Ground Water Hydrology
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Module No # 03
Lecture No # 13
Unsteady Flow into Wells (Contd.)

Welcome to this lecture number 13 and this we will solve 1 or 2 numerical problems in the steady flow into wells and then we will move on to unsteady flow into wells.

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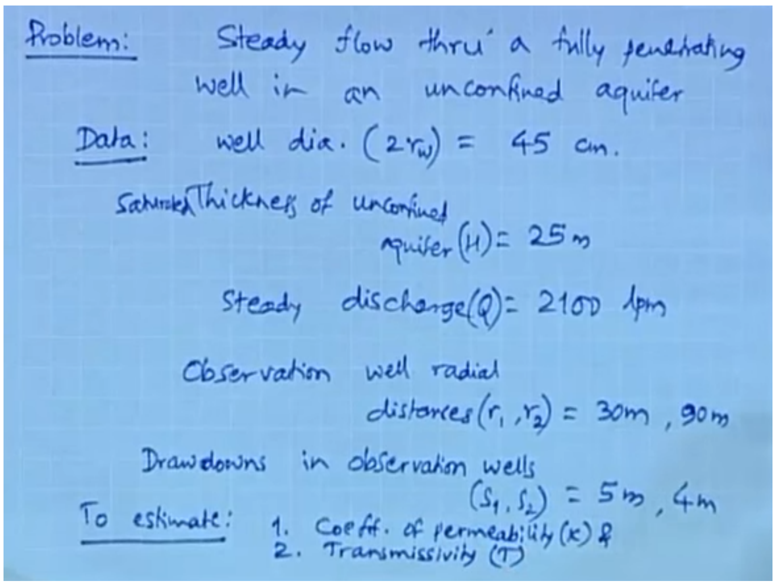

First let us consider one numerical problem of a steady flow through a fully penetrating well in an unconfined aquifer. So the data is well diameter that is 2RW let us say 45 centimeter so this is the given data then thickness of unconfined aquifer which is saturated thickness that is H is 30 meters I am sorry it is 25 meters and the steady discharge that is Q is 2100 liters per minute.

The observation well radial distances that is R1 and R2 R1 comma R2 this is 30 meter comma 90 meter drawdowns in observation wells S1 comma S2 that 5 meter comma 4 meter and to estimate. So based on this data that is well diameter saturated unconfined aquifer thickness steady discharge observation well radial distances. Obviously this is from center of well that is R1 an R2 and drawdown in this observation wells in these two observation wells.

So these are the data given to estimate the coefficient permeability one, coefficient of permeability K and 2 that is transmissivity that is T okay.

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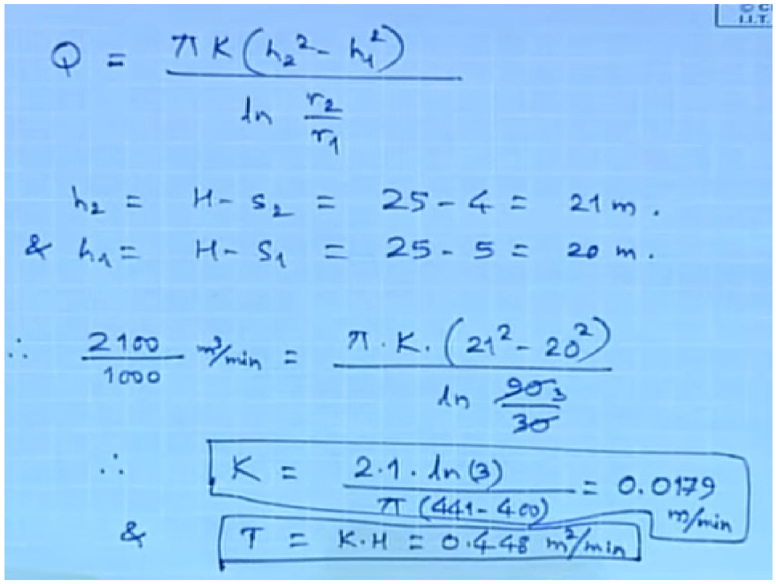

So know we now that for steady flow through an unconfined aquifer we have the relationship for the discharge in terms of the two drawdowns are the this one is given by Pi KH2 square and H1 square divided by LN of R2 / R1 and here so this H2 = H – S2 and in this case this is a H is given which is say 25 – 4 which is 21 meters and H1 = H – S1 which is 25 – 5 which is 20 meters.

So let us substitute the values therefor this Q which is given in terms of liters per minute so that is 2100 and so let us convert this into meter cube divided by 1000. So this will be meter cube per minute this = Pi into K into H2 – H1 square that is 21 square – 20 square divided by natural log off that is R2 / R1 and in this case this R2 / R1 is R2 is 90 and R1 is 30. So therefore K will be given by 2.1 into natural log off 3, 90 / 30 is 3 / Pi into so this is 441 – 400.

So this is 21 square is 441 and 20 square is 400 and this will directly give the value of K in terms of meter Cube per minute let me use here the calculator it is 2.1 into LN of 3 divided by Pi into 41. So this will directly give the answer of coefficient of permeability K is .0179 is this is meter per minute and this the transmissivity T is simply given by K times H. So this into H into 25 so this will be .448 meter square per minute, So these are the answers the K value is .0179 and then T is .448 meter square per minute okay.

So this is how we can use this data from the well and determine to estimate the formation constants of either formation constants of the aquifer or if they are given the steady or rate of discharge.

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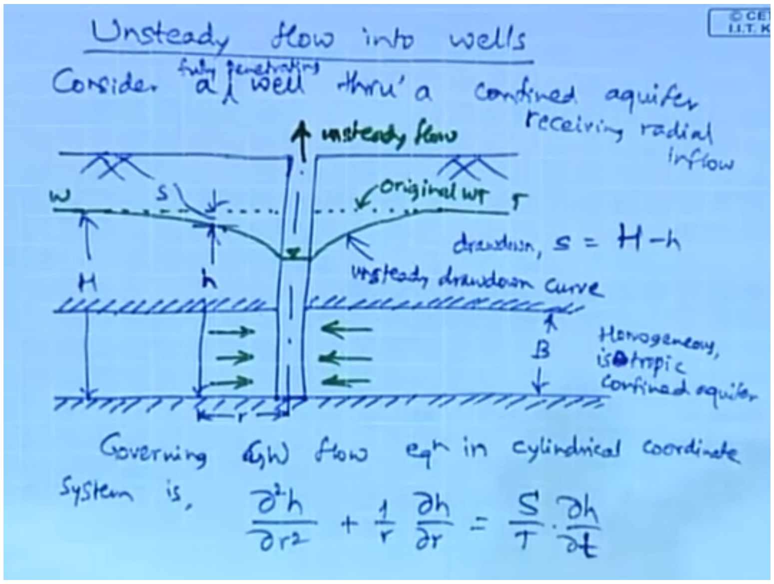

Now let us come to the unsteady flow into wells here so the flow is varying with time and firstly let us consider the unsteady flow through a confined aquifer consider as well through a confined aquifer and that too a fully penetrating well. So were in the there is a unsteady flow so let us draw the diagram so this is the ground level and this is the confined aquifer and this is the well this is the lower impermeable boundary of the confined aquifer.

And this is the upper impermeable boundary of the confined aquifer and for simplicity let us consider the thickness as B and the flow is varying here and this case this is a so this is a unsteady flow through a confined aquifer. Obviously so here so there is a radial flow and so consider a fully penetrating well through a confined aquifer receiving radial inflow okay.

So in such case the equation is given by the equation the ground water flow in the radial coordinates and here obviously so let me also show the water table the original water table which is horizontal. So this is the original W2 and then so this is the so this is the drawdown and obviously so here at a distance for a general section the height of this one above this is S and the drawdown is s and height is H.

So now in such case we can write down the governing of ground water flow equation in radial co-ordiantes in say cylindrical coordinates and here you can say this is the homogenous comma isotropic confined aquifer is D square H / DR square + 1 by R so this is the radial distance R where the head is H and drawdown is S. So this 1 by R into DH / DR so this is the second partial derivative of H the variable head H in the unconfined flow am sorry the unsteady flow through a unconfined aquifer through a fully penetrating well in a unconfined aquifer.

So this must be = S by T storativity by transmissivity into DH / DT an since this is an unsteady flow so this DH / DT is a non-zero value. So this is the governing equation now in this case and here we can write down this the total head the undistributed head or piezometic head is a capital H and here we can write down. So this obviously that we can write down the expression the drawdown small S = the piezometric capital H – variable head in the well along the drawdown curve.

So this is the drawdown curve this is the unsteady drawdown curve so this drawdown curve is given by this capital H – small H.

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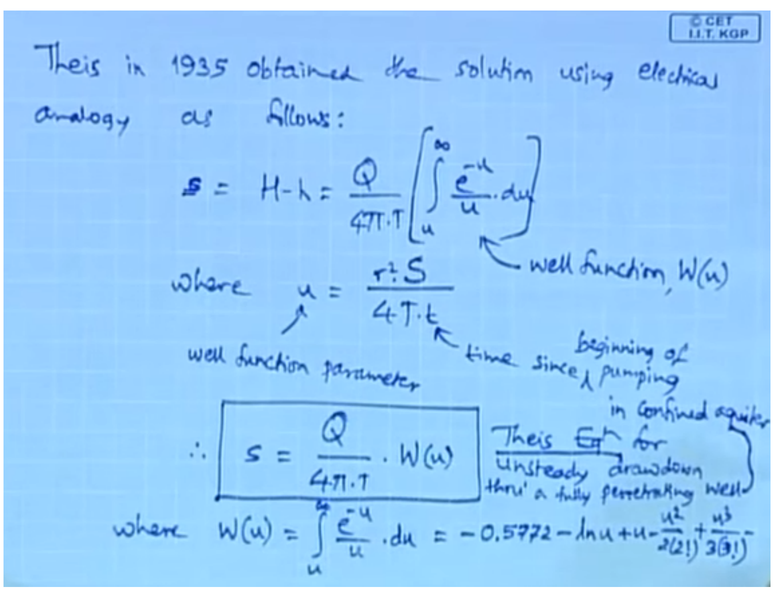

So for this case if we so it is forced in nineteen thirty five obtained a solution so using electrical analogy as follows is that is why this equation is known as the phase equation so the equation is for the drawdown S the variable drawdown = capital H – Small H the peizometric head the constant piezometric head and variable head of the drawdown curve H measured with respect to the bottom impervious layer of the unconfined aquifer.

So this equal to Q divided by 4 Pi into T into integration that is U to infinity E to the power – U divided by U into DU. So here so this term U is known as the well function parameter so this is known as the well function parameter this is equal to R square into s the storativity divided by 4T into T. So this is transmissivity T and then this is the time since pumping since beginning of pumping and this R is the radial distance where this drawdown is S.

So in this case so this R square S / 40 T is denoted as well function parameter and it is integration that is E to the power – U / U into DU between limits U to infinity and this whole thing is known as well function denoted by WU. So therefore the equation for this the solution for the governing equation is the drawdown the variation drawdown S is simply given by Q divided by 4 Pi into T into the well function WU.

And here the well function where this well function this WU = integration U to infinite E to the power – 2 / U into DU. So this has been evaluated as a – 0.5772 – natural log of U + U – U square divided by 2 into 2 factorial + U cube divided by 3 into 3 factorial – and so on. So this is the expansion of this integral so therefore we can write the and this is known as the this is known as phase equation for unsteady drawdown through fully penetrating well and it is in an u confined aquifer.

So this fully penetrating well in an unconfined aquifer okay so this is the Theis equation. And now let us write down say that is an here obviously there are various tables are given for well function value for a different well function parameter and so Theis equation can be solved by this graphical methods.

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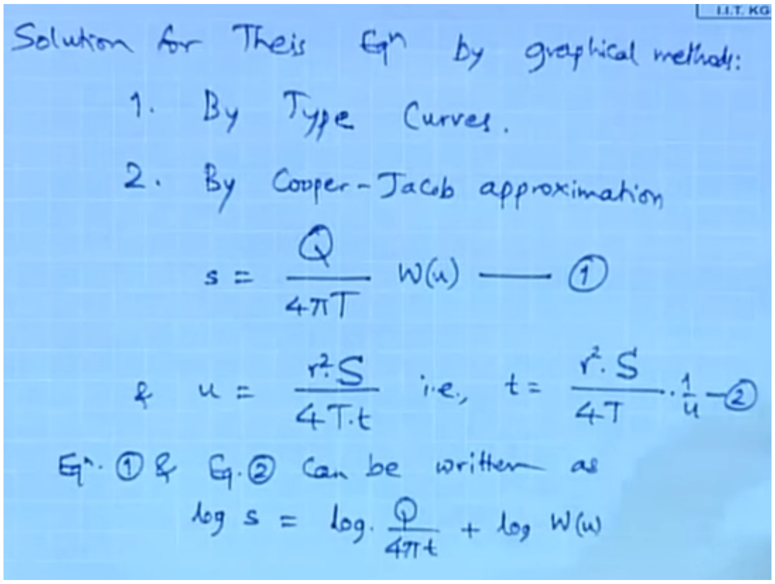

So a solution for Theis equation by let us say this is the graphical methods in this the first one let us consider it as the type curves one is that is by type curves 2 by COOPER JACOB approximation. Now let us consider the solution of Theis equation by the first method which is by type curves the graphical method by type curves. So here what is done is the expression for this unsteady drawdown is given by Q divided by 4 Pi T into the well function W.

And we know that so this well function parameter U is given by R square S divided by 4T into T. So that is we can write this down as a T = R square S divided by 4 T into 1 by U let us denote this as equation 1 and this as equation 2. So this equation 1 and equation 2 can be written can be written as log of S this is unsteady drawdown S obviously here this Q is the steady rate of pumping.

So here the it has not reached the equilibrium so therefore even though this Q is steady the drawdown S is unsteady its varying time so this is equal to log of Q divided by 4Pi T + log of the well function WU.

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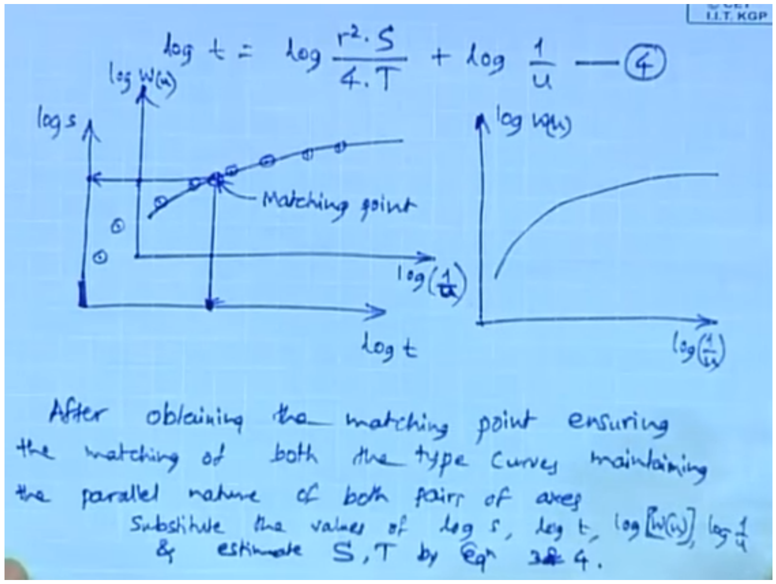

And similarly so for the second equation so we can write this as if you take the logarithm so this log of T = log of R square into S divided by 4T + log of 1 by U. So here these are the two equations the first so let us denote this as equation 3 which is simply the logarithm of equation 1 and this is equation 4 which is the logarithm of equation 2.

So here so these comparison of 3 and 4 will tell us that if we plot S versus T that is the drawdown versus the time since the beginning of the pumping on a log-log scale. And compare that with the plot of this log of W versus log of 1 by U and match them. So that the both the coordinate access or maintain parallel so in that case we get what is known as the first let us say this is the plot of log S versus log T and we get the second plot of log WU versus log u log of 2 by U.

So in this case so this is log S versus log T is based on the pumping test data and obviously as T increases. So this S will increase so this number of data points represent the number of readings in the pumping test taken and then this log of WU versus log of 1 by U. So from the tables so this is the type of curve so here in this type curve this one what is done is so these two plots are matched that means on this the plot of this log S versus log T this log of well function WU and log of reciprocal of function parameter 1 by U.

So is superimpose such that both the pairs of axis are parallel in such a way that so there are the curves more or less match in such case what is done is so this is trial and error this is done and then corresponding to this matching point on the curve. So here say suppose this is the matching so this is the and corresponding to this we have the log S value given along the log S axis the log of WU value given along the log of W axis and then this will be log of 1 by U and this will be log of T.

So if we substitute these values then and of course we have the data that is log S value is and then then log T value is there and then so we get this then we can find out the that is both this log of Q by 4Pi T as well as log of R square by 4T. So then we can solve for this and get the values of this is a so after get the values of the aquifer parameter.

So after obtaining the matching point ensuring the matching of both the type curve so this is type curve one which is basically the plot of drawdown versus time on a log-log scale in type curve 2 the log of well function WU versus log of the reciprocal of the well function parameter 1 by U. And so maintaining the parallel nature of both pairs of axis so we get the matching point and corresponding to any matching point we can get this log of S, log of WU, log of 1 by U and log of T.

So once we get all this so simply substitute the values so then we can solve for so substitute the values of log S, log T, log WU, log 1 by U and estimate storativity S comma T by equation 3 and 4. So this the first graphical method for determining the aquifer parameter using the Theis equation.

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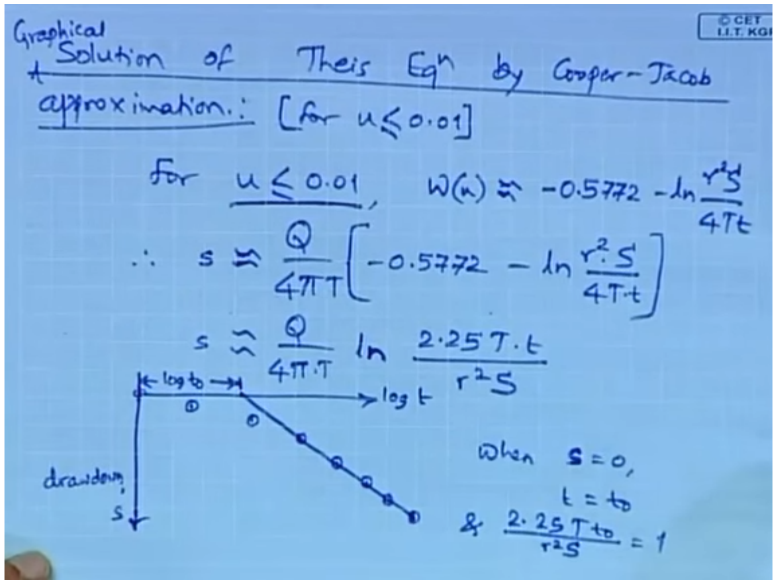

Now let us go for the second graphical method which is the aquifer parameter that is a so this is the graphical solution of this equation by COOPER JACOB approximation. So in this case so obviously here and of course this has some limitation for U the well function parameter less than = .01 so in such cases only this approximation can be used so in such cases the well function.

So here say for U less than or equal to .01 the well function that is WU is given by just the two parameters it has the first two terms well function is approximately = -.5772 – natural log of that is the well function parameters which is R2 square divided by 4T into T. So therefore the expression for the drawdown S will be given by Q divided by 4 Pi into transmissivity T multiplied by – 0.5772 – natural log of R square into storativity S divided by 4T into T.

So this is transmissivity and this is the time since pumping and this is the drawdown so in this case so obviously so this is the approximation or so for the this the value of the well function has to be less that .01 less than or equal to .01. So in such case so this this is S can be approximated as Q divided by four Pi into T multiplied by the natural log of that is 2.25T T divided by R square into S.

So this is the approximation and in this case so what is done is so here a semi log plot of drawdown versus time the drawdown S the unsteady drawdown versus the time. So that is plotted down a log scale log of T this case so obviously it will have to so this is here it starts with it has to start with this one. So here we get a number of this a data points and this one so these data points the data points having the higher values of time so they form a straight line and if we extend this so this is a log of T0.

So in these case basically so when S is the drawdown the unsteady drawdown S = 0 then T = T0 or in other words in this case and here we can say that so that is the in this case the argument of logarithm has to be 1. So that the natural log of 1 = 0 so in that case S is 0 so that is that is 2.25 T T0 divided by R square into S this must be = 1 okay. So therefor in the COOPER JACOB approximation what is done is a semi log plot of drawdown on the linear scale and then this time T on the logarithmic scale.

So that is plotted and then so the data points having higher time values so they will be forming they will be having a linear relationship. So there linear relationship is extended backwards so to intersect the logarithmic scale T and wherever it intersects so that intercept will give the value corresponding to this time that is T0 where in the drawdown is 0. So therefore here what is done is in the COOPER JACOB approximation so this T0 is determined and once this T0 is determined.

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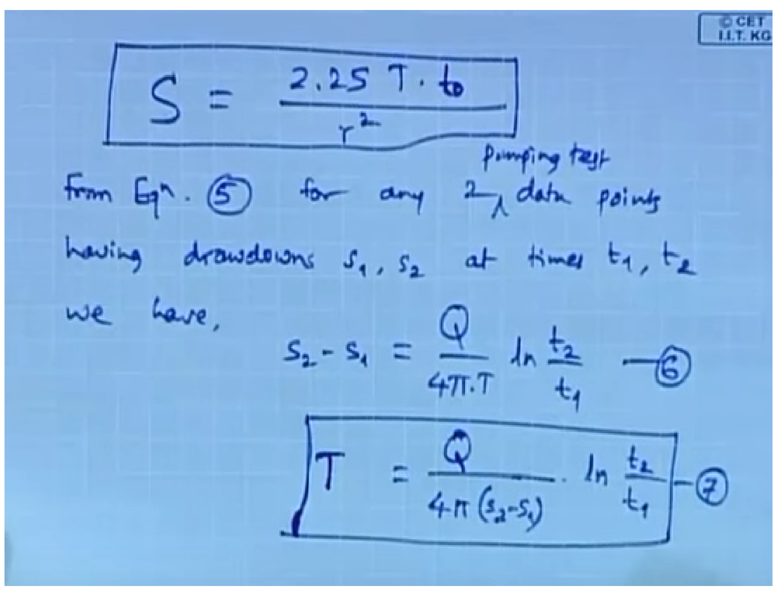

So then we say that so the storativity S is simply given by 2.25 T into T0 divided by R square and before this we need to know this value of T and we know that so the suppose denote this as equation 5 and so based on this equation 5 any two pumping test data points having drawdowns

S1 comma S2 times T1 comma T2 we have S2 – S1 = Q divided by 4 Pi into T into natural log of T2 by T1.

So here this is equation 6 and from this equation 6 we can write down the expression for this transmissivity T. So this T = Q divided by 4 Pi S2 – S1 into natural log of T2 by T1 so this is the equation 7 and by this and here obviously there S2, S1, T2, T1 the are obtained by the pumping test and Q is also obtained by the pumping test so by the pumping test data where the discharge is constant only the drawdowns are variables are unsteady.

So therefore we can estimate the value of transmissivity T and once we estimate the value of transmissivity so then substitute this value of transmissivity and the value T0 is obtained by the extending backwards the linear portion of the semi log plot of drawdown S versus the time T so we get we can estimate the storativity. So this is how we can estimate the parameter the aquifer parameters the storativity S of the aquifer as well as transmissivity T of the aquifer.

So this is this we can get this unsteady this one and here we should know the approximations so the assumptions on which this phase equation is based on.

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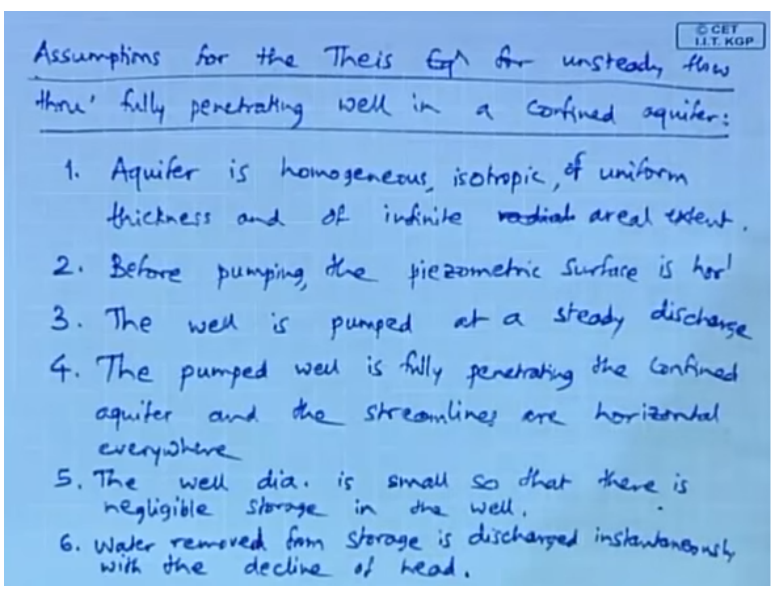

So assumptions for phase equation for unsteady flow through fully penetrating well in a confined aquifer. So let us list this assumptions so the first assumption is aquifer is homogeneous, isotropic of uniform thickness and of infinite radial extent am sorry aerial extent the second assumption is before pumping the piezometric surface is horizontal the third assumption is the well is pumped at a steady discharge rate the fourth assumption is the pumped well is fully penetration the confined aquifer.

And the stream lines are horizontal everywhere the fifth assumption is the well diameter is a small so that there is a negligible storage in the well. In the last assumptions is water removed from the storage is discharge instantaneously with the decline of head. So these are the six assumptions on which the phase equation for unsteady flow is based.

And so in the next lecture we will discuss the unsteady flow through wells in unconfined aquifers as well as leaky aquifer and here we should note that so that of course I have already made it amply clear. So the well is with pump with steady discharge okay and so in the next class we will discuss the unsteady flow through the well in unconfined aquifer. Thank you