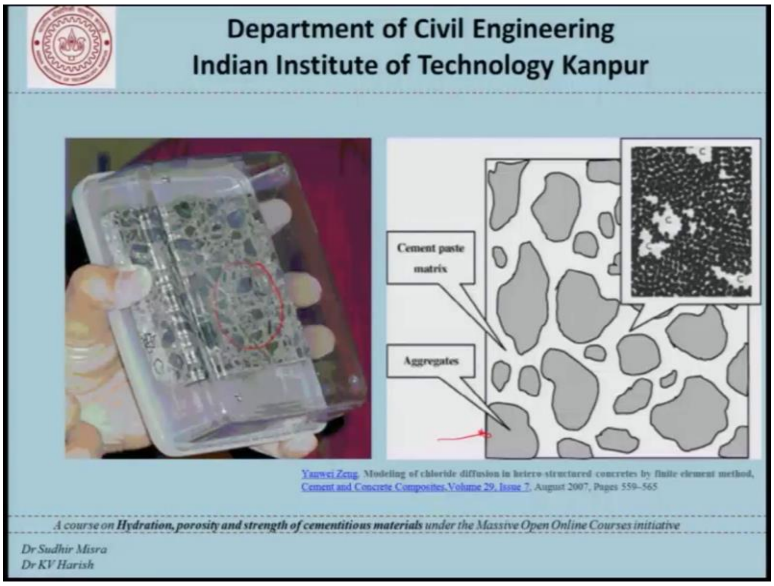
Hydration, Porosity and Strength of Cementitious Materials
Prof. Sudhir Mishra and Prof. K. V. Harish
Department of Civil Engineering
Indian Institute of Technology, Kanpur

Lecture - 37
Concrete
-Stress-Strain Behaviour-

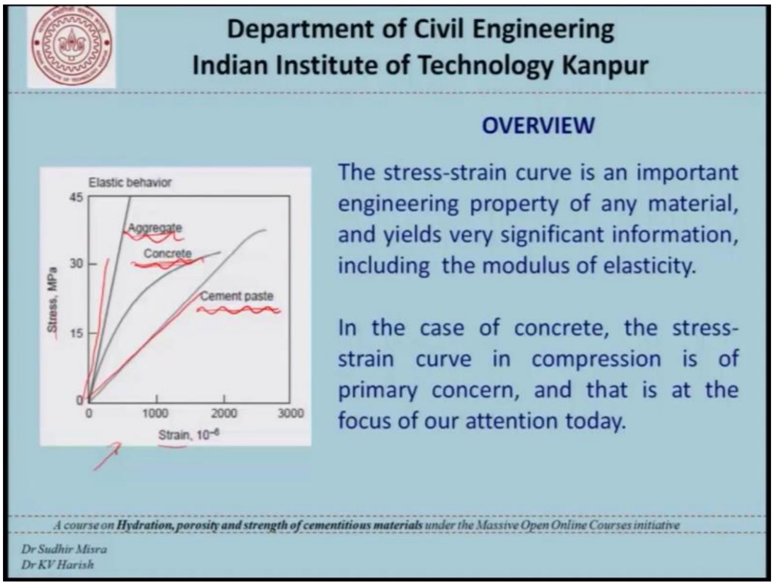
[FL] and welcome to lecture 37 of this course on Hydration, Porosity and Strength of Cementitious Materials, and today we will concentrate on the Stress Strain Behavior of Concrete; that is how does concrete behave under applied stress.

(Refer Slide Time: 00:37)



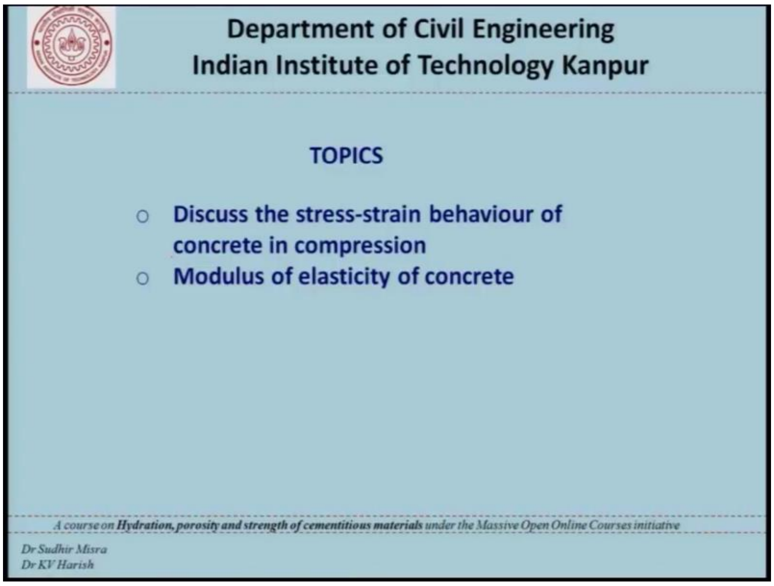
This is our familiar slide where today we will try to model the behavior of this concrete, which has been viewed like this under the action of a stress.

(Refer Slide Time: 00:47)



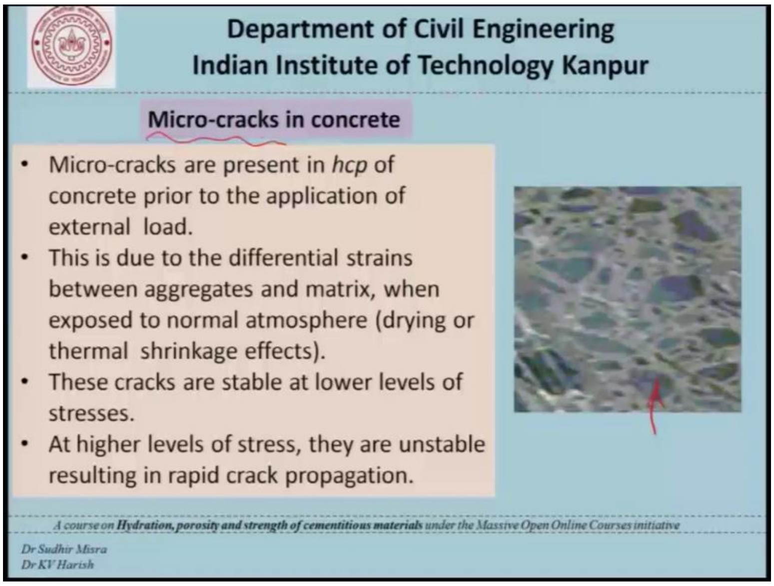
The stress strain curve is an important engineering property of any material, and yields very significant information including the modulus of elasticity in case of concrete the stress strain curve in compression is of primary concern and that is what we will focus our attention today. On the left hand side here is given the stress strain curve of the cement paste the aggregate and the concrete. That is what our model. So, if you look at cement pastes and aggregate the stress strain behavior is largely linear, which is not the case for concrete. And that is something which we need to talk about and discuss in detail and that is what the idea of the discussion today is.

(Refer Slide Time: 01:39)



We have basically discussed the stress strain behavior of concrete in compression, and we will talk about how the modulus of elasticity of concrete can be defined including what the codes and specifications and methods of design help us do how it can be determined how it is normally determined and also how designers estimate those values in design and what is the guide lines that are available to us in different codes and specifications.

(Refer Slide Time: 02:18)

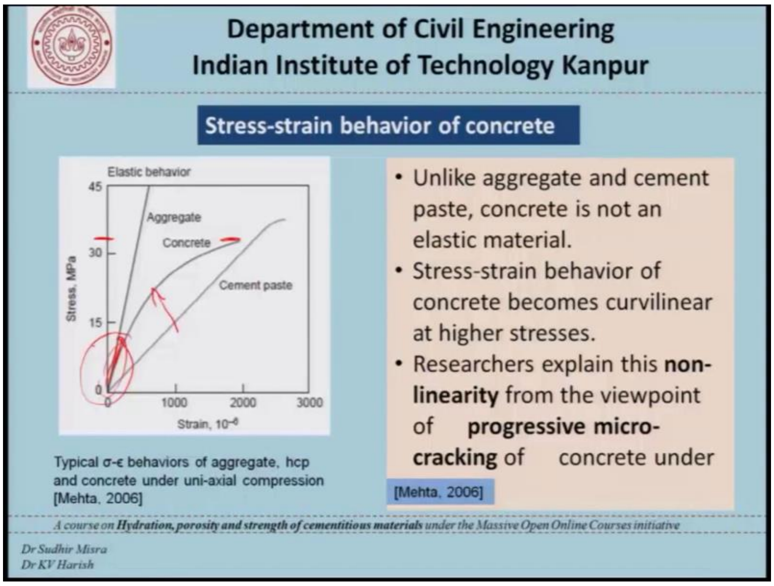


Now, this concrete here is having a lot of micro cracks in the hcp for all kinds of regions and that is something which we have talked about when we are talking about the constitution and the formation of the hydrated cement paste. So, these micro cracks are present in hcp of concrete prior to the application of external load. So, these cracks are micro cracks very small not necessarily connected with each other, but they have part of the concrete to begging with and that is these are not formed on account of load application. These micro cracks could be due to differential strains between the aggregations and the matrix that is the hard and cement paste, when expose to normal atmosphere drying or thermal shrinkage affects these cracks are stable at lower levels of stresses.

But as the structural cracks of formed in the concrete these micro cracks also; obviously, become involved in the whole process of crack propagation, they also play a part when we talk about crack initiation in the concrete. This business or this subject of cracking is.

In fact, the fully developed subject in it is own write and we talk about fracture mechanics, which deals with the formation and propagation of cracks we are not going to look at this crack formation from a fraction mechanics point of view, but you must know that those of you are interested to understand this whole idea of cracking insolence fracture mechanics is something which will help you understand that aspect a lot better. Today our concentration of course, is on the stress strain behavior which gets affected by the presence of these cracks.

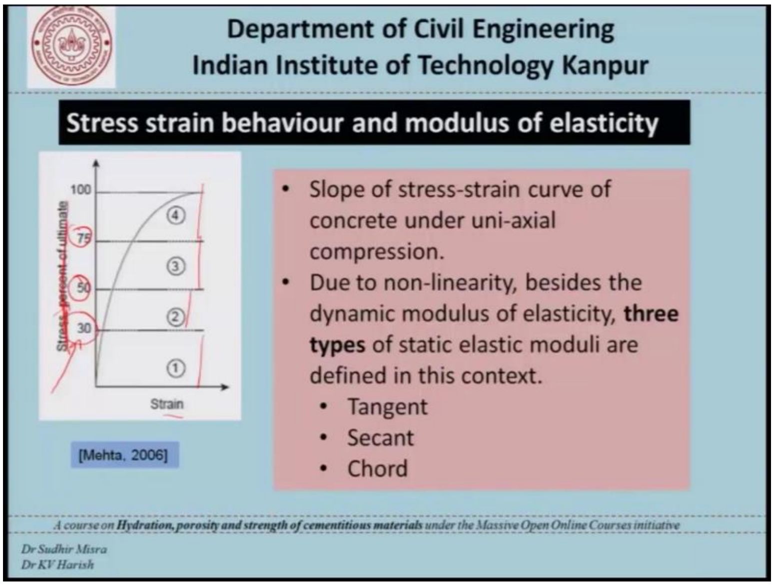
(Refer Slide Time: 04:17)



And at high levels of stress they are unstable resulting in rapid crack propagation. Now with this kind of a back drop we look at this picture once again and we find that unlike aggregates and cement paste concrete is not an elastic material the stress strain behavior of concrete becomes curvilinear at higher stresses. Initially yes we can assume or we can see that the behavior of concrete is also quite linear, but does not remain linear at higher levels of stresses or strains. Researchers explain this non-linearity from the point of view of progressive micro cracking in concrete under the action of loads.

I would like to draw your attention to this particular graph which is of course, an illustrative example it is only a case in point. And we find that if the ultimate load carrying capacity is about 30 to 33 MPa the linear proportion is only about 10 to 12 which is about one third to one fourth of the overall capacity, the ultimate strength of concrete and this is something which we need to understand or keep that back of our mind when we move forward in our discussion.

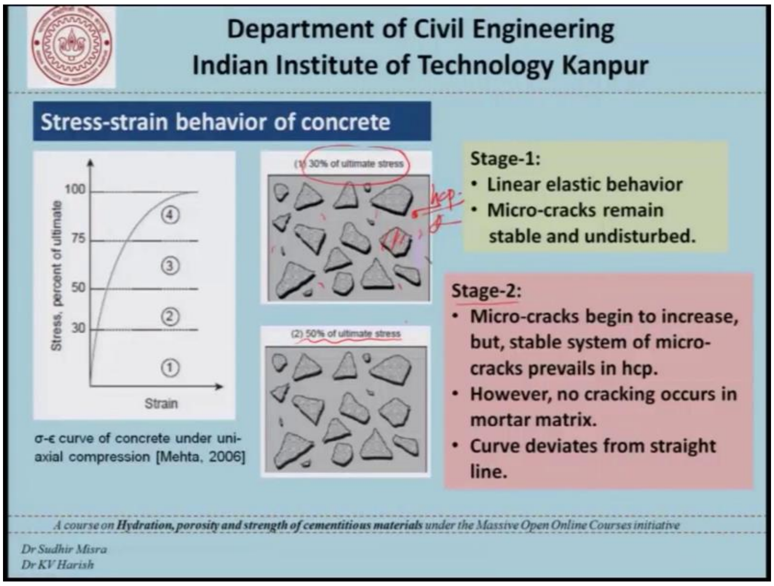
(Refer Slide Time: 05:26)



Now, this is an illustrative example of how we would like to discuss the stress strain behavior in the modulus of elasticity of concrete. This picture shows a stress and strain and this stress levels have been divided into 1 2 3 and 4 where level 1 level 2 level 3 and 4 are given in terms of percentages of the ultimate strength, and that is exactly the kind of thing which I was (Refer Time: 21:53) in to when I try to draw your attention to the stress strain curve of concrete in the last slide.

Now, moving forward with this the slope of the stress strain curve of concrete under uni axial compression that is what is shown here. Due to the non-linearity besides the dynamic modulus of elasticity 3 types of static moduli are defined in this context one is tangent there are the secant and the chord. We will come to this when we come to the definition and the estimation of the modulus elasticity of concrete.

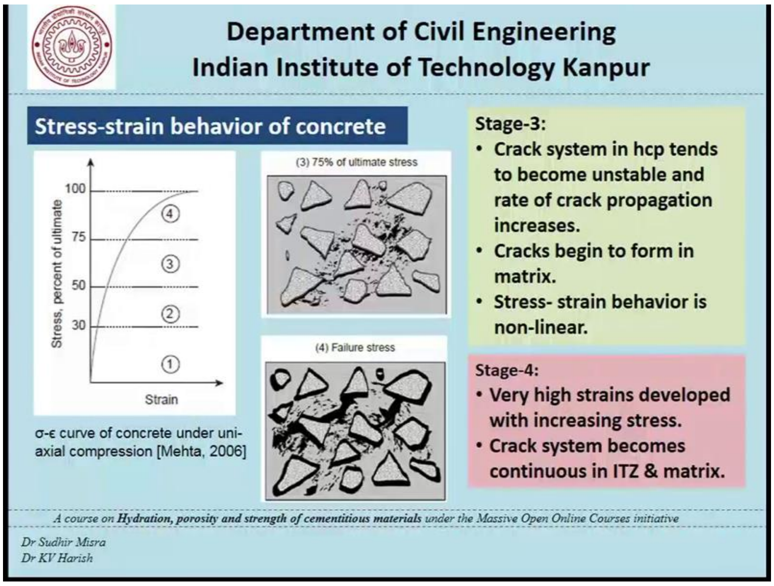
(Refer Slide Time: 06:32)



Now, continuing with our discussion on this stress strain curve is, stage one we have a load level of about 30 percent of the ultimate stress we can assume or we get more or less a linear elastic behavior as for as concrete is concerned. And that is characterized by micro cracks remain in stable and undisturbed. So, this picture here is that of concrete with aggregates suspended in our hcp and So, longer as the low does not exceed 30 percent of the ultimate strength whatever micro cracks are represent in the hcp they remain stable and undisturbed.

However, when it comes to stage 2 and we start reaching about 50 percent of the ultimate load the micro cracks begin to increase, but that a still is the stable system of micro cracks prevailing in the hcp, and no cracking occurs in the mortar matrix the curve deviates from the straight line.

(Refer Slide Time: 07:41)

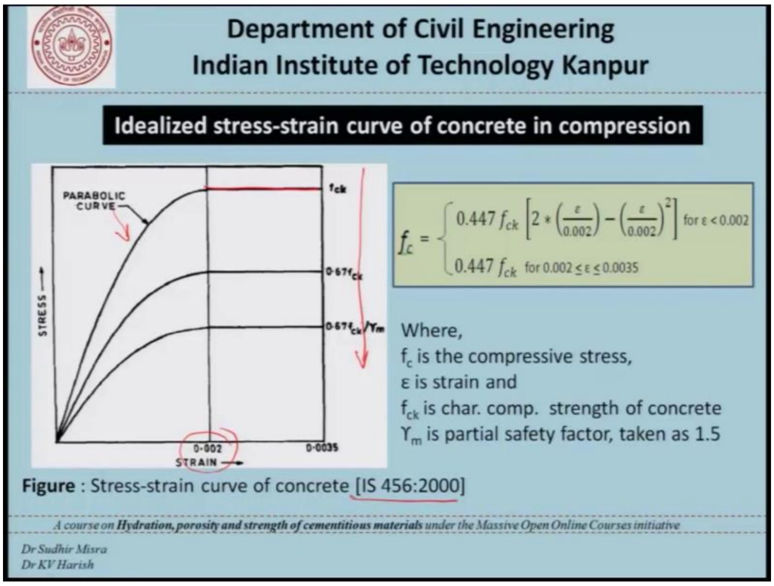


And when we come to stage 3, where the stress applied is closer to 75 percent or is between 15 percent and 75 percent basically exceeds about 50 percent of the ultimate stress the cracks system in the hcp tends to become unstable and the rate of crack propagation increases. That is the time when cracks being too from the matrix and the stress strain behavior are truly non-linear. Progressively when we get into stage 4 where this stressed are closer to failure, there are very high levels of strain and that increases there are very high levels of strain due to which the cracks system becomes continuous in the hcp and the matrix.

So, these pictures here on the left hand side show how we can visually represent the formation of cracks in the concrete as we apply higher and higher levels of stress. So, it is very interesting to visualize that the concrete specimen when subjected to load behaves linearly to begin with when the micro cracks remains stable, which is when the applied road levels are less than about 30 percent of the ultimate strength or the capacity of that concrete, as the stress level increase the micro cracking increases causing the behavior to become non-linear and finally, we have a complete disintegration of the concrete with the formation of lager cracks.

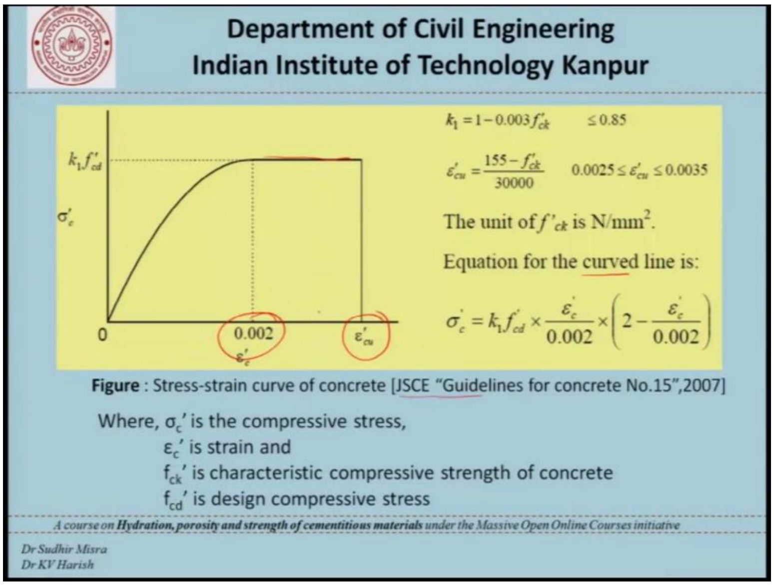
Now, with this back ground let us go and see what kind of a stress strain curves are prescribed or available to us in different cogent specifications. What we have seen in earlier is the actual experimentally determined stress strain curve of concrete which is carried out in the laboratory using a concrete specimen. Now in a lot of cases it may not be possible to carry out that kind of sophisticated experiment to determine the stress strain curve of that concrete or to study the stress strain behavior, also to facilitate design across different structures and different designers the codes of practice for example, the IS 456 give us certain idealized stress strain curves of concrete in compression.

(Refer Slide Time: 10:17)



So, for IS 456 it is as that the upper strain value of 2.002 the variation can be take into be parabolic and following, that it can be assumed that the strain will increase without any change in stress and that is what is depicted here and it is not our intention to discuss with design aspects of this stress strain curve the difference in my intention was only to introduce to you the fact that idealized graphs are available in codes and specifications to help us evaluate the level of the stresses and strains at different locations and So, on as for as the concrete is concerned in a rcd member.

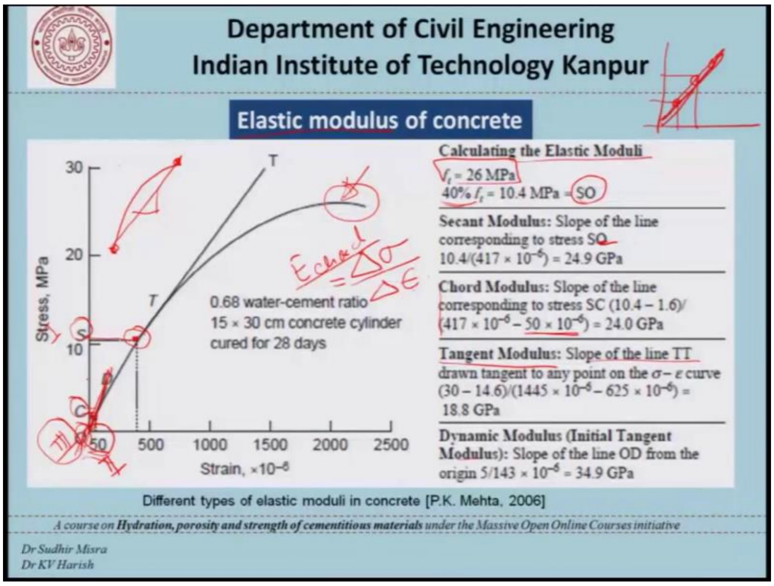
(Refer Slide Time: 11:12)



Now, other than the IS this picture here shows the japan society of civil engineering guidelines for structures. I should say more or less similar graph where there is a curved portion and the flat portion of course, these values may or may not be exactly the same as given in the Indian standards. So, this really shows that difference standards adopt different details as for as any aspect is concerned of course, in this case we taken up the idealized stress strain curve, but that is true more or less for any parameter, if we compare it across different codes we find certain differences, but the principle more or less is quite similar.

Now one of the most important information that comes out of a stress strain curve is the elastic modulus or the modulus of elasticity.

(Refer Slide Time: 12:06)



Now, for a linear material if you plot the stress strain curve the slope of this line does not change depending on the location whether we talk of stresses or we talk of strains. In the case of a non-linear material like concrete as we see here the stress strain curve is changing it is slope at different levels of the applied load. In a situation like this how should the modulus of elasticity we defined and used as for as calculations are concerned for deformation deflection and so on and so forth.

Codes tell us secant chord we can talk in terms of the secant modulus the chord modulus tangent modulus in the dynamic modulus as we will discuss now. For calculating the elastic modulus of concrete if the final strength is 26 MPa as we can see from here, we decide that let us take 40 percent and that helps us the defined this point S that is for this non-linear curve of this stress strain we take 40 percent of the ultimate strength and try to find out whatever the strain value is at this point. The second important point in this graph is a 50 micro strain point where we go upon this stress strain graph a stress strain curve and find out what is the stress level corresponding to that point. Now beginning with this point 1, this point 2 and the third point which of course, is the origin let us try to define what is the secant modulus.

Now, the secant modulus given here corresponds to the slope of the line S 0. That is from the origin we take this point S and even though it is a curve you say that this is S my origin this is my S through the slope of this line is what we are talking of and that is my secant modulus, now when it comes to the chord modulus we say no the origin is not the point which we are talking about, but we will take a point corresponding to the stress level of 50 micro strains, and then we use the formula delta sigma bond delta epsilon and use this to calculate the chord modulus or this is the difference between the secant modulus in the chord modulus.

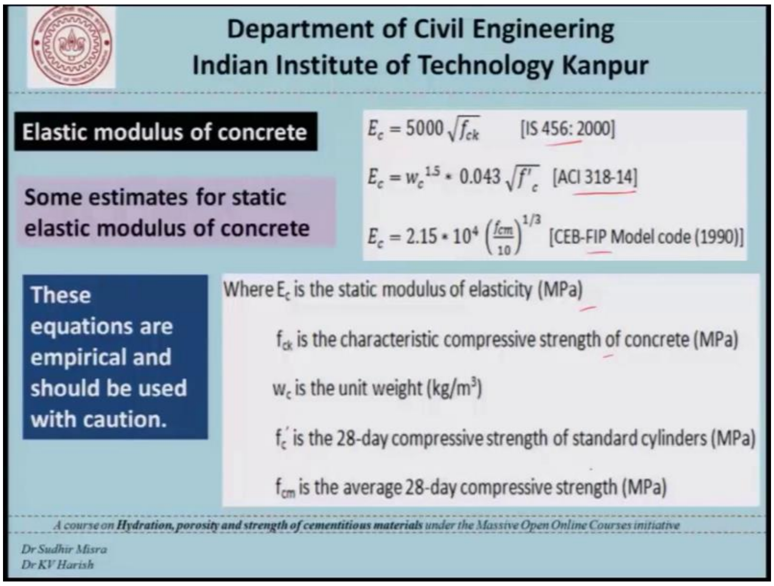
The end point here is the same, but the origin when calculating the delta sigma and delta epsilon is different in one case it is calculated from the origin and the other case it is calculated at 50 micro strains coming to the, tangent modulus in deed by definition the modulus of elasticity of a material is the tangent of a stress strain curve. And as we can see in this non-linear is stress strain curve of concrete the tangent or the value of the tangent changes depending on a stress level that we have it is something here it is something else here.

So, the tangent can be drawn at any point on the stress strain curve and that slope will give us the tangent modulus coming to the dynamic modulus it is also the initial tangent modulus that is we try to draw the tangent at the origin of the stress strain curve and that is our dynamic modulus of elasticity. So, if you take the real values as for as this particular is stress strain curve is concerned we find that the secant modulus is 24.9 gpa the chord modulus is about 24 gpa the tangent modulus is 18.8 gpa for the point which you have taken the tangent and the dynamic modulus which is the slope of the a stress strain curve at the origin that is 34.9 gpa.

Now, this obviously is the highest because we can see from the nature of the stress strain curve of concrete that that is where the slope is the maximum we have increase the application of load the slope changes. We do not see much difference between the secant modulus and the chord modulus, and of course, the tangent modulus will keep changing. So, it is not a matter of which of them is true or which of them is false it is only a matter of how we define it and also keep in mind the fact what is the range of this stress in which most of their structures operate. And then we find that most of the structures actually operate at levels which are lower than 30 to 40 percent of the ultimate stresses for most of the time and that is the basis for using this 40 percent to define the point S on this a stress strain curve are moving forward.

We must remember that this is stress strain curve is also a function of time. If we are talking of early age concrete this stress strain curve will be very different. If we are talking of very old concrete this stress strain curve will be different it will also depend on the moisture conditions and so on as we will see later on. So, we should be care full in interpreting the results that may come along like in the case of the stress strain curve in the case of the elastic modulus of concrete also, without having to go to the stress strain curve and try to determine the modulus of elasticity experimentally and so to that extent evaluating the modulus of elasticity for a given concrete for a given project codes whether it is the IS 456 or the ACI318 or the ceb, fip, they give us some kind of guidelines to say that we can estimate the modulus of elasticity using these equations.

(Refer Slide Time: 18:02)

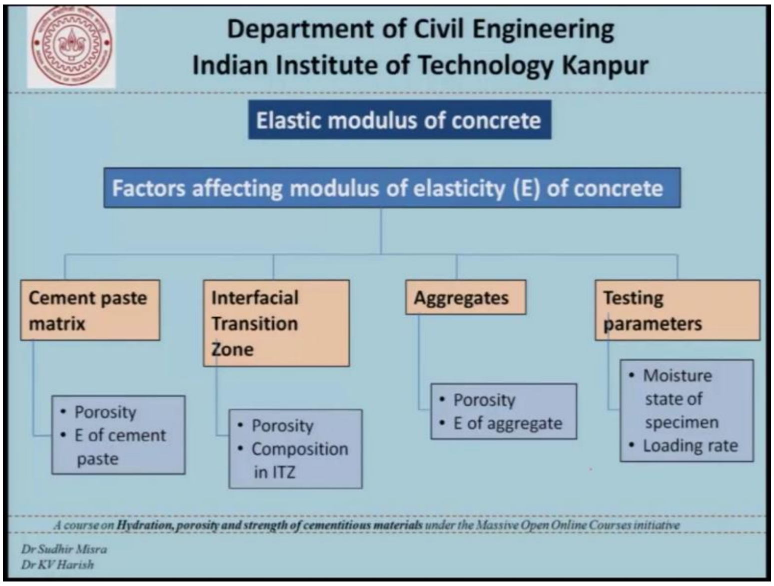


So, for example, IS codes as ec can be taken as 5000 root of ACK the ac, I tell us the that it can be taken as w c to the power of 1.5 into 0.043 root of f c dash and so on, where these parameters are defined here. So, there is some guidelines available to us even before or even without carrying out any tests to determine the stress strain curve of concretem and then using that to formulate the modulus of elasticity, having said that we must remember these codes are applicable only for a certain range of concrete at a certain age and so on.

So, we cannot use these values for example, for very young concrete or very high strength concrete and so on. So, the designers have to make the choice and that is very important to appreciate the even though the code allows you to use these equations to estimate the modulus of elasticity and perhaps. So, many other parameters we must make a judgment that indeed the estimation is within the range which was for seen by the people who wrote that document.

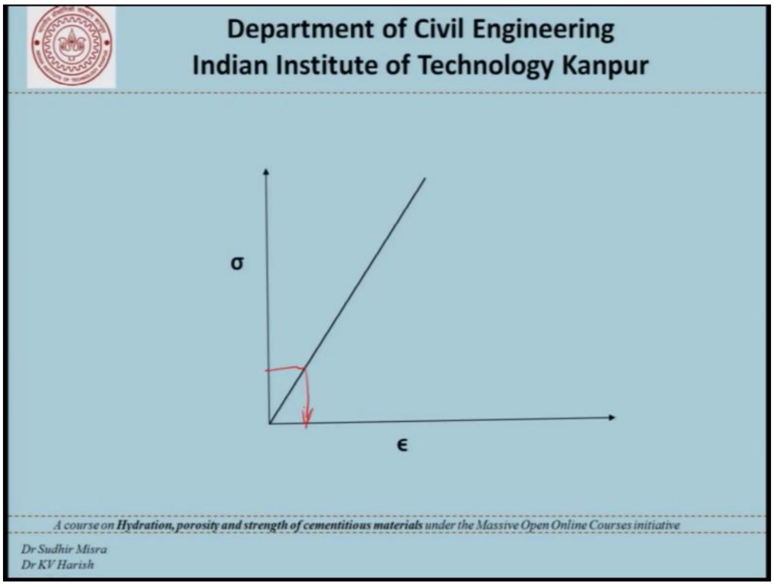
So, how he said that; so we should remember that these equations are largely empirical and should be used to with caution.

(Refer Slide Time: 19:44)



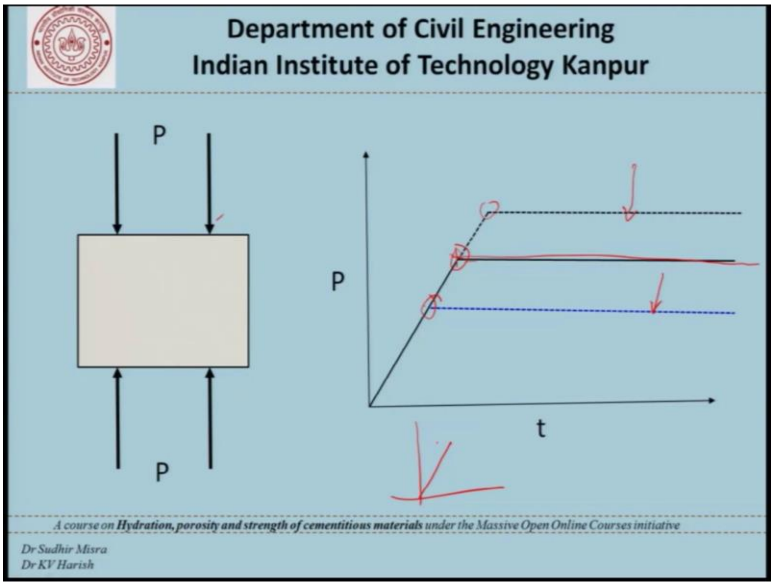
Coming to the kind of factors that affect the modulus elasticity of concrete there is the cement paste matrix which could be the porosity of that matrix as well as the e of the cement paste that ITZ the porosity on the composition of the ITZ the aggregates themselves which is the porosity and the elastic modulus of the aggregates themselves and of course, the testing parameters which could be the moisture state of this specimen the loading rate and so on. So, you will recall that all these parameters are very similar to the parameters that we talked about when we are talking about the factors affecting the strength of concrete and before we close the discussion on the stress strain curve let me just spend some time and talking about this curve a little differently.

(Refer Slide Time: 20:34)



So, now here is the stress strain curve which is linear what it basically tells for a particular level of stress there is the particular unique level of strain and vice versa.

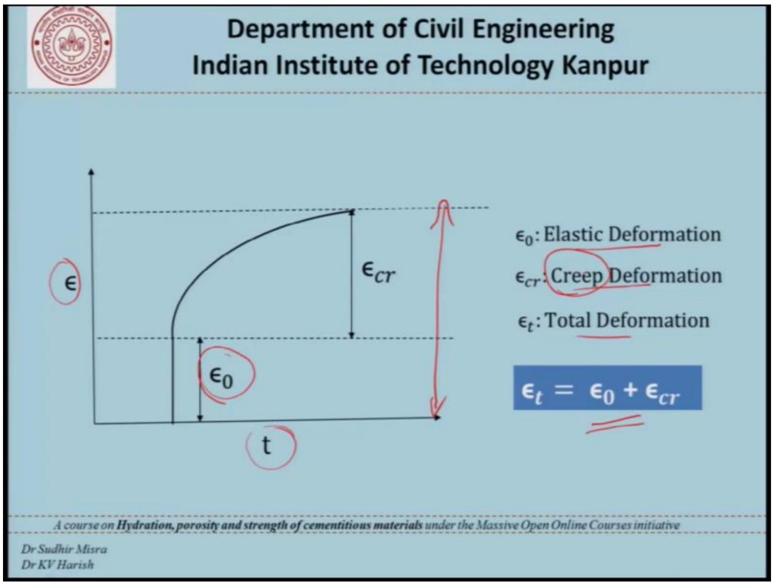
(Refer Slide Time: 20:49)



Now, if we look at the situation where this applied load is taken into certain level and then maintained at that level. Or for that matter the load is maintained at this level which is slightly lower than this level or it is maintained at a higher than this level, then what is be expected response this material in terms of the strain. Do you expect the stress strain to not increase or decrease and remain at the level at which it was measured when the load was initially reaching this level or this level or this level? Now this is the idea of a sustained load. Now the action under sustain load is something different than under a monotonically increasing load. So, the stress strains curve as we normally see; which is a kind of graph which I showed you like this.

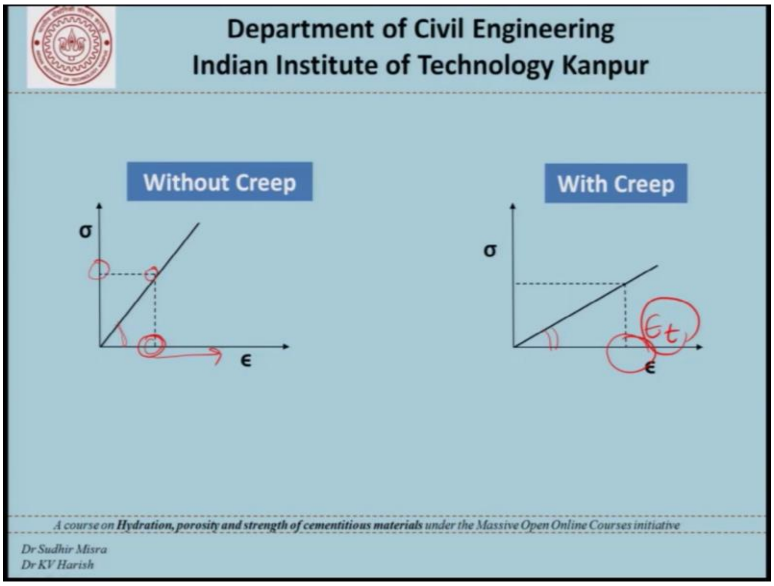
This is determined under monotonically increased not wait for too long, how long we wait and so on that is a material and that is why. In fact, the codes tell us the rate of load application. Now once we talk in terms of sustained load and so on; we obviously, have to talk about the increased strains under sustained loading.

(Refer Slide Time: 22:15)



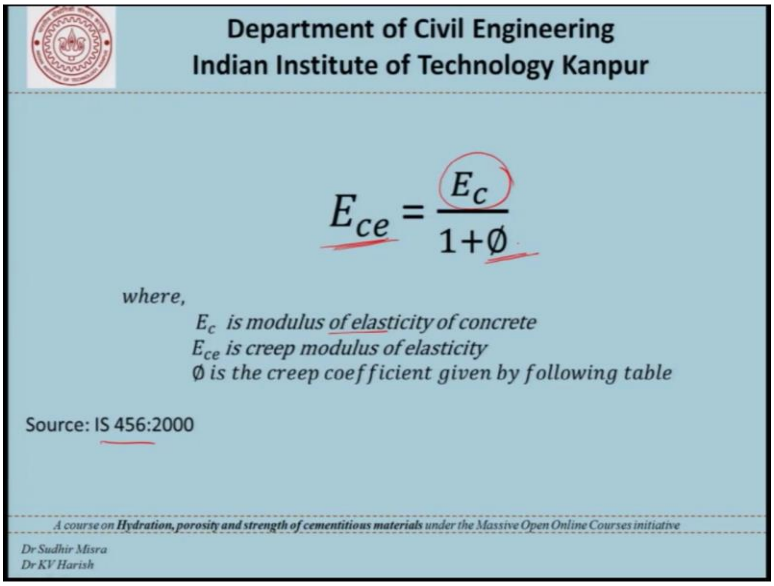
So, now we talk of a strain over time there is what is called elastic deformation then there is something called creep deformation and this is the total deformation. So, this creep basically is the phenomenon that is increased deformations under a sustained load. So, at a given point in time or a given point of load there is an elastic strain and then if the load is sustained at that level we have the creep strain.

(Refer Slide Time: 22:49)



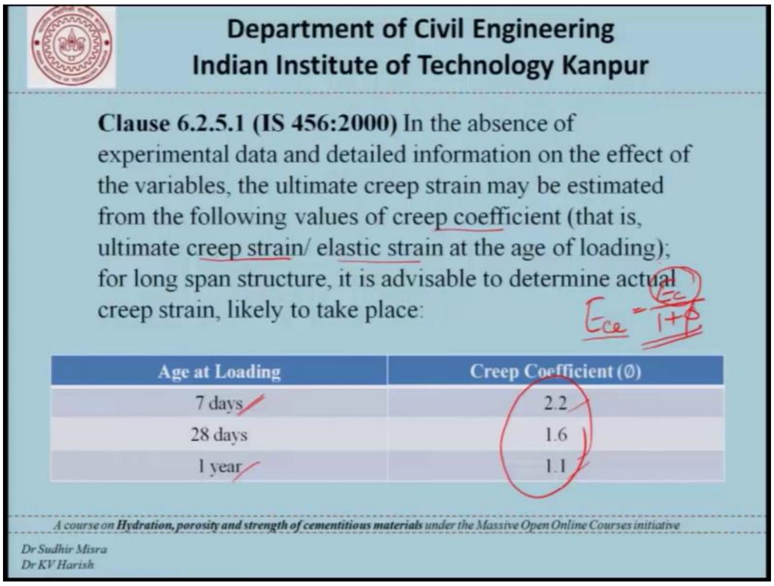
In other words, what is being said is that this level of strain for this level of load is not necessary be final strain under a sustained load. So, that strain is somewhat higher than this level, effectively what we are saying is that slope of this line whatever that is if we somehow reduce the slope. So, this value here would be closer to the total stain that we get if we consider the creep of that material in the deformation.

(Refer Slide Time: 23:23)



Now, this is something which is used in codes and specifications and for example, the Indian specification tell us that we can modify the modulus of velocity of concrete to Ece using a creep coefficient which is given in the following table.

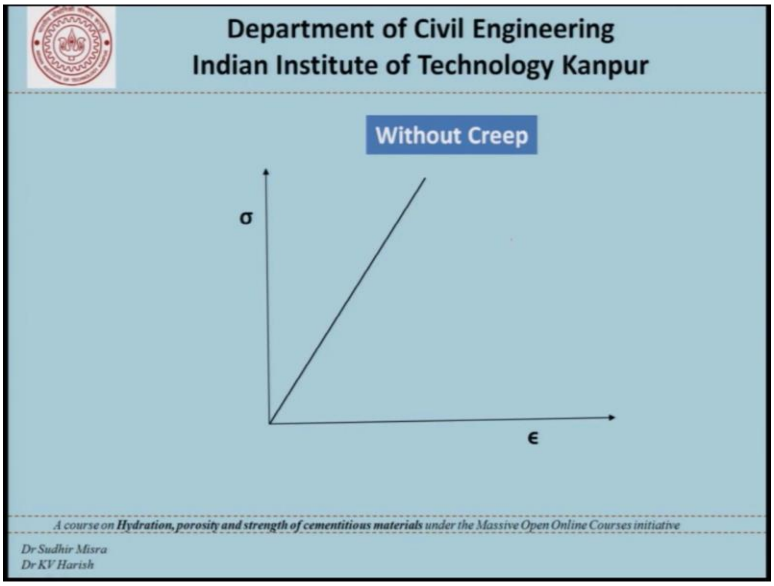
(Refer Slide Time: 23:43)



And that is what it says is in the actions of experimental data and detailed information on the effect of the variables the ultimate creep strain may be estimated from the following values of creep coefficient and the creep coefficient is defined as the ratio of the ultimate creep strain to the elastic strain at the age of loading. So, in the case of concrete it is also important to note that the extent of creep deformation would change depending on the age of loading.

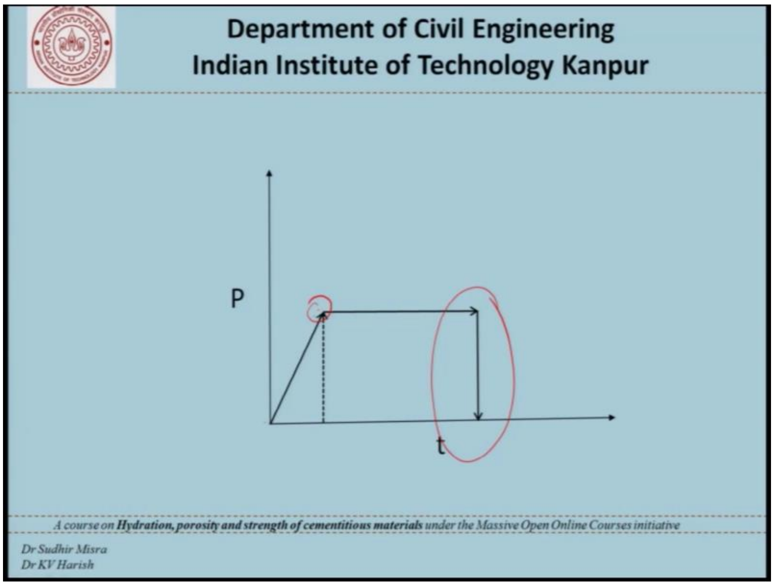
So, if you are loading at 7 days we can expect the creep coefficient to be 2.2 where as if you are loading at one year the total the creep strains would be only one point one times the elastic strain. So, this picture here tells us how the creep strains will vary as function of the age of loading and the relationship that Ece is being taken as e divided by one by phi tells us how much will be the modification in the e that we originally used if we want to take into account creep effects.

(Refer Slide Time: 24:53)



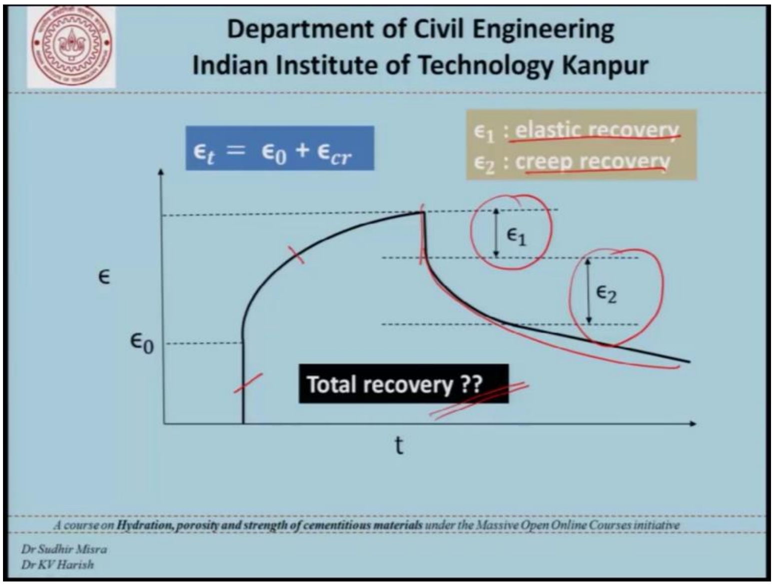
Continue this discussion further what happens that if this is what is our stress strain curve without creep.

(Refer Slide Time: 25:00)



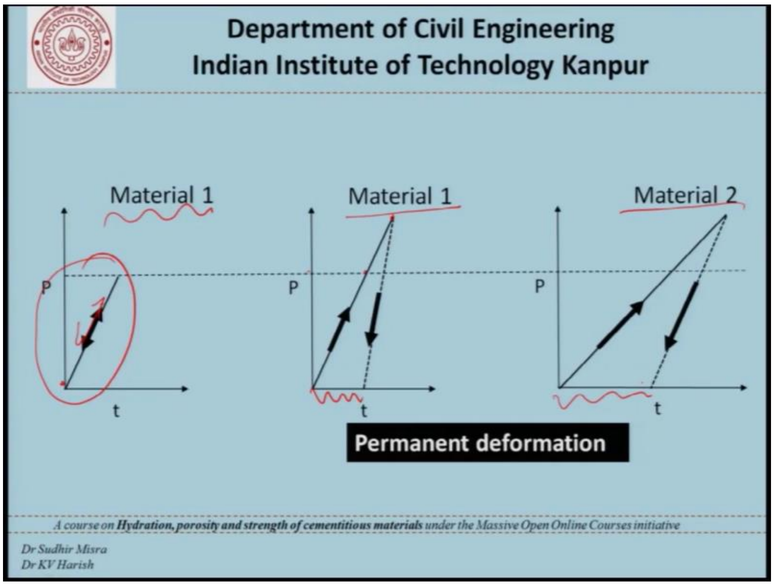
What happens if we unload that is we take the load to a certain point keep it for first sometime and then we unload. What is the response of this strain to this kind of a loading cycle? Now this strain response over time would be similar to that when we saw.

(Refer Slide Time: 25:27)



In the loading cycle in the loading cycle we had a elastic strain and creeps strain. In the unloading cycle we will have an elastic recovery which will happen immediately and we will have a creep recovery which will carry on over a long period of time. Now whether or not the recovery will be total that is whether or not there will be any permanent deformation would depend on what is the material that we are talking about what is the load level to which we had taken the material and then are unloading and so on.

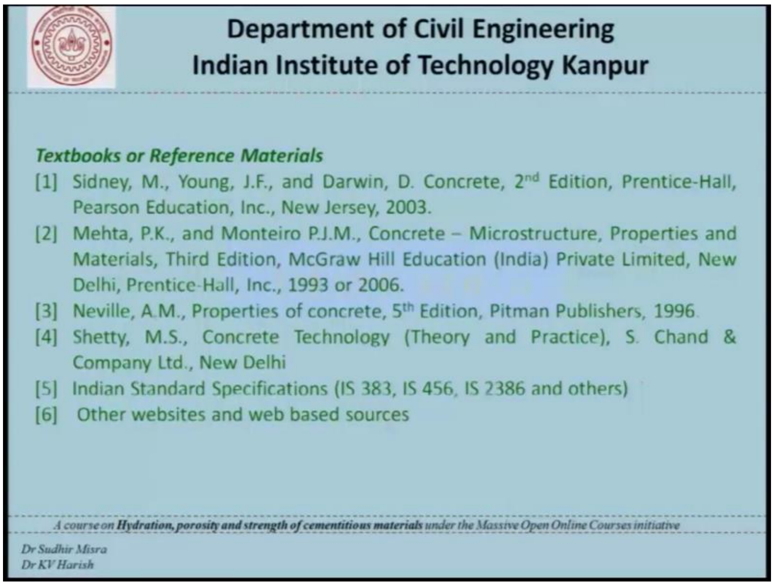
(Refer Slide Time: 25:51)



So, in the case when the loading level is such that the material is in the linear range and we have expected completely recovery this what we get here. In the same material if the load has gone beyond this point then of course, we can expect this amount of permanent deformation. Similarly, in the case of material to or a different material the amount of permanent deformation could be quite different depending on the material characteristics and so on. So, we are of course, not discussing creep in the major way we are not discussing permanent deformation in any great detail, but I thought the discussion on the stress strain curve of concrete would not be complete, unless we have spent some time talking about creep that is in structures which are under sustained loading.

How much is the sustain load may concrete structure is a different matter. There are dead loads which are always acting on the structures in certain structures the dead loads. In fact, govern the designed process. And that is why we thought that it is important to understand this aspect of this stress strain curve.

(Refer Slide Time: 27:13)



This of course, is the list of references and material which we could refer too.

And I would like to thank you for your attention.