Geology and Soil Mechanics
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Lecture - 65
Problems on Earth Pressure Theories - 3

Welcome back. So in the last lecture we have solved a few problems numerical problems rather on earth pressure theory so today we will be taking a couple of numerical problems on earth pressure theory and then we will be moving to the numerical problems for stress distribution in soil.

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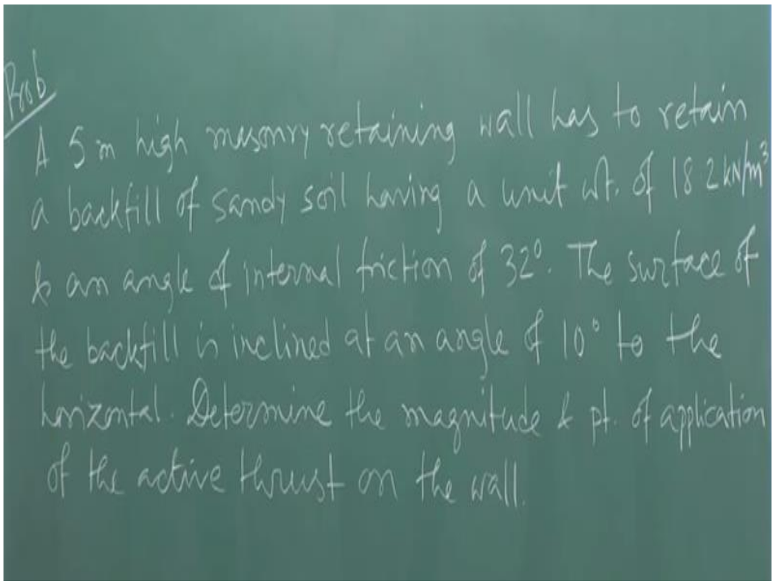

So, first problem we will take today the problem says a 5 m high masonry retaining structure wall has to retain a backfill of sandy soil having a unit weight of 18.2 kN/m cube and an angle of internal friction of 32 degree. The surface of the backfill is inclined at an angle of 10 degree to the horizontal. Determine the magnitude and point of application of the active thrust on the wall okay. So, this is the problem.

The problem says that a 5 m high masonry retaining structure wall has to retain a backfill of sandy soil having a unit weight of 18.2 kN/m cube and an angle of internal friction of 32 degree. The surface of the backfill that is the top surface of the backfill is inclined at an angle of 10 degree to the horizontal. Determine the magnitude and point of application of the active thrust on the wall. So, this is a kind of inclined backfill problem okay so and you know so solution.

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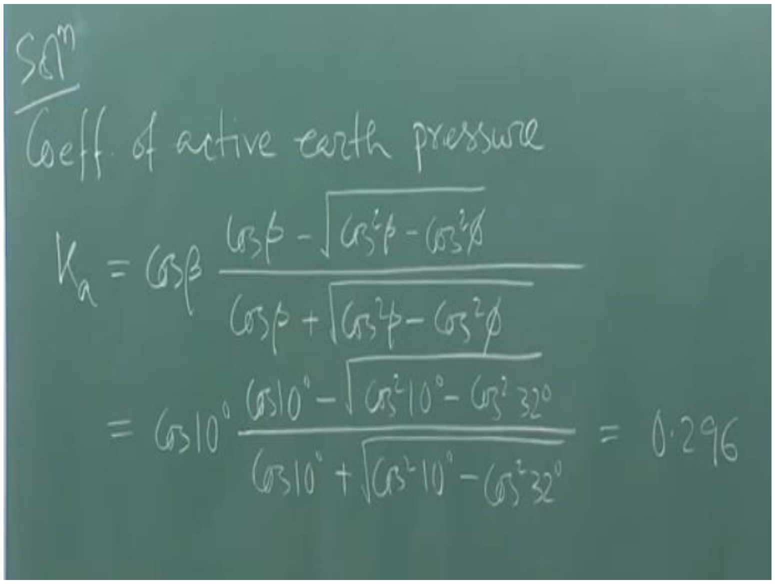

So first we will try to find out coefficient of active earth pressure say K a that is nothing but you know already we have seen this formulation in the lecture cos beta into cos beta - cos square beta - cos square phi divide by cos beta + cos square beta - cos square phi where beta is the angle of inclination of the backfill and phi is the angle of internal friction of soil right. So, in this formula if we put instead of cos beta I can put cos 10 degree into cos 10 degree - cos square 10 degree - cos square 32 degree divided by cos 10 degree + cos square 10 degree - cos square 32 degree. So that gives me 0.296.

That is the only tricky part of this problem otherwise this problem is pretty straightforward. So, you can find out the distribution of the pressure active earth pressure and from there you can find out the active thrust anyway. So, this is your active earth pressure coefficient for this inclined backfill. Now we are going to find out the resultant thrust as asked in the problem.

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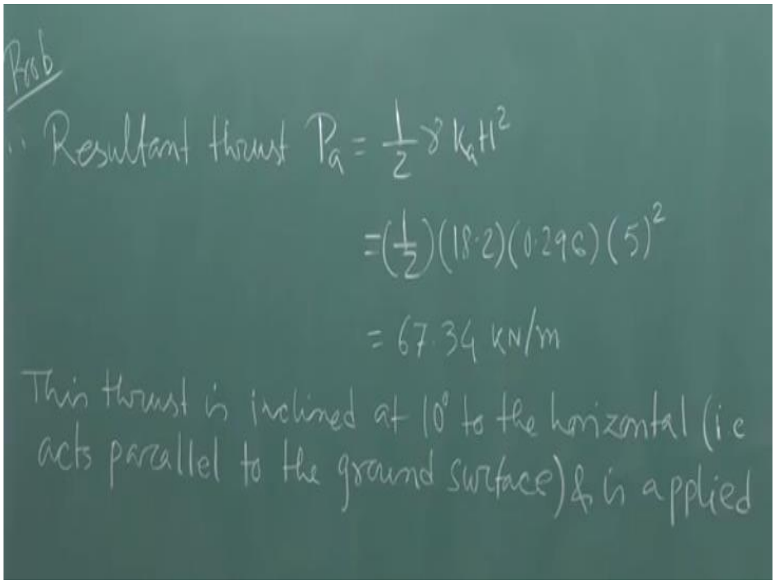

So, the resultant thrust therefore the resultant thrust P a is equal to half into gamma into K a into H square right. So, the formula will be remaining same. Only thing is that K a expression will be different right. So here if you put the values half gamma is 18.2 K a is 0.296 and H is the total height of the retaining wall is 5 m. So, this gives me 67.34 kN/m and this thrust is inclined this thrust is inclined at 10 degree to the horizontal. That is acts parallel to the ground surface and is applied at a height of and is applied at a height of what 5 by 3 right so that will be the triangular distribution again.

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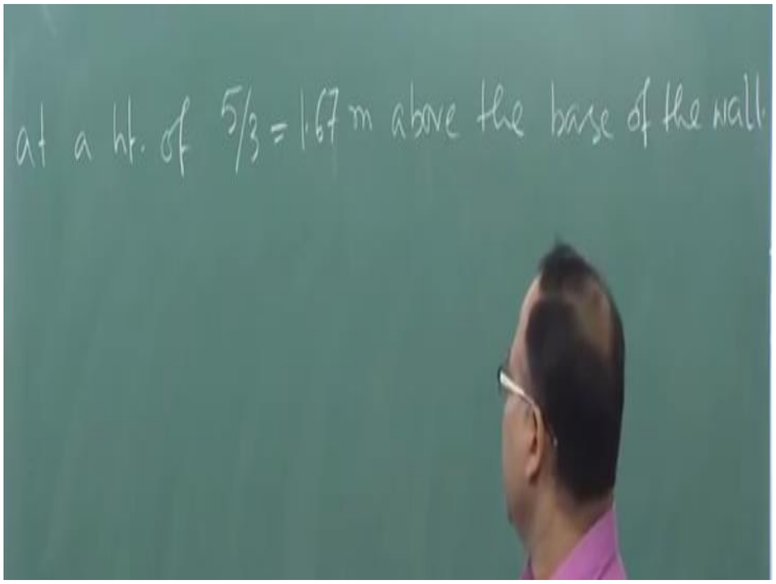

So, 5 by 3 that is equal to 1.67 m above the base of the wall. So, this is your whole problem okay. So, you have got the resultant thrust equal to 67.34 which will be inclined right which will be inclined at an angle 10 degree with the horizontal that means parallel to the inclined backface okay already we have seen this thing when we talked about this inclined backface problem right. So, it will be inclined at an angle 10 degree with the horizontal and is applied at a height of 5 by 3 because this is a triangle this will be the triangular distribution so 1.67 m above the base of the wall. So, we will take the next problem.

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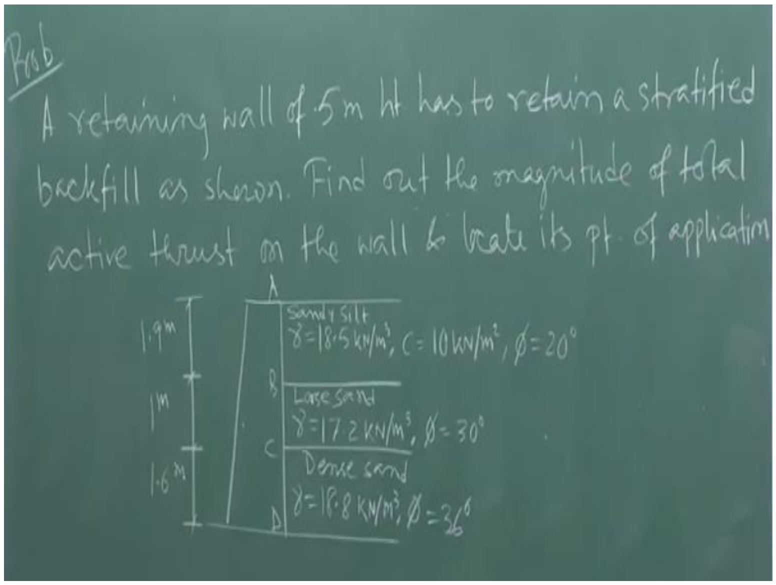

So, next problem says a retaining wall of 5 m height has to retain a stratified backfill as shown. Find out the magnitude of total magnitude of total active thrust on the wall and locate its point of application okay. So, let me draw the wall. So, this is point A, B, C, and D. So, you have 3 different stratum okay. Depth of stratum is 1.9 m. Then second layer is 1 m and third layer is 1.6 m and this is your sandy silt which will be having gamma equal to 18.5 kN/m cube.

C equal to 10 kN/m square and phi equal to 20 degree and second stratum is loose sand which has gamma equal to 17.2 kN/m cube and phi equal to 30 degree and third is your dense sand which has gamma equal to 18.8 kN/m cube and phi equal to 36 degree okay. So, this is the problem. The problem says a retaining wall of 5 m height has to retain a stratified backfill as shown.

Find out the magnitude of total active thrust on the wall and locate its point of application. Now if you see this problem this problem is a little bit different than the problems whatever we solved in the earlier classes. Now here actually the top layer is having the cohesion. Instead of I mean earlier whatever problem we solved we solved the problems where the all the layers will be having cohesionless soil.

Now you have cohesion in the top layer. So, you have to consider that thing. That means you may have the possibility of getting tension cracks at the top layer right as well as you will be getting some tension at the top of the wall. So that thing you have to you need to consider in the pressure diagram and in the active thrust I mean total active thrust calculation okay. So, let us let us see how we can solve this.

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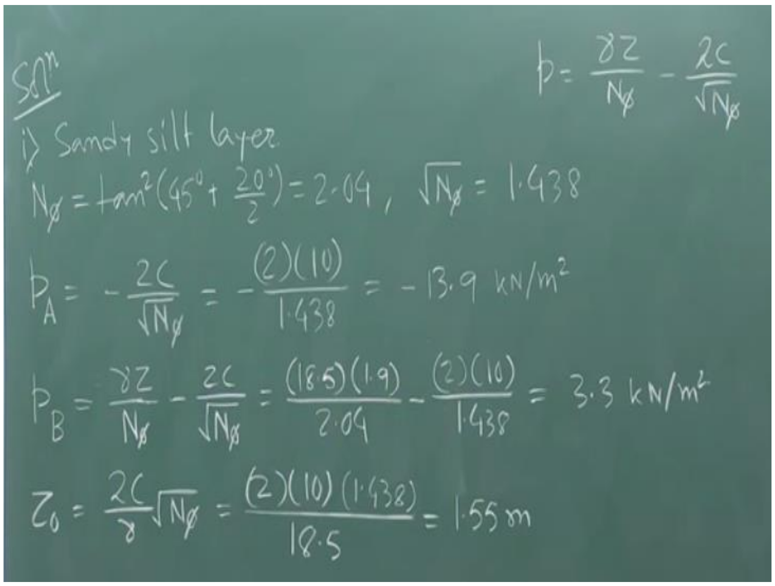

So, solution. Now first one is sandy silt layer okay. So, we calculate N phi because we need to know N phi and root over N phi that is there in the equation that you know that. So, tan square 45 degree + 20 by 2 degree which gives me 2.04 and subsequently root over N phi is 1.438. So, this things we can calculate beforehand before going to the actual analysis okay. So therefore, p A that means active earth pressure at point A okay that means at the top of the wall.

So, what is the magnitude of p A? p A is I mean how do you know that? So, you know that gamma z by N phi - 2c by root over N phi that is the actual expression right. So, in case of cohesionless soil p is gamma z by N phi - 2c by root over N phi. So, this is the expression right. Already you have seen this equation several times in the numerical problems whatever we have solved in the last classes okay.

So, p A in this equation when we are considering point A at that time z equal to 0 that means at the top of the wall. So, it will be simply 2c by root over N phi. If you put the values - 2 into c is

10 kN/m square divide by 1.438 is equal to - 13.9 kN/m square okay. Then this top layer is spanning from point A to point B so I will get p B that is active earth pressure at point B when you are considering stratum 1 that means at the interface right. So, p B is equal to gamma z by N phi - 2c root over N phi which is equal to 18.5 into what is z? z is 1.9 m 1.9 divide by N phi is 2.04 - 2 into 10 divided by 1.438.

So that gives me 3.3 kN/m square okay. So, we have got the active earth pressure at point A and active earth pressure at point B as long as we are considering stratum 1. Now because you have the you have the cohesion in the top layer so you may expect the tension crack. So that zone of tension crack is nothing but that will be defined by the depth z 0 right. So, we need to calculate z 0. So, z 0 will be equal to 2c by gamma into root over N phi which is equal to 2 into 10 into 1.438 divided by 18.5. It gives me 1.55 m. So, we can draw the active earth pressure distribution plot. Let us see how it will look like.

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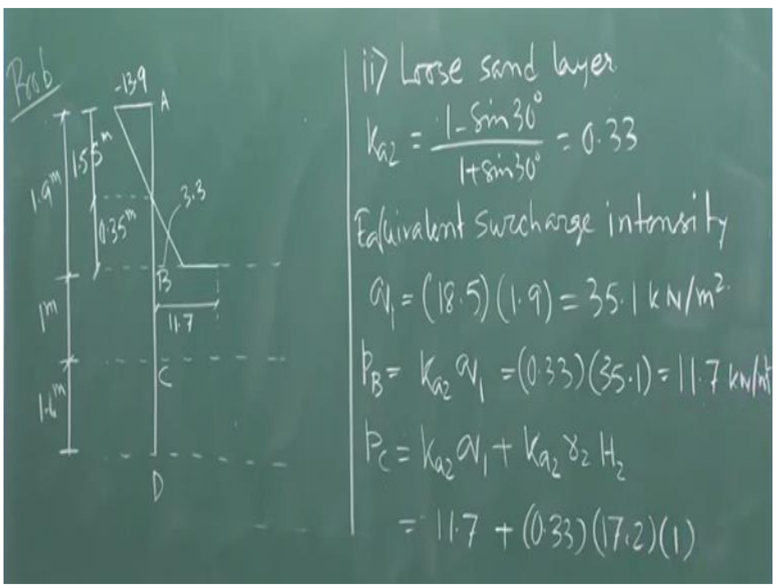

So, suppose this is your whole wall okay this is your point A say this is your point B this is your point C and this is your say point D okay. So as given it is 1.9 m, it is 1 m, it is 1.6 m okay. So, at point A what active pressure you have got? - 13.9. So, it will be on this side - 13.9. I am not writing kilo newton per meter square everywhere. So, it will be quite understood okay. Then where it is crossing over where or pressure is becoming 0.

The active earth pressure is coming from negative side to positive side. Where it is becoming 0 at z 0 right. So, z 0 is the distance or the depth at which your active earth pressure is becoming simply 0. So that is say this point say. This is your 1.55 m and this is your 0.35 m okay. So, you are getting the distribution like this okay where this is your 3.3 kN/m square. That is active earth pressure at point B.

Now we will consider stratum 2 that is your loose sand and we will try to find out the pressure distribution. So, second one is loose sand layer okay. So there basically your K a 2 can be defined as 1 - sin 30 degree by 1 + sin 30 degree which comes as 0.33 okay, equivalent surcharge intensity because now your stratum 1 will be acting as equivalent surcharge right uniform surcharge on top of layer 2 right.

So, surcharge intensity say q 1 is equal to 18.5 that is your gamma of the top layer multiplied by the depth of top layer that is 1.9 it comes as 35.1 kN/m square okay. So therefore, p B that means active earth pressure at B as far as you are considering layer 2 that is starting from point B to point C. So, when you are in stratum 2 at that time what is active the earth pressure at point B and that will give you the discontinuity that is the jump right.

P B is equal to simply K a 2 into q 1 right so that gives me 0.33 into 35.1 is equal to 11.7. So, you will be getting some increase and that is your 11.7. That is the jump right at point B okay. So similarly, I can calculate p C that means active earth pressure at point C which is the interface which is eventually or evidently the interface between stratum 2 and stratum 3 right? So, p C is equal to K a 2 q 1 + K a 2 gamma 2 H 2 which gives me K a 2 q 1 is nothing but 11.7 + K a 2 is 0.33 gamma 2 is 17.2 and what is H 2 that is the depth of second layer is simply 1 m.

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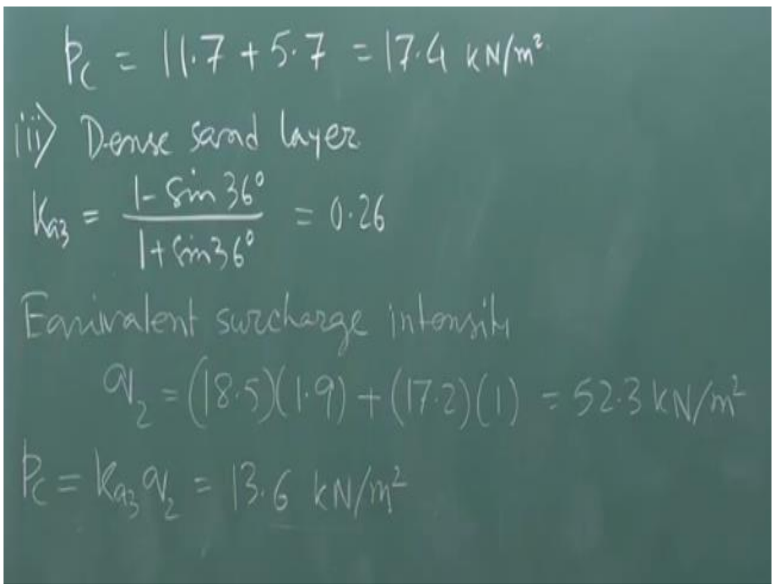

So that gives me that gives me p c is equal to 11.7 + 5.7 is equal to 17.4 kN/m square okay. So therefore, we can complete here. Now this point we know. So, from there so this is your 5.7. This is your 5.7 okay. So, I have got the pressure up to point C. That means we have covered stratum 1 and stratum 2. Now we are going to stratum 3. So, stratum 3 is nothing but dense sand layer okay. So there your K a 3 is equal to 1 - sin 36 degree by 1 + sin 36 degree that comes as 0.26.

So, equivalent surcharge intensity q 2 is equal to what? 18.5 into 1.9 that is for top layer plus for second layer 17.2 into 1 that gives me 52.3 kN/m square okay. So, I have got the equivalent surcharge intensity. Now when basically your top 2 layers first layer and second layer will be acting as surcharge on top of third layer okay. So therefore, I can calculate p c. How I can calculate p c? p c is simply K a 3 into q 2. So, K a 3 you know 0.26 and q 2 you know 52.3. So, if you put those values you will be getting 13.6 kN/m square okay.

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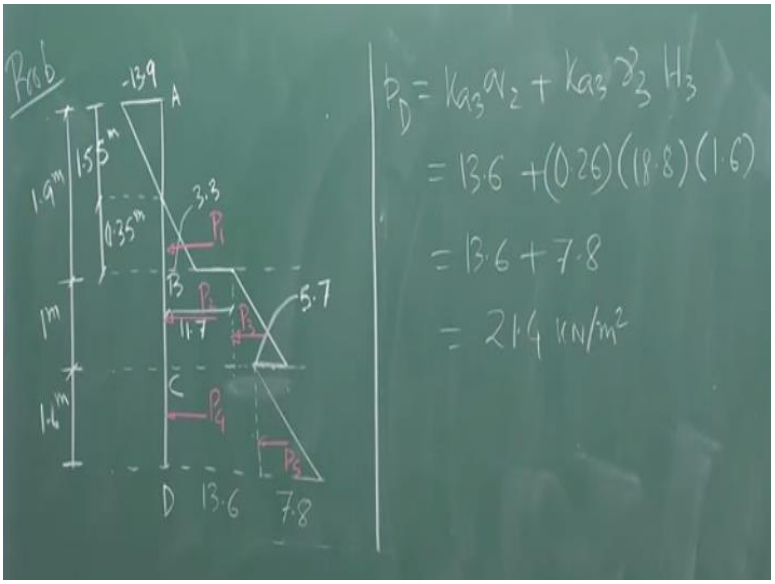

So similarly, I can find out p D that is the active earth pressure at point D that is the bottommost point of the wall. P D is equal to K a 3 q 2 + K a 3 gamma 3 into H 3. So that is nothing but 13.6 + K a 3 is 0.26 gamma 3 is 18.8 and H 3 is 1.6. That gives me 13.6 + 7.8 and total is 21.4 kN/m square. So therefore, I can complete the figure now. So, this is 5.7. Now if you see p C from stratum 3 you are getting p C equal to 13.6.

So that means 11.7 + 5.7 that was equal to 17.4 from here to here it was 17.4 but when you are considering stratum 3 at that time you have to come back to 13.6. So basically, you are coming back this is your 13.6 and then so this is your 13.6 from this point to this point and this point is

7.8 okay. So, this is your pressure distribution. This is the active pressure distribution on the wall. So now what are the different forces because tension crack is happening so we will be neglecting the tension zone.

So, we are not considering the tension zone. So, we will be considering from the point 0 to the positive pressure right. So, P 1 is acting here okay. P 2 is here which is nothing but this the area under this rectangle. Then P 3 area under this triangle again. P 4 area under this rectangle and P 5 area under this triangle. Now we are going to find out all these forces. So how we can find out all these forces? So, these forces will be nothing but the area under the respective area curves okay.

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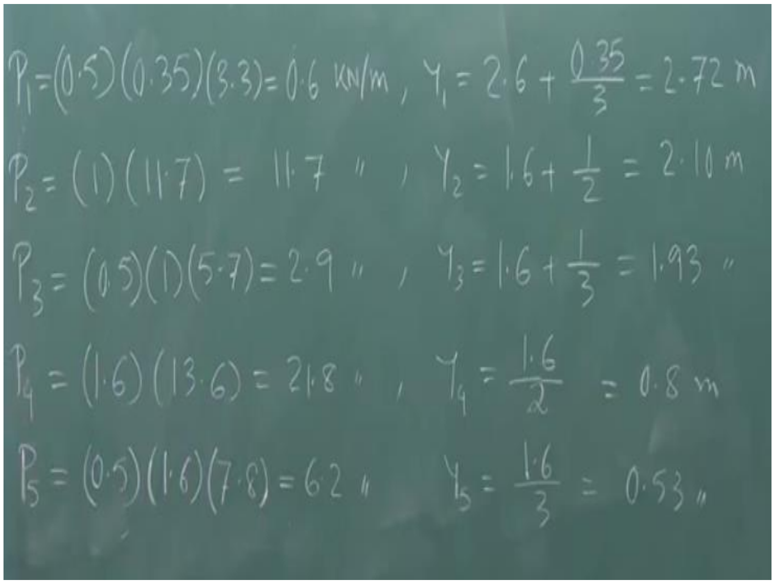

So, suppose I want to calculate P 1. How I will calculate P 1? P 1 is equal to area of this small triangle right. So, P 1 is nothing but 0.5 into 0.35 that is the height multiplied by 3.3 which comes as 0.6 kN/m run of the wall and it acts at a point which is Y 1 distance above the base of the wall. So, Y 1 is equal to 2.6 + 0.35 by 3 is equal to 2.72 m okay Similarly you can calculate P 2. P 2 is equal to 1 into 11.7. So that is this is the area.

So, 11.7 is this and 1 is the depth. So, 1 into 11.7 that comes as okay. Similarly, you can calculate P 2. P 2 is equal to 1 into 11.7. So that is this is the area. So, 11.7 is then and 1 is the depth. So, 1 into 11.7 that comes as 11.7 kN/m run and Y 2 is equal to 1.6 + 1 by 2 okay because that will be acting at the mid depth of this rectangle. So that comes as 2.10 m. Then P 3 is equal to 0.5 into 1 into 5.7 which is equal to 2.9 and Y 3 is equal to 1.6 by 1 by 3 is equal to 1.93 m.

Similarly, P 4 is equal to 1.6 that is the area under this triangle okay, not triangle this rectangle sorry this rectangle, so 1.6 into 13.6 that comes as 21.8 and Y 4 is equal to 1.6 by 2 that is nothing but 0.8 m and finally P 5 is equal to the area under this last triangle 0.5 into 1.6 into 7.8 is equal to 6.2 kN/m run and Y 5 equal to 1.6 by 3 is equal to 0.53 m. So, we have got all the forces and we have got all the locations or the point of application of each force right.

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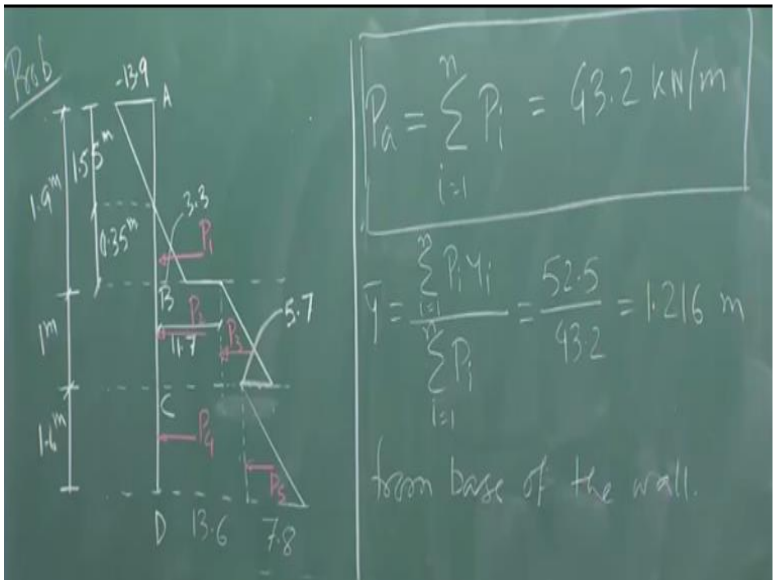

So, I can calculate the total thrust P a is nothing but summation of P i, i is equal to 1 to n which gives me 43.2 kN/m run P 1 + P 2 + P 3 + P 4 + P 5 okay and Y bar that is nothing but summation i is equal to 1 to n P i Y i divided by summation P i i is equal to 1 to n which gives me 52.5 by 43.2 that is nothing but 1.216 m from base of the wall okay. So, I hope that you have understood.

So, the main tricky thing is that to get the pressure distribution plot. So, once you get this pressure distribution plot the rest of the things will be pretty simple okay I mean pretty say mechanics oriented that means you will be calculating the forces calculating the center of gravity and then find out the total thrust and point of application of the total thrust okay. So now with this basically I will conclude the numerical problems for earth pressure theory. Now we will be solving or we will be taking few numerical problems on stress distribution in soil.

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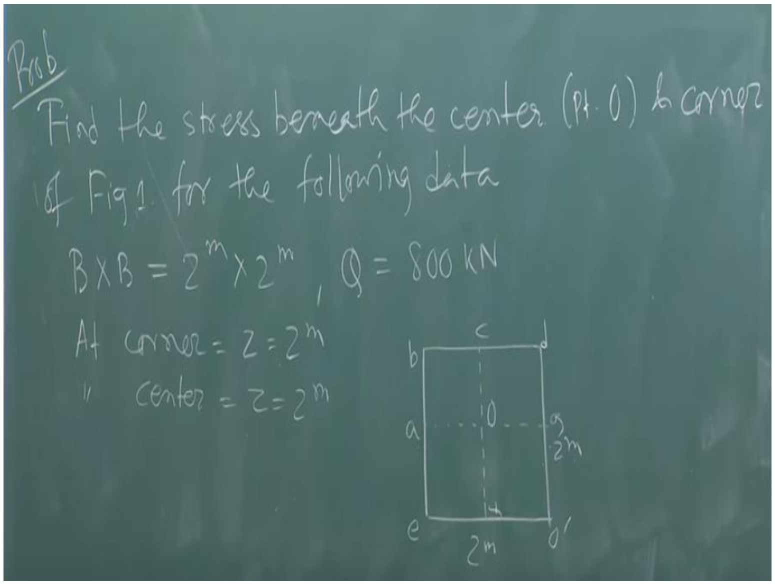

So, the problem on stress distribution. Find the stress beneath the center that is point O and corner of say figure 1 which I will be showing now for the following data. What are the following data? That is the area is B by B which is nothing but 2 m by 2 m and total load on that area capital Q is 800 kN okay and the depth at corner that is z equal to 2 m that means at depth z equal to 2 m you need to calculate the stress right and at center z equal to 2 m and the area looks like this.

This is your figure 1 square area a, b, c, d, e, o prime okay so this is the area. Now on this area this area is 2 m by 2 m okay. So, on this area on this square area basically your total load is acting as 800 kN that is the concentrate load say force okay. Now you need to find out the stress beneath the center, center means beneath the point O at which depth at depth z equal to 2 m.

So, at 2 m depth just below the center you need to calculate the stress by using the formula okay by using the concept whatever you have learnt so far and you need to find out the stress beneath the corner, beneath the corner means at any corner you consider beneath the corner also at which depth same depth the depth equal to 2 m okay.

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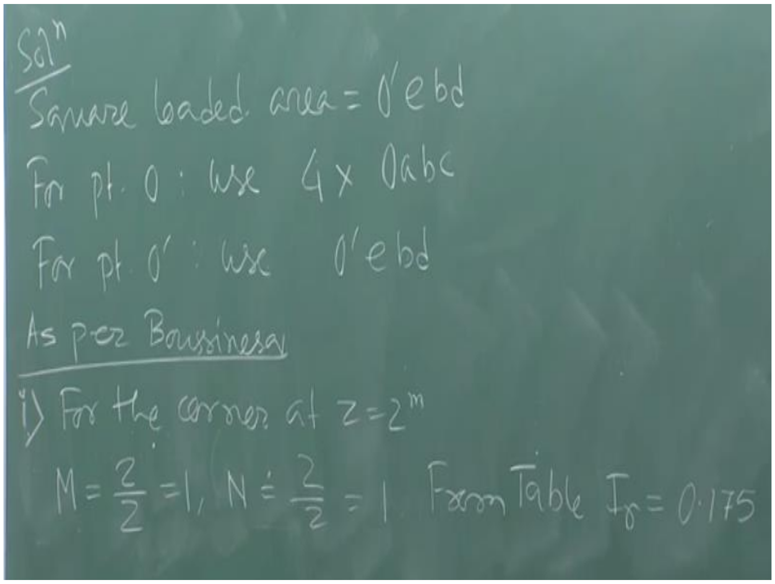

So, let us solve this problem. So, square loaded area is equal to O prime e b d for point O and we know if you recall whatever we have learnt in the lecture class basically any rectangular area you consider you have the formula that is sigma v equal to or the q v equal to q 0 into I sigma and this I sigma when you are calculating that will give you the stress at the corner of any rectangular area.

So, if you consider the mid-point or the center of this square basically this point should belong to some corner of the rectangle or square then only you will be able to use those that formula right. Similarly, this corner is fine. So, you need to find out the stress just below the corner of this square area so the whole square area if you consider this is the corner it is fine. So, you can use the formula directly.

But when we are talking about the center of the square at that time you will not be able to use the formula directly. So, you have to make the whole area or you have to discretize the whole area in such a way that O will be coming as one of the corner of that loaded area okay. So that we are making. So, total square loaded area is O prime e b d. That is the total square area. Now for point O now if I consider point O for point O basically what we can use 4 times of O a b c.

What is O? O a b c. So, we are discretizing the whole square area in 4 different square area and once we consider area O a b c so O is coming as the corner of this area. So, I can find out the stress coming just below point O because that point is now belong to the corner of this square area right. So now this corner this point will be having the participation of all 4 different squares right. So, what are the 4 different squares?

O a b c, O a e and if you say this one is your f so O a e f, O f O prime some point this and O c d so this point is say g so O c d g right. So, these 4 things and you can use the method of superposition so each part will be giving you some stress at this point and the 4 times of this area will be the stress at point O because point O is not the corner point in the actual problem in the actual area. So, we have made the point O in such a way that it is coming as the corner of one of the square small square right.

So, for point O use 4 times into O a b c. So, you consider area O a b c, you find out the stress just below point O. That you can find out by using the formula and 4 times of that will be the total stress at the point O right. Similarly, for point O prime, O prime is not a problem O prime is the corner point of the actual area so point O prime use O prime e b d okay. So as per Boussinesq first one is for the corner at z equal to 2 meter what is the magnitude of M? M is what? M is B by z so what is the value of B? Both B and L both are same in this problem right.

B equal to 2 m and l equal to 2 m because this is the square area. Now M is B by z so 2 by 2. B is 2 m and z the depth at which you are interested to find out the stress that is also 2 m. So that is giving me M equal to 1. What is the value of N? L by z right. So that is also 2 by 2. That is also 1 is it not? So, you have got M and N. So, once you know M and N from the table whatever I have given in the lecture so from that table you can find out I sigma. So, from table I sigma you can find out. So, you can check it. That comes as 0.175. So, if you are not getting I sigma directly you need to do the linear interpolation.

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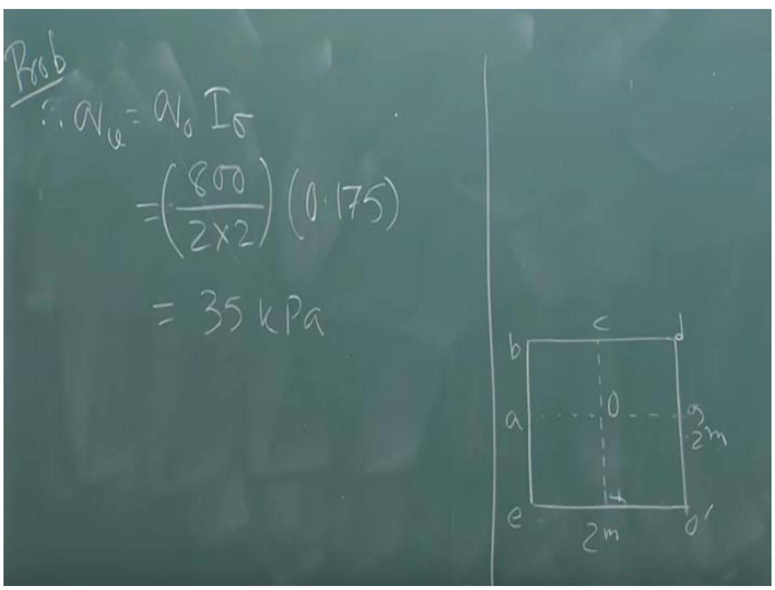

So, your q v your q v is equal to q 0 I sigma which is what is q 0? Total load is acting as 800 kN okay. On what area? On 2 m by 2 m square area. So, q 0 is nothing but 800 by 4, 2 into 2 okay into I sigma is 0.175 that comes as 35 kPa. So, what is that? So, 35 kPa is the stress just below point O prime. Just below the corner of this loaded area either below point O prime or below point e or below point b or below point d at some depth 2 m. So, this is the stress or the pressure happening due to the application of 800 kN load on this square area okay.

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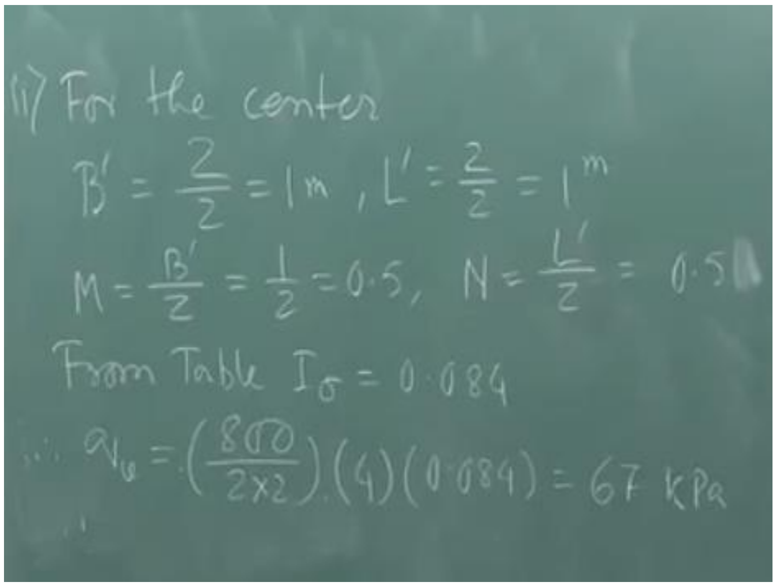

Now for the center what is happening? Now for the center your modified B prime that is modified B is 2 by 2 is 1 m. So, center when you are considering center at that time you are considering this small this small square right. At that time, what is the magnitude of B modified B that is nothing but B prime is half of that 2 by 2 is equal to 1 m. Similarly, L prime is also 2 by 2 is equal to 1 m. So therefore, your M is nothing but B prime by z, z is same 2 m. So, 1 by 2 that comes as 0.5. Similarly, N equal to L prime by z is equal to 0.5 m 0.5 okay. Now from table now go back to the table from table I sigma equal to 0.084 okay.

So, from this I can find out q v is equal to pressure will be remaining same into 4, 4 times you must multiply to get the total because this is the pressure if I do not multiply 4 then I will be getting the pressure due to this area only but I have all the 4 areas right so I have to multiply with 4. So, 4 into 0.084 that gives me 67 kPa. Significant increase of pressure. That means you are getting higher pressure at the center than the corner and which is quite obvious right. Now we will be taking the next example.

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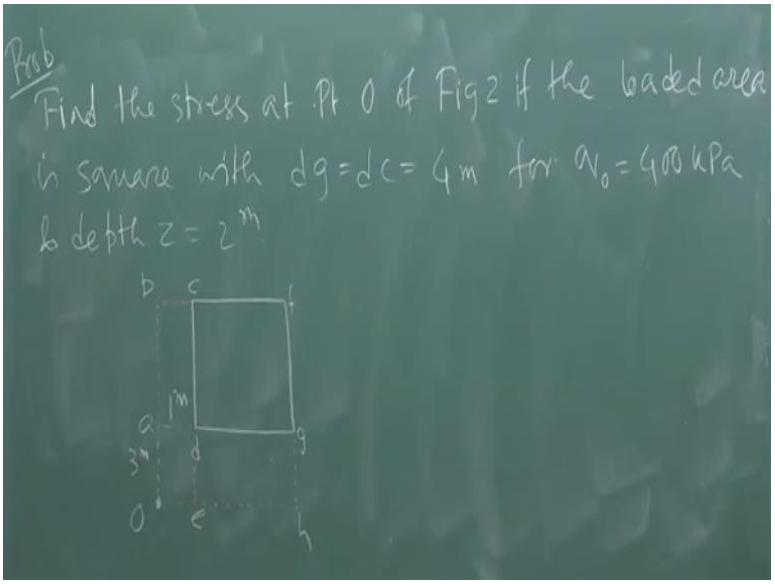

Next problem. Find the stress at point O of figure 2 say if the loaded area is square with dg equal to dc equal to 4 m for q 0 equal to 400 kPa and depth z equal to 2 m okay. Now let me show this area. This is the square area c d f g and the point is here this is your point O which is from d it is 1 m on left side and 3 m below. So, this point is outside the loaded area. That is the interesting thing okay.

This point is outside the loaded area. The loaded area is c d g f. This is your loaded area okay that is 4 m by 4 m and on which you are having the pressure 400 kilo Pascal okay and at depth 2 m you need to find out the stress at point O okay just below point O you consider some depth 2 m and you find out the stress due to the application of stress on this loaded area because this point is outside the loaded area.

Now how to do that okay. So how to do this? So, you must discretize this thing more I mean very intelligently okay. So, we will discretize this thing like this. This is the loaded area. So, I am extending, virtual extension okay some fictitious extension of this I am extending this, I am extending this, I am extending this. Now I am giving the names like this. This is b, this is a, this is e, this is h okay.

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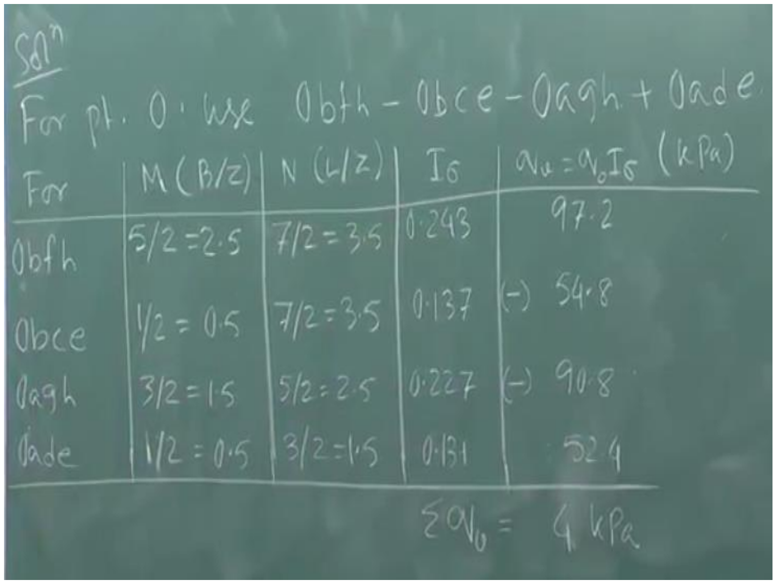

Sol^n

For pt. O: use Obfh − Obce − Oagh + Oade

For	M (B/z)	N (L/z)	I_σ	$q_v = q_0 I_\sigma$ (kPa)
Obfh	5/2 = 2.5	7/2 = 3.5	0.243	97.2
Obce	1/2 = 0.5	7/2 = 3.5	0.137	(−) 54.8
Oagh	3/2 = 1.5	5/2 = 2.5	0.227	(−) 90.8
Oade	1/2 = 0.5	3/2 = 1.5	0.131	52.4

$\Sigma q_v = 4$ kPa

Now for point O solution will be for point O, O has to be the corner right otherwise you cannot use this formula that is q v equal to q 0 into I sigma. That is only valid that will only give you the stress at the corner of the rectangular area right. So, O has to be corner of some rectangular area then only you can use that formula use you can take the help of that formula okay. So, for point O I can use Obfh. Please look at this figure Obfh that means the whole rectangle.

If I consider whole rectangle O belongs to the corner of that rectangular rectangle okay so I can use that formula minus Obce because actually that much loaded area is not there. What is the loaded area? Cdgf. So, I need to deduct the dotted part. So, what I am doing? I am doing Obce, Obce that I am deducting fine. So, minus Obce minus Oagh minus Oagh but whatever we are considering all the times O is coming as the corner.

Please try to understand, please try to see the beauty of this problem okay. So, O is always coming as the corner. Now when I am considered when I have considered Obce and when I have Oagh and when I deducted these 2 areas from Obfh basically I am deducting this part twice. So therefore, it has to be added at least 1 part should be added, Oade okay. So, if you do this operation then you will be getting the stress at point O okay.

So, let us do that. For that we will be using some table. That will be convenient for us. M which is nothing but B by z, N which is nothing but L by z. I sigma we will be getting that thing from the table directly and q v which is nothing but q 0 I sigma in kPa say okay. Now for Obfh, for Obfh what is the value of M. For Ob Obfh what is the value of B and what is the value of L? The value of b is 5 m so B by z will be 5 by 2 is equal to 2.5.

How it is coming 5? 4 this is your 4 m by 4 m, 4 + 1. So, 5 m is coming as B and what is L? 7 by 2, 4 + 3 right so that comes as 3.5. So, for that value of M, M equal to 0.5, N equal to 3.5 for that I will be getting I sigma equal to 0.243. So therefore, this comes as q 0 I sigma is 97.2. What is q 0? Q 0 is given that is 400 kPa already it is given in the problem. Then coming to Obce. For Obce what is the value of B. O Obce, what is the value of B?

That is the shortest dimension. That is 1 m, 1 by 2 is coming as 0.5 and what is the value of N? 7 by 2 that is the longest dimension 7 by 2 is your n value okay. So that gives me 0.137 as I sigma from the table and ultimately this will be subtracted it is coming as 54.8. Similarly, Oagh M will be 3 by 2 that is 1.5 and n will be 5 by 2 that is 2.5 just check it from the figure with the same logic. This is also subtractive 90.8 and finally Oade, Oade.

What is your B? 1 by 2 will be your M that is 0.5. What is the value of L? That is 3 by 2 is your N and I sigma will come as 0.131 and that is additive. So, summation of q v is equal to 4 kPa okay. So that means due to this loaded area at point O okay below the depth of 2 m you will be getting the stress as 4 kPa okay. So, in this way you can find out the stress if it is not even coming within the loaded area okay.

So, in that way you can find out the stress in anywhere. Only thing is that you have to discretize the thing in more intelligent way okay. So, I will stop here today. So, in the next lecture we will be talking about we will be taking more numerical problems on the stress distribution. Thank you very much.