Ground Water Hydrology
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Module No # 03
Lecture No # 12
Steady Flow into Wells (Contd.); Unsteady Flow into Wells

Welcome to this lecture number 12 so in which we will continue with our previous lectures article on steady flow through wells, So in lecture number 11 we discuss the steady through a confined into well through a confined aquifer.

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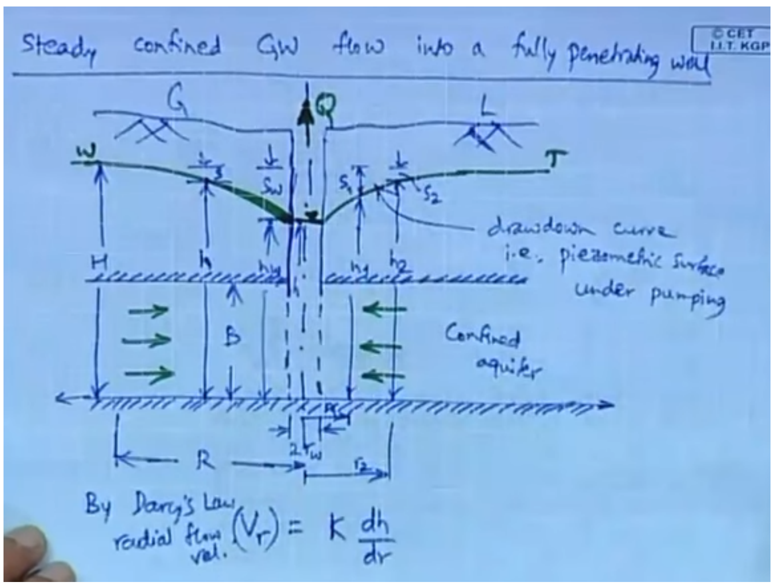

So in which the confined aquifer thickness was B and the well was penetrating the entire thickness of the confined aquifer and the water was horizontal before the pumping began which resulted in steady flow into the aquifer into the well and which also resulted in steady discharge of Q from the well.

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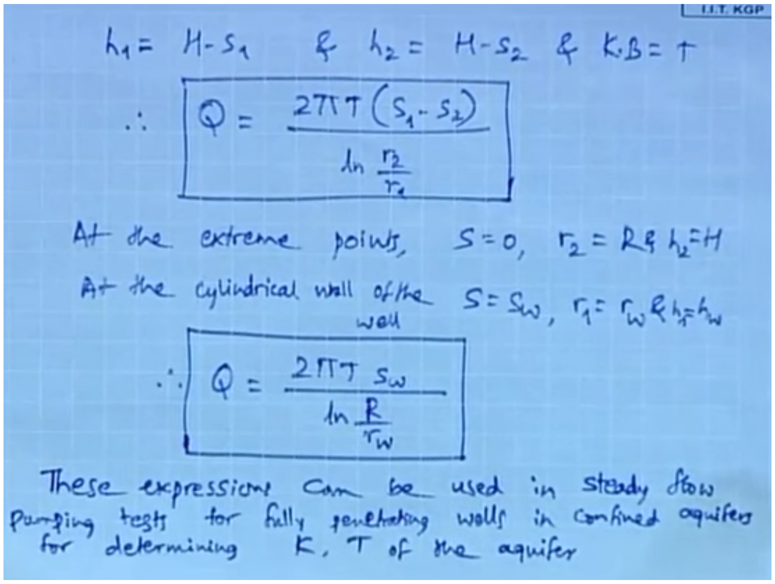

And we got the expression for the discharge which is popularly known as the yeah this is the expression.

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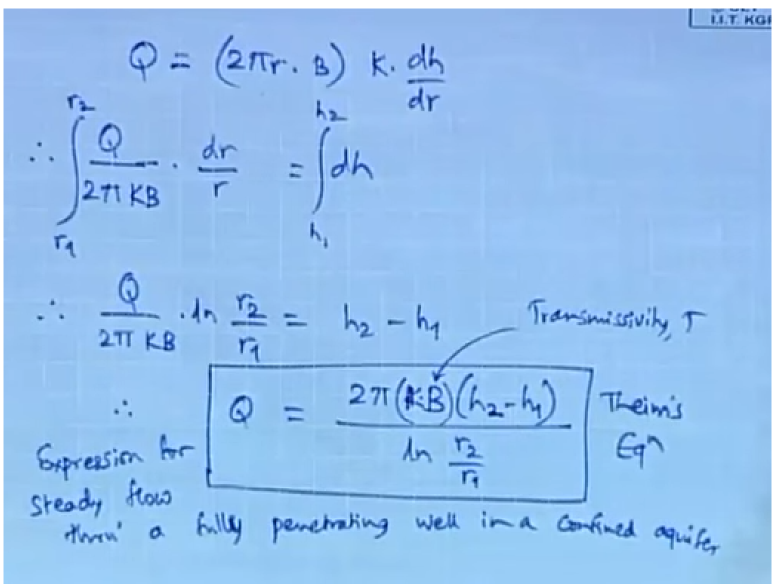

And yeah the original expression in terms of the hydraulic conductivity is Q = 2 PIE and K into B H2 – H1 divided by natural log of R2 / R1. So this is popularly known as themes equation and this when we simplify and assume applying the boundary conditions at the extreme points that is the extreme point when in the there is no drawdown and the radius is equal to radius of influence R capital R.

And H the head is equal to the total this head or the piezometric head and that so this is the upstream most extreme the downstream most extreme is the cylindrical where this S the drawdown is equal to drawdown in the well equal to SW R1 = radius of the well that is RW and H1 the head is equal to the depth of the water in the well in the confined aquifer.

So we get this expression and these expression can be used for with in case of pumping test to determine the hydraulic and aquifer parameters such as the hydraulic conductivity and tranmissivity. So therefore so these expressions can be used in steady flow pumping test for fully penetrating wells in confined aquifer for determining the hydraulic conductivity K and transmissibility or transmissibility T of the aquifer.

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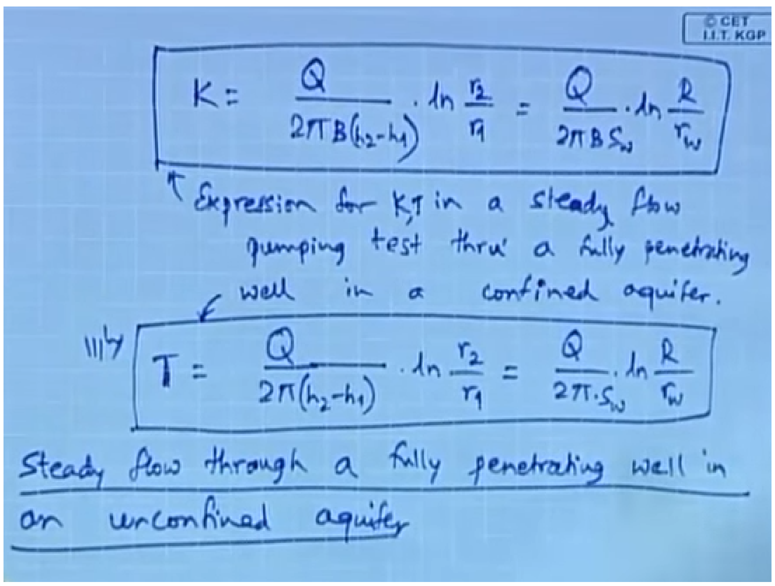

So here we get an expression for the hydraulic conductivity K for the steady pumping test through the confined aquifer having a fully penetrating well as ah well as Q divided by 2 PIE B into H2 - H1 into natural log off R2 by R1. And the same thing can also be written as Q divided by 2 PIE B and in this case that is SW and natural log off R divided by RW. Now here so this is the expression for K in a steady flow pumping test through a fully penetrating well in a confined aquifer.

Similarly the same expression using this equation you can write down the expression for T as Q divided by 2 PI H2 – H1 into natural log of R2 / R1 this can also be written as Q divided by 2 PI into SW into natural log of R the radius of influence divided by RW the well radius. So this is the expression for K comma T so this is the expression for K and this is expression for T. So in a steady flow pumping test through a fully penetrating well in a confined aquifer.

So we need to know to determine this hydraulic conductivity K in transmissivity T we need to know the steady flow rate from this fully penetrating well in a confined aquifer. We need to know the thickness of the confined aquifer we need to know either the depths between two the observation wells the depths of water in the two observations wells that is H1 and H2 and their radial with respect to the their radial distance with respect to the well access the vertical well axis R1 and R2.

And either we need to know this H1 and H2, R1 and R2 or we need to know the well radius RW the drawdown in the value SW and the radius of influence in the capital R and so in this case so we can easily determine easily estimate the value of the two of the three aquifer parameters formation consents that is the hydraulic conductivity K and transmissivity or transmissibility T. Now let us go to the case of steady flow through a fully penetrating well in an unconfined aquifer.

So far I have the seen the steady flow through a fully penetrating well in unconfined aquifer and now let us g to this steady flow through a fully penetrating well in a unconfined aquifer.

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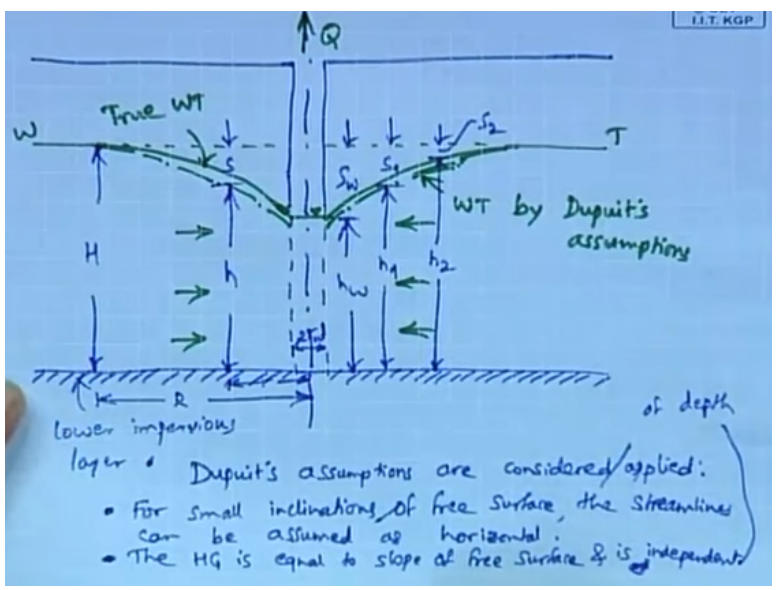

So for this the diagram would be there is a well which is fully penetrating through an unconfined aquifer. And in this case this is the water surface this is the water table before the pumping started and after the pumping started so this water table will show the steady draw down. So this is the well axis and this is the steady discharge from this well in the unconfined aquifer and this is the lower impervious strata for the unconfined aquifer.

And this is the radius of the well is RW and the diameter is 2RW and this is the radius of the influence or radius of the circle of the influence is R and the head their measured above the bottom impervious layer as capital H and at any two points and here what happens is so this is the we need to apply the Dupuit's assumptions. And so this is the water table by Dupuit's assumption and this is the true water table.

And here this is a H1 and drawdown there is H1 and similarly for the second observation well the head is H2 and the draw down there is S2. And obviously here the Dupuit's assumptions which we discussed in the previous lecture or have considered that is applied. So the Dupuit's assumptions accordingly that is a say for small inclinations of free surface the stream lines can be as approximated or assumed as horizontal.

And second assumption is the hydraulic grade the hydraulic gradient is equal to slope of free surface and is independent of depth is independent let me write here independent of depth. So here the first assumption that is the streamlines are assumed to be horizontal so this is a more or less true to accept close the well where this stream lines are significantly deviating from this one but still with these assumption the that means the error in estimating the discharge is a quite less.

So therefore because if we consider the inclination of stream line near to the well s then we have to modify our expression and for the discharge and it will make it more complicated. So therefore we are making the to Dupuit's assumption and this one. So here the this one so let me show you the flow that is radially into the well and here it is a HW and this is a SW okay and it any general point this is H at distance of R from the well axis the drawdown is S okay.

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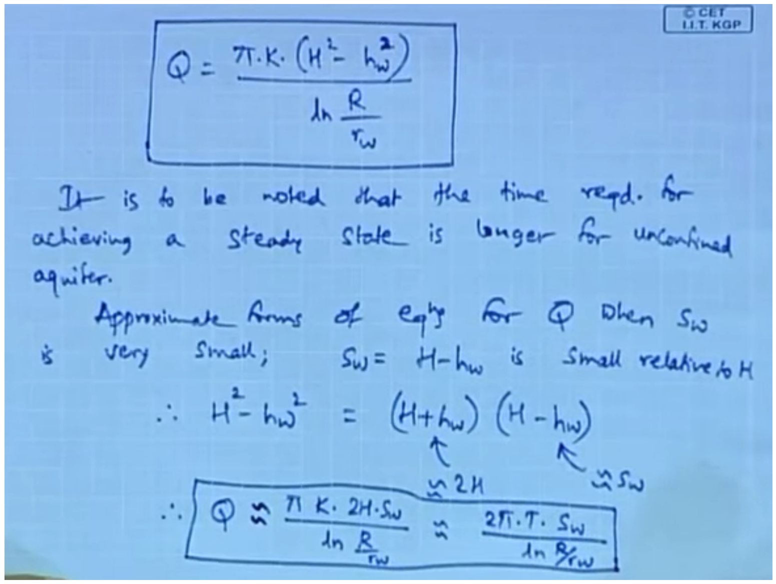

So now we will derive the expression for this steady flow so we know that the radially involved velocity VR is given by K into DH / DR this is the Darcy's Law and so this is a by Darcy's law.

So therefore the study flow rate through the well is given by this VR into the area which contributes and that area which is contributing in this case will be = 2 PI R which is the circumference of the circle like that radial distance R multiplied by H which is the depth of water at measured with respect to the horizontal impervious flow multiplied by this is the area multiplied by the velocity that is VR and this is = 2 PI RH into K into DH / DR.

So therefore we can write this as Q divided by 2 PI into K into DR / R this is = HDH okay. So this expression needs to be integrated that is between the limits R1 the lower limit R1 and the upper limit R2 and for the H the lower limit is H1 the upper limit is H2. So therefore we get Q = PI K H2 square – H1 square divided by natural log of R2 / R1. And in this case so this is the expression or the flow rate so at the extreme points so here this case that is H2 = H R2 = R and H1 = HW, R1 = R2.

So therefore this expression can be re-written as Q = PI into K into H square – HW square divided by natural log of R divide by RW. So these are the expressions for the steady flow rate in terms of hydraulic conductivity KH which is the depth of water above the impervious layer the in the unconfined aquifer.

HW is the depth of water in the well and R is the radius of the influence RW is the well radius and of course if you express a same this for two observation wells in that case so this is the expression to observation wells at a distance of R1 and R2 at a radial distance of R1 and R2 where the depth of water is H1 and H2. So these are the this is the expression so now let us so in this case it should be noted it is to be noted that the time required for achieving a steady state is longer for unconfined aquifer.

And so these two expressions let us write down the approximate forms approximate forms of equation the steady flow rate Q. So when SW that is the drawdown in the well very small so in that case that is the SW = H – HW is a small relative to H therefore H, H square – HW square you can write this as H + HW into H – HW. So this H + HW is approximately = 2H and H – HW is approximately = SW.

So therefore the approximate expressions for the steady flow rate = 2Pi that is Pi into K into 2 H SW divided by natural log of R divided by RW and this can be written as 2 Pi into T into SW divided by natural log of R divide by RW. So these are this is one of approximate expression for Q when the SW is very small.

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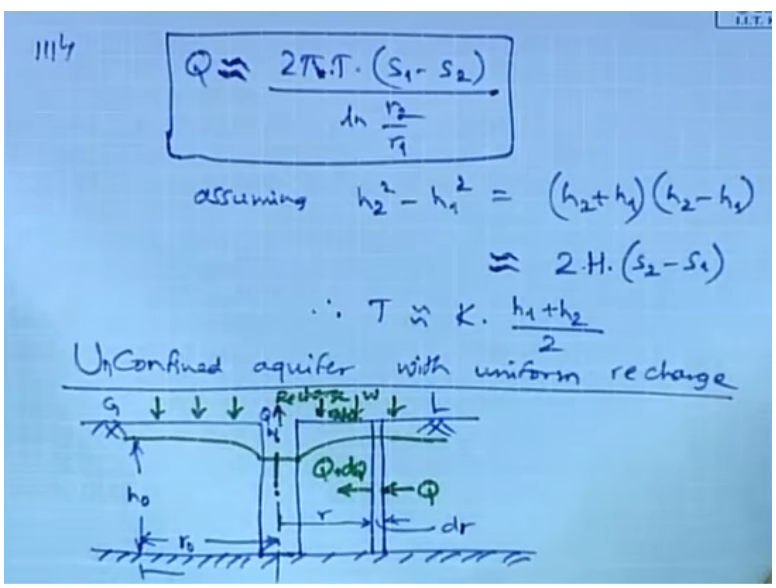

Similarly this we can also write this Q is approximately = 2 Pi into T into S1 – S2 divided by natural log of R2 / R1. So in this case that is so here that is H2 square – H1 square so here that is assuming H2 square – H1 square = H2 + H1 into H2 – H1 and this H2 + H we can take this to the approximately equal to that is 2T 2 that is H into and this S2 so this S2 H2 – H1 = S2 – S1.

So therefore we get this expression that is so these two expression will give the approximate expression for steady flow rate through this fully penetrating well in an unconfined aquifer and obviously their almost same as the expression for steady slow through a fully penetrating well in a confined aquifer. So now we have discussed the steady flow through a penetrating well in an unconfined aquifer as well as a fully penetrating well in unconfined.

And now in this case so here we can also write down so therefore this T is approximately approximated as K into H1 + H2 by 2. So therefore this K into H1 + H2 = 2T so this is with this assumption so we get this approximate expression for this Q. Now let us consider the unconfined aquifer that is unconfined aquifer with uniform recharge. So in this previous two cases we did not consider any recharge into the well so now let us consider the unconfined aquifer with uniform recharge.

So in this case that is the this is the ground level and this is the well in the unconfined aquifer and this is the water table and of course here we are getting the uniform recharge and let us take this as a W recharge rate W and this is the this steady flow through this well this Q, QW and here so this is the impermeable strata and let us consider a strip wherein to the inflow is Q and outflow is Q + DQ.

So this DQ is the one which gets contribution from this recharge the constant recharge and in this case let us consider the water table and let us consider this steady height as say H0 and the radius of influence as R0 and here so this radius is R and then this elemental strip of radial distance DR okay.

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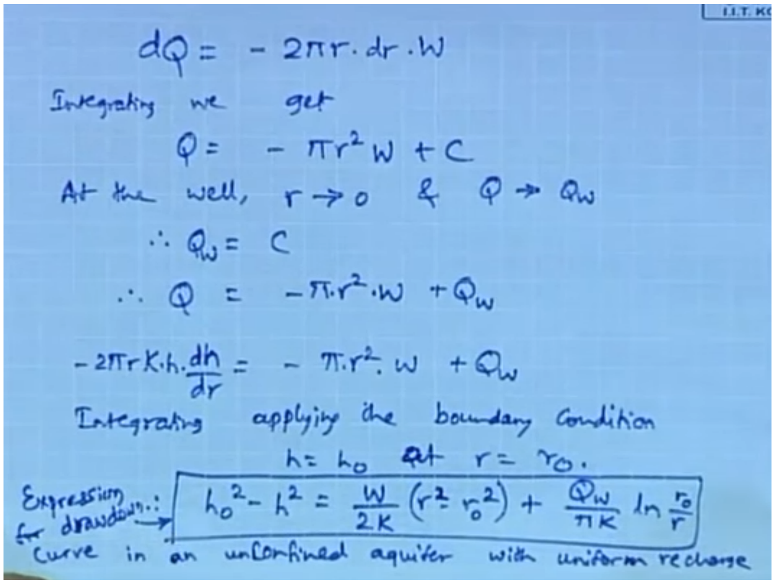

So now we can write down for this case we can write down the expression for the discharge DQ is given by the area that is -2Pi R into DR into the uniform rate of recharge. So if we integrate this one we get that is Q = - Pi into R square into W + constant of Integration and the boundary conditions are at the well R tends to 0 and Q tends to QW. Therefore QW = C therefore we can write down this Q = - Pi R square into W + QW okay.

If you substitute the expression for Q which is – 2 Pi R K into H DH / DR so here this K into DH / DR is the velocity as per Darcy's law and 2 Pi R into H is the area which contributes flow so this is = - Pi R square into W + QW so if we integrate this expression integrating applying the boundary condition it is a H = H0 the upstream most boundary condition and at R = R0.

So therefore we can write down after integration we will get and expression for the drawdown curve H0 square – H square will be = this is W divided by 2 K into R square – R0 square + QW divided by Pi K into natural log of R0 / R. So this is the so the expression for this draw down curve in an unconfined aquifer with uniform recharge okay.

So if you know the value of the depth of water above the lower impervious layer the uniform rate of recharge hydraulic conductivity and the radius influence R0 QW which is the steady flow through the this well fully penetrating the unconfined aquifer. So in that case this will be the expression for the drawdown.

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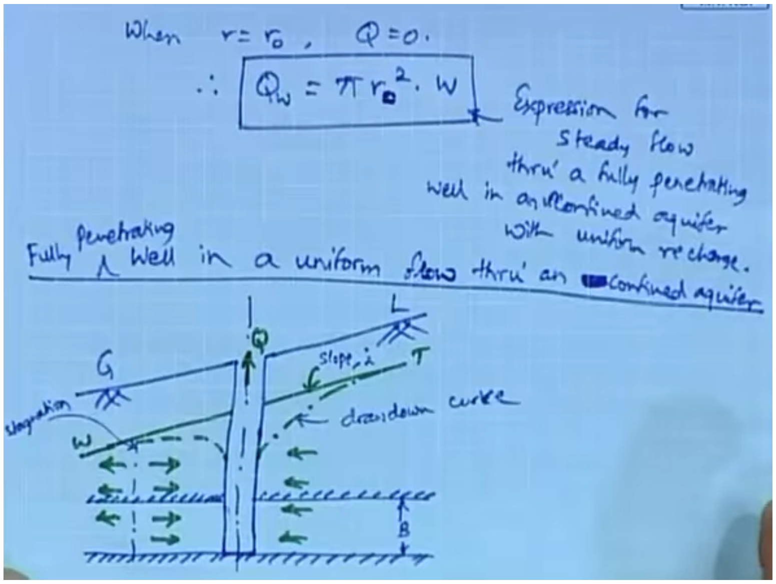

And we know that when R = R0 Q = 0 that means beyond the cone of influence there is no flow contribution. Therefore substituting this values we get QW = Pi into R0 square into W in this case so that is a so when R =R0 that case so this Q = 0 so therefore if you apply this expression so in that case this QW into Pi into R0 square into W.

So therefore so this is the expression for the steady flow through this well in an unconfined aquifer having a uniform recharge and this is the so this is the expression for steady flow through a fully penetration well in a confined aquifer in an unconfined aquifer with uniform recharge. So it is fairly simple so simply the area of the circle of influence multiplied by the uniform rate of recharge so that will be given by that will give the expression for the steady flow through the well.

And now let us consider a well in a uniform flow through and unconfined aquifer so again this is a say fully penetrating well and uniform flow through. So far we have seen we have consider the water table to be horizontal so that only after the pumping starts the well starts getting contribution from the aquifer. Now instead of that suppose we consider a well which is drilled through an unconfined aquifer having a sloping water table below the sloping ground.

In this case we consider so this is the sloping water table so with slope of I okay and let me denote the well axis here and the uniform discharge through this well and here what happen is after the pumping starts then the water table will show a drawdown curve like this. So this is the drawdown curve and in this case obviously here so up to this point only there will be contribution to the well and obviously in this right side it is fully flowing this one.

And if you draw the so in this case let us I am sorry this is let us say this is the unconfined aquifer and here this is the top confining layer and the confined aquifer thickness is B and now in let me so this represents the stagnation this vertical line represent stagnation and let us also draw the top view for this.

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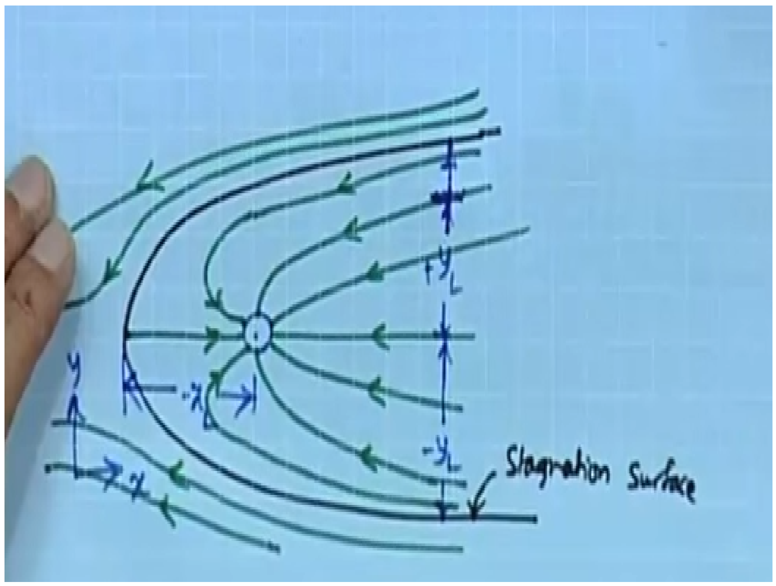

If this is the let me draw view in this case this is the well and let me so this represents stagnation surface and within this. So these are the streamlines and so here at any let us consider this as + YL and let us consider this as a +5 goes all the way up to here this is – 5L and this is this distance this say – XL and obviously this is the X and Y coordinate and beyond this the flow lines will be like this okay.

So now for this case so there is an stagnation surface because of original slope that is original hydraulic gradient due to an inclined water table.

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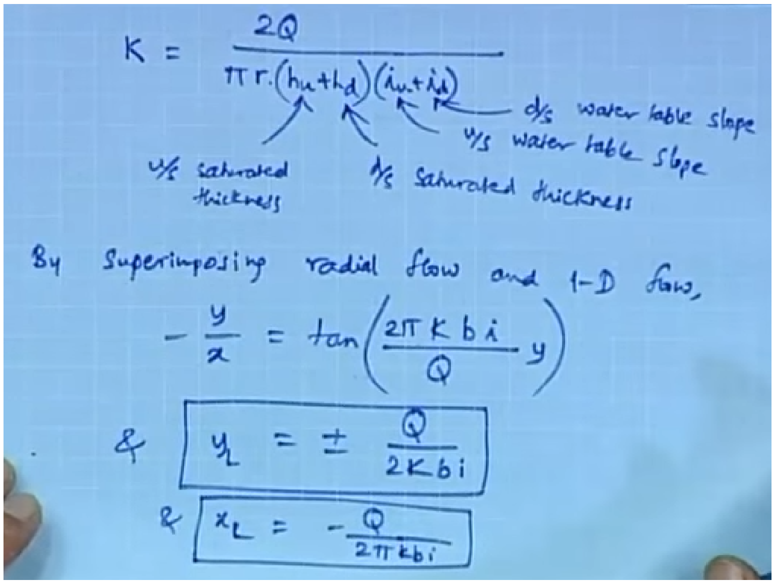

So in this case we can write the expression for the hydraulic conductivity as 2 Q k = 2Q divided by Pi into R into HU + HD into IU + ID and for here this HU is the upstream saturated HD is the downstream saturated thickness and this IU is the upstream water table slope and ID this is the downstream water table slope.

So by super imposing a radial flow and 1D flow we get we can write down the expression for this stagnation surface that is Y / – Y / X = Tan of 2Pi KBI divided by Q into Y. So this is the expression so this B is the saturated that is the aquifer thickness and Q is the steady discharge of course am not going into the derivation of this so therefore and this Y L is given by + or – Q divided by 2KBI so this is the distance of the stagnation surface at distance and okay.

And excel is given by – Q divided by 2Pi KBI so we will stop here and we will continue in the next lecture thank you