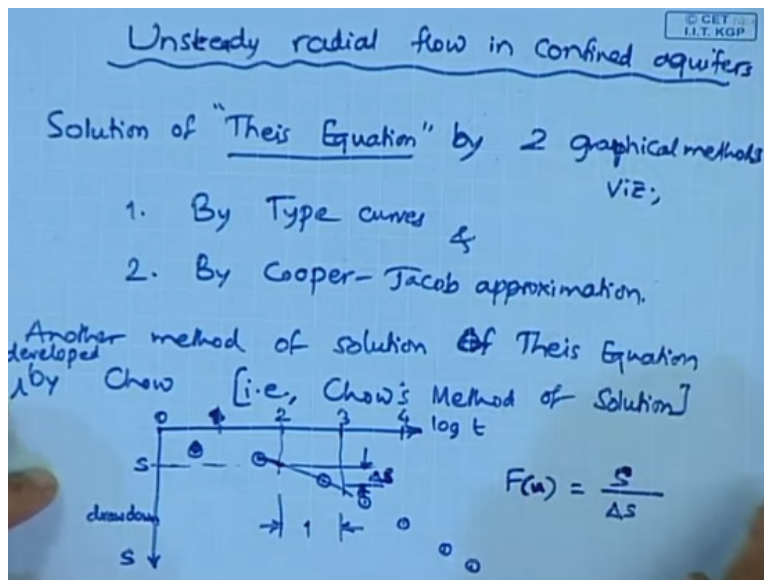


Ground Water Hydrology
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Module No # 03
Lecture No # 14
Unsteady Radial Flow in Confined and Unconfined Aquifers

Welcome to this lecture number 14 on unsteady radial flow in confined and unconfined aquifers. Here in this lecture in the previous lecture discussed on unsteady flow into the wells and in this lecture we will moving on to unconfined as well as unconfined aquifers and of course the flow is radial and it is unsteady. And here in the in the previous lecture so there was this unsteady flow equation which was solved by two methods that is two graphical methods.

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One is the time drawdown method as well as the am sorry this unsteady radial flow and confined aquifers. So in the previous we had this we discussed the solution of Theis equation by two graphical methods namely 1 by type curves and 2 B COOPER JACOB approximation and in this lecture we will discuss another method for the solution of the Theis equation which is used in the unsteady radial flow in unconfined aquifers.

And that method is the another method for solution of Theis equation by this develop by CHEW so which is that is why this method is known as the CHEW's method. CHEW's method of

solution so here what is done is so in a pumping test so the curve is plotted and semi logarithmic plot observable observation observed data is plotted on a semi-log plot.

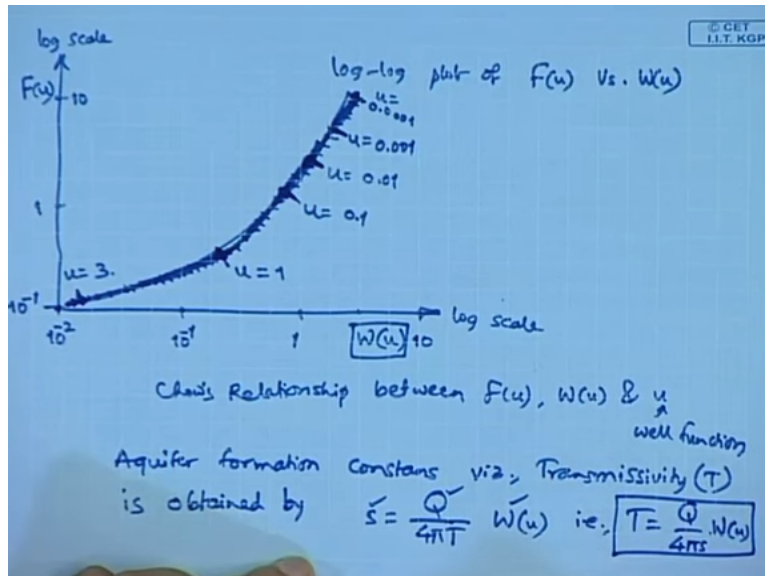
That means the drawdown axis is linear this one scale and the time axis is on logarithmic scale and here so this is on the plotted curve. So the arbitrary points have chosen and the coordinates that is the time coordinate T as well as the draw down S . So they are determined just like say here so this is drawdown S and then this is log of time T .

And here so this is a tangent to the curve at a chosen and determine to drawdown difference ΔS in feet per log cycle of time. So here actually say that is say suppose this is the say these are the points suppose we are getting. So what should be done is so this is a log at axis and here say this is may be this is 1 this is 0, this is 1, this is 2, 3, 4 and so on. So here say two points are chosen on this plot such that the difference in $\log T = 1$.

And so that the corresponding difference in this say for examples these are the two points and here and this difference is 1 where and this is so this difference is ΔS that is the change in drawdown that is one scale difference of the log value in time. And here so this FU is a parameter which is function of this well function U well function parameter. So this function = S the drawdown divided by ΔS .

So drawdown divided by ΔS so this is ΔS and then the original value of this is S here so this is a the value S the drawdown the typical value of S and then so this is ΔS . so this $F / \Delta S / \Delta S$ is a denoted as FU then what is done is the corresponding value of W and U are obtained from the figure.

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So this figure is so here we have this $F(u)$ which is $S / \Delta S$ and here we have the W that is well function. So here so the corresponding value of W and U are obtained from this figure. So here this in this so this $F(u)$ so this is also one logarithmic scale so this is a log scale and W is also log scale. So this is a log-log plot of $F(u)$ versus $W(u)$ and here so this curve will have a shape of like this so initially the slope will be flat and eventually the slope increases.

So this will be the type of curve and so these along this the curve so the value of well function U are plotted are denoted and here say typically so this is say this $F(u)$ axis may start with 10 the power - 1 and then this is 1, this is .1 and then this is 1 and this so this is 10. And similarly so this is a this is the 10 to the power -2 and then this is 10 to the power -1 and then this a 10 to the power 0 and this is 1 and then so here this is the 10 and then so on.

So basically both these scales are on this is log-log plot and here typically the values of U so they start with say here it at the top it may start with say point this $U = .0001$ that is 10 to the power - 4 and so here 10 to the power $U =$ to here $U = .001$ and somewhere here you will get $U = 0.01$ and here this is a $U = .1$ then this is here where the slope changes.

So rough this U is 1 and here this all the way it is close to this one so this $U =$ say 3 so typically this is how the value of the well function U changes and then using this. So here what is done is so they using this equation that is $F(u) = S / \Delta S$ so this 1 so this $F(u) = S / \Delta N$ let us denote

this equation as 1 and then so here the corresponding W and U are obtain from the figure that is this figure okay. So this is the relationship between FU and W.

So this is CHEW's relationship between FU WU and U the well function U so this U is the well function and so here what is done the is the formation constants that is the transmissivity the aquifer formation constants with transmissivity T is obtained by so the equation that is the typical drawdown = $Q / 4 \text{ Pi } T$ and into this W and here this W is known, S is known Q is known and so this T can be determined.

So that is T the transmissivity $T = Q / 4 \text{ Pi } S$ into WU okay and so this is the first this one the and then next this storativity.

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Storativity 'S' is obtained by the eqn

$$\frac{r^2}{t} = \frac{4T}{S} u$$

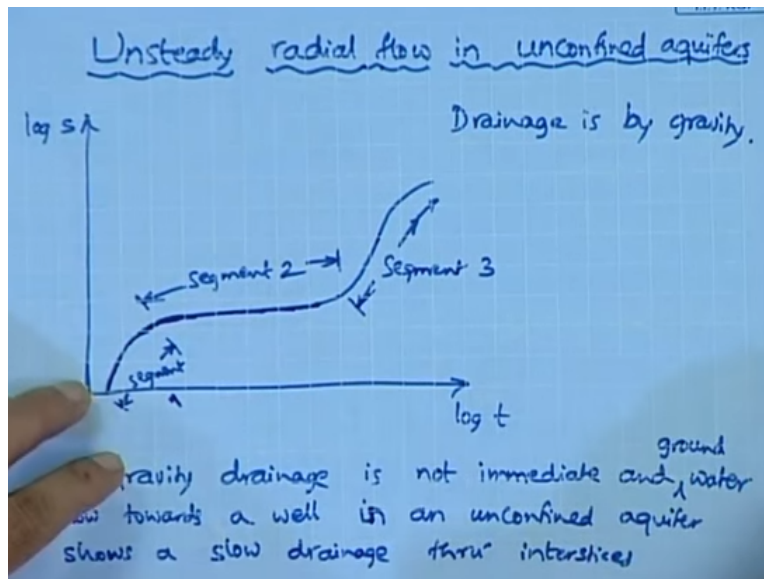
$$u = \frac{r^2 S}{4T \cdot t} \quad \text{well function equation}$$

$$S = \frac{u \cdot 4T \cdot t}{r^2}$$

The storativity S is obtained by the equation S is obtained by so this is the storativity S obtained by the equation that is $R \text{ square} / T = 4T / S$ that is the well function equation. So we know that well function = $R \text{ square } S \text{ divided} / 4T \text{ into } T$ by this expression we get that is so this is the well function equation from this the storativity $S = U \text{ into } 4\text{Pi}$ and $U \text{ into } 4 \text{ transmissivity } T \text{ into time since pumping divided by } R \text{ square}$.

So like this using this CHEW's relationship between this FU and WU so we can determine the formation constants of the aquifer okay. So now we will move on to the unsteady radial flow in unconfined aquifers.

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So this CHEW's method is a third method by which we can solve the Thies equation and now we will move on to unsteady radial flow in unconfined aquifers. So far we have the confined aquifer where in the it is under pressure the wherein the things are somewhat more straight forward I should say as compared to the unconfined condition and here in this case so this there will be it represents three types of behavior suppose we plot the drawdown S versus time T on log scale.

So this is log of S the drawdown and then the log of T the time since the pumping so here so it indicates three different kinds of nature. So the first one is this segment wherein the slope is a quite steep. So this here we can denote so this as the segment one wherein the slope is you can say it is relatively steep then so this is what happens is as a time in increases the as time further increases the drawdown just marginally increase.

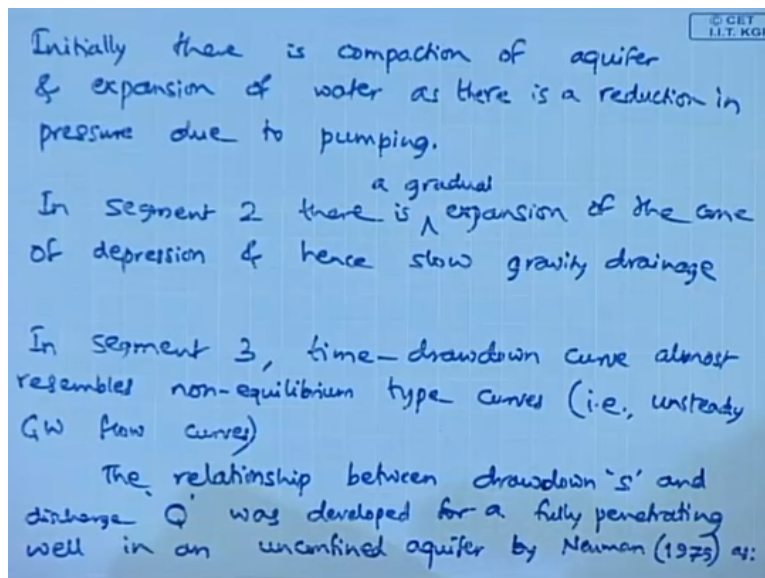
And so this one here you can you can denote this as segment 2 or stage 2 segment having a relatively steep slope then segment 2 having a relatively flat slope then again in the third this one. So this is here you can say so this is segment three so here what happens is so the gravity drainage and obviously in an in this unconfined aquifer so this drainage is by gravity and this gravity drainage is not immediate.

And obviously that is why it constitutes an unsteady flow condition and so the water flow towards well in unconfined aquifers is characterized by slow drainage of interspaces. So

basically and water flow are at the ground water flow towards well is towards a well in an unconfined aquifer shows a slow drainage through the interspaces or the pores.

That is why initially what happens is when the pumping starts so then so this drawdown increase and the this is drawdown increase relatively steeply in this first segment and then once this it has increased so when what happen is so the here so the that is the cone of depression. So here the compaction of the aquifer as well as expansion of the water as pressure reduced from pumping.

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Initially there is compaction of aquifer & expansion of water as there is a reduction in pressure due to pumping.

In segment 2 there is ^{a gradual} expansion of the cone of depression & hence slow gravity drainage

In segment 3, time-drawdown curve almost resembles non-equilibrium type curves (i.e., unsteady GW flow curves)

The relationship between drawdown 's' and discharge 'Q' was developed for a fully penetrating well in an unconfined aquifer by Neuman (1975) as:

So initially is there a compaction of aquifer and expansion of water as pressure is as there is a reduction in pressure due to pumping. So this the first segment that is so the first segment having a steep slope our steep draw down so here what happens is so this will continue for a very short while and here so the drawdown reacts similar to an unconfined aquifer. So that means so here this is the gravity here in this region it more or less behaves like a confined or artesian aquifer or pressure aquifer.

And after wards what happens is so this gravity drainage so this is basically here you can say this is segment one here you can say this is analogous to so here this segment one is analogous to say confined aquifer or artesian aquifer or pressure aquifer confined flow next here the segment two. So here so this is slow gravity flow in this segment 2 and here so this is because of the expansion of the cone of depression.

So in the in segment two there is expansion of the cone of depression here we can say is a gradual expansion of the cone of depression and hence slow gravity drainage. And so next continues and next is in the third segment in segment three so the time drawdown curve almost resembles non equilibrium type curves that is unsteady ground water flow curves. So and therefore so there are three distinct segment.

Segment one having analogous to confine confined flow segment two having slow gravity flow and then in segment three. So there is this is a unsteady flow again which is maybe again somewhat like segment one and then again this slope flattens like that.

So here so therefore in such case so this relationship between the drawdown and the discharge was drawdown S and discharge Q was a developed for a fully penetrating well in an unconfined aquifer by NEWMAN in nineteen seventy five.

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The image shows handwritten mathematical equations on a blue grid background. At the top right, there is a small box containing the text "I.I.T. KGP". The main equation is $s = \frac{Q}{4\pi T} \cdot W(u_a, u_y, \eta)$. A bracket above the $W(u_a, u_y, \eta)$ term is labeled "unconfined well function". Below this, the parameter u_a is defined as $u_a = \frac{r^2 S}{Tt}$. The parameter u_y is defined as $u_y = \frac{r^2 S_y}{Tt}$ with a note "(applicable for higher t values)". The parameter η is defined as $\eta = \frac{r^2 K_v}{b^2 K_r}$, with arrows pointing to K_v labeled "vertical hydraulic conductivity" and to K_r labeled "horizontal hydraulic conductivity".

As $S =$ the drawdown $S = Q$ divided by $4 \pi T$ the discharge divided by 4π into transmissivity and here so this is a so well function as a 3 parameters that is U_A , U_Y , η .

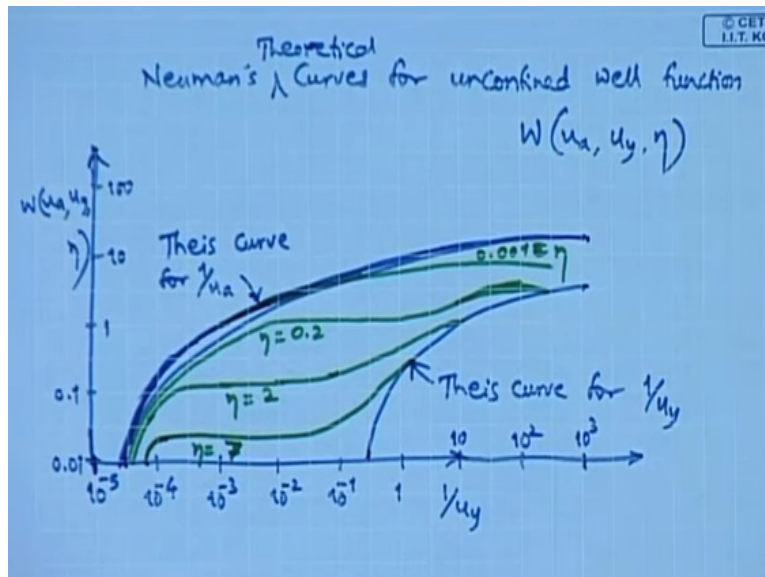
And here where this U_A so each of them represent one segment $U_A = R$ square into storativity divided by Tt . So in case of the unsteady flow in a confined aquifer it was the well function $U = R$ square $S / 4Tt$ whereas in this case so this U_A is R square $S /$ simply T the transmissivity

multiplied by the time since the beginning of pumping. And here so this is a so this is W of UA, UY and so this is denoted as the unconfined well function.

UA is given by $R^2 S / 4T$ and then UY is given by $R^2 SY / 4T$ and this is applicable for higher T values higher values of time. So it represent segment three and this ETA, so ETA is given by $R^2 KZ / b^2 KR$. So here this this KZ is the vertical hydraulic conductivity and KR is the horizontal hydraulic conductivity. And obviously R is the radius and the B that is the unconfined aquifer thickness.

So using these three parameters are that is this the unconfined well function is more complicated as compared to a well function in case of confined aquifer wherein there is only 1 parameter that is well function parameter that is $U = 4$ square that is $R^2 S / 4T$. Where as in this case it is a function of three parameters that is UA, UY as well as ETA and here the theoretical curve for this UA and UI as well as ETA are given by this NEWMENS curves.

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So the NEWMAN's curve for unconfined well function that is WUA comma UY comma ETA. So here in this NEWMAN's curve we have along the vertical axis of course here also this is log-log plot and so here we have so this is a W UA, UY and ETA and here it is starts with say .01 and then .1 and this is 1, 10, 100. So this is the unconfined well function which is plotted along the vertical axis and then here we have that is a 1 by UY along the horizontal axis.

So it starts with say 10 to the power -5 then 10 to the power -4, 10 to the power -3, 10 to the power -2, 10 to the power -1 then this is 1 then further extends so this is 10 then this is a 10 square of hundred and then this is 10 cube or 1000 and here what happens is somewhere between this 10 to the power -1 and 1 so here this is the curve goes up to so this is 1. And then similarly somewhere between 10 to the power -5 and 10 to the power -4 and so this curve goes up to say little over 10.

And here so these are the so this is the here actually let me so this is the Theis curve for 1 by UA and this is the face curve for 1 by UY and in between we have color this one that is so this is a 7 and say this 2 this is 2 and so here this is .2 and next here this is .001. So this is .2 and this .001. So these are the so this is ETA values so this is $ETA = 7$, $ETA = 2$, $ETA = .2$ and this this ETA. So this is a $ETA = .01$ so this is how theoretical curves for a the can say this is NEWMAN's theoretical curves for unconfined well function.

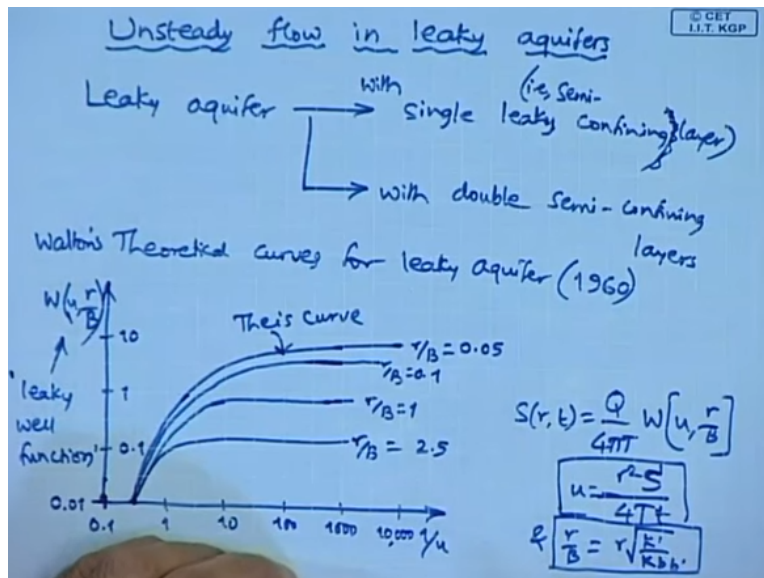
So like this here we will get the it is more complicated as compared to the unconfined flow radial flow in a in a confined I am sorry un steady radial flow in a confined aquifer where in so it is a there it is only there is a well function parameter that is $U = R^2 S / 4T$ into T. Whereas in this case so there are there are three parameters one is UA which is given by $R^2 S / TT$ representing the first segment UY which is $= R^2 SY / TT$ which is applicable for the higher values of T representing the third segment.

And then ETA which is a ratio of square / B square that is the distance from the well axis the radial distance from the well axis square of the that divided by the square of the unconfined aquifer thickness of course that is the itself is a variable multiplied by the ratio of vertical hydraulic conductivity and the horizontal hydraulic conductivity.

So like this so the in this the unsteady radial flow in an unconfined aquifer is even though it is a unconfined aquifer is the one which is much which is the first aquifer as you as we encounter when we go from the ground surface. But there so because it is at the top and then sot here it is more it is subjected to more fluctuations because of the natural as well as naturally ground water recharge and it is the reason and many times if it is a this one.

In some cases where the even evaporation may also predominant role at if they okay if there is a tropical desert kind of situation like OSS or anything. so therefore it is represents more intensive so unlike the unsteady radial flow in confined aquifer. So this unsteady flow in unconfined aquifer represents more this one so now with this we will go to we will just briefly start with these leaky aquifers.

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So that is the unsteady flow and here so this leaky aquifer so they may have either bottom confining layer which may be leaky and of course one if the bottom layer is having is having more perforations or in that case what happens is there will be aquifer will be losing water whereas on the other hand if the top confining layer is more leaky as compared to the bottom confining layer.

So in that case this leaky aquifer may gain in terms of ground water so therefore so they it represents the entirely different this one and here so this is the WALTON presented the theoretical curves for leaky aquifer. So this is a so this leaky aquifer may have say with single leaky confining or other confining means says semi confining layer say leaky that means semi confining layer.

Or with double semi confining layers so double means this confining layer may be at the top or as well as at the bottom so in this case the theoretical curves were developed by WALTON in nineteen so theoretical curves for leaky aquifer. So this is the WALTON's theoretical curves for

leaky aquifer. So they were developed in the year nineteen sixty and here so similar to the well function for the unconfined aquifer.

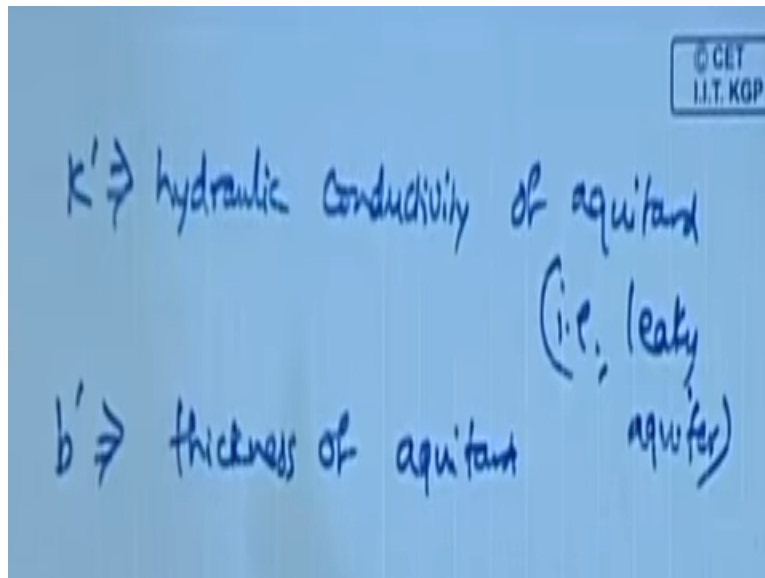
Here we have a well function for the leaky aquifer and that is denoted by $WU, R/B$ and here we have this is $1/U$ and of course both are on this one and here this axis to the $WRU, R/B$. So that is this is denoted as leaky well function so this is in case of confined aquifer it is WU this is well function that is W . Whereas in case of unconfined aquifer we have unconfined well function.

So that is $W, U, R/B$ whereas in this case the leaky aquifer so this is a somewhere in between a confined aquifer and an unconfined aquifer. So here there are two parameters the first parameter is U and the second parameter is R/B and so this is the leaky well function that is $W, U, R/B$ and it is the theoretical WALTON theoretical curve. So it is here the WU axis will start at .01 and then this is .1 so this is 1 and then 10.

Then similarly here the $1/U$ axis will start at .1, 1, 10, 100, 1000 and 10,000 or say 10 to the power 4. And here the Theis curve is the one which starts somewhere in between that is .1 okay that is .1 and 1 and it starts here and this is the Theis curve. So here this is $R/B = .05$ and this is the Theis and here for this form same point so this is $R/B = 2.5$. And in between so there are different this one so this $R/B = 1$ and here this is $B = .01$ s.

So like that so in this case the drawdown which is function of R and T is given $Q / 4 \pi T$ into the leaky well function that is W of $U, R/B$ and again so here this U is the same as the confined well function parameter that is $R^2 S / 4 T T$ okay and this and $R/B = R$ into under square K dash / $K B$ B dash okay. And this U is same as this one and here this is the here this B dash is the aquitard thickness.

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So this is the K' is the hydraulic conductivity of aquitard or leaky aquifer so that is leaky aquifer and b' is the thickness of aquitard that means leaky aquifer and K and B are the for the regular aquifer okay. We will stop here and we will continue in the next lecture on we will move on to the further topics in this well hydraulics thank you.