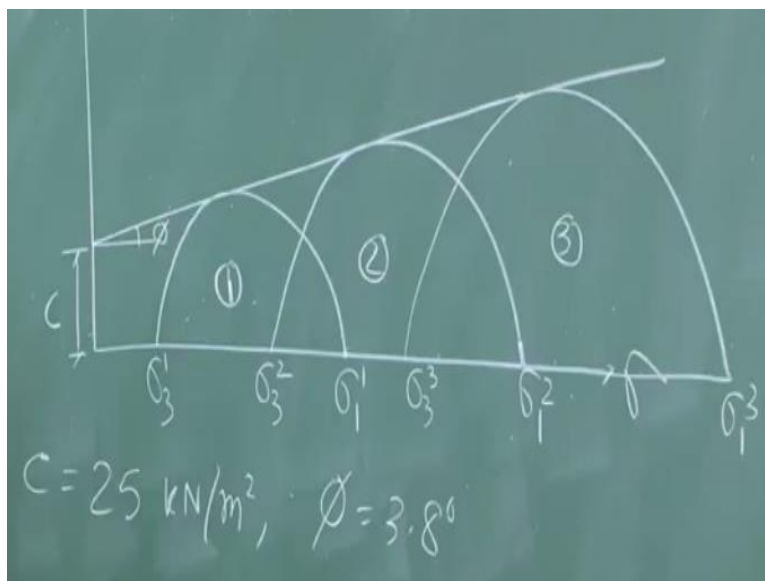
Sample	σ_3 (kN/m^2)	F_d (kN)	ΔV (cc)	ΔL (mm)	A_c (cm^2)	σ_d (kN/m^2)	σ_1 (kN/m^2)
1	50	0.0711	-0.9	5.1	12.03	59.10	109.10
2	100	0.0859	-1.3	7.0	12.36	69.50	169.50
3	150	0.0956	-1.6	9.1	12.65	75.61	225.61

So, therefore sigma 1 is equal to sigma 3 + sigma d which is 50 plus, 50 is the cell pressure that is nothing but sigma 3 for sample 1 you see that thing from the table plus 59.1 is equal to 109.10

kN/m square. So, now we will complete the calculations for other 2 samples. So, if I make the table something like this where you have sample then sigma 3 in kilo newton per meter square then deviator load in kilo newton then volume change in cc then axial deformation in millimeter then corrected area in centimeter square then sigma d that is the deviator stress in kilo newton per meter square and then sigma 1 kilo newton per meter square okay. So, for sample 1 already we have calculated.

We are just putting the values so I did not calculate the values for other 2 samples however you can calculate all those things by following the similar procedure whatever we followed for sample 1 so sample 2 is 100, 0.0859, - 1.3, 7, 12.36, 69.50, 169.50 and for sample 3 cell pressure is 150, deviator load is 0.0956, volume change is -1.6, axial deformation 9.1 mm, corrected area 12.65 cm square, deviator stress is 75.61 kN/m square, and sigma 1 is equal to 225.61 okay. So, this is the table we have completed. So, once we get this table now you can calculate the shear strength parameters by graphically as well as analytically. So, I will give you the hints for the analytical solution however we will see that how you can get this graphically.

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So, tau this is sigma. So, basically you will be getting 3 Mohr circles. So, if you draw all the Mohr circles okay, this is for sample 1, this is for sample 2, this is for sample 3. This is your sigma 1 for sample 1. This is for sigma 1 for sample 2. This is for sigma 1 for sample 3 and so on right. This is for sigma 3 for sample 1, sigma 3 for sample 2, sigma 3 for sample 3 okay. So, you

will now this is the these are 3 Mohr circles for 3 different samples but they are identical so they should give you the same shear strength parameters right.

So, you will be getting a common tangent okay which will be acting as the Mohr-Coulomb failure envelope. So, this is your common tangent. This is the phi and this is the c value. So, graphically if you solve that means to the scale if you plot so you will be getting c equal to 25 kN/m square and phi equal to 3.80 okay. The same problem can be solved analytically also. Analytically also means say you know sigma 1 and sigma 3 combination right for all 3 samples.

So, you will be able to I mean you know sigma 1 is equal to sigma 3 tan square alpha + 2 c tan alpha that equation you know. So, in that equation if you put okay sigma 1 and sigma 3 for sample 1 for sample 2 and sample 3 you will be getting 3 simultaneous equations and if you solve these 3 equations you will be getting c and phi value. So, already we have solved similar kind of problem earlier. So, if you follow that thing without graphical plot you can get the value of c and phi. So, I will stop here today. Thank you very much.