

**Geology and Soil Mechanics**  
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**Lecture - 61**  
**Earth Pressure on Retaining Wall - 1**

Welcome back. So, in the last lecture we discussed about the active earth pressure and passive earth pressure distribution not passive earth pressure distribution, active earth pressure distribution when you considered the cohesion of the soil and there we have seen that if you consider the cohesion of the soil basically you will be getting some amount of tension at the top of the wall right so and that tension will be extending from say  $z$  equal to 0 to  $z$  equal to  $z_0$ .

So, up to that depth basically you will be getting some amount of tension on the wall and beyond that so at  $z$  equal to  $z_0$  your pressure active pressure will be becoming simply 0 and from that point onwards that means  $z$  equal to  $z_0$  to  $z$  equal to  $H$  basically you will be getting the compressive pressure whatever is generally you see for cohesionless soil right. So, that is the main difference already we discussed and based on that we got I mean few different parameters one is the critical depth right so that critical during that critical depth critical depth  $H_c$  is nothing but  $2z_0$  okay so that within that depth basically your negative pressure and your positive pressure positive pressure means the compressive pressure so negative pressure means the tensile pressure.

So, these 2 pressure will be getting nullified and because of that you will be getting or you generally get and that you have seen in your practical from your practical experience that if you consider some cohesive  $C$   $\phi$  soil basically you can excavate some amount of soil with some certain depth right without any support. So, that certain depth without any support you can excavate in cohesive soil because that depth is less than  $H_c$  that is critical depth that is  $2z_0$  right. So, those things we saw in the last lecture.

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## Earth Pressure on Retaining wall

### Total active earth pressure

- Soil does not stand any tension and as such it is quite unlikely that the soil would adhere to the wall within the tension zone of depth  $z_0$  producing cracks in the soil

*Tension cracks*

And we have seen another thing that is soil does not stand any tension right and as such it is quite unlikely that the soil would adhere to the wall within the tension zone of depth  $z_0$  producing cracks in the soil so and that is known as tension cracks so already we have seen in the last lecture that you will be getting tension cracks and that is known as tension cracks right.

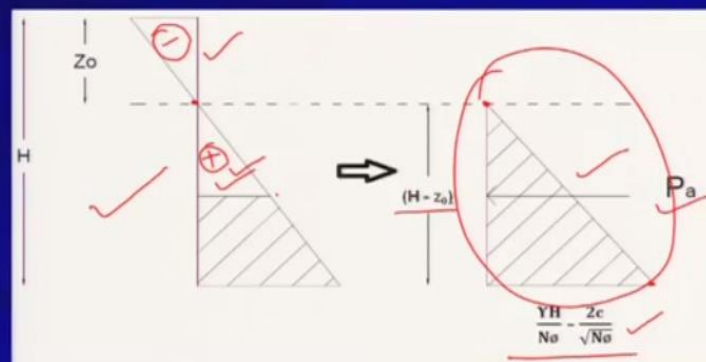
So, the area where the tension is coming okay within that depth basically your soil cannot pull the wall right soil is weak in tension so I mean you cannot expect some pulling effort from the soil so that it can pull the wall back so that is not possible in soil and therefore what will happen you will be getting some cracks because of tension the soil will be experiencing some cracks in the tension zone and that is known as tension cracks as I wrote here okay.

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## Earth Pressure on Retaining wall

### Total active earth pressure

- The active earth pressure is represented by



So, now based on that okay how can you modify the active earth pressure distribution plot in case of cohesive soil. So, this is the actual say active earth pressure distribution plot for cohesive soil for  $C \phi$  soil so this part is experiencing some tension so this is your negative pressure okay. So, this part  $z$  equal to  $z_0$  within that zone you will be getting tension and from at  $z$  equal to  $z_0$  you will be getting say  $0$  pressure okay and then from that point onward you will be getting compressive stress right.

So, ultimately what is happening here so this negative pressure and this positive pressure they are symmetrical so this much I mean up to  $z$  equal to  $z_0$  whatever negative pressure you will be getting that will be simply equal to the compressive pressure or the compressive stress okay that is coming within this zone right and they are cancelling each other right and that is why you are not getting any pressure virtually you are not getting any pressure on the wall but frankly speaking so we when we are going to design okay the wall we do not consider this nullifying effect or this cancellation effect okay in the active earth pressure distribution.

What we consider so instead of this kind of say this is the shaded part which is actually acting on the wall which is compressive in nature and this is nothing but the active thrust active pressure on the wall right but we do not consider like this kind of diagram rather we consider the diagram like this where your compressive pressure so we are simply neglecting the tension tensile pressure and we are not thinking or we are not telling that the tension is going to cancel the positive some amount of positive pressure.

So, that thing we are neglecting and we are considering that the total compressive pressure or total compressive thrust active thrust is acting on the wall starting from  $z$  equal to  $z_0$  so this is the point at  $z$  equal to  $z_0$  and it is going to end at point  $z$  equal to  $H$  okay and this is the if you if you see the previous expression previous equation for  $P_a$  that is the active earth pressure distribution active earth pressure so at  $z$  equal to  $H$  okay so this is the active pressure  $\gamma H$  by  $N \phi - 2c$  by root over  $N \phi$  where  $1$  by  $N \phi$  is nothing but  $K_A$  right already we have seen that.

So, this is the now basically this is the total pressure distribution we will be considering when we are going to design some wall some retaining wall okay under the active pressure in cohesive soil in  $C \phi$  soil okay. So, we are not considering we are not cancelling any part of so this part this part we are not cancelling or we are not ignoring. Rather we are considering the total triangular

distribution of the active earth pressure. Now based on that we define or we calculate the total active thrust  $P_a$  okay.

Now this depth okay along which you have actually the pressure on the wall so that depth is nothing but  $h - z_0$  because  $z$  to  $z_0$  you have the tensile pressure or the tensile I mean pulling or tensile nature of active pressure okay so that we are neglecting so therefore at  $z$  equal to  $z_0$  at  $z$  equal to  $0$  to  $z_0$  we do not have any pressure so all of sudden the pressure is getting started from  $z$  equal to  $z_0$  to  $z$  equal to  $H$ . So, the total depth along which the pressure is acting that is nothing but  $H - z_0$ .

So, that means that the wall if you think physically the wall up to depth of  $z$  equal to  $z_0$  is not experiencing any kind of pressure okay because we are neglecting the tension or the tensile pressure and tension is not I mean it is not possible for soil to carry the tension right and that is why you are getting the tension cracks. So, the top portion of the wall from  $z$  equal to  $0$  to  $z$  equal to  $z_0$  that portion of the wall will not be experiencing any pressure from the active thrust rather the pressure will be starting from  $z$  equal to  $z_0$  and it will extend up to  $z$  equal to  $H$  and that you are getting from this diagram right.

So, it is very unlikely I mean if you compare with your cohesionless soil in the cohesionless soil the top part or the top portion of the wall was experiencing  $0$  pressure and it is gradually increasing with the linear distribution but in this situation up to a certain depth it is there is no pressure and then from  $z$  equal to  $z_0$  your pressure is getting increased from the wall right. So, you can design the wall in such a way that I mean the top portion is not experiencing any pressure.

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## Earth Pressure on Retaining wall

Total active earth pressure

➤ Total pressure/active thrust,

$$P_A = \frac{1}{2} \left[ \frac{\gamma H}{N_\phi} - \frac{2c}{\sqrt{N_\phi}} \right] (H - z_0)$$

So, therefore total pressure active I mean the total pressure or the active thrust so whatever that is the force that is  $P_A$  right. So, this  $P_A$  is nothing but if you see this so this is at  $z$  equal to  $z_0$  okay from there we are getting this pressure okay that is  $\gamma H$  by  $N_\phi - 2c$  root over  $N_\phi$  right and this depth is  $H - z_0$  so we have just seen in the previous slide right. So, total thrust will be nothing but the area under this triangular plot.

This is your total thrust  $P_A$  okay. So, that  $P_A$  is nothing but the area of this triangle so that means half into  $\gamma H$  by  $N_\phi - 2c$  by root over  $N_\phi$  into  $H - z_0$ . So, this is the total thrust, where it is acting? It is acting  $H - z_0$  by 3 distance above the base. So, this is nothing but this okay. So, this is the point of application of this active thrust  $P_A$  is nothing but  $H - z_0$  by 3 okay. So, I hope that you have understood the basic difference between the cohesionless soil and the cohesive soil okay.

So, in the cohesive soil the pressure the active earth pressure can be calculated by this expression whereas in case of cohesionless soil you calculated active earth pressure  $P_A$  is equal to half into what half into  $K_a$  into  $\gamma$  into  $H$  square right. So, this is the thing I mean here in this equation if you put  $c$  equal to 0 suppose you want to I mean find out the active earth pressure for cohesionless soil. So, that means  $c$  equal to 0.

So, if you put  $c$  equal to 0  $z_0$  will be 0 okay and  $c$  will be 0 so this part will be going this part will be going and this part is nothing but half into  $\gamma H$  into  $K_a$  into  $H$ . So, you are getting the same expression whatever we have got earlier right if  $C$  is 0 fine. So, I hope that you have understood this concept.

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**Earth Pressure on Retaining wall**

**Rankine's passive pressure of cohesive soil**

$$\sigma_1 = \sigma_3 N_\phi + 2c\sqrt{N_\phi}$$

Here,  $\sigma_1 = \sigma_h = p_p$

$$\sigma_3 = \sigma_v = \gamma z$$

$$p_p = \gamma z N_\phi + 2c\sqrt{N_\phi} = \gamma z K_p + 2c\sqrt{K_p}$$

$N_\phi = \frac{1}{K_A} = \frac{1 + \sin \phi}{1 - \sin \phi}$   
 $= K_p$

Now based on this concept we are going to calculate the Rankine passive pressure for cohesive soil okay. So, we have seen how to find out active thrust, active pressure for the cohesive soil that is C phi soil that means if C is there then what will be the expression for active pressure. Now we are going to find out the expression for passive pressure if C is there. So, basically your sigma 1 and that is the relation already we know from our earlier discussion that sigma 1 equal to sigma 3 N phi + 2c root over N phi right.

Now in case of passive pressure already we have seen that in case of passive pressure what is sigma 1, sigma 1 is nothing but the lateral pressure that is the horizontal pressure that is sigma H which is nothing but p p that is the passive pressure am I right. So, in case of active sigma 1 was the vertical pressure. In case of passive the horizontal pressure will be your major principle stress. Similarly, sigma 3 in case of passive pressure what will be the sigma 3, sigma 3 is nothing but the vertical pressure that is nothing but sigma v which is equal to gamma z.

So, if I put these things in this expression okay so I will get an expression like this. So, p p is equal to gamma z into N phi + 2c root over N phi where N phi already we know N phi is nothing but 1 by K A which is nothing but 1 + sin phi - 1 - sin phi which is nothing but K p right. So, gamma z N phi is nothing but gamma z K p plus 2c root over N phi instead of that I can write 2c root over K p. So, this is the pressure. This is the passive pressure along the wall along the wall depth of H okay.

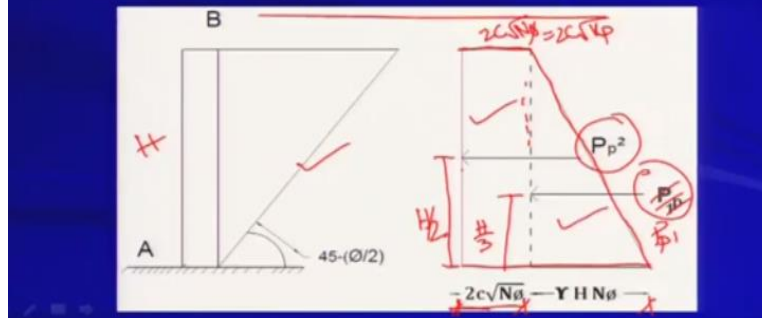
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## Earth Pressure on Retaining wall

### Rankine's passive pressure of cohesive soil

At  $z = 0$ ,  $p_p = 2c\sqrt{N_\phi} = 2c\sqrt{K_p}$  ✓

$z = H$ ,  $p_p = \gamma H N_\phi + 2c\sqrt{N_\phi}$



So, now what we are going to find out at  $z$  equal to 0 that means at top of the wall what is the passive pressure what is the  $z = 0$  so it was 0 for cohesionless soil earlier we have seen that if  $z$  is 0 then immediately you are getting 0 passive pressure in case of cohesionless soil. Now whether that will be same for cohesive soil is it so or not so we are going to see that okay. So, at  $z$  equal to 0 so  $p_p$  from this from the previous expression given in previous slide so  $p_p$  is nothing but  $2c$  into root over  $N_\phi$  is equal to  $2c$  into root over  $K_p$  okay.

So, this is the passive pressure we are getting at  $z$  equal to 0. That means instead of becoming 0 passive pressure 0 pressure at the top of the wall you are getting some amount of pressure which is positive which is compressive that is okay, okay. That is not like active pressure where you are getting some tension. Here you are not getting any tension but you are getting some amount of pressure, positive pressure at the top of the wall.

So, your top of the wall is not free from any pressure. It has got or it has experienced some amount of pressure. So, at  $z$  equal to  $H$  what is the value of  $p_p$  that is the passive pressure  $p_p$  is equal to  $\gamma H N_\phi + 2c$  into root over  $N_\phi$ . So, in the previous I mean equation if you put  $z$  equal to  $H$  you will be getting this. So, now if I plot this so I will be getting this plot. So, this is the wall of depth say  $H$  okay.

This is the failure surface linear failure surface. Now I am getting this part. So, this is nothing but  $2c$  into root over  $N_\phi$  which is nothing but  $2c$  root over  $K_p$ . So, this is the so this is the pressure at top of the wall. So, it is not stress free okay. At top of the wall you are getting some

amount of pressure and from that point basically you are getting the linear distribution and the total I mean the pressure at the base of the wall you are getting  $\gamma H N \phi + 2c \sqrt{N \phi}$ .

So, this is this part is  $\gamma H N \phi$  and this part is  $2c \sqrt{N \phi}$ . So, this is the total pressure at passive pressure distribution. So, the passive pressure distribution curve will look like this okay. So, now the total thrust total passive thrust if you want to calculate what you can do? You can get the area of this whole trapezoid. So, this is a this is looking like a trapezoid right. So, you can find out or you can determine the area of the trapezoid and that area of the trapezoid will give you the total passive thrust.

Now for sake of convenience we are not going to determine the total area of the trapezoid rather we are distributing or we are dividing the total area of this trapezoid in 2 parts. In one part we are getting this rectangle shown by this dash line ok and another part we are getting some triangular distribution. So, in the rectangular distribution the total thrust the area of the rectangular distribution is say  $P_1$ ,  $P_2$  if it is  $P_2$  and the this is your say  $P_1$  okay this is say  $P_1$ .

So, the total thrust on the triangular part will be say  $P_1$ . So, that means the area of the triangular zone will be  $P_1$  and the area of the rectangular zone is say  $P_2$  and now we can calculate this  $P_1$  and  $P_2$  and we find out the point of application of these 2 I mean these 2 forces separately okay. So,  $P_2$  is acting at  $H/2$  from the base right. This is your  $H/2$  and  $P_1$  will be acting  $H/3$  yes or no right. So, we can calculate each individual forces thrust as well as the point of application of each individual forces and if I want to find out the resultant force as well as the resultant force point of application then we can take the moment with respect to point A and we can find out that okay as before we have done.

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## Earth Pressure on Retaining wall

### Rankine's passive pressure of cohesive soil

✓  $P_p^1 = \frac{1}{2} \gamma H^2 N_o = \frac{1}{2} \gamma H^2 K_p$ , acts at  $\frac{H}{3}$  from base

✓  $P_p^2 = 2cH\sqrt{N_o} = 2cH\sqrt{K_p}$ , acts at  $\frac{H}{2}$  from base

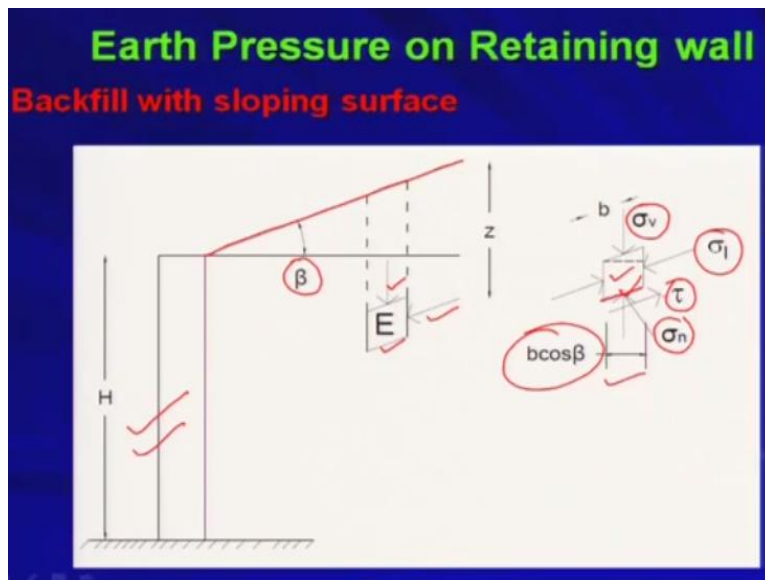
(  $P_p = P_p^1 + P_p^2$  )

So,  $P_p^1$  okay so  $P_p^1$  is equal to half  $\gamma H^2$  into  $N_\phi$  right. If you look at this figure so this is  $\gamma H N_\phi$  and total depth is  $H$  so half  $\gamma H^2$  into  $N_\phi$ . So, that is your  $P_p^1$ . That is nothing but half  $\gamma H^2$  into  $K_p$  which will be acting at  $H$  by 3 from the base as I told you okay. Whereas  $P_p^2$  that is the area of the rectangular portion that is nothing but  $2cH$  into root over  $N_\phi$  that is nothing but  $2cH$  into root over  $K_p$  right.

So, that is the area of the rectangular portion and that will be acting as I told you at  $H$  by 2 distance from the base. So, from this  $P_p^1$  and  $P_p^2$  you can find out the total passive thrust that is total passive thrust is nothing but  $P_p^1 + P_p^2$  okay and the point of application of  $P_p$  if you want to find out you have to take the moment equilibrium with respect to point A for these forces for these forces okay.

Now I hope it is clear. So, that means now you are well equipped to get to find out the active pressure in completely cohesionless soil as well as passive pressure in cohesionless soil. Now also you can find out the active and passive pressure in cohesive soil as well okay and you know that what will be the difference if you deal with some cohesive soil okay and if you deal with the cohesionless soil okay.

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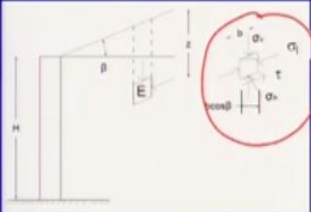
Now if you have the backfill with sloping surface. So, it is very common right. So, all the times you will be getting the backfill which is horizontal. It is it may not happen in the real life right. So, if you want to retain the hilly slope okay so suppose the hilly slope is like that and if you want to retain that thing with your retaining wall okay. So, this slope so if you have this kind of retain I mean inclined backfill then what will happen okay. Let us see that.

So, if you have the inclined backfill that means so this is the wall okay which is having the inclined backfill which is inclined at an angle beta okay. Now we are considering one soil element so E like this which will be experiencing some inclined lateral pressure sigma L and vertical pressure sigma v therefore if you take out this soil element so this is the soil element say. So, this is your sigma L which is acting parallel to the sloping surface sloping backfill okay. So, that is nothing but your lateral pressure okay so inclined lateral pressure. Now sigma v is the vertical pressure okay and b where b is the say this inclined length okay b is the inclined length say so therefore the horizontal length will be b cos beta okay because the angle of inclination is beta okay. Now we can calculate sigma n and tau. What is sigma n? Sigma n is the normal stress on this sloping surface and tau is the shear stress along this sloping surface okay.

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## Earth Pressure on Retaining wall

Backfill with sloping surface



$$\sigma_v = \frac{\gamma z \cdot b \cos \beta}{b} = \gamma z \cos \beta$$

$\sigma_l = \text{lateral pressure} = p_a \text{ (active)}$

$$\sigma_n = \sigma_v \cos \beta = \gamma z \cos^2 \beta$$

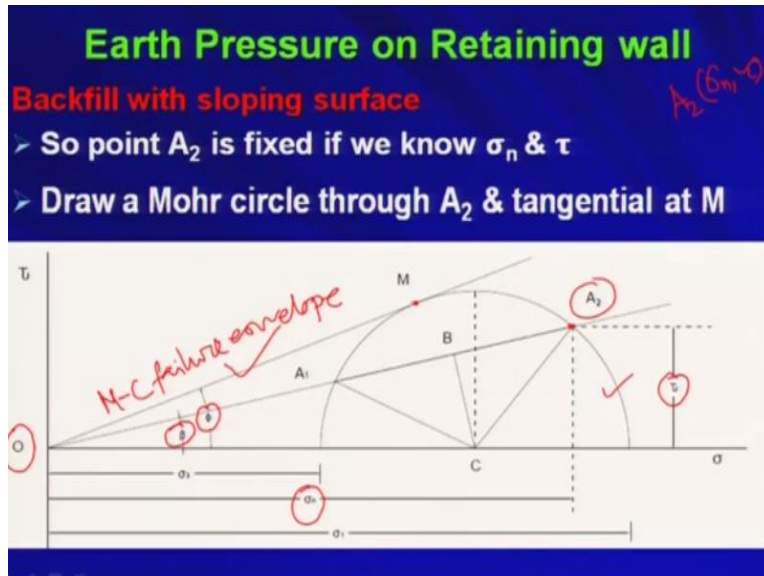
$$\tau = \sigma_v \sin \beta = \gamma z \sin \beta \cos \beta$$

So, now for this I can calculate sigma v, what is sigma v? Sigma v is nothing but the vertical stress. So, that is nothing but gamma z into b cos beta right so that is the total force yes or no? That is the total force, Gamma z into b cos beta divided by b will be your vertical stress gamma z cos beta right. Similarly, sigma L that is the lateral pressure that is nothing but p a that is our objective to obtain to find out okay. So, sigma L is this p a.

Now we are going to find out sigma n and tau. So, sigma n sigma n is nothing but sigma v into cos beta. The component of sigma v along the normal direction. So, sigma v cos beta is nothing but sigma n which is nothing but gamma z into cos square beta because sigma v itself is equal to gamma z cos beta. Similarly, tau is equal to sigma v sin beta okay. So, now if you look at look at this figure basically you will be getting all those things.

So, tau is equal to sigma v sin beta. So, sigma v sigma v is nothing but gamma z cos beta into sin beta. So, you are getting tau equal to gamma z sin beta cos beta. So, at any depth okay you can find out sigma n by this expression if you know the slope of the backfill the depth which you want and of course you if you know the unit weight of the soil you can find out sigma n that is the normal stress at any depth. Similarly, you can find out the shear stress at any depth that is nothing but gamma z sin beta cos beta okay.

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So, once you know this okay so if you know  $\sigma_n$  and  $\tau$  that is the normal stress and the shear stress so point  $A_2$  is fixed in  $\tau$   $\sigma$  space we are now plotting this point okay in  $\tau$   $\sigma$  space the point  $A_2$  is fixed if we know so this is the point okay. That point is fixed in  $\tau$   $\sigma$  space if I know  $\sigma_n$  and  $\tau$ . So, because  $A_2$  the coordinate of  $A_2$  is  $\sigma_n$  and  $\tau$  okay so that is shown here okay.

So, once point  $A_2$  is fixed and if I join point  $A_2$  with the origin  $O$  that will make an angle  $\beta$  with the horizontal with the  $\sigma$  axis agreed? It has to be yes or no it has to be because the inclination already you considered that inclination is  $\beta$  inclination of the sloping surface or the sloping backfill right. So, based on that you calculated  $\sigma_n$  and  $\tau$  right. So, the point this point is now fixed if I if you know  $\sigma_n$  and  $\tau$ .

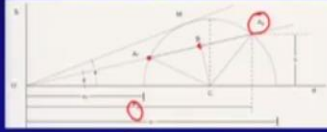
Now if you join this point with the origin  $O$  then it will make an angle  $\beta$  with the  $\sigma$  axis with the horizontal right okay. Now draw a Mohr circle through  $A_2$  and tangential at  $M$ . So, now you are drawing one Mohr circle through point  $A_2$  and tangential at point  $M$ . Now what is point  $M$ , point  $M$  is lying on the Mohr-Coulomb failure envelope because this angle is  $\phi$  and you are considering completely cohesionless soil.

So, this will be your Mohr Coulomb failure envelope okay. So, this is the Mohr circle which is passing through point  $A_2$  and which is touching at point  $n$  which is telling you the failure right. When the Mohr circle is touching at point  $n$  so point  $n$  is telling you about the failure okay.

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## Earth Pressure on Retaining wall

**Backfill with sloping surface**  
Applying equation (6.1),



$$A_1B = A_2B = \frac{\sigma_1 + \sigma_3}{2} \sqrt{\sin^2 \phi - \sin^2 \beta}$$

$$\sigma_v = OB + BA_2$$

$$= \frac{\sigma_1 + \sigma_3}{2} \cos \beta + \frac{\sigma_1 + \sigma_3}{2} \sqrt{\sin^2 \phi - \sin^2 \beta}$$

$$p_a = \sigma_1 = OB - A_1B \rightarrow OA_1$$

$$= \frac{\sigma_1 + \sigma_3}{2} \cos \beta - \frac{\sigma_1 + \sigma_3}{2} \sqrt{\sin^2 \phi - \sin^2 \beta}$$

Now CM, now CM so this is the point C, C is the center of the Mohr circle, M is the point okay at the I mean at the Mohr circle but it is touching with the failure Mohr-Coulomb failure envelope. So, that is point M. So, if you join these 2 points C and M basically it will give you the radius right? It will give you the radius of the Mohr circle. So, radius of the Mohr circle is nothing but OC into sin phi.

What is OC? OC is the distance between origin, origin O and center of the Mohr circle. So, several times we have seen this thing when we are discussing about the Mohr circle and the shear strength and all those things several times we have seen that. So, CM is equal to OC sin phi. Now what is CM? CM is the radius of the Mohr circle. Now how can I express that? Sigma 1 - sigma 3 by 2 agreed okay is equal to what is OC?

OC is the distance between the Mohr circle center and the origin. So, that can be expressed as sigma 1 + sigma 3 by 2 into sin phi okay. Now therefore I can write sigma 1 - sigma 3 is equal to sigma 1 + sigma 3 into sin phi okay. So, got it. Now we are trying to find out few points or the few references. So, now what is OB? OB is nothing but this the point this is the line and this is the point B.

Now what is B point? B is the point if you draw a normal okay from point C to the line OA 2 okay so you will get the point B. What is B? B is the point if you draw a normal from C on the line OA 2 you will be getting point B okay. So, OB is nothing but OC into cos beta agreed? Very simple from the geometry from the figure itself okay. So, what is OC? OC we have got sigma 1 + sigma 3 by 2. So, therefore I get sigma 1 + sigma 3 by 2 into cos beta.

Similarly, I am going to find out BC okay. So, BC is equal to OC sin beta okay. So, again that will be equal to  $\frac{\sigma_1 + \sigma_3}{2} \sin \beta$  okay. Now A<sub>1</sub>B what is point A<sub>1</sub>? A<sub>1</sub> is the point on the Mohr circle where OA<sub>2</sub> line is cutting the Mohr circle or intersecting the Mohr circle, so A<sub>1</sub>. So, I am getting 2 points A<sub>1</sub> and A<sub>2</sub> already we have established. Based on that we have drawn the Mohr circle.

So, A<sub>1</sub>B is equal to A<sub>2</sub>B from the geometry or from the property of your triangle and the circle okay. So, A<sub>1</sub>B must be equal to A<sub>2</sub>B is equal to what?  $A_1C = \sqrt{A_1C^2 - BC^2}$  okay. So, from this what is A<sub>1</sub>C? A<sub>1</sub>C is nothing but the radius of the Mohr circle which is nothing but equal to CM so that I am getting here okay and BC what is BC? BC just now we have calculated  $\frac{\sigma_1 + \sigma_3}{2} \sin \beta$ .

So, that we are putting here okay. Now what is this  $\sigma_1 - \frac{\sigma_1 + \sigma_3}{2}$ . That is nothing but  $\frac{\sigma_1 + \sigma_3}{2} \sin^2 \phi$  okay. So, A<sub>1</sub>B is equal to A<sub>2</sub>B therefore  $\sigma_v$  is equal to  $\frac{\sigma_1 + \sigma_3}{2} \cos^2 \phi - \frac{\sigma_1 + \sigma_3}{2} \sin^2 \beta$ . Now what is  $\sigma_v$ ? If you look at this figure  $\sigma_v$  is nothing but OB plus BA<sub>2</sub> because this is your  $\sigma_n$  because the A<sub>2</sub> the coordinate of A<sub>2</sub> is  $\sigma_n$  and tau that is normal stress and shear stress. So,  $\sigma_v$  is not the normal stress.

Please try to understand.  $\sigma_v$  is not the normal stress and  $\sigma_L$  is not also  $\sigma_L$  is not also the normal stress right. So, if you want to find out  $\sigma_v$   $\sigma_v$  is nothing but the component of  $\sigma_n$  in the vertical direction okay. So, that is nothing but OB from the graphical I mean representation OB + BA<sub>2</sub>. So, OB + BA<sub>2</sub> will give you  $\sigma_v$  yes or not, yes right. So, that is nothing but we can get  $\frac{\sigma_1 + \sigma_3}{2} \cos \beta + \frac{\sigma_1 + \sigma_3}{2} \sin^2 \phi$  that is nothing but BA<sub>2</sub> already we have got it here okay.

That we have got,  $\sigma_v$  we have got. Now  $\sigma_v$   $\sigma_v$  is not the matter of concern for us. We are going to find out the  $\sigma_L$  that is the lateral pressure yes so  $\sigma_L$  is nothing but the active earth pressure that is what OB - A<sub>1</sub>B because see I mean you can understand.  $\sigma_v$  is acting vertically and  $\sigma_L$  is acting I mean parallel to the sloping surface right? So,  $\sigma_L$  will be OA<sub>1</sub> basically.  $\sigma_A$  will be OA<sub>1</sub>.

So, how we will get OA<sub>1</sub>? OB - A<sub>1</sub>B. So, OB already you have got it okay from the previous calculation and this is your A<sub>1</sub>B which is nothing but A<sub>2</sub>B okay. So, this is your lateral pressure. So,  $\sigma_L$  you have got.  $\sigma_v$  you have got. So, your KA will be  $\sigma_L - \sigma_v$

v. You have got lateral pressure, you have got vertical pressure. The ratio of these 2 will be your  $K_A$ . So, in the next slide we are going to see that.

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**Earth Pressure on Retaining wall**

**Backfill with sloping surface**

$$K_a = \frac{\sigma_1}{\sigma_v} = \frac{p_a}{\gamma z \cos \beta}$$

$$p_a = \cos \beta \times \frac{\cos \beta - \sqrt{\cos^2 \beta - \cos^2 \phi}}{\cos \beta + \sqrt{\cos^2 \beta - \cos^2 \phi}} \gamma z = K_A \gamma z$$

$K_A$

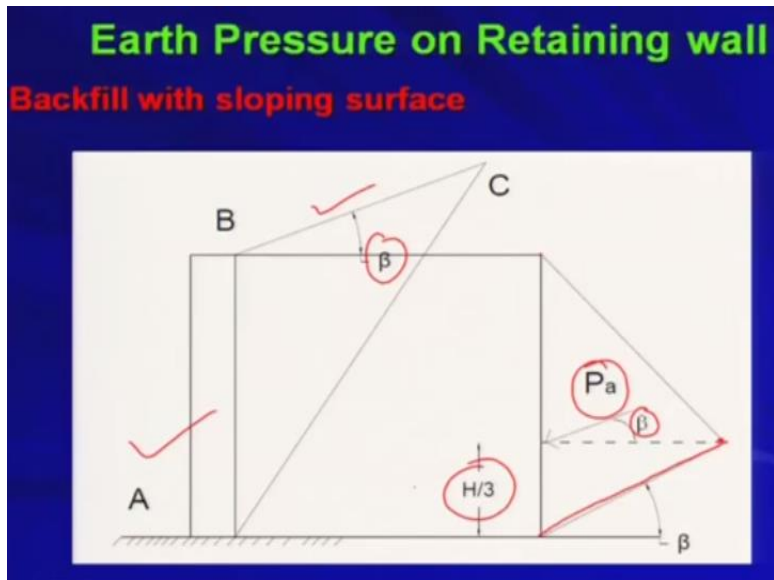
$$\text{Similarly, } K_p = \cos \beta \times \frac{\cos \beta + \sqrt{\cos^2 \beta - \cos^2 \phi}}{\cos \beta - \sqrt{\cos^2 \beta - \cos^2 \phi}}$$

So,  $K_a$  is nothing but  $\sigma_1$  by  $\sigma_v$  and which is nothing but  $p_a$ ,  $p_a$  is the  $\sigma_1$  that is active earth pressure divided by already we have calculated do you remember already we have calculated  $\sigma_v$  which is equal to  $\gamma z \cos \beta$  right. Now this is the thing we have got earlier. Now this is now  $\sigma_1$  and  $\sigma_v$  just now we have got the expression for that from the Mohr circle so that if I put here so I will be getting one expression big expression like this  $p_a$  is equal to  $\cos \beta$  into this part into  $\gamma z$ .

So, that we can write as  $K_A$  into  $\gamma z$ . So, therefore this whole part is your  $K_A$ . Now you put  $\beta$  equal to 0 here you will be getting simply  $1 - \sin \phi$  by  $1 + \sin \phi$  okay. So, when  $\beta$  is there right at that time your active earth pressure coefficient will be taking the form like this. So, you have to calculate the active earth pressure coefficient from this expression and then it will be remaining same.

So,  $p_a$  will be equal to  $K_A$  into  $\gamma z$ . That  $K_A$  automatically will take care of the inclination effect from this expression okay. So, similarly I am not going in much detail for the passive thrust. In a similar fashion, you can find out the passive earth pressure coefficient in the sloping backfill okay. So, that will be equal to  $\cos \beta$  into this part. So, that will give you the passive earth pressure coefficient for inclined backfill okay.

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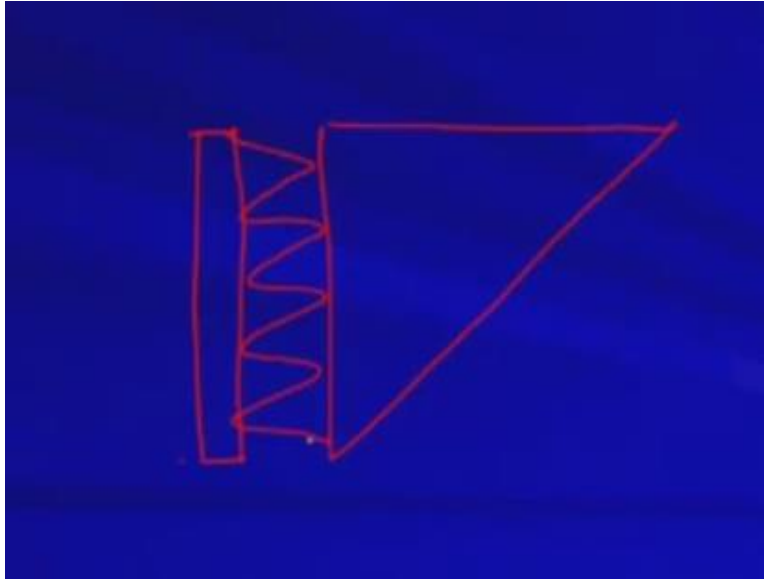


So, now you can see how the pressure distribution is working so this is the inclined backfill. So, BC is your inclined backfill which is inclined at an angle  $\beta$ . So, AB is the wall okay. So, now the triangular distribution is this. It is 0 at the top and at  $z$  equal to  $H$  this is the pressure okay inclined pressure so inclined total thrust is  $P_a$  which is inclined at an angle  $\beta$  with the horizontal and the point of application is  $H/3$ .

Please remember  $P_a$  is acting with an inclination of  $\beta$  with the horizontal okay. So, now basically so far what we have seen whatever active earth pressure coefficient or passive earth pressure coefficient we have calculated so those are based on Rankine's analysis. So, Rankine did not consider the friction of the wall. That means he considered wall is very smooth and there is no friction in between wall and soil right.

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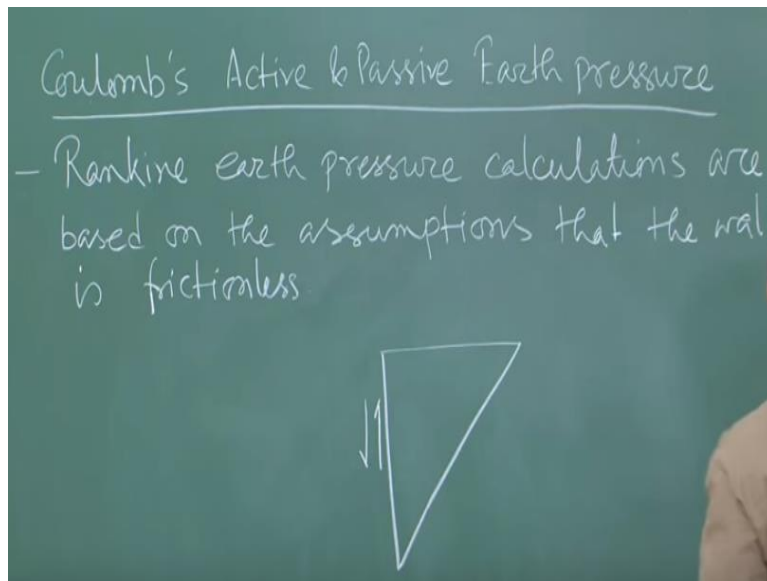
So, that means say suppose if this is my backfill okay and if this is my wall. So, if I want to separate it out so this is wall say. So, I do not have any interface friction okay at the backfill and wall interface. So, that smooth condition or the frictionless condition was considered by Rankine. Now it is not always true. Suppose you are using concrete wall which is more common in real life right.

So, I mean smooth wall generally you will be seeing that if you use stainless steel wall or something like that which will be having smooth surface so there is no friction I mean you can consider there is no friction at the interface. But if you use a rubble masonry wall or brick wall or even concrete wall you will be getting enormous amount of or I mean significant amount of friction at the interface right.

So, that friction must be taken care when you are calculating the active and passive earth pressure coefficient and that was proposed by Coulomb. So, we will see now that Coulomb's I mean method for calculating active and passive earth pressure coefficient which will consider the wall soil interface friction okay.

Now we will go to the board and we will see that how we can write down or we can we will give the I mean complete say expression for active and passive earth pressure coefficient by Coulomb. So, as I told you that Coulomb's mechanism that Coulomb generally considered the wall soil interface friction. So, so far you have seen in the Rankine's method that you do not consider the interface friction that means you considered the wall is completely frictionless that is smooth wall right.

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But Coulomb's proposed so Coulomb's active and passive earth pressure we are going to see that okay so how we will be getting different expression for active and passive earth pressure coefficient? So, now what is the basic difference between Coulomb and Rankine. So, Rankine earth pressure calculations are based on the assumptions that the wall is frictionless. This is the basic difference between Rankine and Coulomb's mechanism okay.

Rankine did not consider any friction at the interface of the wall and soil. So, Rankine earth pressure calculations are based on the assumption that the wall is completely frictionless okay but which is not the case in the real life. You will be getting some interface friction. That means if soil so when in case of active case active case the soil wedge is coming down it will try to come down right along with the wall and you will be getting some resistance at the wall soil interface. So, suppose if this is the wall. This is the backfill.

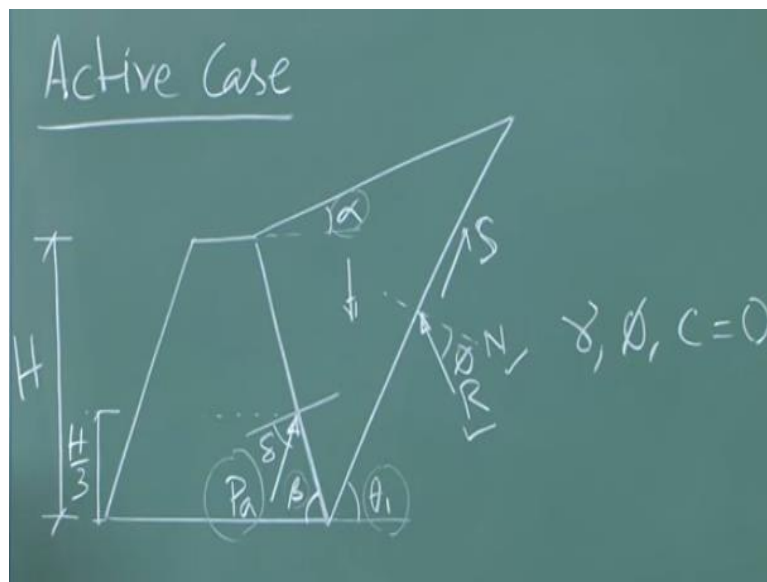
So, if I say this is the backfill. In case of active situation, it will try to move in the downward direction am I right? So, you will be getting some resistance in the upward direction right and this resistance is basically based on the interface friction okay. So, if soil wall interface friction is not there then this kind of resistance will not be developed okay. So, in case of passive so this block is will try to move in the upward direction so you will be getting the resistance in this direction right.

So, this resistance is very significant and this resistance will be there if you consider the I mean friction I mean interface friction if it is there then of course this resistance will be developed and

to calculate this resistance you need to know that how much interface friction is there. So, whatever maybe the case if interface friction is there so I mean you cannot use Rankine earth pressure theory.

So, whatever you have seen  $K_A$  value or  $K_P$  value whatever you have calculated so far for cohesionless soil or cohesive soil those things will not be valid if you consider wall is not frictionless okay. If wall is frictionless it is okay. If wall is not frictionless you have to use Coulomb's earth pressure theory okay. So, Coulomb's theory takes care of the friction soil wall interface friction okay. That is the basic difference between Coulomb's and Rankine's theory okay.

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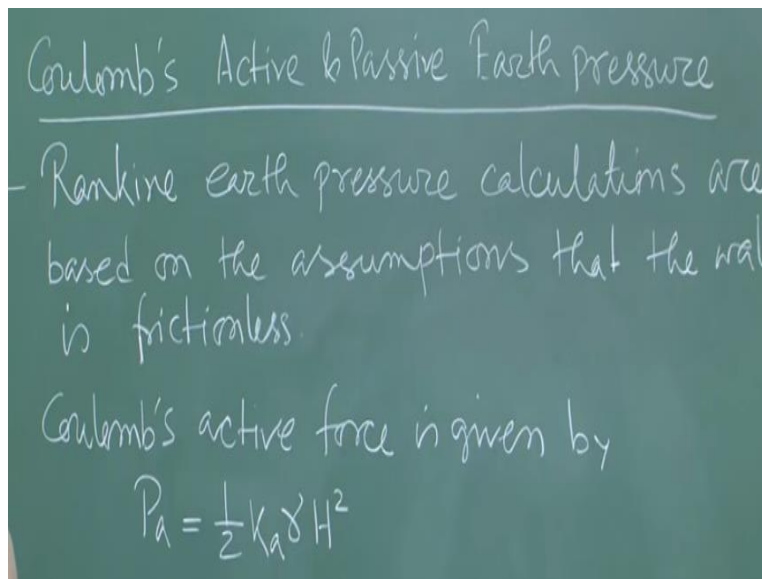


Now in case of active case let me draw it. So, I am showing the figure for most general situation okay where you have the wall inclination that is defined by say beta. You have the inclined backfill that is defined by alpha okay and this is your say failure wedge okay in the backfill side and which will be experiencing some because this is the active state. So, this block this wedge will try to move in the downward direction so you will be getting the development of the shear resistance S okay along this surface in the upward direction and this is the normal force and this is the resultant force R.

So, this will be developed based on the shear resistance mobilized at the failure surface okay fine. So, theta 1 is the angle of inclination of the failure surface with the horizontal okay. So, now you need to calculate or need to find out  $P_A$  so I am not going in much detail that how you

will find out all those things so this is based on the limit equilibrium method if you consider the force equilibrium as well as the moment equilibrium of the system then you can find out the magnitude of  $P_a$ . So, I will be giving you the direct expression for  $P_a$  rather going for the detailed calculation of  $P_a$  okay. So, and this  $W$  is the weight of the wedge. So, if you consider all the forces and if you consider the force equilibrium as well as the moment equilibrium then you can find out the value  $P_a$  okay.

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So,  $P_a$  will be given by so Coulomb's active force is given by  $P_a$  that is the active thrust  $P_a$  is equal to half into  $K_a$  into  $\gamma H^2$ . This expression is similar. Only thing is that that the difference will be happening in the expression of  $K_a$ . So, instead of  $1 - \sin \phi$  by  $1 + \sin \phi$  you will be getting something else. Or maybe if you are considering the inclination of the wall whatever you considered earlier in case of Rankine already we have derived that right from the Mohr circle I hope that you remember. So, that will not be valid for this Coulomb's analysis okay. So, this  $K_a$  value will be the Coulomb's active earth pressure coefficient.

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$$K_a = \frac{\sin^2(\beta + \phi)}{\sin^2\beta \sin(\beta - \delta) \left[ 1 + \frac{\sin(\beta + \delta) \sin(\phi - \delta)}{\sin(\beta - \delta) \sin(\alpha + \beta)} \right]^2}$$

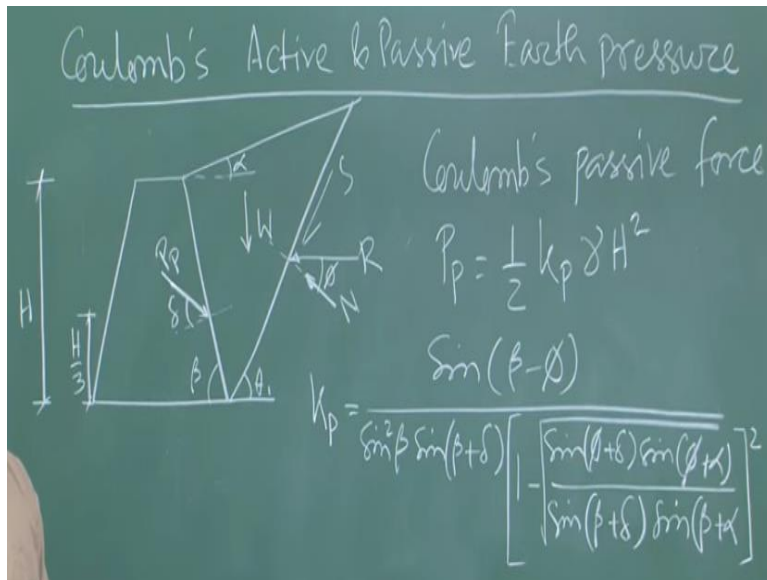
- In the actual design of R.Ws the value of  $\delta$  is assumed to be between  $\frac{\phi}{2}$  to  $\frac{2}{3}\phi$

So,  $K_a$  is your Coulomb's active earth pressure coefficient and is given by  $\sin^2(\beta + \phi)$  divided by  $\sin^2\beta \sin(\beta - \delta)$  into  $1 + \frac{\sin(\beta + \delta) \sin(\phi - \delta)}{\sin(\beta - \delta) \sin(\alpha + \beta)}$  whole square okay. Now everything is defined. What is  $\beta$ ?  $\beta$  is the wall inclination wall face that is the back face of the wall inclination.  $\alpha$  is the inclination of the backfill soil backfill slope okay and  $\delta$  what is  $\delta$ ?  $\delta$  is nothing but the angle of interface friction.

So,  $\delta$  could be two-third of  $\phi$ , one-third of  $\phi$ . Generally, it has been seen that it is two-third of  $\phi$  right. So,  $\delta$  if it is soil interface then of course it will be  $\phi$ . So, as it is the wall and soil interface so  $\delta$  will be less than  $\phi$  or maximum it can go up to  $\phi$ . So,  $\delta$  could be 0 and it could be maximum it could be  $\phi$ .

So, if  $\delta$  becomes 0 you will see this expression will be same whatever expression you have got just now from the Rankine's analysis okay but if  $\delta$  is not 0 that means if the interface friction is there that means the soil wall interface friction is not 0 then in that situation the active earth pressure coefficient must be calculated from the Coulomb's analysis and that will take the form like this okay. So, I can write down this in the actual design of retaining walls the value of  $\delta$  is assumed to be between  $\frac{\phi}{2}$  and two-third of  $\phi$  okay. Now in case of passive case what will happen?

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So, in case of passive case what will happen? Now let me draw it okay. In case of passive case this S will be going in the downward direction because the whole block will be trying to be lifted in the upward direction right. So, N is the normal force and R is the resultant force which will be acting at an angle phi with the normal force okay so that these forces will be developed along this failure surface.

So, these details I am not talking about this is kind of little bit more advanced details okay so any book if you want to find out how to find out all these forces you can refer any book on the advanced topic okay. W is the weight of the backfill wedge or the failure wedge okay. So, beta is the angle of inclination of the back face of the wall, alpha is the again same that is the angle of inclination of the sloping surface.

Now P p, P p is the active thrust now which will be making an angle delta because delta is the angle of internal friction right. So, which will be making an angle delta with the normal to the back face. Now you see because wall I mean the wedge failure wedge will be trying to be lifted up will be moving in the upward direction. So, P p will be acting in the in this direction whereas P a was acting in this direction so of you look at this okay.

So, this is the basic difference between the direction of P a and P p. So, if in this situation so Coulomb's passive force P p therefore is equal to half K p into gamma H square where K p is nothing but Coulomb's passive earth pressure coefficient and that is given by  $\frac{\sin^2 \beta \sin(\beta + \delta)}{1 - \frac{\sin(\frac{1}{2}(\beta + \delta)) \sin(\frac{1}{2}(\phi + \alpha))}{\sin(\frac{1}{2}(\beta + \delta)) \sin(\frac{1}{2}(\phi + \alpha))}}$  whole square okay.

So, this is the expression. Now you put delta equal to 0 you will be getting the same expression whatever we have got for passive case from the Rankine's analysis okay. So, anyway so this is all about your earth pressure theory on the retaining structure. So, I hope that you have enjoyed this part this topic because this is very important so I am not going to the design part of the retaining wall because they are actually by using this earth pressure coefficient you can calculate the pressures and based on that you can find out the stability of the retaining wall stability means external stability as well as internal stability. So, external stability there are several external stability sliding or overturning or bearing capacity all those things.

So, those things are purely on the design aspect of the retaining wall so which are beyond the scope of this course. So, I will restrict myself at this point where we have come across the earth pressure okay developed by the soil on the retaining wall. So, this much is sufficient to know okay to understand the earth pressure concept okay.

So, I will stop here today. So, in the next class we will talk about the stress distribution. That means if you have any kind of foundation or any kind of say structure okay beyond that part what will be the distribution or how much stress will be coming so that will be we will be seeing next and then of course the numerical problems will be solved for earth pressure as well as stress distribution together at the end. So, thank you very much.