

**Geology and Soil Mechanics**  
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**Lecture - 60**  
**Earth Pressure on Retaining Wall - a**


Welcome back, welcome back to the course Geology and Soil Mechanics. So, due to some unavoidable circumstances I was not able to give the lecture in few weeks last few weeks. So, last few weeks I hope that you have enjoyed the problems okay whatever numerical problems was solved by the student tutor. I hope that you have you must have enjoyed that.

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Earth Pressure on Retaining wall

Active state of stress

From Mohr circle  $C_a$



> Major principal stress,  $\sigma_1 = OP_1 = \sigma_z = \gamma z$

> Minor principal stress,  $\sigma_3 = OP_2 = \sigma_h$

$OO_1 = \frac{\sigma_1 + \sigma_3}{2}, \quad O_1C_1 = \frac{\sigma_1 - \sigma_3}{2}$

From  $\Delta OO_1C_1$ ,  $\frac{\sigma_1 - \sigma_3}{2} = \frac{\sigma_1 + \sigma_3}{2} \sin \phi$

$\sigma_1 = \left( \frac{1 + \sin \phi}{1 - \sin \phi} \right) \sigma_3$

So, now again we are coming back to this particular topic. So, we stopped here so when we were talking about the active earth pressure on the retaining wall right. So, already we have seen that how the pressure lateral pressure will be getting developed on the wall and what are the different kinds of say pressures generally developed on the wall like active earth pressure earth pressure at rest condition and passive earth pressure right.

So, this is purely based or purely dependent on the movement of the wall. That means if the wall is moving away from the backfill you will be getting the active state and whatever pressure will be getting developed on the wall that will be known as the active earth pressure whereas if the wall is not moving at all, it is completely under rest condition then whatever pressure will be exerted on the wall that will be known as active I mean passive earth pressure at rest condition and if the wall is moving towards the backfill that means you are pushing the wall towards the

backfill then you need more movement and because of that whatever pressure will be getting developed that will be known as passive earth pressure.

So, with that we stopped. So, and we talked about this thing in the last lecture that this is the Mohr circle representation for active and passive earth pressure so first one we will be talking about the active state and we will be trying to obtain the active earth pressure coefficient and earth pressure coefficient is nothing but  $\sigma_h$  by  $\sigma_v$  where  $\sigma_h$  is the horizontal pressure and  $\sigma_v$  is the vertical pressure.

So, this is the constant say expression for the earth pressure coefficient. Now whatever maybe the case we will try to find out the lateral earth pressure lateral pressure that is  $\sigma_h$  and we will try to find out the vertical pressure that is  $\sigma_v$  and we will take the ratio to get the earth pressure coefficient at the corresponding state okay. So, suppose if I want to find out the earth pressure coefficient at active state that means in the active condition we will try to find out  $\sigma_h$  that is the horizontal pressure and  $\sigma_v$  that is the vertical pressure and the ratio of these 2 will be giving you the earth pressure in active condition okay.

So, now basically this  $C_e$  is your the Mohr circle representation for the active earth pressure. So, from the Mohr circle major principle stress  $\sigma_1$  now what is the major principle stress in case of active state? Suppose you are you have the retaining wall okay and the wall is moving away from the backfill so this is the backfill soil okay which is lying behind the wall. Now wall is moving away from the backfill so slowly or gradually you have you might have seen that thing in the previous lecture that gradually when the wall moves your pressure is decreasing right.

So, your pressure is decreasing on the wall. Now when the wall is moving so that means your at any section at any section of the wall if you consider at that point whatever vertical stress will be there that will be nothing but your major principle stress right because on that plane you do not have any shear stress. So, that horizontal plane will be the major principle plane and on which your major principle stress is acting and that will be in the vertical direction right in case of active state okay and so as wall is moving so your pressure is decreasing in the lateral direction am I right? You have the wall at rest condition.

Wall is moving away from the backfill so slowly your earth pressure that is the lateral earth pressure is getting reduced right. so that means you are starting at that time there is some vertical stress on any horizontal plane at any depth below the ground surface on the wall okay so that is your vertical stress so that will be becoming the major principle stress and as the stress is

decreasing in the lateral direction as the wall is moving stress is also decreasing so that will be becoming your minor principle stress right.

So, the horizontal stress or the lateral stress will be becoming the minor principle stress in case of active state whereas the vertical stress will be the major principle stress in case of active state agreed okay. So, the major principle stress  $\sigma_1$  is nothing but  $OP_1$  so this is your O means origin and this is the point  $P_1$  okay. So,  $OP_1$  is the major principle stress and that is nothing but your  $\sigma_z$  that is the vertical stress at that point okay and is equal to  $\gamma z$  and where  $z$  is the any depth below the ground surface along the wall okay so  $z$  is 0 means top of the wall  $z$  is  $h$  means bottom of the wall okay.

So, it varies from 0 to  $h$   $z$  is varying from 0 to  $h$  okay. So, now minor principle stress is nothing but the lateral stress as I told you. So, the lateral stress that is  $\sigma_3$  is nothing but  $OP_2$  so  $P_2$  is here so  $OP_2$  is equal to  $\sigma_h$  that is the horizontal stress okay. Now from the Mohr circle representation whatever Mohr circle we have drawn now and this Mohr circle because already we have seen that in case of active state you are you are getting the failure right.

Of course, the Mohr circle must touch the failure envelope okay agreed? So, Mohr circle that is the maximum I mean state of stress possible state of stress in case of active state okay. So,  $OO_1$  okay  $OO_1$  is nothing but  $\frac{\sigma_1 + \sigma_3}{2}$  that is the distance between the center of the Mohr circle and the origin of the plot okay. So,  $\frac{\sigma_1 + \sigma_3}{2}$ . Similarly,  $O_1C_1$ , what is  $O_1C_1$ ?

$C_1$  is the point where the Mohr circle at active state is touching the failure envelope. So,  $O_1C_1$  is nothing but the radius of the Mohr circle and that is nothing but  $\frac{\sigma_1 - \sigma_3}{2}$ . So, we have seen these things several times when we talked about the shear strength of soil okay. So, from triangle  $OO_1C_1$  so if you if you form a triangle with these points  $OO_1$  and  $C_1$  from there we can write  $\frac{\sigma_1 - \sigma_3}{2}$  that is nothing but the radius of the Mohr circle is equal to  $\frac{\sigma_1 + \sigma_3}{2} \sin \phi$  where  $\phi$  is nothing but the angle of internal friction so this angle is  $\phi$  okay. So, from this we can write  $\sigma_1$  is equal to  $\frac{1 + \sin \phi}{1 - \sin \phi}$  into  $\sigma_3$  from this relation we can get it.

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## Earth Pressure on Retaining wall

Active state of stress

Or,  $\sigma_h = \left( \frac{1 - \sin\phi}{1 + \sin\phi} \right) \sigma_v$

$\sigma_h = K_A \sigma_v$

Or,  $K_A = \frac{1 - \sin\phi}{1 + \sin\phi}$

$K = \frac{\sigma_h}{\sigma_v}$

So, what is what is sigma 3 already we have discussed. Sigma 3 is nothing but the minor principle stress. In which direction it is acting? It is acting in the horizontal direction. So, sigma 3 is nothing but sigma h and what is sigma v sigma v is nothing but major principle stress that is sigma 1 right. So, sigma 1 is nothing but sigma v and sigma h is nothing but sigma 3. So, from the previous expression we can write sigma h is equal to 1 - sin phi by 1 + sin phi into sigma v.

Now if you see this relation if you see this relation now any situation earth pressure coefficient is nothing but sigma h by sigma v. So, this is this relation is caused and this relation is valid for all kind of state of stress right whether it is active whether it is passive whether it is rest whatever may be the case. So, k is nothing but sigma h by sigma v. So, now from this expression I can write so sigma h this is nothing but sigma h is equal to K A into sigma v yes or no right where K A is nothing but the earth pressure coefficient at active condition or simply active earth pressure coefficient and that is nothing but 1 - sin phi by 1 + sin phi right.

So, we have seen this thing in the last lecture so once again due to the for the continuation we just started from this so earth pressure coefficient in active condition is nothing but 1 - sin phi by 1 + sin phi. So, if you know the angle of internal friction of the soil okay so you can find out K A agreed? So, no other parameter is coming into the picture. Please try to understand. So, whatever may be the soil if you know the phi okay from some laboratory experiment or field experiment you need to find out phi so once you know phi you can find out the active earth pressure coefficient from this expression okay.

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## Earth Pressure on Retaining wall

**Passive state of stress**

- > Major principal stress,  $\sigma_1 = OP_3 = \sigma_h = p_p$
- > Minor principal stress,  $\sigma_3 = OP_1 = \sigma_v = \gamma z$

> Here,  $\sigma_h = \left( \frac{1 + \sin\theta}{1 - \sin\theta} \right) \sigma_v$

**Passive earth pressure coefficient,**

$$K_p = \frac{1 + \sin\theta}{1 - \sin\theta}$$

$\sigma_h = K_p \sigma_v$

Now we are going to the passive state of stress. Now in case of passive state of stress so we are having or already we know in case of active state whatever pressure is exerted on the wall okay in the passive state you will be getting more pressure on the wall right as compared to the active state. So, the and whether you consider active state or passive state your vertical stress will be remaining same. What is the vertical stress gamma into z.

At any location z your vertical stress sigma v is constant that is gamma z. Now in case of active state you start this is your vertical stress P 1. P 1 is the point which will denote the vertical stress okay. Now in case of active state that P 1 point was designating was representing the major principle stress because in the active state your lateral pressure is getting reduced but in case of passive state your lateral pressure will be getting enhanced getting increased right.

As you push the wall towards the soil you need more pressure on the wall to push it to have the movement in the backfill side okay. So, you will be getting more pressure and still your vertical plane or the horizontal plane will be remaining as principle planes. So, now when you are pushing the wall towards the backfill try to understand physically at that time at any location z your vertical stress will be same gamma into z.

But your horizontal stress or the horizontal pressure will be getting increased right. So, that will be becoming the major principle stress. In case of active state the vertical stress was the major principle stress and due to the reduction in the lateral pressure you got the lateral pressure as the minor principle stress. But in case of passive it will be just reverse that means in case of passive your vertical stress that means the on the horizontal plane whatever stress is acting that vertical

stress will be becoming the minor principle stress whereas you are pushing the wall and you will be getting much more increased lateral pressure that will be becoming the major principle stress okay. So, just reverse okay.

So, now basically the thing is that in case of passive state if you consider any soil element so this will be the direction of major principle stress. This will be the direction of minor principle stress. This is this is for passive. In case of active this was your  $\sigma_3$  and this was your  $\sigma_1$ . That was for active right already we have discussed. So, now we are going to consider this configuration okay. Now the major principle stress  $\sigma_1$  is nothing but  $\sigma_3$ .

Now which is the Mohr circle for the passive state. So, this is the Mohr circle. This  $C_p$  is the Mohr circle which will be representing increased stresses on the soil element okay. So, this point is fixed because you are not going to change the vertical stress rather you are going to change the lateral stress. So,  $\sigma_3$   $\sigma_3$  is the major principle stress that is nothing but  $\sigma_h$  that is acting in the horizontal direction as we have seen here and is nothing but the passive pressure  $p_p$ . That is nothing but your passive pressure right.

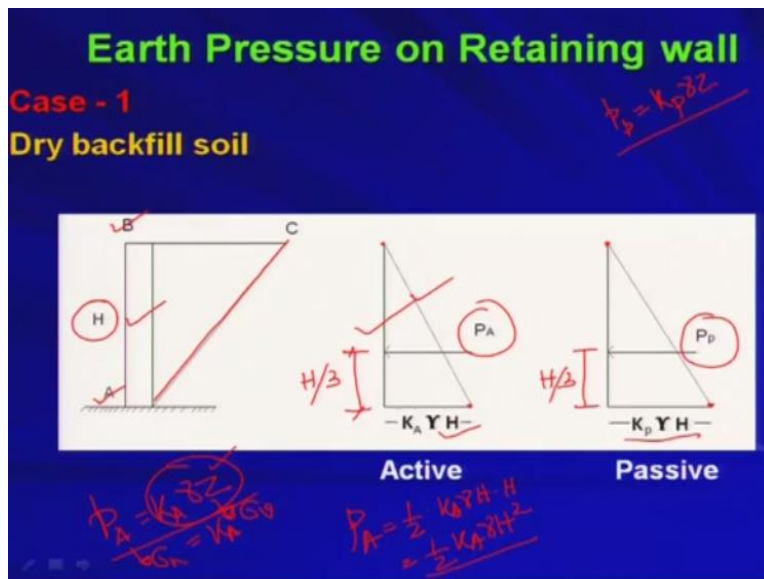
Lateral pressure is nothing but your earth pressure. So,  $\sigma_h$  is nothing but your passive pressure okay. So, what I mean to say that  $\sigma_1$  I do not know what is the magnitude of  $p_p$  right. I know the vertical stress only and which is eventually  $\sigma_3$  in case of passive state. So, minor principle stress is nothing but  $\sigma_3$  that is  $\sigma_1$  that is constant that was constant for active as well as for passive and that is nothing but  $\sigma_v$  which is equal to  $\gamma z$  okay. So, I have got  $\sigma_1$  I have got  $\sigma_3$ . So, from this relation the  $\sigma_h$  is equal to  $\frac{1 + \sin \phi}{1 - \sin \phi} \sigma_v$ . So, now if you see this Mohr circle so basically this is the point where the passive Mohr circle is touching the failure envelope that is  $P_3 C_2$  okay  $P_3 P_3 C_2$  I mean this is the plane whereas the  $C_2$  point is the point where the Mohr circle for the passive case is touching the failure envelope.

So, similarly I mean by similar exercise whatever we have done for the active state by considering the radius by considering the distance from the origin to the center of the Mohr circle all those things if we satisfy we will be getting a relation between the lateral pressure and the vertical pressure like this  $\sigma_h$  is equal to  $\frac{1 + \sin \phi}{1 - \sin \phi} \sigma_v$ . Now again we know  $\sigma_h$  is nothing but  $K_p$  into this is for passive case right.

$\sigma_h$  is nothing but  $K_p$  into  $\sigma_v$  where  $K_p$  is nothing but the passive earth pressure coefficient and which is nothing but  $\frac{1 + \sin \phi}{1 - \sin \phi}$ . Now you see  $K_p$  is much more

larger than  $K_A$  and which will tell you that active earth pressure will be always lesser than your passive earth pressure okay. I hope you have understood this thing. So, whenever you will be getting this kind of situation whether you need to calculate the pressure under active state or under passive state you need to find out the soil friction angle that is angle of internal friction of the soil  $\phi$ . Once you know  $\phi$  immediately you can calculate  $K_p$  or  $K_A$  as per your requirement okay.

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Now we will see few cases where you have different soil conditions in the backfill side and based on that whatever pressure you will be getting on the wall so that we are going to find out. Now if you have dry backfill soil now your backfill is completely dry there is no water at all. So, if that situation persists then basically this is the wall okay. This is the height of the wall is capital  $H$  and  $A$  is the bottom of the wall  $B$  is the top of the wall and this is the failure surface okay.

This is the failure surface ( $()$ ) (16:39) failure surface already we have seen in the previous lecture. So, now if I want to find out the active earth pressure okay on this or the distribution of active earth pressure on this wall then how I will find out? So, as you know your active earth pressure  $P_A$  is equal to  $K_A$  into  $\gamma z$  where  $\gamma z$  is nothing but  $\sigma_v$  and  $P_A$  is nothing but  $\sigma_H$  right.

So, you know already we have discussed enough about this. So, now this when  $z$  is 0 what is the active earth pressure on the wall  $z$  is 0 means the top of the wall so active earth pressure is

simply 0. So, that is nothing but this point okay. Now when  $z$  is equal to  $H$  that means at the base or at the bottom of the wall at that time what is the active earth pressure coefficient active earth pressure value that is  $P_A$  is equal to  $K_A \gamma H$  right.

So, this is the point and it will be connected with a linear variation. So, this is a linear variation right. With  $z$   $P_A$  is having the linear relation. So, this  $K_A \gamma h$  is equal to the active earth pressure which is acting at the base of the wall and from 0 to  $H$  this is the variation of the active earth pressure triangular and the distribution triangular pressure okay. Now I mean what is the difference between water pressure and this active I mean soil pressure.

So, water pressure as you go on increase the depth basically you will be getting  $H \gamma$  right  $\gamma H$ . So, at that time at any location you consider at that point so you will be getting same amount of pressure that is the hydrostatic pressure. So, instead of this  $\gamma H$  in case of soil you are getting  $K_A \gamma H$  one more term you are getting that is taking care of the earth pressure coefficient okay.

So, that is the only difference okay. So, now if I want to if you if I want to find out the total thrust active thrust or the total force active force acting on the wall say  $P_A$  what is  $P_A$  then  $P_A$  is nothing but the area of the triangle this triangle is nothing but the pressure distribution. So, area of this triangle will give you the total active thrust on the wall. That is nothing but area of the triangle is half into  $K_A \gamma H^2$ .

So, half  $K_A \gamma H^2$  okay. So, that is nothing but your  $P_A$ . So, this I show with small  $p$  that is nothing but the pressure and capital  $P_A$  is the total thrust. Now once I calculate this. So, that is the total force acting on the wall and your wall should be designed based on this total force. Now you may think of that as the pressure distribution is not uniform this is a linear distribution right so you will be having less section at the top if you think of your reinforced concrete design aspect so you will consider less thickness at the top and higher thickness at the bottom because maximum pressure has to be resisted at the base okay so these are the things will give you the idea that how you can design your retaining wall okay. Now this  $P_A$  is the total active thrust where it is acting?

It is acting at the center of gravity of this triangular area or CG of this triangular area okay. Now this distance is what  $H/3$  okay. So, this  $P_A$  will be acting at  $H/3$  from the base agreed? So, you know the whole thing now. You know how to calculate active earth pressure. You know how to obtain the active earth pressure distribution. You know how to calculate the total active



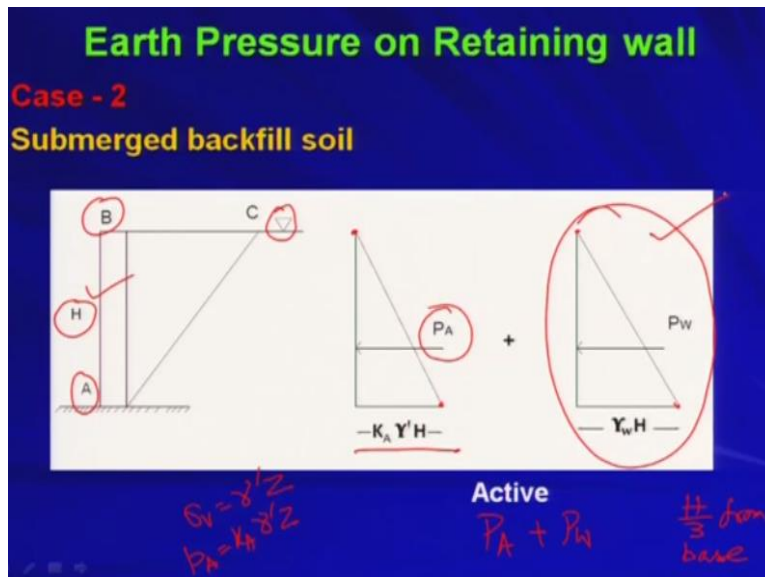
thrust from the distribution itself and you know the line of action of the total active thrust right. So, everything is known to you.

Similarly, in case of passive pressure okay so passive pressure will be generally larger. So, in case of passive pressure it will be remaining same. So, in case of passive pressure what is the passive pressure expression  $p$  is equal to  $\gamma z$  right. So,  $K p$  if I know I can find out the passive pressure. Now here also you see the relation between the passive pressure and the  $z$  is nothing but linear.

So, at  $z$  equal to 0 what is the magnitude of the passive pressure simply 0. At  $z$  equal to  $H$  what is the magnitude of the passive pressure  $K p$  into  $\gamma$  into  $H$  so that is here and within these 2 points you will be getting the linear variation linear distribution am I right, as very simple as active state okay. So, and the total thrust  $P$  will be nothing but the area of this triangle that is nothing but  $\frac{1}{2} \times K p$  into  $\gamma$  into  $H^2$  which will be acting at  $\frac{H}{3}$  distance from the base agreed.

So, if you have dry backfill there is no water at all. If water is there water pressure has to be considered separately. So, if there is no water at all then basically this is the pressure distribution under active state and under passive state and you will be getting all information like what is the total thrust what is the line of action of the total thrust all those things from this distribution itself. I hope you have understood now okay.

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Now another case we will be considering when you have submerged backfill that means water table is at the top of the wall okay water table is at the top of the wall, completely submerged backfill. Now what is happening here okay. Now we will be considering only active state. The passive state will be just same okay like active state only difference you will be considering that is the earth pressure coefficient instead of active earth pressure coefficient you have to consider passive earth pressure coefficient in case of passive strain that is all.

So, we will be talking about only active state let us say and passive state already you can find out okay. So, same thing the same wall say the height of the wall is capital H A is the base of the wall B is the top of the wall. Now where you have you do not have I mean earlier case we had the dry backfill that means water was not there. Now in this situation you have the backfill which is completely submerged in water okay.

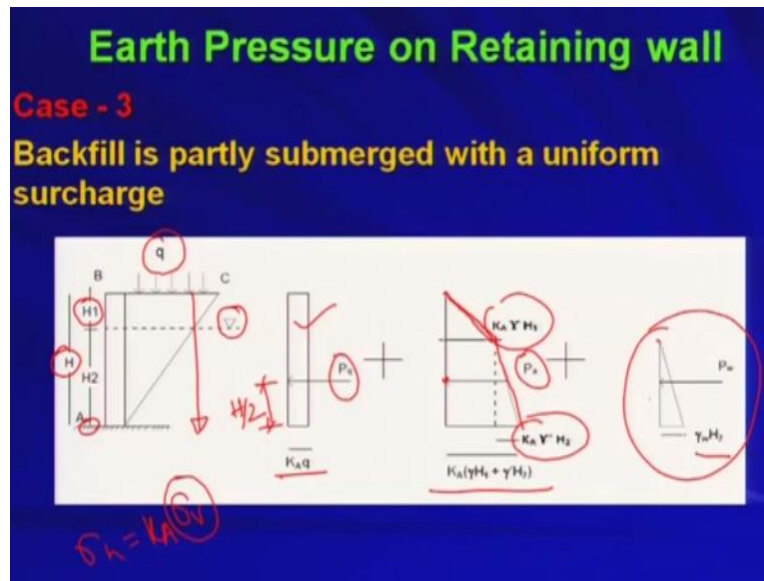
That means you have soil as well as water. So, you will be getting the lateral pressure under active state you will be getting the pressure due to water as well as due to the soil. Now when you are considering the soil then you will be getting the active thrust or passive thrust depending on the situation but in case of water it will be giving you the hydrostatic pressure simple. There is no active there is no passive in case of water pressure right.

So, it will be giving you the all-round hydrostatic pressure okay. So, in case of active state still we can consider that okay so your  $\sigma_B$  is nothing but  $\gamma' z$ . What is gamma prime? That is the submerged unit weight right in case of submerged backfill you have to consider submerged unit weight. So,  $\sigma_v$  is nothing but  $\gamma' z$ . So, if you know this you can find out  $P_A$  that is  $K_A$  into  $\sigma_v$  that is nothing but  $\gamma' z$ .

So, at  $z$  equal to 0  $P_A$  is 0. At  $z$  equal to H  $P_A$  is  $K_A$  into  $\gamma' z$  okay and distribution is again linear. The total thrust will be the area under this distribution under this triangular distribution so that is nothing but  $P_A$  and you know the line of action. So, this was the situation in case of dry state. Now you are considering submerged backfill. So, in addition to this pressure you will be getting another pressure which is coming from the water simple from the water.

So, water pressure is nothing but you know at top it is 0 at bottom it is  $\gamma_w H$  and that distribution again is triangular. So, total thrust on the wall is nothing but  $P_A + P_W$  okay. Both are acting at  $H/3$  distance from base right agreed. So, this is the term this is the thing which is coming extra because you are considering a submerged backfill understood okay.

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Now we will see the next case where you have some amount of soil is submerged some amount of soil is not submerged. Let us see what is happening. Backfill is partly submerged with a uniform surcharge. Now you have the surcharge as well as your submerged condition. Now let us understand this problem. The problem is the total height of the wall is capital H. Up to this up to depth H 1 capital H 1 the soil is not submerged.

From H 1 to H okay that means the total depth H 2 this soil is completely submerged. So, water table is here okay. So, when you are considering H 1 at that time you will be considering the dry soil and when you will be considering H 2 soil H 2 portion then the soil will be completely submerged and apart from that you have some surcharge load that is q that any kind of pressure it could be any structure or any foundation any kind of say road or embankment whatever that is on top of this surface and which is giving you the uniform surcharge pressure that is q okay. So, now for q what will be the earth pressure? Q is the vertical pressure.

Now whatever depth you consider along this wall surface whatever depth you consider q will be remaining same, q will be remaining q itself right, but what will be the magnitude of the lateral earth pressure in case of active state that will be  $K_a$  into q.  $K_a$  is your active earth pressure coefficient multiplied by q will be giving you the lateral pressure because your  $\sigma_v$  is equal to  $K_a$  into  $\sigma_v$ .

Here  $\sigma_v$  is q. So,  $\sigma_H$  that is the active earth pressure due to surcharge will be equal to  $K_a$  into q simple okay and that will be nothing but a rectangular distribution. It is constant. At

top it is also  $K A \gamma$  at bottom it is also  $K A \gamma$  okay and the total thrust  $P \gamma$  will be acting at the midpoint. So, this is nothing but your  $H/2$  okay, so at the midpoint of the wall  $P \gamma$  okay. Now coming to the actual soil earth pressure.

So, from 0 to  $H_1$  okay you have the dry soil. So, at 0 you will be having active earth pressure equal to 0 at  $H_1$  you will be having active earth pressure equal to  $K A \gamma H_1$  okay. So, this is the distribution fine. Now from  $H_1$  to  $H_2$  at  $H_1$  you are having  $K A \gamma H_1$ . Now at  $H_2$  what will be your pressure? At  $H_2$  you will be having the pressure at  $K A \gamma' H_2$  because you have to consider the submergence right.

The soil is submerged from  $H_1$  to  $H_2$  right. So, at the at the depth  $H_2$  the the whole depth the soil is completely submerged. So, that depth is basically  $K A \gamma' H_2$ . So, that will be the pressure. So, this is the total pressure distribution okay bilinear distribution. So, the total thrust if you want to calculate the total active thrust basically that will be nothing but the area of this bilinear distribution okay and at the bottom the total pressure will be  $K A \gamma H_1 + K A \gamma' H_2$ .

That is the total pressure acting at the base of the wall. So, this  $P A$  is the total thrust. Now how I will find out the line of action of this  $P A$  or the point of application of this  $P A$ . So, point of application if you want to find out you have to find out the CG of this bilinear distribution and you know how to do that from your mechanics of solid or strength of materials course right. So, you find out the CG and that will be your point of application of  $P A$  okay.

Now that is not enough because soil is partly submerged. So, you have to have the water pressure plus this is the water pressure. So, water is from  $H_1$  to  $H_2$ . So,  $H_1$  to  $H_2$  is your water pressure and at  $H_1$  you have water pressure is 0. At  $H_2$  your water pressure is  $\gamma_w H_2$  okay. So, that is your water pressure that is  $P_w$  and that is acting at the CG of this triangular distribution.

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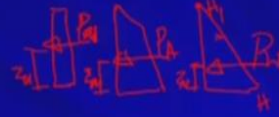
## Earth Pressure on Retaining wall

### Case - 3

Backfill is partly submerged with a uniform surcharge

$$\bar{P}_A = P_q + P_A + P_W$$

❖ Point of application of  $\bar{P}_A$  above the base of the wall can be found out by taking moments of all the forces acting on the wall about A



So, backfill, when the backfill is partly submerged so your total thrust  $\bar{P}_A$  say is equal to  $P_q$  that is the active thrust due to the surcharge plus  $P_A$  that is the active thrust due to the soil only plus  $P_W$  that is the pressure, water pressure okay due to the submergence okay that during that I mean within that zone. So, point of application of  $\bar{P}_A$ . So, you can calculate the point of application separately that is  $P_q$  as I told you  $P_q$  is acting at the midpoint of the rectangular distribution,  $P_A$  is acting at the CG of this bilinear distribution, and  $P_W$  is acting at  $H/3$  distance from the base of the triangular distribution.

That is the line for the point of application of different active thrust. Now if you want to find out the point of application of the total active thrust  $\bar{P}_A$  above the base of the wall so then it can be found by taking moments of all the forces acting on the wall about A. So, that means you have this, you have this, you have this. So, you have this for water okay so which is starting from  $H/3$  to  $H$  okay. So, you are getting  $P_q$  you are getting  $P_A$  and you are getting  $P_W$  okay. Now this so you know the point of application of each individual thrust or each individual force okay.

So, you can take the moment of all the forces because if you know this, this is say  $z_q$ , this is say  $z_A$ , and this is say  $z_W$ . So, if you know the distances if you know the point of application of each individual force you can find out the point of application of the total thrust  $\bar{P}_A$  okay total thrust  $\bar{P}_A$  by considering the moment with respect to the base of the wall okay and you can say okay so this is my wall  $\bar{P}_A$  is acting here which is the distance  $z$ . So,  $z$  can be calculated by taking the moment with respect to the base of the wall.

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## Earth Pressure on Retaining wall

### Rankine's active pressure of cohesive soil

$\sigma_v = \gamma z$   
 $\tau_f = c + \sigma \tan \phi$

We know,

$$\sigma_1 = \sigma_3 N_\phi + 2c\sqrt{N_\phi}$$

Where,  $N_\phi = \tan^2 (45 + \phi/2) = \frac{1 + \sin \phi}{1 - \sin \phi}$

$\sigma_1 = \gamma z$ ,  $\sigma_3 = \sigma_h = p_a$

$$p_a = \frac{\gamma z}{N_\phi} - \frac{2c}{\sqrt{N_\phi}} = \gamma z K_A - 2c\sqrt{K_A}$$

*Handwritten notes:*  
 $\sigma_1 = \frac{\sigma_3}{N_\phi} - \frac{2c}{\sqrt{N_\phi}}$   
 $\sigma_1 = \sigma_3 \tan^2 (45 + \phi/2) + 2c \tan (45 + \phi/2)$   
 $N_\phi = \frac{1}{K_A}$   
 $K_A = \frac{1}{N_\phi}$

Now so far we have seen okay if you have the cohesionless soil that means  $c$  was 0 in that situation if you have the cohesionless soil then basically how we have got the distribution, pressure distribution on the wall and how you could calculate the total active thrust or passive thrust on the wall right. Now most of the situation you will be having soil which is  $c$   $\phi$  soil so you will be having cohesion as well as  $\phi$  right.

So, if you have the cohesive soil that means  $c$  is there as well as  $\phi$  is there at that situation how you can find out the active earth pressure or passive earth pressure okay so that we are now going to look at okay. Now in case of this whatever maybe the case  $\sigma_v$  is  $\gamma z$  that is the vertical pressure at any depth  $z$  will be  $\gamma z$ . There is no issue for that okay if the soil is completely dry okay.

Now  $\tau_f$  that is the shear strength already we have seen that thing in the shear strength chapter that  $\tau_f$  is equal to  $c + \sigma \tan \phi$  where  $c$  is the cohesion  $\sigma$  is the normal stress on that particular plane and  $\phi$  is the angle of internal friction of the soil. Now we know from our earlier say calculation or derivation when we talked about the shear strength of soil so  $\sigma_1$  is equal to  $\sigma_3$ .

So, we got this kind of expression  $\tan^2 \alpha + 2c \tan \alpha$ . Instead of  $\tan \alpha$   $\tan^2 \alpha$  I am writing  $N_\phi$ . So,  $N_\phi$  is nothing but  $\tan^2 (45 + \phi/2)$  which is nothing but  $\frac{1 + \sin \phi}{1 - \sin \phi}$ . So, that is  $\sigma_1$  is equal to  $\sigma_3 \tan^2 \alpha + 2c$

tan alpha so instead of tan square alpha I am writing  $N \phi$  and instead of tan alpha I am writing  $\sqrt{N \phi}$  okay where  $N \phi$  is given by this expression.

Already we have seen that already we have discussed enough on this expression. Now in case of active case in case of when you are talking about the active state  $\sigma_1$  what is the major principle stress that is nothing but the vertical stress already we have seen. That is nothing but  $\gamma z$ . Now what is the minor principle stress? That is nothing but  $\sigma_3$  which is nothing but the horizontal or the lateral stress lateral pressure and that is nothing but your active earth pressure  $P_A$  okay.

So, therefore  $P_A$  is equal to  $\gamma z$  so I am putting this thing in this expression okay. So,  $P_A$  what is  $P_A$  that is nothing but  $\sigma_3$  in case of active state. So,  $P_A$  is equal to  $\gamma z$  by  $N \phi$  minus or what is  $\gamma z$  that is  $\sigma_1$  right. So, from this expression if you want to write  $\sigma_3$  is equal to  $\sigma_1$  by  $N \phi - 2c$  by  $\sqrt{N \phi}$ . So, same thing we are applying here okay. So,  $P_A$  is equal to  $\gamma z$  by  $n \phi - 2c$   $\sqrt{N \phi}$  okay.

So, which is nothing but  $\gamma z$  into  $K_A$  you look at here. What is  $N \phi$ ?  $N \phi$  is nothing but  $1$  by  $K_A$  or  $K_A$  is equal to  $1$  by  $N \phi$ . Already  $K_A$  already we have calculated. That is  $1 - \sin \phi$  by  $1 + \sin \phi$  that is not going to change. Only thing is that pressure distribution will be going to change and that we will see okay. So,  $K_A$  or  $K_P$  value will be remaining same okay.

That is nothing to do with whether it is you are considering cohesive I mean it could be a good question right if you deal with cohesive soil what is the magnitude of your earth pressure coefficient in the active state or passive state. That will be remaining that will be completely dependent on the angle of internal friction soil that is  $\phi$  okay. So,  $\gamma z$  into  $K_A$  this is nothing but this and  $2c$  by  $\sqrt{N \phi}$  is nothing but  $2c$  into  $\sqrt{K_A}$  okay.

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## Earth Pressure on Retaining wall

**Rankine's active pressure of cohesive soil**

> The active pressure,  $p_a = 0$  when

$$\frac{\gamma z}{N_o} - \frac{2c}{\sqrt{N_o}} = 0$$

Or  $z = z_0 = \frac{2c}{\gamma} \sqrt{N_o}$

> At depth  $z = 0$ , pressure  $p_a$  is

$$p_a = -\frac{2c}{\sqrt{N_o}}$$

i.e. active pressure  $p_a$  is tensile between depth 0 and  $z_0$

Now the active pressure  $P_A$  is equal to 0 when? So, this was the expression. This was the expression for  $P_A$ .  $P_A$  is equal to  $\gamma z$  into  $K_A - 2c$  into root over  $K_A$ . Now when the active earth pressure will be becoming 0 let us see. Earlier case in case of purely cohesion cohesionless soil okay at that time the active earth pressure or passive earth pressure was becoming 0 at the top of the wall because  $P_A$  or  $P_p$  was equal to  $K$  into  $\gamma$  into  $z$ . If  $z$  is 0 simply it was becoming 0.

Now whether that situation still valid or not that we have to examine right. So, in case of active pressure say when  $P_A$  becoming  $P_A$  is becoming 0 at that time what is happening say  $\gamma z$  by  $N_{\phi} - 2c$  by root over  $N_{\phi}$  is equal to 0 from the previous expression okay. From this we can find out at which depth  $P_A$  is becoming 0. So,  $z$  is equal to some depth say  $z_0$  okay is equal to  $2c$  by  $\gamma$  into root over  $N_{\phi}$  okay.

So, that means if you have the wall like this so at  $z$  equal to 0 it is not becoming 0. Earlier case in case of cohesionless soil we have seen that earth pressure was becoming lateral earth pressure was becoming whether it is active or passive it was becoming 0 at the top of the wall when  $z$  equal to 0. But here when you are considering the cohesion of the soil at that time you cannot say simply that if  $z$  becomes 0 my pressure is also becoming 0 no.

Rather in case of active state you are seeing this when  $z$  is equal to some value of  $z_0$  okay which is nothing but  $2c$  by  $\gamma$  into root over  $N_{\phi}$ . At that distance, some distance say this is your  $z_0$  at some distance from the top of the wall okay your earth pressure active earth pressure is



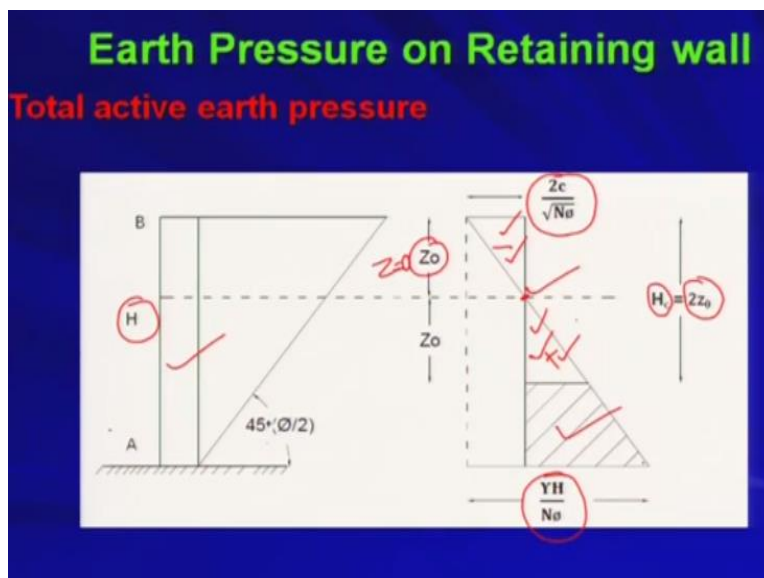
becoming 0. This is the basic difference okay between the cohesionless soil and cohesive soil.

So, in case of cohesive soil just at the top of the wall you are not getting 0 active earth pressure. You will be getting something else. We will come to that point later on but you will not be getting 0 active pressure at the top of the wall rather you will be getting 0 active pressure at some depth below the top of the wall okay. So, at depth  $z$  equal to then what is happening at  $z$  equal to 0? So, you are saying that at  $z$  equal to  $z_0$  you are getting active earth pressure is 0.

Now what is happening at the top of the wall okay. Let us see, let us examine. So, at  $z$  equal to 0 the pressure  $P_A$  is minus  $2c$  by root over  $N \phi$ . Now  $c$  is positive  $N \phi$  is positive so  $P_A$  is becoming negative. That means active pressure  $P_A$  is tensile because already we have considered the compressive stress is positive in soil mechanics as I told you several times previously.

So, when you are getting negative pressure that means it indicates that you are getting or you are I mean developing some tensile pressure or the tensile stress okay at the top of the wall. So, active pressure  $P_A$  is tensile between depth 0 and  $z_0$  understood. So, if I want to this is say  $z_0$  so if I want to obtain the pressure distribution so this is my negative okay. So, it is 0 at  $z$  equal to  $z_0$  okay and this is negative  $2c$  by  $N \phi$  this is negative. So, from top to  $z_0$  distance  $z_0$  depth you will be getting tensile stress or the tensile pressure understood okay.

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Now you see the distribution okay. So, this is the wall. This is the total height of the wall H okay. So, as I told you that when z equal to z 0 so z equal to z 0 you are getting active earth pressure is 0. At z equal to 0 that means at the top of the wall you are getting 2c by root over N phi which is nothing but tensile minus. Now from this point onward you will be getting K A into I mean this the distribution whatever distribution whatever magnitude you will be getting okay so that is valid from this. So, from this to this you will be getting this.

So, at bottom you will see gamma H into N phi. So, at z equal to H what is happening? So, from the expression from the expression itself you can see at z equal to H that will be gamma into H into K A - 2c into root over K A right. So, that I am showing here okay. So, this is nothing but the total - 2c so this is the total - 2c so this much will not be there so this is the total pressure distribution. So, this hatched part is the total pressure distribution that is the compressive stress or the compressive pressure thrust okay.

So, and so what does it mean. So, this is compressive, this is positive, this is negative. Now if you consider and they are symmetric right. so this triangle and this triangle both are symmetric agreed. Now from z equal to 0 to z equal to 2z 0 which is nothing but say H c which will be coming later on. So, from z equal to 0 to twice z 0 within this distance this negative triangle or the negative pressure will nullifying this positive pressure agreed? So, there will be no pressure at all from z equal to 0 to z equal to twice z 0 you do not have any pressure okay.

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**Earth Pressure on Retaining wall**

**Total active earth pressure**

$$P_A = \int_0^H pz \, dz = \int_0^H \left( \frac{\gamma z}{N_o} \right) dz - \int_0^H \left( \frac{2c}{\sqrt{N_o}} \right) dz$$

$$= \frac{1}{2} \gamma H^2 \frac{1}{N_o} - 2c \frac{H}{\sqrt{N_o}}$$

- The shaded area gives the total pressure  $P_A$
- If total earth pressure is equal to zero

i.e.  $P_A = 0$

or,  $\frac{1}{2} \gamma H^2 \frac{1}{N_o} - 2c \frac{H}{\sqrt{N_o}} = 0$

$P_A = \frac{\gamma z^2}{2N_o} - \frac{2c}{\sqrt{N_o}}$

So, now total active pressure I mean this  $P_A$  okay the total thrust if you are going to find out that is nothing but  $\int_0^H p \, dz$  which is nothing but  $\int_0^H (\gamma z - c) \, dz$ . So, this is nothing but  $P_A$  right. So, if I say this is  $P_A$  this is  $P_A$  okay. So, now  $P_A$  into  $z$  that and  $P_A$  already we have seen that  $\gamma z - c$   $\gamma z - c$  already we have seen that  $2c$  by root over  $N \phi$  okay. So, into  $K A$ .

That should be so  $1$  by  $N \phi$  is nothing but your  $K A$  already we have seen. So, this already we have seen that is nothing but  $P_A$  okay. So, if you do this integration both the integrations basically you will be getting the total thrust total thrust that is  $P_A$  is equal to half into  $\gamma H^2 - 2cH$  by root over  $N \phi$  okay. Now the shaded area gives the total pressure  $P_A$ .

The previously whatever shaded area we have seen this is the shaded area. So, this area if you calculate if you calculate this area this area will give you the total active thrust in case of cohesive soil okay and which is obtained from this expression from this integration right. So, total thrust you can calculate so at this is the total thrust acting at some points that point of application needs to be calculated that is different issue.

But the thing is that this total thrust how you will find out this total thrust that is the total pressure? That is nothing but the shaded area whatever shaded area I have shown you in the previous slide. Now if total earth pressure is equal to  $0$  if this total thrust is becoming equal to  $0$  what is happening? So, if it is becoming  $0$  so this is also becoming  $0$ .

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## Earth Pressure on Retaining wall

Total active earth pressure

or,  $H = H_c = \frac{4c}{\gamma} \sqrt{N_o}$

$z = 2z_0$   
 $H_c = 2z_0$

- >  $H_c$  is called as **critical depth**
- > This indicates that a vertical bank of height smaller than  $H_c$  can stand **without lateral support**

Then I can find out because already we talked about that right when  $z$  equal to twice  $z_0$  right do you remember right. So, at that time the negative part negative triangle and the positive triangle they will cancel each other right. So, the total thrust from 0 to that depth twice  $z_0$  you will be getting 0 you will be getting some negative stress negative pressure and you will be getting some positive pressure they will be cancelling each other and total thrust will be equal to 0. That is physically you can explain but mathematically or the I mean expression or when you are developing the expression at that time at that depth I mean up to that depth actually 0 to  $H_c$  okay is equal to  $H_c$  where  $H_c$  is nothing but twice  $z_0$  say already we have seen that.

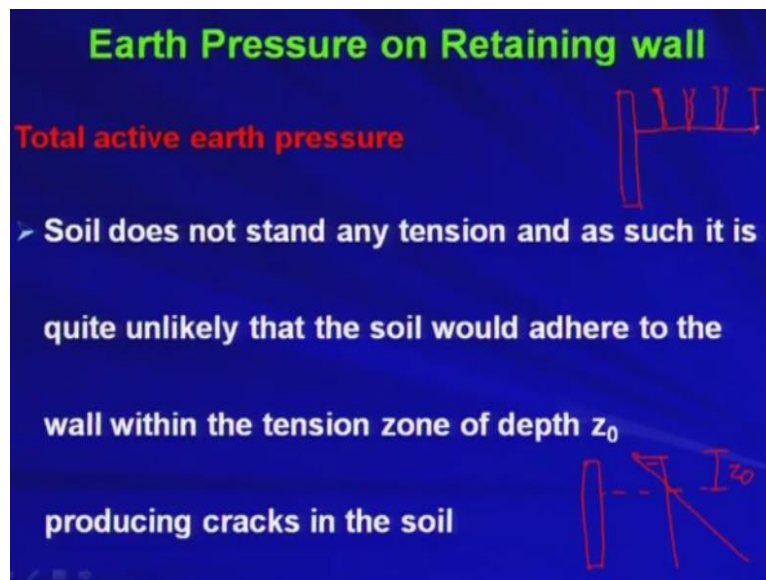
So, when  $H_c$  is equal to  $\frac{4c}{\gamma} \sqrt{N_{\phi}}$  at that time you will be getting total thrust equal to 0 because the negative part and positive part will be cancelling each other. So,  $H_c$  is very important in soil mechanics. So,  $H_c$  is known as critical depth okay. So, this is very important. Now this indicates that a vertical bank of height smaller than  $H_c$  can stand without lateral support.

That is why if you want to excavate okay in sandy soil in cohesionless soil okay basically you cannot excavate without giving any vertical support. So, you need to give the vertical support. Otherwise soil will collapse in the excavation. But if you I mean you might have seen that thing in your say day to day life or in your physical or the practical experience you have gathered that if you want to excavate anything in  $C_{\phi}$  soil okay when you have some cohesion you can get some amount of say vertical curve which can stand without any support.

You need not to give the lateral support that means bracing or all those things right. So, those things are not required. So, certain depth will stand without any support. Now what is that certain depth? That certain depth will be lesser than the critical depth  $H_c$  because you do not have any pressure so if there is no pressure that soil will be standing without any pressure there the earth pressure at rest condition.

So, there is no wall required at all okay. Soil is standing without any active or active thrust. So, it will consider it will be considered as active earth pressure at rest condition kind of thing right. So, there is no movement. So, that is why in case of  $C\phi$  soil any certain depth or certain bank of height smaller than  $H_c$  can stand without any lateral support okay.

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So, soil does not stand any tension already we have talked about this thing. Soil can take care of compression but it is very weak in tension so soil cannot take care of tension. Soil does not stand any tension and as such it is quite unlikely that the soil would adhere to the wall within the tension zone of depth  $z_0$  producing cracks in the soil okay. So, what does it mean what is the meaning of this statement.

Say you have the wall like this okay and you have seen so this is your  $z_0$  so within this depth 0 to  $z_0$  basically you are getting tension and I right. Soil is experiencing some tension. Now soil cannot take care of any tension agreed? Soil cannot take care of any tension. So, that means the tension means it will try to adhere the wall right. That soil cannot take care. Soil cannot produce this kind of adherence right.

So, this I mean if you want to adhere the wall I mean something like your tie rod or something like your rope you are just I mean just pulling the wall back. So, that pulling or that kind of situation may not have I mean will not happen in soil. Soil cannot take care or soil cannot suck or soil cannot pull the wall back okay. So, therefore what happens you will be getting some tension crack within this zone 0 to  $z_0$  tension crack means you might have seen this thing.

So, if the wall is constructed you will be getting some cracks okay. So, this is known as tension crack okay and this will be only up to depth  $z_0$  because soil cannot take any tension so it will be having the crack or the failure okay. It cannot pull the wall back. So, this is very important and that is why you might have observed this thing okay but you observed this thing without knowing. Now you know why this kind of tension crack is really happening in the soil. So, you will not be getting this kind of tension crack in cohesionless backfill.

You will be getting tension crack in cohesive backfill when you have cohesion because you are getting some development of tension at the top portion or the top part of the wall okay. So, I will stop here today. In the next lecture, we will be continuing this thing to understand this phenomena in a in more precisely and then we will be talking about in case of rough wall, so far we are considering the smooth wall right. The wall surface is smooth okay. So, if you consider the rough wall what will be happening? So, that was proposed by Coulombs so that we will be discussing later on. So, I will stop here today. Thank you very much.