Ground Water Hydrology

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Module No # 07
Lecture No # 35
Modeling and Management of Ground Water Ground Water Stimulation Models, Ground Water

So welcome to lecture number 35.

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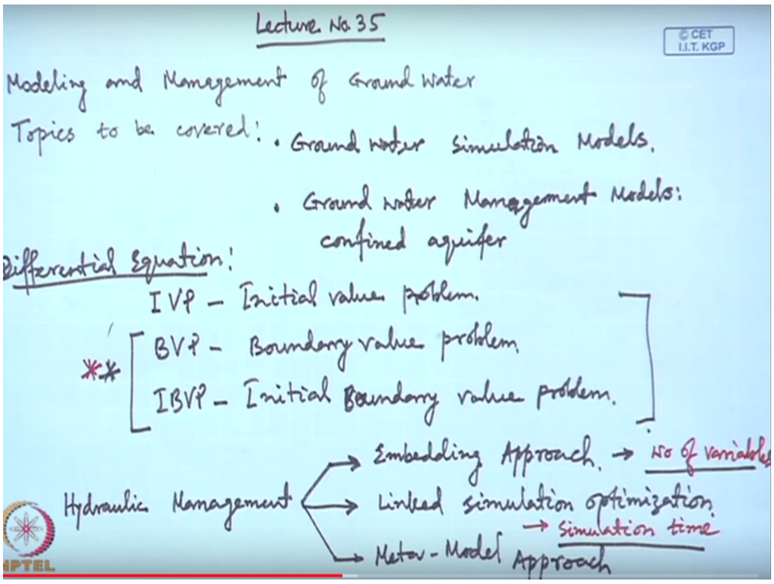

In this particular lecture we will start this modeling and management of ground water. And under this we will cover ground water simulation models and ground water management models. And particularly for confined aquifer. So in under this modeling and management of ground water it has got two aspects.

One is simulation aspect. Another one is management aspect. In simulation aspect we try to see the physical processes by using our governing equations that we have derived in our previous lectures. And in management model we try to see what if kind of scenarios. If we do certain kind of management, or we put certain kind of restriction on the ground water use or restriction on variables then what will be the situation in near future or in long term aspect?

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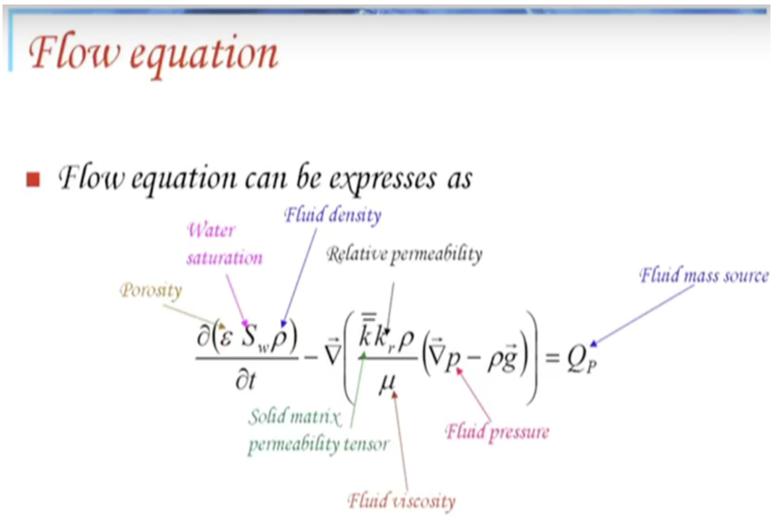

$$\frac{\partial(\varepsilon S_w \rho)}{\partial t} - \vec{\nabla}\left(\frac{\bar{\bar{k}} k_r \rho}{\mu}\left(\vec{\nabla} p - \rho \vec{g}\right)\right) = Q_P$$

Ok so in flow equations under modeling that has got two components. One is the temporal component. The other two parts are spatially based components. So in this one the first one this epsilon. This is basically porosity.

Then we have water saturation and fluid density. So basically if this combined term, the multiplication of porosity, water saturation, fluid density this is temporally varying, then we can say that either we can model that density dependent or density independent phenomenon. For density independent phenomenon, the equation will be slightly modified because it will not be varying with the density aspect.

So other parameters in this equation, solid matrix permeability tensor we have fluid viscosity. This is basically dynamic viscosity of the fluid. Then relative permeability, and fluid pressure. We have fluid mass source. So in the flow equation this rho value or this g value this is having only component in the vertical direction. So this is basically a modified form of our Darcy's equation. If we consider it with the fluid pressure term.

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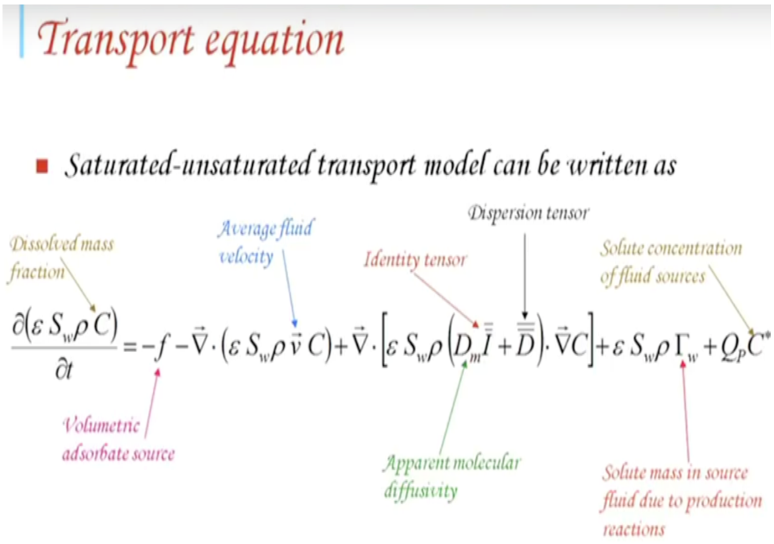

Next one is saturated unsaturated transport model. So this is basically porosity water saturation density. And this is dissolved mass fraction. Then volumetric adsorbate source. Other things that is average fluid velocity. Then we have apparent molecular diffusivity and identity tensor. And this is dispersion tensor. And this is our mass solute mass in source fluid due to production reactions.

And this is Qp into C star, this is C star is basically solute concentration of fluid sources.

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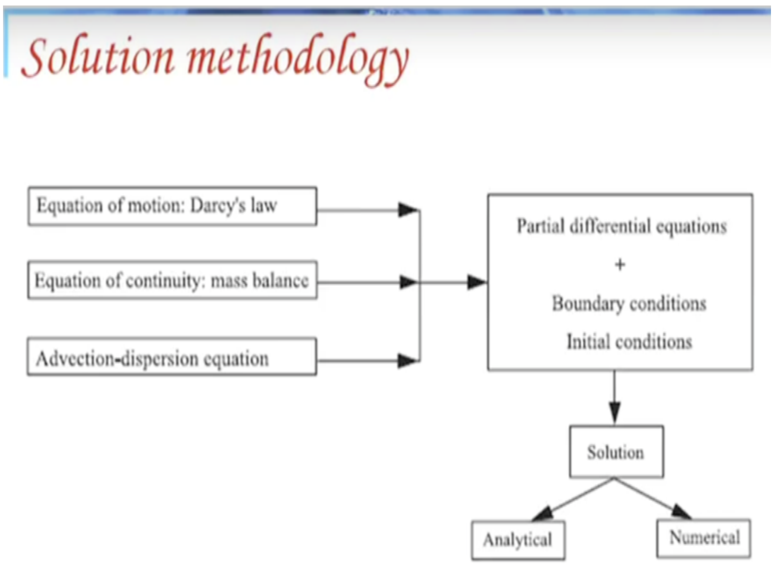

So our solution methodology is basically solving the basic governing equations. So what are the basic governing equation we have covered in this particular course? First one is Darcy's law. This is our most fundamental equation. That is relates the equation of motion, or if you say that is one form of our Navier-Stokes equation if we compare it with fluid mechanics.

Then we have equation of continuity. That is uh mass balance equation. And finally we have solute transport equation or advection dispersion equation. In advection differ dispersion equation also we can have other components like absorption including isotherms, or our radioactive decay. And these terms will be there to model the whole thing.

Now these three equations are our fundamental equations. These are basically if you consider it in one-dimentional then it will be mostly if you are considering a steady state case, then it will be normal differential equations or ordinary differential equations. But most of the cases depends on both space and time. So these equations are basically partial differential equations. Then we need boundary conditions. So boundary conditions and initial conditions.

So for a particular problem we can have either a initial value problem, IVP, partial different differential equation, IVP or initial value problem. Or BVP or boundary value problem. Or IBVP, or initial boundary value problem. So our equations are mostly in these two forms. In these two forms, so solution can be obtained for these two equations using either analytical or numerical methods.

To solve these two or solve this equation in form of analytical solution we need to have some kind of simplified form of the equation. Otherwise it is difficult to find out the complex analytical solution for a complex hydrogeological system. Numerical solutions are easier to find out because we generally used a discretized form of the equation. And we try to solve it with algebraic equations.

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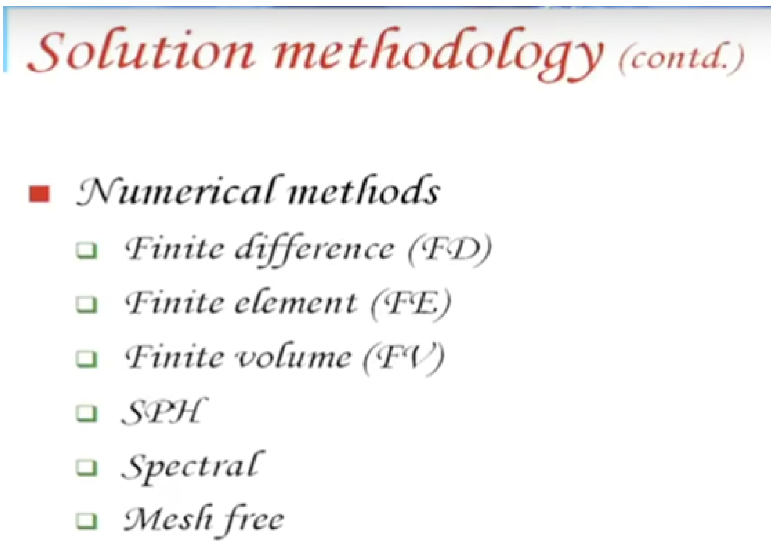

So in case of numerical methods either we can use this finite difference, finite element, finite volume. SPH that is uh smooth particle hydrodynamics, spectral methods or any mesh free methods for this purpose.

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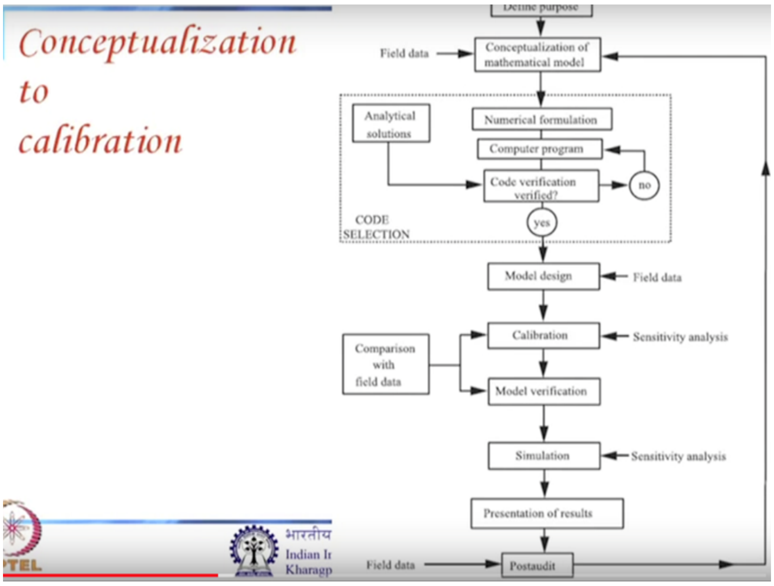

So what is the conceptual form of calibration? So first we need to have some kind of definition or we should have proper definition of our purpose. What is our objective? Whether we want to model the thing or we want to model it for some kind of management strategy. So we will have some kind of field data.

And from field data we can create the conceptual model that is conceptualization of the mathematical model. So we need to identify the governing equations, that will be required for a particular problem. So identification of governing equation then boundary condition, initial condition. So next thing is numerical formulation. Numerical formulation, we can discretize the equation.

We can write the computer program, code verification. This part is important because we need to some kind of verification of the code with the existing solutions. So solutions can be either existing analytical solutions. So existing solution for a two-dimentional problem. But two-dimentional problem or three-dimentional problem the problem is that if we start with the reality thing, then there will be certain kind of errors.

So it is better to use that analytical solution for code verification. So if the code is not verified, then we can go back to the computer program and we can try to rectify or correct our errors. So code selection is important. So either one can write his or her own code or one can select a proper code which is either a open source or a commercial one. So if code verification is complete or code identification or code selection is complete, then we can go to this model design part.

So with the field data we can design the model, then calibration. Calibration comparison with the field data. So one way model verification is done and calibration is done. And this calibration should be supported by the sensitivity analysis. With the sensitivity of the parameters there should not be much change in the model results. Because if we are identifying the parameters in such a way that it is not a proper identification for which there will be not much variation, then it will be a problem.

So we should have uh proper identification and sensitivity analysis with our problem. And then after model verification we can go for simulation. With simulation also we can correct our sensitivity analysis. And final is presentation of the results. If there is some kind of monitoring network, then finally we can monitor the thing.

And we can correct our assumptions or conceptualization with those monitoring data. And again we can start from the initial part and we can conceptualize it the thing. And we can redo the whole process so that we can correctly adjust the model to get the proper results. So code selection in this case is an important part of the modeling exercise. Either one can have own code one can have their own code or they can readily select some codes which are available in public domain.

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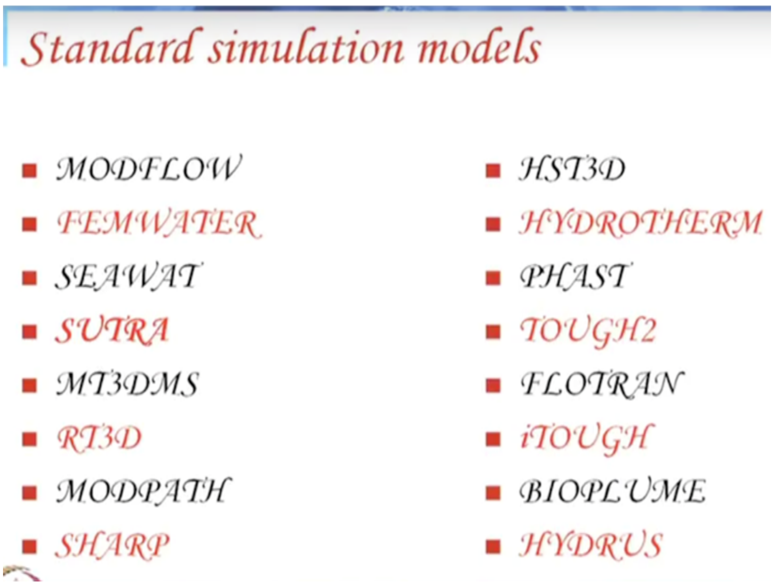

So standard simulation models that is available in the public domain or with some restriction or in commercial domain. These this is one comprehensive list of those models. So first model is the MODFLOW this is basically three-dimentional finite difference based simulation flow simulation model. This is FEMWATER.

This is 3-D finite element model for saturated unsaturated flow through a porous medium. SEAWAT, this is saturated ground water modeling or density dependent ground water modeling in coastal aquifers. SUTRA, this is saturated unsaturated flow and transport model. With this we can model it the ground water flow and transport in any kind of aquifers. MT3DMS, this is for contaminant transport with multiple contaminants. RT 3D, this is reaction with reaction.

MODPATH is flow path identification thing in ground water. Then SHARP this is a two-dimensional quasi 3-D simulation model for salt water intrusion modeling. And this is based on sharp interface. Then HST 3D, this includes also that heat transport part. Other models that can be named here are this HYDROTHERM, PHAST.

This also include the chemical reaction part. TOUGH2, flow thru fractures medium, FLOWTRAN, iTOUGH, the inverse tough thing. BIOPLUME, this is one. HYDRUS this is also important software to the model flow in unsaturated porous medium. So the identification or selection wise we can select any of these models for our modeling purpose. Or we can discretize our governing equations and we can use our standard discretization methods to get the solution.

So this is the starting of our modeling exercise. So in modeling and management of ground water we have two aspects. One is the modeling aspect. In modeling aspect we will see what is the simulation results. And the policy aspect or management aspect we try to formulate some kind of management strategy.

So for hydraulic management I am talking about the hydraulic management because if we are talking about the flow part only, then only we say it as hydraulic management. Otherwise we need to have certain kind of management models with transport mechanism. So in case of hydraulic management, hydraulic management we can have equations in the form of embedding technique, embedding approach.

In embedding approach, the governing equations are directly discretized. And these are used within the optimization model or decision models to get the solution. Next is our linked simulation optimization, simulation optimization. In linked simulation optimization the simulation model is directly linked with the optimization model or decision model to get the solution.

So first approach, in the embedding approach we generally write the equation in discretize form and we directly use it in the decision models to get the results. But the problem with the embedding approach is that as we are discretizing it uh for a number or points or for a number of grids. So number of decision variables, that is crucial for this kind of approach.

But in case of linked simulation optimization the simulation model is doing the simulation part separately and only with limited number of variables, we can solve the decision model. So first approach concern is variable, number of variables. The second approach the concern is simulation type.

If we are setting up a complex simulation model then for each simulation, it will take a significant amount of time and that will be a limitation for this linked simulation optimization. Then the third approach which can overcome these two limitations that is called meta model approach. Meta model approach.

In this meta model approach basically meta models are trained with the simulation results from original simulation model. And we try to minimize the meta model errors between the meta model output and the output from original simulation model. So at each iteration there will be correction in terms of correction of errors. And that can be assessed with certain kind of indicator reference functions.

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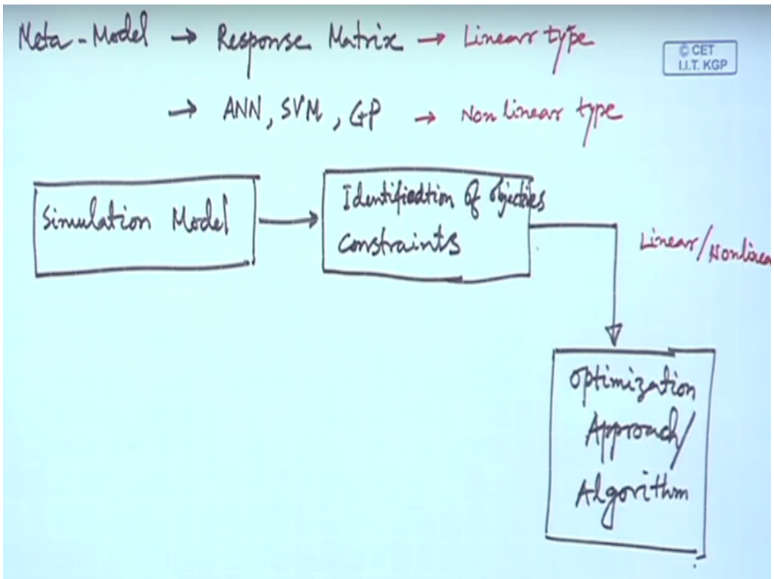

So in meta model approach the most primitive approach is response matrix. Most primitive approach is this response matrix. So response matrix approach. What is the advantage? Advantage is these are basically linear type.

With the linear type of response matrix thing we can easily form some kind of linear model based decision model. And we can quickly get the solution. But the problem is that response matrix will always give you some kind of linearized results. And one way we are compromising with the accuracy of the simulation model. So for a complex nonlinear general equations or general problem we cannot represent it properly with our response matrix approach.

So we need to have certain kind of approach in which we can consider that linear model thing nonlinear model thing. So now a days people are using this ANN, SVN, GP based models. So what are these? ANN is artificial neuron network. SVN is support vector machine. And GP is genetic programming.

These models can consider or can represent the nonlinear behavior of the complex hydrogeological system. So now we can have the approach where we should be defining our equations. Then some optimization methodology and then we should have the solution. So first part is simulation. Simulation model, that is original simulation model or uh a meta model.

Then identification of objectives, identification of objectives. Constraints, so from simulation model through identification of objectives, constraints. Then based on constraints and objectives, whether they are linear or nonlinear we can find out the optimization approach slash algorithm. So this is a process where first identification of simulation model. Then identification of objectives constraints, then depending on linear or nonlinear behavior of the objectives or constraints we can identify the optimization algorithm.

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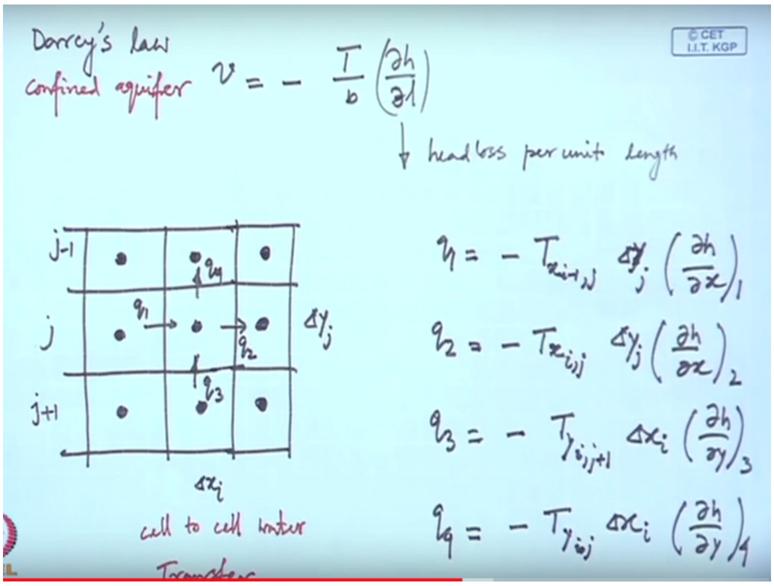

So let us start with the simulation model. So for as per Darcy's law, we know that Darcian flux V for any confined aquifer, that is for any confined aquifer can be defined as V del H / del l. So this is basically head loss per unit length. So now with this we can have one finite difference grid. So these are basically nodes. So black dots are nodes.

This is level say J + 1, this is j, this is J - 1. This is ith cell so del XI And let us say that Q3 is entering then Q2 is leaving, Q4 is leaving and Q1 is entering. And this is your del YJ. Basically this is cell to cell water transfer. So here we can say that this Q1 is basically minus TXI - J del YJ / del X this is 1.

This is Q2 is basically TXIJ del YJ del X second one. This is Q3 - YIJ + 1 del xi del H / del Y third one. And the fourth one and the final one is TYIJ that is del XI del H / del Y4. So in this case TXJ is the transitivity in the X direction of the element IJ. So TI, TIJ and this is 2 element I + 1 J. So from this point to this point, so in this direction this TXIJ that is active.

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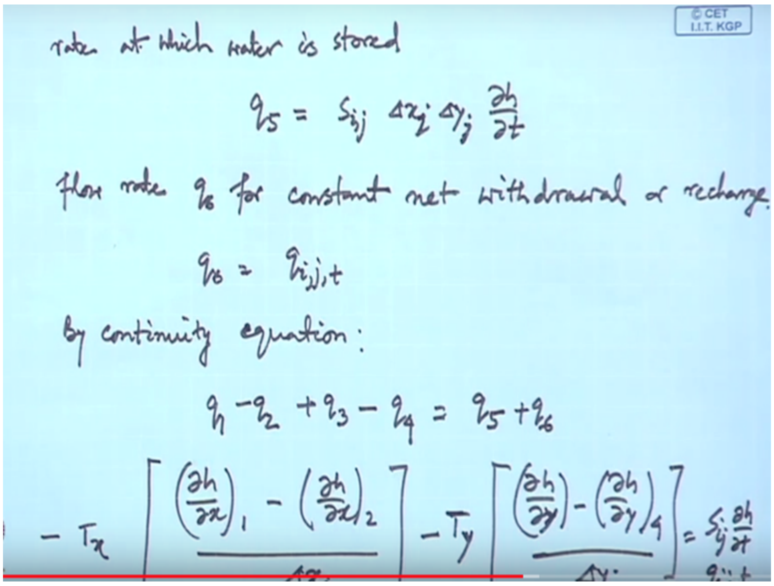

So that is rate at which water is stored. That is considered as Q5 so SIJ. Where SIJ is your storage coefficient this is del YJ and this is del H / del T. And in addition to that flow rate, flow rate Q6 for constant net withdrawal or recharge. So Q6 is basically QIJ and T. This is varying with T.

So by continuity equation maybe with the continuity of the flow we can write it as q that is entering, Q2 that is leaving + Q through Q3 that is entering - Q4 that is leaving the thing. And Q5 is again that is the rate at which it is stored so it is equivalent to the storage + Q6. So with this if we replace all this previous expressions for Q1, Q2, Q3 and q4, we will get this is TX.

This is del H / del X1 this is del H / del X2. This is del XI and this is TY with this del H / del Y del H / del Y 4 and this is del YJ. So SIJ del H / del T + QIJT / del XI and del YJ.

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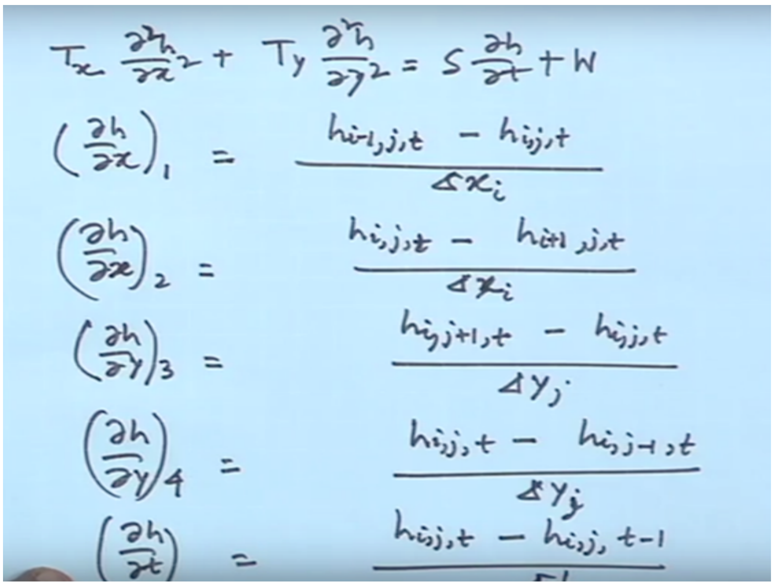

So in finite values of del X and del Y we can write the equation as TX del 2 H / del X2 + DY del 2 H, del Y2 S del H / del T W. So if we talk about finite difference thing, then we can write this equation as del H / del 1. And this is basically JT and HIJT del XI del H / del X for second phase that is IJT - I I + 1 JT / del XI. So del H / del Y third one, or third phase that is IJ + 1 T - IJT / del YJ.

And the last one that is the fourth one is IJT - HIJ - 1T del J. So with this and del H / del T is HIJT - HIJT - 1 / del T.

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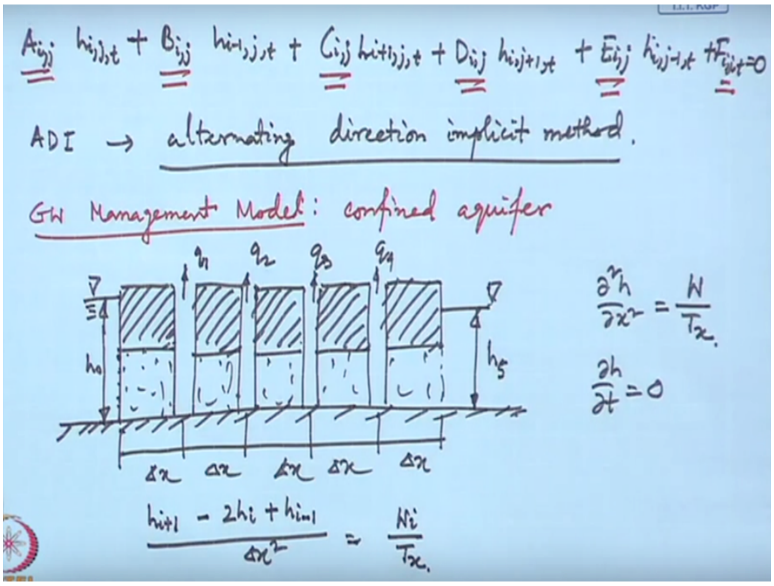

So if we substitute these five expression in our previous equation, then we will get a compact form of the equation in terms of AIJ HIJT + BIJ H - 1 JT + CIJ and HI + 1 JT + DIJ IJ + 1 T + EIJ HIJ - 1 T + FIJT = 0. Where these terms are basically coefficients involving our known values or provided information. So this can be solved using any standard numerical technique like ADI or alternating direction implicit method, alternating direction implicit method. Or so this is the form of the discretized equation. These can be directly used for embedding purpose.

Now we are concerned about the management model and ground water management model. There we are concerned about the confined aquifer. So in case of confined aquifer let us say that our configuration is like this. This is the base or lower portion. Then we have in between the impermeable part Q1; this is Q3, and this is Q4. So basically this is the aquifer material there.

And this is the water height which is H5 at this level. And all are having equal del X difference. And on the left-hand side we have h naught value of the water level. So it is bounded problem. Both cases we have specified boundary condition. This is also called as kind of boundary condition.

So for this kind of equation we have del H / del T del H / del X = W / TX. Where for a steady state condition we have T = del H / del T = 0. So for this kind of equation let us discretize it using our normal finite difference method for two. Second order finite difference we will get del X squared WI TX, so this is the discretized form of governing equation. And we have 1, 2, 3, 4 pumping from this particular aquifer. And this h naught and this H5 are two water levels on left and right-hand side.

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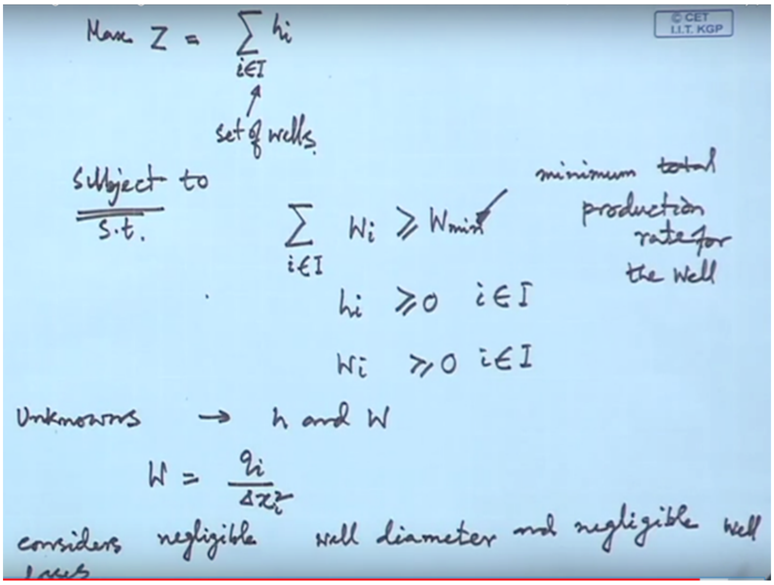

So our objective is to maximize. So first we have completed the simulation part. Now we are concerned about identification of our objective function. So our objective function is maximization of our total head, total head for i that belongs to the set of wells, set of wells. And this is subject to or subject to or ST we can write it as this I this WI is greater than W min.

And HI is greater than equals to 0. Wi is greater than equals to zero again i belongs to that set I. That W min is the minimum total production rate. So rate for the well. So the total rate should greater than or equal to the minimum rate that is specified. So unknown for this problem are unknowns H and W.

So once the model is solved uh this can be determined, W can be determined as QI divided by XI del X squared. So head objective that thing is for managing uh the aquifer. So the above formulation, one limitation is there. That is it considers negligible well diameter and negligible well losses. So for any example problem we can define the same thing we can define the same thing using our this particular approach.

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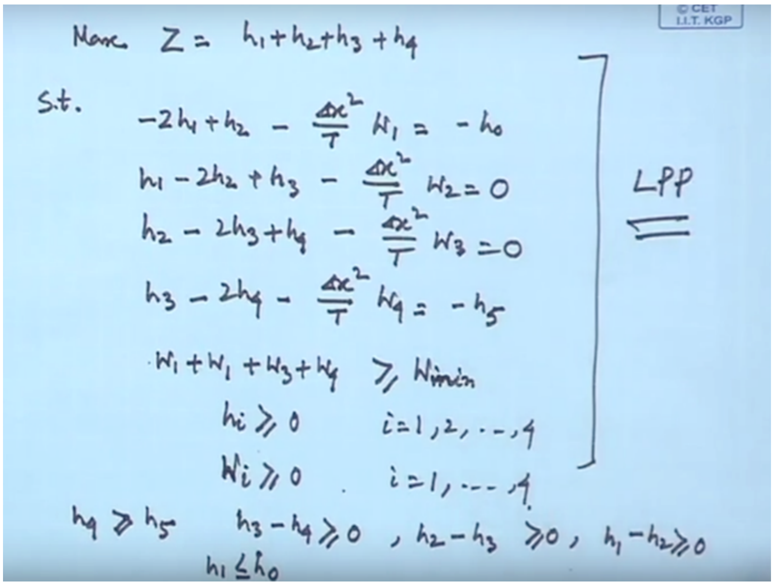

So for our original problem we can write the equation as the maximize Z = H1 + H2, H3, and H4. And this is subject to our finite difference equations and that is - 2 H1 + H2 - del X square / T W1 - H naught. This is H1 - 2 H2 + H3 - WX square / T W2 = 0. 2 H3 H4 - del X square Q3 0. This is H3 - 2 H4 del X square T W4 this is - H5.

So we have defined our constraints. We have defined our constraints and the final constraint is this one that is there is minimum value of total pumping in production wells. And HI greater than 0 that is I = 1 to 4. And Wi that is also positive I4. So in this model, additional constraints we can put that H4 should be greater than H5.

Then H3, H4 greater than 0. That means H2 H3 greater than 0. And H1 - H2 that is greater than 0. And finally H1 is greater than or less than H naught. That means your head on the upstream direction or you can say that where we have a higher head value that cannot exceed or that cannot be a lower one compared to your down gradient value.

So these are the additional constraints that will be required for solving the thing. And the full equation as because this is linear in nature, we can solve it using LPP. So this is all about the management of confined aquifers. Ok thank you.