Numerical Methods in Civil Engineering
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Lecture No - 22
Analytical Methods for Solving the Wave Equation

In lecture twenty two of our series on numerical methods in civil engineering, we will continue with our discussion of analytical methods for solving the wave equation, we illustrate last time we talked about the eigen function approach by solving the wave equation. And we illustrated the use of the eigen function approach by solving the one d wave propagation problem in a ring of unit circumference.

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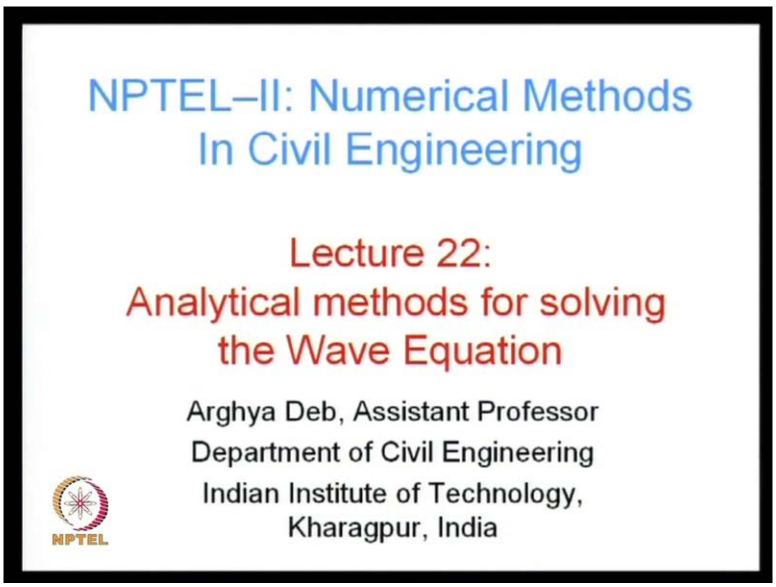

We solve del two phi del t square c square del to phi del x square over the domain shown in the figure. So, the shaded region is the ring, and here i'm showing that it is of unit circumference, because of the nature of the geometry [noise] the boundary (refer time: 01:00) condition is periodic that is phi at x is equal to zero that is it start. If we start from here if this is our starting point phi at x is equal to zero must be equal to phi at x is equal to one must be equal to phi at x is equal to two and so forth .

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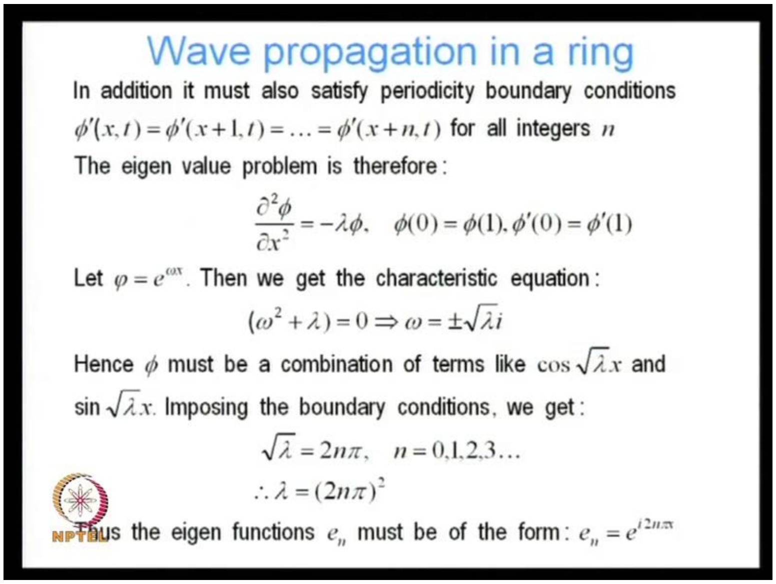

So, the phi must be periodic similarly the slope of phi that too must practiced by the periodicity condition. So, phi prime x t must be equal to that x prime if phi prime x plus one t, and so forth. And we said that to solve this problem, we have to first solve the eigen value problem. This is the eigen value problem, which resolved we have obtained the eigenfunctions of this form n is equal to e to the power I 2 pi n x, and we said that since the eigenfunctions are complete.

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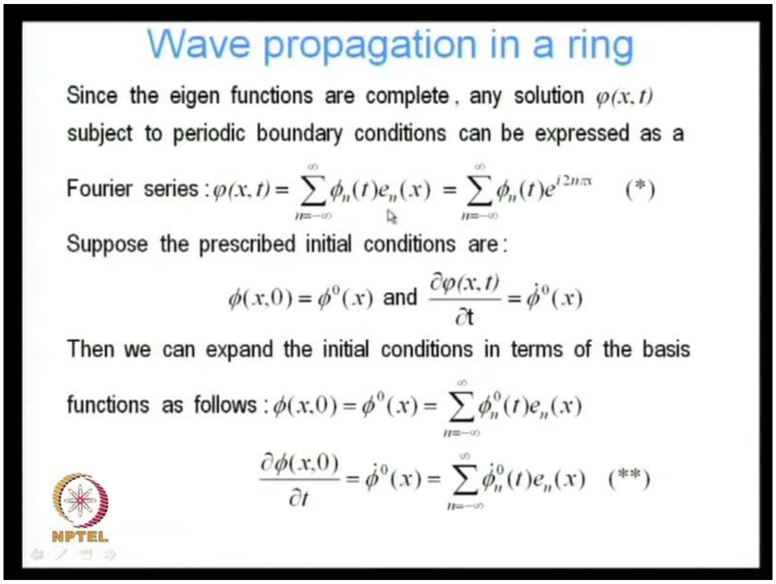

Any solution can be expanded as a fourier series in the eigenfunctions coefficients phi n [vocalized-noise], and you can evaluate (refer time: 02:00) the coefficients phi n taking into account the fact that [vocalized-noise], we have our boundary you have initial conditions right. And if we substitute them in the differential equation this expansion for phi m right, if we [vocalized-noise] phi if we substitute that in the differential equation [vocalized-noise], we can [vocalized-noise] we get [vocalized-noise], and taking into account the fact [vocalized-noise] that eigenfunctions are orthonormal. I can get a differential equation in phi n, which we then solved and finally we got an expression like this.

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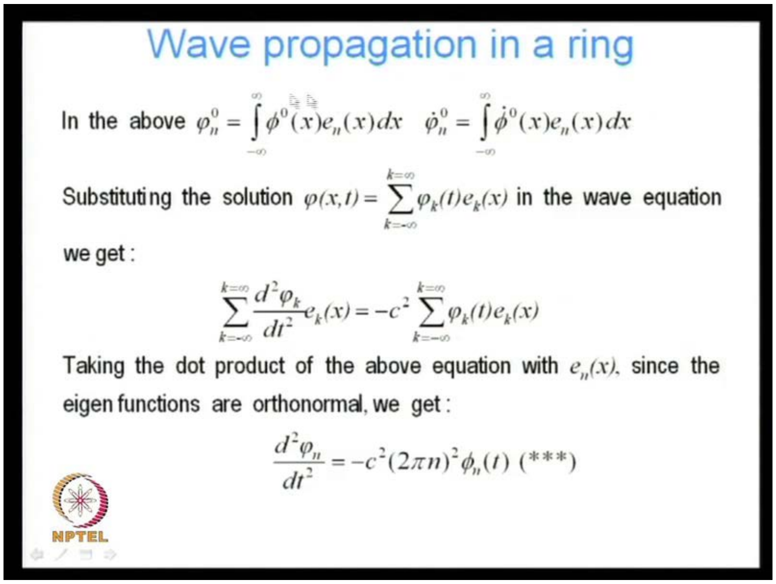

And this is, where we stopped last time substituting this back this expression for phi n, which we got here after solving the initial value problem, we solved we substituted that back in the fourier series. So, now everything is known the coefficients have also known

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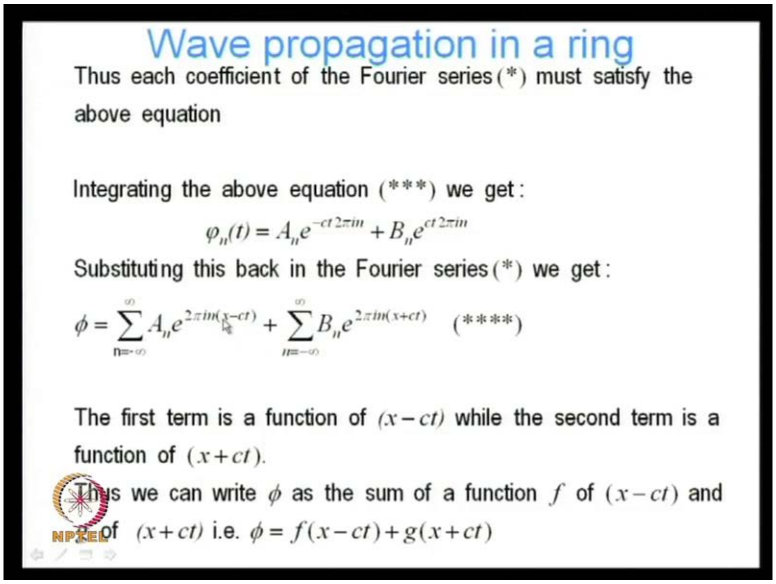

the eigenfunctions are known. So, we have a complete solution for the wave equation (refer time: 03:00) for the ring.

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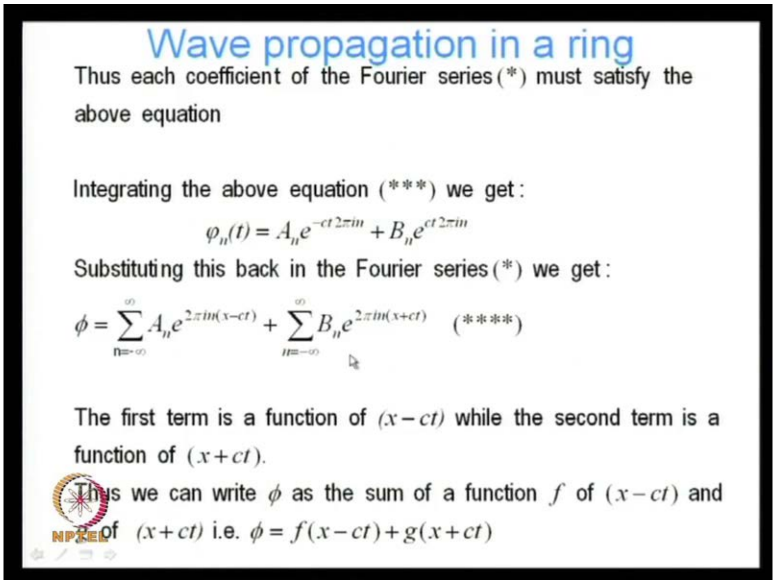

Now, if we look at this expression the solution phi is equal to n e to the power two pi i n x minus c t plus. This term the first function the first term in the solution is a function of x minus c t, while the second term is a function of x plus c t. Thus we can write phi as the sum of a function f of x minus c t and g of x plus c t. That is phi is the sum of two parts one part depends on x minus c t the other part depends on x plus c t.

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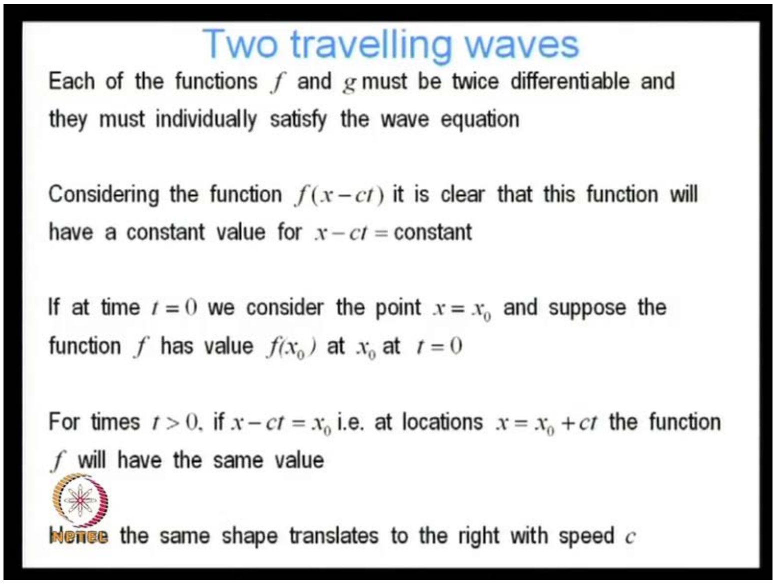

Each of the functions f and g must be twice differentiable that is obvious right, because I have in my wave equation involves secondary (( )) right. So, f and g must be twice differentiable, and since its linear then [vocalized-noise] my equation is linear they must individually satisfy (refer time: 04:00) the wave equation. Both f and g must individually satisfy the wave equation.

Let's look at the first part of the solution f of x minus c t. It is clear [vocalized-noise] that this function will have a constant value, wherever x minus c t is a constant right. If at time t is equal to zero, we consider the point x is equal to zero, and suppose the function f has value f of x zero at x zero at t equal to zero. So, at time t is equal to zero [vocalized-noise], we are considering the point x is equal to x zero. So, at time t equal to zero the function f has value f of x zero right. So, for all times t greater than zero if x minus x t is equal to x zero right. Then at all locations, which I given by x is equal to x zero plus (refer time: 05:00) x t since x minus x t is equal to x zero at all locations s is [vocalized noise] x is equal to x zero plus s x c t the function f will have the same value right. So, what does this mean physically? It means that at least for the first part of the solution what it means that the same wave translates to the right with the uniform speed of c right. The information at t is equal to zero, which was f is equal to x zero at point x zero on the t is equal to zero axis right. If I draw an x t axis at t is equal to zero at point x zero, whatever the value of f was right, f of x minus c t was that is going to get as time increases as t goes from zero to infinity that information is going to propagate [vocalized-noise] (refer time: 06:00) with the constant speed of c right.

So, this is exactly why the equation is called the wave equation and the parameter c in the equation is known as the wave speed. You can see that what the information the f of x zero right the initial [vocalized-noise] the initial condition right initial solution on f of x zero is going to move to the right with the speed of c right, because it is going to be f of x zero at all points x is equal to x zero plus c t. So, that means it is moving to the right, because x is increasing right. If I have x [vocalized-noise] x t x is increasing. So, it is moving to the right. It's moving to the right the information to f of x zero is getting propagated to the right with the uniform velocity of c that is why this equation is known as the wave equation.

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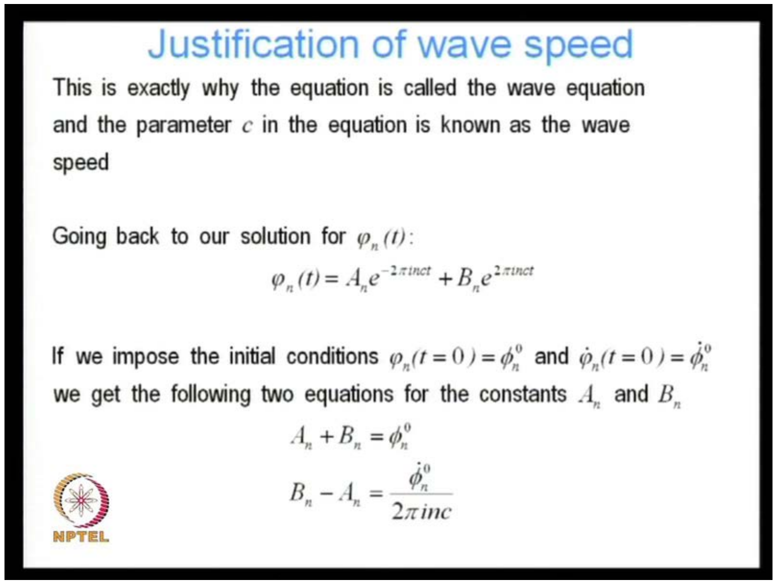

So, [noise] let's go back to a solution for phi and t (refer time: 07:00), which with which we got right there, and we can see that phi n t is of this form right. If we impose the initial condition phi n at t is equal to zero is phi n zero, and phi dot n t is equal to zero phi dot [vocalized-noise] n zero, we get the following conditions for the constants a n and b n. So, we impose the initial conditions to evaluate these constants.

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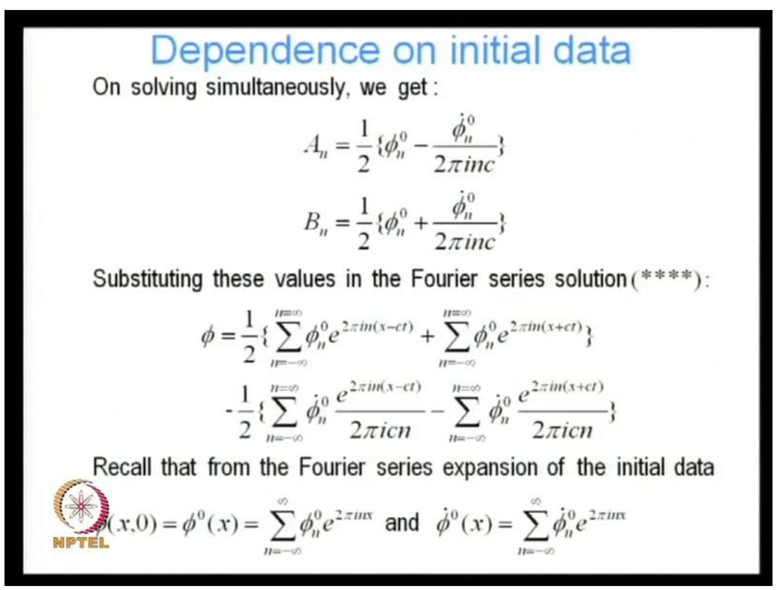

We find these constants, we solve this two equations simultaneously we get our a n and b n of this form. Substituting these values in the fourier series solution, which was this equation right the final fourier series solution [vocalized-noise], we get an expression for phi of this nature right of this nature when these are quantities, which dependent entirely on the initial conditions (refer time: 08:00) right [vocalized-noise].

We call that from the [vocalized-noise] fourier series expansion of the initial data, we had phi zero is equal to [vocalized-noise] this is how we define phi zero n, and phi dot zero. And so, because we did that the first term in the series is nothing, but phi zero x minus c t, because phi zero x [noise] look at this phi zero x is equal to sigma n to n equal to minus infinity to infinity phi zero n e to the power two pi n x right. [vocalized-noise] So, what is the first term? The first term is phi zero x minus c t, because this is exactly the same except that here I have x minus c t, and here I have x right. So, the first term is phi zero x minus c t the second term is phi zero x plus c t right.

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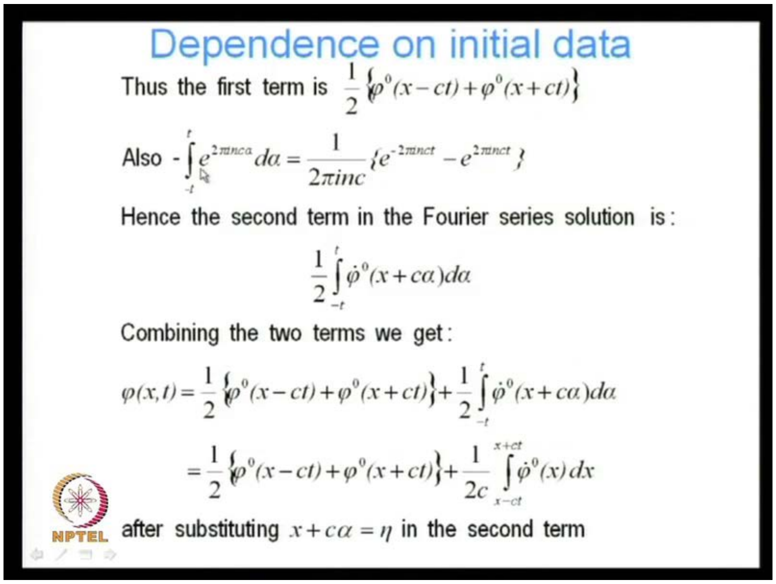

Also this integral if we evaluate (refer time: 09:00) this integral e to the power two pi i n c alpha d alpha, where alpha is an [vocalized-noise] intermediate integration variable I can show that this is equal to that right. So, the second term in the fourier series solution this term is actually can be expressed in terms of phi dot zero of [vocalized-noise] at x, and what does it become? It becomes half integral minus t to t phi dot zero x plus c alpha d alpha.

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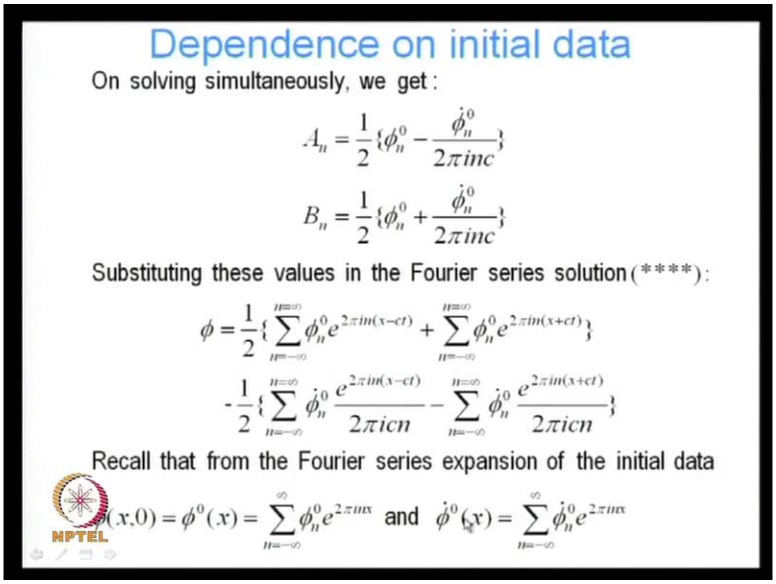

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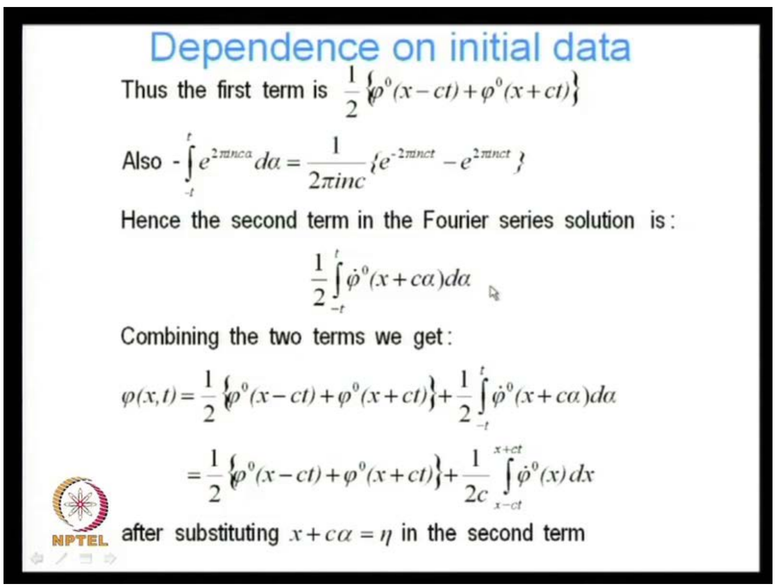

So, we can see we are we are able to express the entire solution.

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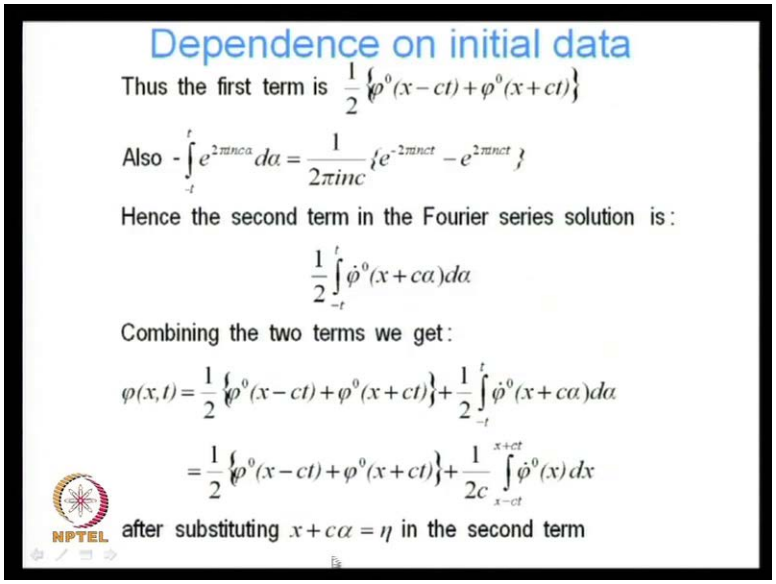

The entire solution of this problem this wave problem. In this [vocalized-noise] if this in this ring of unit circumference entire solution. In terms of the initial conditions right, phi zero and phi dot zero, and finally we can get it in this form after doing this substitution x plus c alpha is equal to eta [vocalized-] (refer time: 10:00), we get it in this form [vocalized-noise]. So, what is interesting about this form? It is clear. That it is a function of x minus c t and x plus c t number one that is part of the solution, and more over the part the function is [vocalized-noise] the solution is totally dependent on the initial conditions [vocalized-noise].

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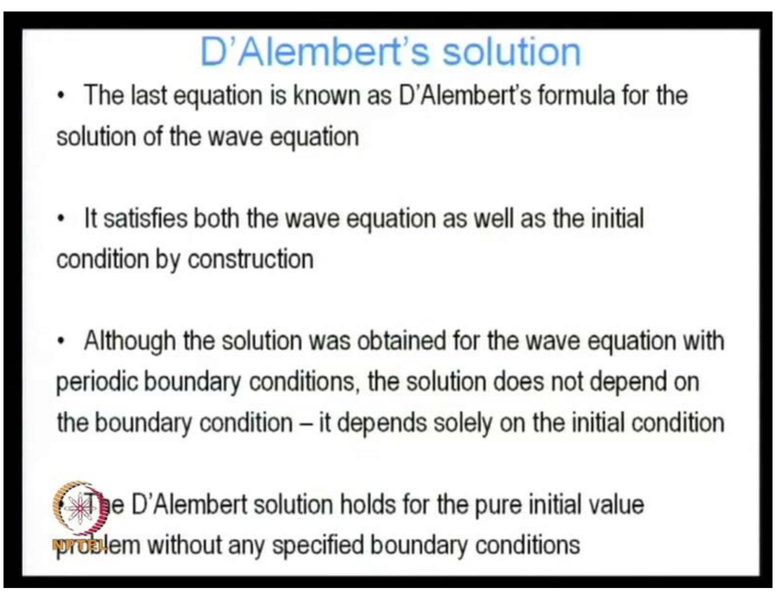

This last equation is known as d'alembert's formula for the solution of the wave equation. It satisfies both the wave equation as well as the initial [vocalized-noise] initial condition by construction right. Although the solution was obtained for the wave equation with periodic boundary condition, it is clear that the solution does not depend on the boundary condition right. It depends totally on the initial condition. So, d'alembert's solution holds for the pure initial value problem without any specified boundary conditions right, because you can (refer time: 11:00) see here there is no influence of the boundary conditions there is no influence of the boundary conditions at all it is totally dependent on the initial conditions.[vocalized-noise]

So, we have solved [vocalized-noise] wide using the initial the boundary conditions are not involved in the solution. Well it turns out that when we satisfy the initial conditions, we automatically satisfy the boundary conditions right for this geometry and for this periodic boundary condition right.

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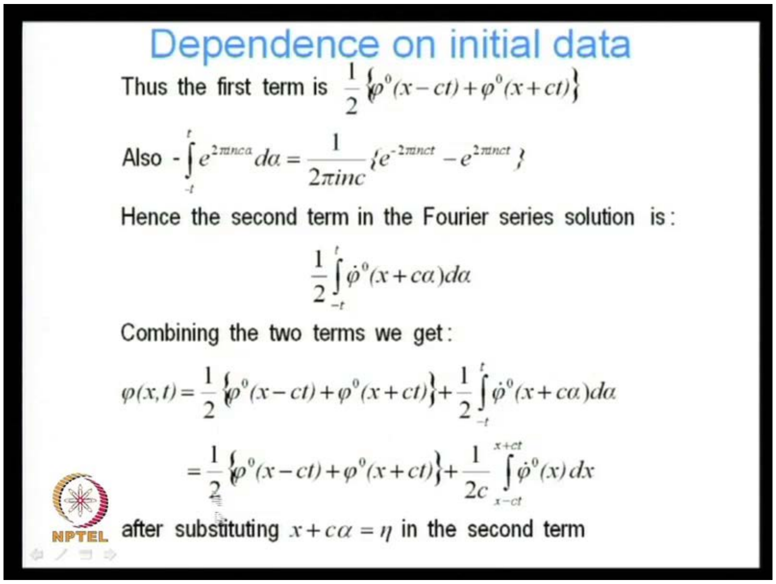

[vocalized-noise]

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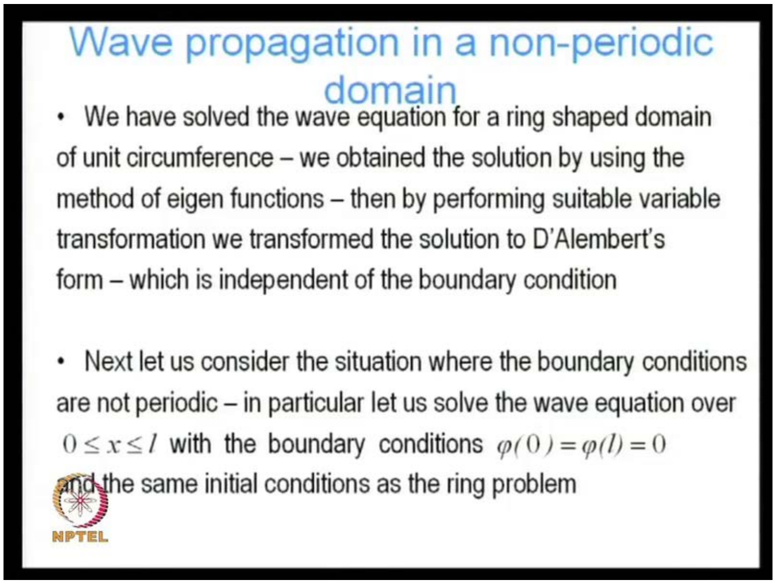

The initial condition this solution automatically satisfies the boundary conditions. So, not only does it satisfy the initial condition [vocalized-noise], but it also satisfies the boundary condition, but the boundary condition does not appear here, because of the nature of the solution right. (refer time: 12:00)it does not, because of the nature of my eigenfunctions and things like two pi i n c t and things like that, because of that it does not really appear in my solution right.

So, the initial condition is such that it automatically satisfies the boundary [vocalized-noise] the initial condition is consistent with the boundary condition. This problem has [vocalized-noise] an if you have an [vocalized-noise] this is [vocalized-noise] the solution depends only on the initial condition, we find that right from our solution, but the initial condition is totally consistent with the boundary condition. So, when it satisfies the initial condition it also satisfies the boundary condition right that is why you do not see the boundary condition explicitly in the solution [vocalized-noise]. So, we have [noise] solved the wave equation for a ring shaped domain to unit circumference, we obtained the solution by using the method of [vocalized-noise] (refer time: 13:00) eigenfunctions. Then by performing suitable variable transformations, we transformed the solution to d'alembert's form, which is independent of the boundary condition right.

So, now let us consider a situation, where the boundary conditions are not periodic well I said that in this case the boundary conditions do not appear in the solution, because the initial conditions automatically satisfy the boundary conditions right, but now let us consider a situation, where the boundary conditions are not periodic what happens in that case? Is the solution dependent on the boundary conditions. Let's take a look so in particular, let us solve the wave equation. Now over a domain in a [vocalized-noise] the one dimensional wave equation, we are going to solve the one dimensional wave equation over a domain zero to l x goes from zero to l with the boundary conditions that phi of zero equal to phi of l is equal to zero. So, it is [vocalized-noise] (refer time: 14:00) it is fixed at the two ends right the primary variable is zero at the two ends, and it has the same initial conditions as my previous problem.

So, at t is equal to zero I am giving phi as well as phi dot right. So, those are my initial conditions, which are the same as the wave problem. And in addition I have now non-periodic boundary conditions, which is phi of zero and phi of l equal to zero [noise]. So, again we are going to adopt the eigenfunction approach. So, we are interested in phi. So, this is my operator l u right l u and this must be equal [vocalized-noise] linear operator l u and I want to find the eigen values in eigenfunctions of this operator. So, I solve this problem right d two u n d x square, where u n are the eigenfunctions lambda n are the eigen values subject to these boundary conditions, which are actually identical to the boundary (refer time: 15:00) conditions and phi, because the boundary conditions, and phi are homogenous boundary conditions.

If the boundary conditions and phi were non homogenous boundary conditions then the boundary conditions for the eigen value problem is going to be different, because the eigen value problem always must have homogeneous boundary conditions right. So, since the boundary conditions are homogenous the solution does not contain any cosine terms, you can see that right, because u n at x must be at x the solution must be equal to zero right at x is equal to zero the solution must be equal to zero. So, it can't contain cosine terms right. So, it is going to contain sine terms only and lambda n, we can find out after solving the eigen value problem after find writing of the characteristic equation like, we did last time for my wave equation in a ring, we will find that lambda n is equal to n square pi square by (refer time: 16:00) l square again. Since the eigenfunctions form an orthonormal basis for the function space I can always represent any solution phi as a sum of these basis functions of this infinite basis functions, and the coefficient is dependent on time c n is dependent on time. So, that is my solution expanded in terms of this basis functions [noise].

Now, we note that the sum in this case is from n is equal to one to infinity and not from minus infinity to infinity as per the ring problem, because the eigenfunctions are odd in this case right, we have sine [vocalized-noise] eigenfunctions involved sines. So, again substitute, we do the exactly the same operation that we did for the ring problem right. Once we obtain our eigenfunctions, you write out a solution in terms of a coefficient and [vocalized-noise] coefficient, which is a function of t times the eigenfunction (refer time17:00) summed over the infinite number of terms in that series right. Substitute that in my differential equation substitute that in my wave equation right, and then from that impose the [vocalized-noise] once I substitute that I get something like this right, which is a second order ordinary differential equation in the coefficients c n .Then I use my orthonormality property of the eigenfunctions right. I take [vocalized-noise] inner product with my eigenfunctions right, and once I do that I reduce this to this form right.

So, this extra terms go away so only the term involving n survives the coefficient c n survives. So, I get something like this right. An equation a second order ordinary differential equation in c n and this is since if the second order equation I need two initial conditions to solve (refer time: 18:00) the second order equation in time. I got get those initial conditions from my initial conditions of the original problem. [vocalized-noise] How do I get the initial conditions? Again by expanding the initial data, which I know phi at x g phi of x zero and phi dot of x zero. In terms of the eigenfunctions since my eigenfunctions form a basis I can expand any function in terms of the eigenfunctions. So, I can as well expand phi of x zero and phi dot of x zero in terms of the eige.nfunctions right. [vocalized-noise]

So, I do that here and these coefficients again I am going to evaluate using the orthonormality property of my eigenfunctions. So, imposing the initial condition in terms of phi zero n and phi dot zero [vocalized-noise] n on the solution of star, which is this right [noise], we can get an expression for c n (refer time: 19:00) in terms of the initial data. So, we solve this [vocalized-noise] second order differential equation for c n impose the initial condition, we get c n like this right, c n as a function of time [noise] that's like this I substitute that in my series solution for phi basically, I substitute now I know c n I substitute that right here right. I substitute that c n right here [noise] and I get my phi, which is the final solution right this is the final solution.

As you can see as you can see this definitely has the boundary conditions explicitly in there right comes from the [vocalized-noise] eigenfunctions. So, these boundary conditions you can see at x is equal to zero x is equal to l, it is going to (refer time: 20:00) [vocalized-noise]. It is satisfying the boundary conditions right. So, now we want to consider the solution for various types of initial conditions, let's suppose that my initial velocity is zero right. So, my phi dot zero is equal to zero that is [vocalized-noise] one of my initial condition. So, the second term here drops out right the second part of the solution drops out. So, I have phi equal to sigma n is equal to one to infinity phi zero n cos of n pi c t by l sin of n pi x by l. Then using some trigonometric identities I write cos of sin n pi c t by l sin of n pi x by l in terms of a [vocalized-noise] of sum of sines [vocalized-noise], and if I do that I get phi is equal to this.

You can see why I am trying to do that, because I want to get it in the form, where it is a function of (refer time: 21:00) x plus c t and x minus c t [noise]. So, if the initial displacement so right. Now we have only conditions only, we have only initial conditions on phi zero right, we do not have the [vocalized-noise] the initial there is no [vocalized-noise] initial velocity. The initial velocity is identically zero right. So, suppose I define since my domain of interest is from zero to l, and if I define my initial [vocalized-noise] displacement phi zero x in the interval zero to l, and if we assume now this is a big f right. So, because my domain of interest is from zero to l right. So, I am going to give initial conditions only in that region right, but suppose my initial conditions can be expended along the x axis right from x is equal to minus infinity to infinity as (refer time: 22:00) a periodic function as a periodic function. So, that [vocalized-noise] whatever the function I get from [vocalized-noise] whatever the shape of the function phi at x is [vocalized-noise] at x as t at t is equal to zero from x is equal to zero to l whatever shape. It has I know that at x is equal to zero [vocalized-noise] and at x is equal to l. It is got to be zero right, because it has to satisfy the boundary conditions right.

Now, if I extend that right I suppose that my initial I assume that my initial condition is extended for the entire x axis right, stretching from minus infinity to infinity, but extended in such a manner that it satisfies this periodicity boundary condition right. So, that it is zero at x is equal to zero x is equal to minus l x is equal to minus 2 l x is equal to minus 3 l x is equal to 2 l x (refer time: 23:00) is equal to 3 l and so on. And so, forth right if that happens then the solution is given by this phi of x t is equal to half of phi zero x plus c t plus half of phi zero x minus c t.

[noise] So, basically I can expand this solution for throughout x is equal to minus infinity to infinity in terms of phi of x zero plus c t phi of x zero minus c t why can I do that? Well I can do that, because again again again. Because my initial conditions are satisfying the boundary conditions. So, my initial condition now is of this form right my initial condition is of this form (refer time: 24:00) and it is periodic. So, it satisfies the boundary conditions like [vocalized-noise] like that right. So, that is why I can extended to the entire x axis? I mean I can extend the solution to the infinite domain.

The reason for this is by extending the initial data to the entire domain in the way we shown here right. In a periodic fashion when sure that the pure initial value problem, and the boundary value problem have the same solution in the interval zero x l right. So, I am extending my initial condition, but I am doing it in such a way that if the boundary conditions are also satisfied [noise] this is, because the solution of the pure initial value problem ensures that phi x is [vocalized-noise] the pure initial value problem solution given by (refer time: 25:00) d'alembert's formula that ensures that phi at x is equal to zero, and phi at x is equal to l is automatically equal to zero right, which means that the boundary condition is satisfied.

[vocalized-noise] This extension procedure is known as the method of images, basically; it extends it takes images of the solution right, images an extensive right image of the it can be extended to the full problem. If phi dot zero is also extended as periodic function that is odd at the points x is equal to zero and x is equal to l we extend we could extend that, because my initial condition which was only on phi, because I assume that phi dot is equal to zero phi dot at x at t is equal to zero is equal to zero. If so I had [vocalized-noise] my initial condition was only on the displacement right, and I extended that in a periodic fashion right satisfying the boundary condition.

(refer time: 26:00) If I do the same thing for my velocities phi dot then i can extend that a full initial value problem right, where I m prescribing both the displacements both phi and phi dot I can also extend that over the infinite domain [vocalized-noise]. So, it can be extended to the full problem if phi dot zero is also extended as a periodic function. That is odd at the points x is equal to zero and x is equal to l. Similarly, that d'alembert's solution which is independent of boundary conditions can be made valid for the finite [vocalized-noise] interval right, because if we make d'alembert's solution if we give initial conditions in such a way that it satisfies the boundary condition. Then the d'alembert's solution is it's just a converse right, it can d'alembert's solution can also be made valid for the finite interval [vocalized-noise]. If both phi zero x (refer time: 27:00) and phi dot zero x are redefined for minus infinity x to infinity. In such a way that they are two l periodic and odd [noise].

[vocalized-noise] So, this is this is that a point to [vocalized-noise] since we are looking at geometry right. Now we are looking at how the solution can be extended right from a finite domain to an infinite domain making sure that we take mirror images of the solution of the initial conditions right. [vocalized-noise] This is the point to look at a very important method for solving [vocalized-noise] for solving partial differential equations particularly hyperbolic partial differential equations that is the method of characteristics. The method of characteristics is a very important and very powerful, and very revealing method, because it gives a lot of physical insight (refer time: 28:00) into the solution of the wave equation and it is based on geometry purely based on geometry right.

So, to do that if we look for a geometrical interpretation for d'alembert's formula [vocalized-noise] and this helps us in getting at the method of characteristics. [vocalized-noise] So, this is again d'alembert's solution phi x [vocalized-noise] x t half of f x minus x c t plus [vocalized-noise] plus f of x plus c t plus this right, this is my d'alembert's solution, we can see that as in the limit as t goes to zero phi of x zero is equal to f of x. why? Where t is equal to zero this part is two f x that is f of x this at t is equal to zero the integration limits go from x to x. So, this integration vanishes. So, I am left with phi of x zero is equal to f of x also if I take the derivative of this solution of d'alembert's solution with respect to time. (refer time: 29:00) I get half minus c f prime x minus c t plus c f prime x plus c t, which at t is equal to zero is going to cancel out that's going to give me zero, and if I look at this term. If I take the time derivative [noise] I have to evaluate the derivative of the [vocalized-noise] of the limits of the integration time is the function value here right, because the integration limits are dependent on time, and if I do that I get something like this, which is basically g of x plus c t by two and since t goes to zero this is g of x right.

[vocalized-noise] Now, let's consider what are known as characteristic lines as shown in the following figure right. So, basically I am interested in knowing the solution at some point this is my x t axis, where x is the distance right. It is the one (refer time: 30:00) dimensional problem x is the distance in the x direction t is my time, and I am plotting it like that, and suppose I want to find the solution at time x zero and t zero right. And these lines these blue lines are what are known as characteristic lines. These blue lines are known as characteristic lines [noise] [vocalized-noise].

So, far any point x zero t zero and the x t plane, we construct lines passing through that point [noise] with slopes of plus c and minus c respectively, you can see that this line has a slope d x d t is equal to c right, and this line has a slope you can see it passing through x zero t zero x zero minus c t zero. So, it has the slope of d x d t is equal to c and this line has a slope of d x d t equal to minus c [noise] (refer time: 31:00) and this characteristic lines have the following equation x minus x zero by t minus t zero is equal to plus c, and minus c right. So, one of the equation is x minus x zero by t minus t zero is equal to c. The other equation is x minus x zero by t minus t zero is equal to minus c.

Let us consider again the first term of d'alembert formula [noise] [vocalized-noise], which is this term a half of [noise] f x minus c t [noise].

It is clear that at all locations x t along the characteristic lines f of x minus c t has the same value as it has as at x zero minus c t zero since along these lines x minus c t is equal to x zero minus c t zero. [noise] So, the characteristic lines have the equation (refer time: 32:00) x minus x zero by t minus t zero is equal to plus c and minus c. Let us suppose we take plus c right. Then what is the equation of the characteristic line? x minus c t is equal to x zero [vocalized-noise] minus c t zero right x minus c t is equal to x zero minus c t zero right so along these lines. So, this is the equation of this line right x minus c t is equal to x zero minus c t zero [noise] of this line right of this line, so along this line by definition at any point along this line x minus c t has got to be equal to x zero minus c t zero right ,because that is the equation of that line right.

If that is true then f of x minus c t has got to be equal to f of x zero minus c t zero right. So, whatever is the solution at t (refer time: 33:00) is equal to zero and x is equal to x zero minus c t zero is going to be [vocalized-noise] f of x minus c t is going to have that value right, at all times along these characteristic lines so the [vocalized-noise] the [vocalized-noise] the initial information here. The initial [vocalized-noise] initial condition here is going to get proper all points on this line all points on this line for the first term of the solution that is going to be identical to what it was here? Right if I look at all points on this line right all values of x and t, which lie on this line right at each of these points the first part of the solution of the wave equation is going to be the same as it was here right as it was here.

[noise] So, (refer time: 34:00) [vocalized-noise] so [vocalized-noise] f of x minus t t has c t has the same value. As it has at x zero minus c t zero since along these lines x minus c t is equal to x zero minus c t zero thus to find this value of this term in the solution at any point along the characteristic line, we only need to evaluate f of x minus c t at t is equal to zero right, and when x is equal to x zero minus c t zero is that clear [noise] [vocalized-noise] similarly, the second term of the d'alembert equation f of x plus c t has the same value along the characteristic lines with slope of minus c. [noise] So, the second term right the second term along these lines that term at any point on this line. If I look at phi and I look at the part of (refer time: 35:00) phi, which is given by f [vocalized-noise] f of x plus c t that part of phi is going to be the same here here here here everywhere right. That part is going to be the same along this line, where the f of x minus c t if I look at the first part of the solution of phi that is going to be the same as here at all points on this line. [noise]

Thus at any point along this characteristic lines the value of the second term of d'alembert's solution is known. If we know the value of the location, where this line intersects the t is equal to zero axis that is at x zero plus c t zero right that the root (( )) intersects. Thus at any point x t the first two terms of d'alembert's solution are known entirely from the values of f at t is equal to zero (refer time: 36:00) that is when x is equal to x zero minus c t zero, and x is equal to c zero plus t c so at any point. So, this is very [vocalized-noise] important right, what is saying?Is that at any point in the x t domain right. If I know the solution at x zero minus c t zero and x zero plus c t zero. I know the solution the first part the first part involving f f of x minus c t and f of x plus c t is totally known [vocalized-noise] what does this mean? This means if I know the initial conditions. If I know the initial conditions at those two [vocalized-noise] at those two locations on the x axis. I know the final solution at any point at least the first part of the final solution at any point right.

So, I know the initial conditions at two points [noise].So, I know the initial values of f of x [vocalized-noise] x zero minus c t zero (refer time: 37:00) here that is going to be the same as the first part of the solution of d'alembert's solution right. If I know f of x zero plus c t zero here that is going to be the second part of the [vocalized-noise] f f of x plus c t term that part that part of d'alembert's solution is known so that part of d'alembert's solution is entirely known. If I know the value of the function here and the value at the function here right f at x minus x zero minus c t zero, and f at x zero plus c t zero [noise].

[vocalized-noise] Thus at any point x t the first two terms of d'alembert's solution are known entirely from the values of f at t is equal to zero. That is when x is equal to x zero minus c t zero and x is equal to x zero [noise] plus c t zero so at any point along the characteristic lines. Through x zero t zero the value of the (refer time: 38:00) integral term in d'alembert's equation depends on the value of g between x zero plus c t zero, and x zero minus c t zero why let's go back? And look at the solution again [noise], you can see what is the solution it says that this part we already know how to solve? what about this part? This part depends on x minus c t at any point x and t in my x t domain right. This part depends on the integral since I know I can write x minus c t as x zero minus c t zero, because from my equation of the characteristic lines right, since I can write x x minus c t as x zero minus c t zero x plus c t as x zero plus c t zero by integrate along my t is equal to zero axis between x zero minus c t zero and x zero plus c t zero [noise] between this, and this if I (refer time: 39:00) integrate this g s right. Then know the entire solution right. So, I know the entire solution.

[vocalized-noise] So, at any point depends on the value of g between x zero plus c t zero and x zero minus c t zero why, because x plus c t is equal to x zero plus c t zero x minus c t is equal to x zero minus c t zero. So, the integral in the limits of the integral change from x minus c t to x zero minus c t zero and x plus c t to x zero plus c t zero [vocalized-noise] thus the solution at x zero t zero has a domain of dependence on the initial conditions that lies in [vocalized-noise] in the interval x zero minus c t zero and x zero plus c t zero on the t is equal to zero line. So, it depends so the solution here the [vocalized-noise] (refer time: 40:00) the solution here depends on the initial conditions, but only on a part of the initial conditions. The solution here does not depend on the initial conditions, yet cannot depend on the initial conditions, yet the solution here [vocalized-noise] here cannot depend on the initial conditions beyond x zero plus c t zero. It cannot depend on the initial conditions less than x zero minus c t zero.

So, this is the domain of dependence on the initial conditions right. So, the solution at x zero t zero depends on whatever is the initial condition for the values of x, where between x zero minus c t zero. So, if I have some function right along for the initial condition I have some function along this axis right at t is equal to zero. Since it is initial condition must be at t is equal to zero. So, I have some arbitrary function of x right for my phi right, and then I have some arbitrary (refer time: 41:00) function of x right for my phi right, and then I have some arbitrary function of x for my phi dot right at t is equal to zero. So, whatever be the arbitrariness here that is not going to affect the solution here right only [vocalized-noise] whatever the values that [vocalized-noise] lie between x zero minus c t zero [noise] and x zero plus c t zero are going to affect the solution here [noise].

So, in fact all points that [vocalized-noise] in the shaded area in the figure will have a domain of dependence on the initial conditions that is a subset of the interval x zero minus c t zero x zero plus c t zero. So, what I mean? Is this I am [vocalized-noise] so here. This point has a domain of dependence between this, and this if I look at a point here. It is also going to have a domain of dependence, but that domain of dependence (refer time: 42:00) will be something like this it will be smaller right, it will be within this interval x [vocalized-noise] sub interval right. If have a pointer what will be it is domain of dependence on the initial conditions, where I draw characteristic lines with slope of minus c and plus c wherever it intersects line t is equal to zero axis those two points the [vocalized-noise] region within those two points within those two intercepts that is going to be my domain of dependence. So, the initial values only lying between those two points are going to affect the solution at that point right [noise].

So that it is important to emphasize that the solution at x zero t zero may depend not just on the values at the initial line t is equal to zero, but on initial values anywhere (refer time: 43:00) inside the shaded region. So, basically what I mean? Is this [noise] x zero t zero I know depends on the initial conditions between here, and here right x zero minus t zero depends on initial conditions here, but then suppose then suppose [vocalized-noise] I think we will come to that little later, but let's go through this first [vocalized-noise]. So, the domain of dependence of x zero t zero not only lies on the line t is equal to zero, but encompasses the entire space time domain enclosed by the characteristic lines passing through x zero t zero. What i want to emphasis? Is this I want to emphasis this is that the solution at x zero t zero depends on the initial conditions between x zero minus c t zero and x zero plus c t zero, but it is not it may not exclusively (refer time: 44:00) depend on that right.

It can depend on the [noise] if I apply disturbance here at a certain time beyond t is equal to zero. So, my solution I start my solution from t is equal to zero right, and I am continuing to infinity so at t is equal to zero. I prescribe initial conditions and those initial conditions are going to affect the solution. I know right, but then suppose during at some point I have a forcing function right I give an impulse I apply impulse load or somehow excite [noise] at a certain point in time and at a certain point. In space I apply an excitation and that point in time and space is suppose denoted by suppose somewhere here right that is somewhere here right at (refer time: 45:00) inside the domain I apply a certain excitation right.

My solution here my solution at x zero t zero may depend on that excitation as well right, and how is that going to happen, let's see [vocalized-noise] all point that lie in the shaded area in the figure the [vocalized-noise] that, we looked will have a domain of dependence on that we have already seen right, but the domain of dependence of x zero t zero not only lies on the line t is equal to zero, but encompasses the entire space time domain enclosed by the characteristic lines passing through x zero t zero. Suppose that for a particular problem I have quiescent initial conditions what does quiescent initial condition mean? That I do not if any initial displacement any initial velocity right f and g (refer time: 46:00) are both zero for all points lying [vocalized-noise] for all points lying between x zero plus minus c t zero and x zero plus c t zero right. So, basically I am seeing that [noise] along this line my initial conditions as phi is equal to zero at time t is equal to zero phi is equal to zero phi dot is equal to zero initial condition says there is absolutely zero displacement, zero velocity does it mean that the solution here. Will be zero for all time well not necessarily that's what I am trying to get that it may not mean it just because the initial condition at time t is equal to zero it's domain of dependence along the line t is equal to zero shows that everything there is no excitation at all right.

No displacement, no velocity does it mean that the solution here will be zero for all time. (refer time: 47:00) Well not really right why [noise] suppose at a subsequent time t one, which is greater than zero, but less than t zero right a disturbance occurs at a point in the [vocalized-noise] shaded region say a point marked in red in the figure. So, basically suppose at time t, which is greater than zero, because I am moving along the time axis right. So, that is after my comments my solution right after I comments my solution I have an excitation here like this red point right I apply an excitation here. In that case [noise] the disturbance is going to propagate out from this point along the characteristic lines passing through this point right. So, since I applied disturbance here from this point (refer time: 48:00) along the characteristic lines passing through this point with slope c and c and minus c this is going to move out right. This is going to move out and then it goes, and reaches this blue points right [noise].

So, when that information reaches points marked in blue on the characteristic lines through x zero t zero. It will influence the solution at that point right. So, once it this disturbance propagates to here right, these points it is going to influence the solution here right, and if [vocalized-noise] if the time and it may subsequently affect the solution at x zero t zero provided that the time taken to traverse the entire path shown in yellow is less than t zero, let's look at it again. So, I have a disturbance (refer time: 49:00) here, which is occurring at time t one right this disturbance is going to propagate along the characteristic lines with slope minus c and c right, [noise] and it is going to suppose it reaches these points right, which in turn lie on the characteristic lines of x zero t zero right. So, one if the solution reaches here right. [noise] If this propagates [vocalized-noise] this influences the solution here that is again going to propagate from here along the characteristic lines right, and it may actually affect the solution at x zero t zero provided the time it takes the time it takes for that disturbance to travel from here through here and from here through here through here is less than t zero right, because we are interested in the [vocalized-noise] solution at time t zero.

So, if the disturbance (refer time: 50:00) takes a longer time to reach that, then it is not going to affect right. Even though it might have occurred this disturbance might have occurred at a time t one, which is less than t zero it might have occurred at an earlier time than the time of interest, but it is not going to affect the solution at the time of interest t zero if the path along the characteristic lines to reach that point is longer than it can travel with the wave speed c is that clear [noise]. So, when that information reaches points marked in blue. It will influence the solution at that point and hence may subsequently affect the solution at x zero t zero provided that the time taken to traverse the entire path shown in yellow is less than t zero. [noise]

[vocalized-noise] The notion of domain of dependence is explained best with reference to (refer time: 51:00) sound waves the domain of dependence originates in the fact that at any [vocalized-noise] a point in time at a special location. I can only receive signals from sound waves that were generated at a previous point in time. So, I have a point in time and space I can only receive signals from signals, which were generated at a previous time right also the time between the generation of those waves, and the current time must be sufficient to allow the sign [vocalized-noise] the sound waves to traverse the special distance between the source, and the point at, which I am interested right.

So, not only must it occur at a at a previous time, which is the first second, which is the second point here right not only must it occur at a previous time, but the time it takes, where the sound wave to travel must from the source to my point of interest, where I am observing the solution must be [vocalized-noise] (refer time: 52:00) less than the time at, which I am observing the solution right than the time at, which I am observing the solution [noise]. So, similar to the domain of dependence points on the initial line say t is equal to zero have a domain of influence comprising the part of the space time domain were the solution is affected by the value of f and g [noise] at the point. So, this is the domain of dependence this is the domain of dependence of x zero t zero. Similarly, for points on the [vocalized-noise] t is equal to zero line. I have what is known as a domain of influence right. This is the domain of influence of so whatever be the initial condition at x zero. It can only affect this part of my space time domain that is shown in the shaded region. It is not going to affect any part outside this region right. So, this is the domain of influence and the domain of influence is limited by the fact that a wave (refer time: 53:00) generated at some time at a point cannot influence events at an another point in less time, then it takes to travel the distance between those points right

So, that is my domain of dependence I have point in x t right not on the [vocalized-noise] t is equal to zero axis. At some point in the [vocalized-noise] in the course of the solution right, and the future right not in the initial time right, and I am interested in what part of the space time domain can influence my solution there right. So, that is giving me my domain of influence then I have the domain of [vocalized-noise] domain of dependence sorry that is that gives my domain of dependence, and then I have my domain of influence, which tells me that I have a point on the t is equal to zero axis what are the possible space time configurations that the [vocalized-noise] the initial condition at that [vocalized-noise] at that point can ever affect right.

So, the total domain of (refer time: 54:00) influence, we will continue our discussion of the method of characteristics in next class and then we will try to move on to analytical techniques for the solution of parabolic and hyperbolic equations, but they are they as I said last class they some of them are very common to those for [vocalized-noise] method of eigenfunctions can be used over all these classes of all [vocalized-noise] all three types of canonical differential partial differential equations. So, we will it will probably be a little repetitive and I think my (( )) going through thank you. [noise]