Computational Hydraulics
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Lecture 36
Gradually Varied Flow

Welcome to the lecture of the course computational hydraulics. We are in module 4, surface water hydraulics. And this particular lecture class I will be discussing gradually varied flow, open channels. And this is unit number 1.

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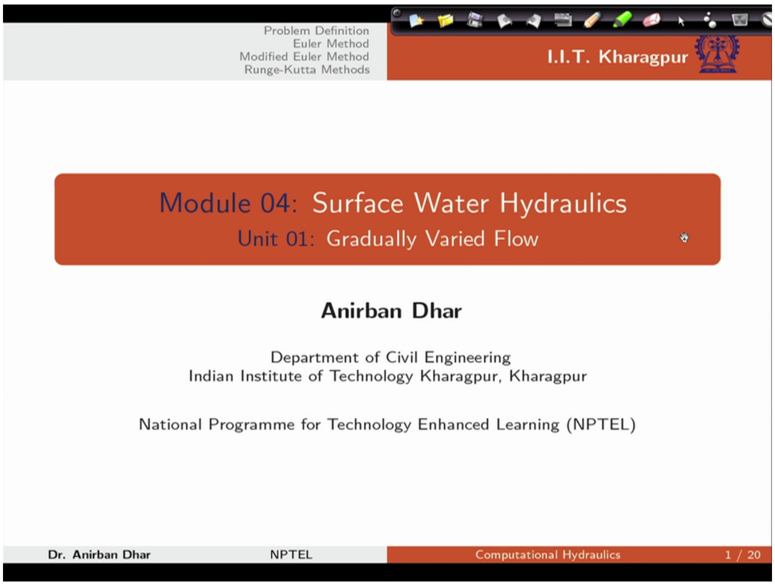

Learning objective for this particular lecture. At the end of this lecture students will be able to solve gradually varied flow problem for open channels.

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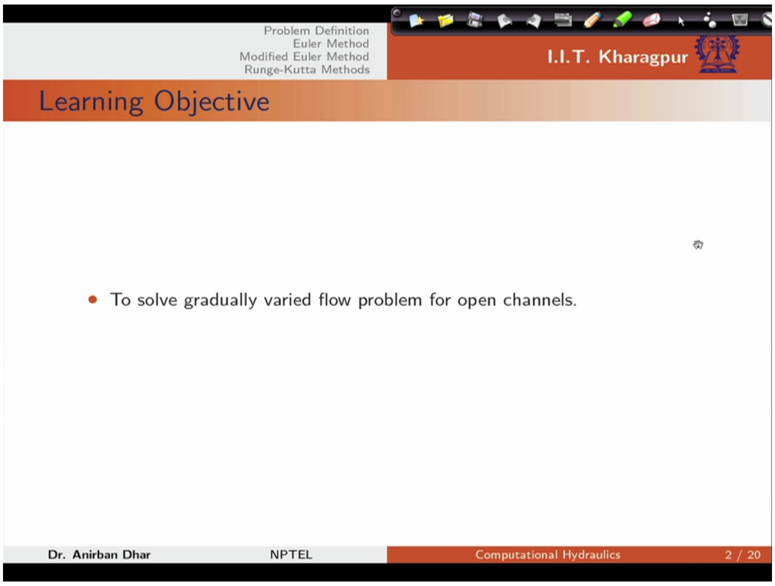

We have seen this basic structure of our computational hydraulics course. We have problem definition, mathematical conceptualization, domain discretization, then numerical discretization, algebraic forms, solution process.

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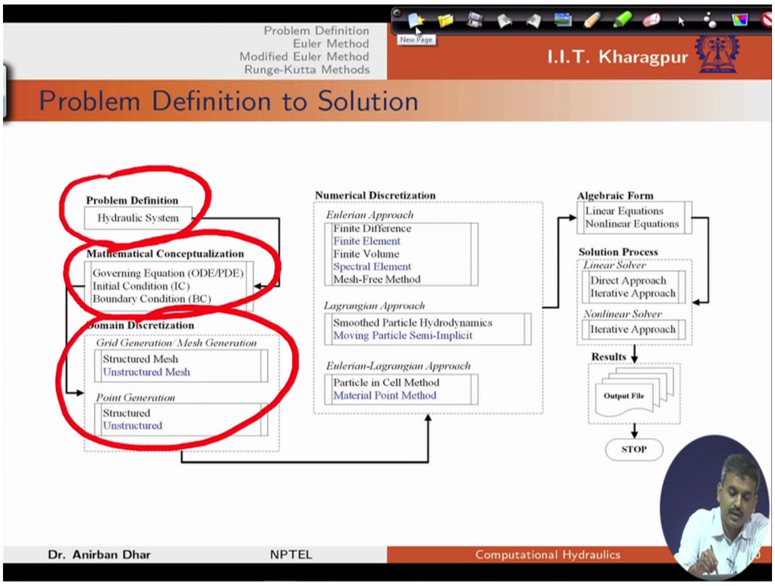

In our groundwater hydraulics we have started from this one dimensional two dimensional case and finally derived the hydraulic head distribution for 2D space and 1D time.

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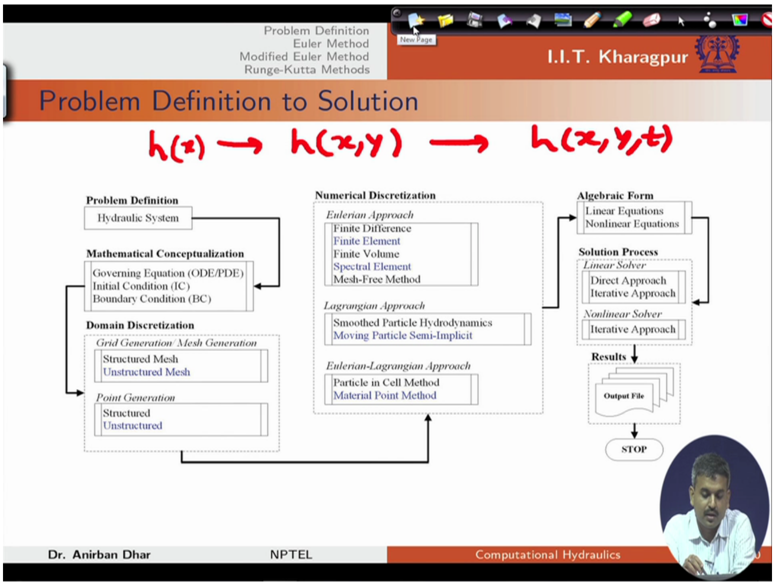

Now in surface water hydraulics first we will start with this channel flow. Channel flow is essentially one dimensional flow due to conceptualization of the problem. And in channel flow, flow depth is one of the important parameter and flow that varies with x if y is the flow depth in a particular floor channel. This is the channel bed and this is the water surface.

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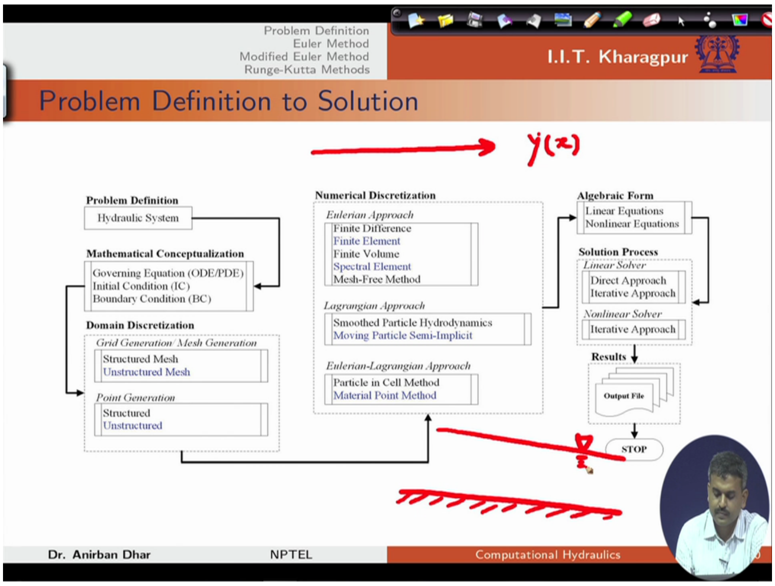

Then flow depth at any particular location y is a function of x. Now we need this variation of y with x for this particular unit.

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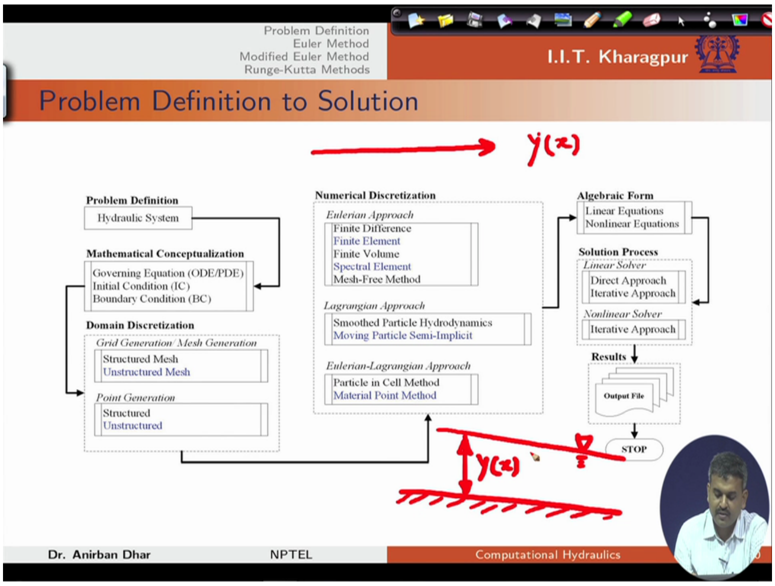

Now governing equation for gradually varied flow in prismatic channel can be written as dy by dx and this is S not minus SF 1 minus Fr square. S not is bed slope, SF is energy slope and Fr is the Froude number.

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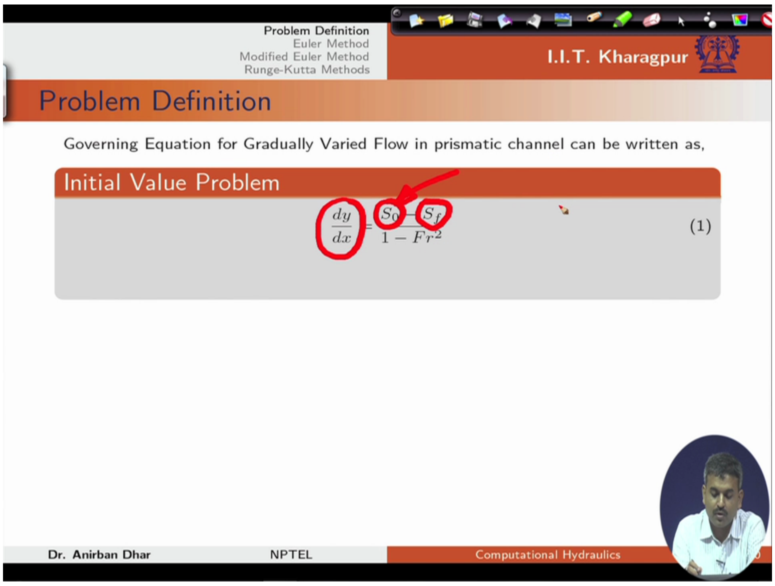

Now to solve this problem we need initial condition. When I talk about initial conditions we may think this particular condition like zero time condition. If the flow depth is specified at a particular section in the channel then we can find out the variation of y with x in the channel. So this is initial value problem and this is first order ODE equation.

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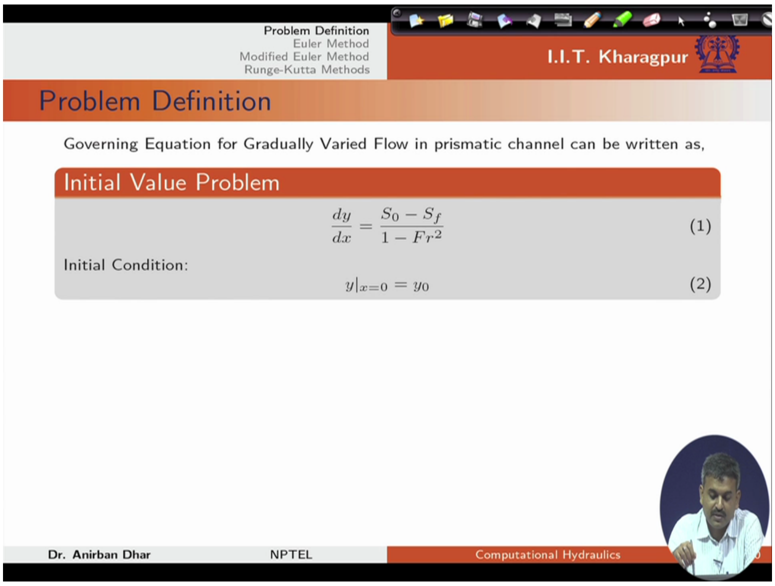

In this case y is the flow depth or depth of flow, x is coordinate direction, S not is bed slope, SF is friction slope. Now SF can be calculated using this expression and Froude number can be calculated using this particular expression. Q is the discharge, T is the top width, G is acceleration due to gravity and R is hydraulic radius, A is cross-sectional area.

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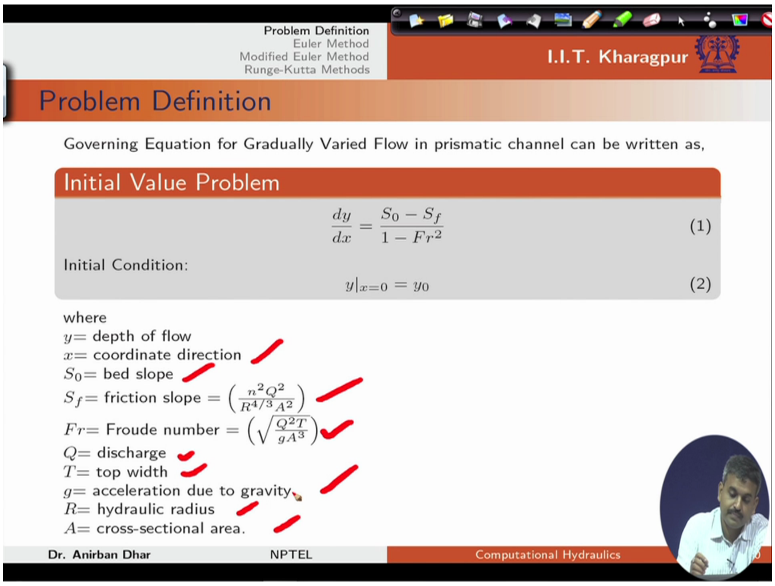

Now as per our general format that we have discussed in lecture 7, we have discussed the format where d phi by d t equals to psi t and phi. This format was there. Now in our problem format we can write it as dy by dx and psi is the general function which is varying with x and y.

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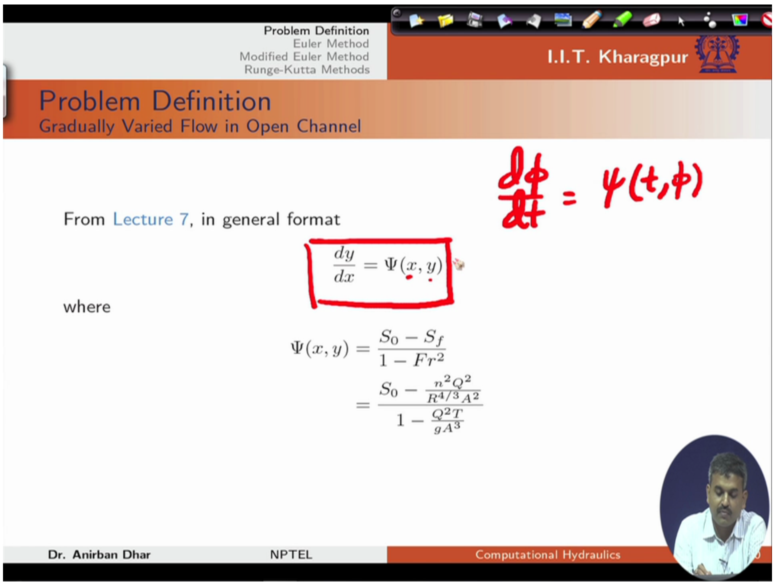

Now if we consider this GVF or gradually varied flow condition then it is not given parameter. Then n, n is Mannings roughness coefficient, Q is discharge, R is hydraulic radius, R can be calculated as A by P. But P is the weighted perimeter, A is cross-sectional area, T is the top width.

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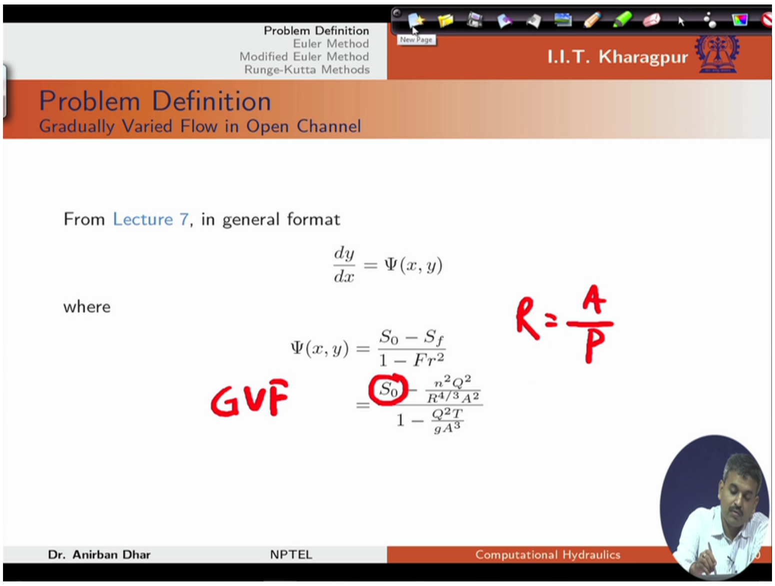

Now we can express this problem as psi as a function of y only. Because there is no x in these expressions. So we can directly calculate the psi xy using the variation of y only.

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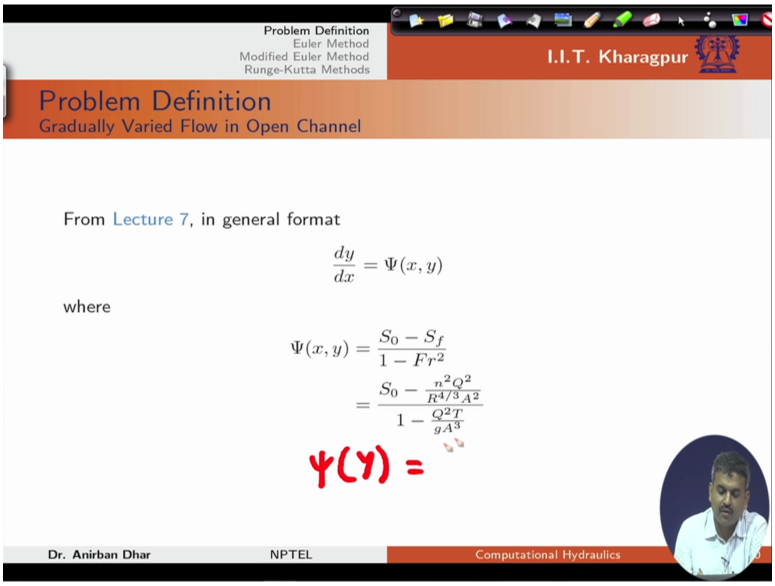

Now if we consider a general trapezoidal section then area we can divide it into two parts. This is flow depth, this is bed width and left hand side we have 1 is to m1, 1 is to m2 slope. Now this intermediate area is By and on both sides if we consider then this will be the area.

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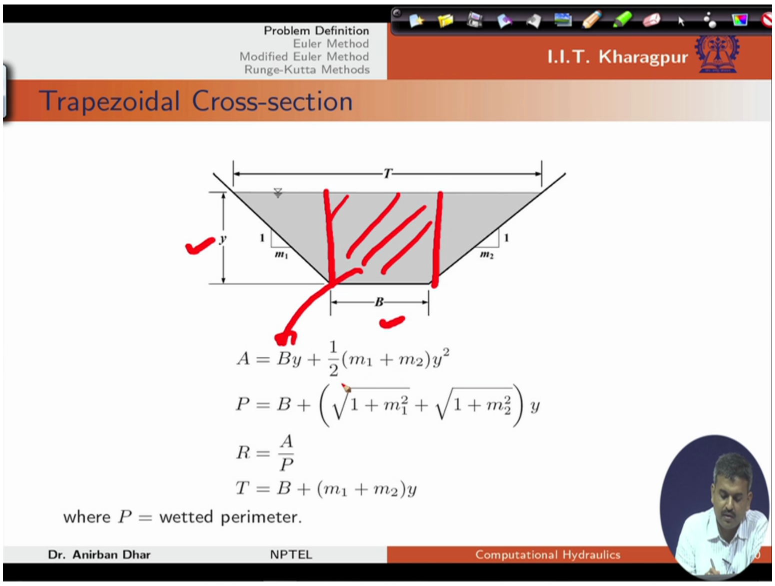

Perimeter is this one, bed width or this part, this is the weighted perimeter or P, we need to calculate.

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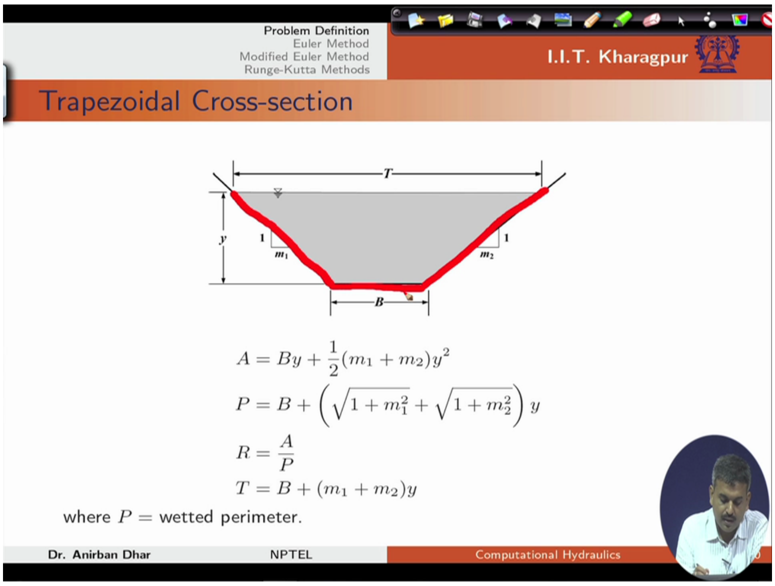

Then we need top width. Top width is this one. And R, obviously R we can calculate based on A and P values.

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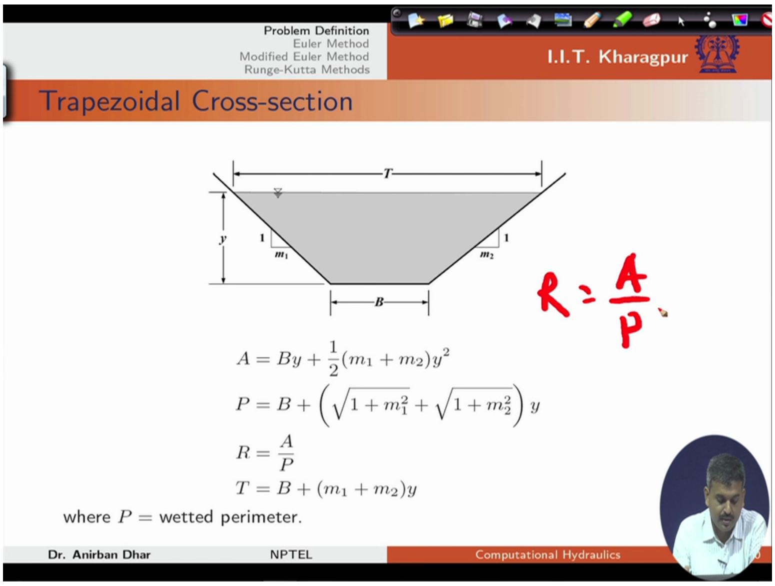

Similarly if we have circular cross section we can calculate this area using this formula where D is the diameter of the section, y is the depth of flow, theta is this particular angle and T is the top width and P is again weighted perimeter, R is hydraulic radius, T is top width.

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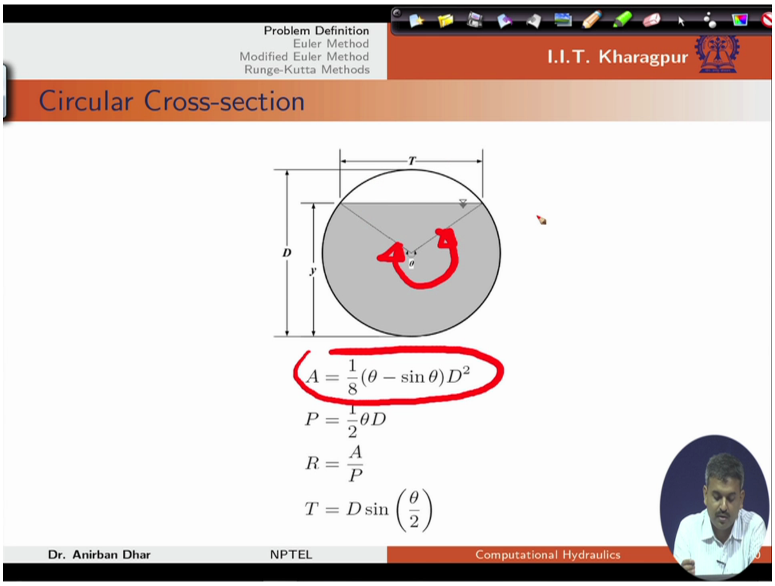

From our problem definition we need to find out the nature of the flow. In gradually varied flow we can classify the flow based on two flow depths, one is critical flow depth, another one is normal flow depth. Now let us see how to find out this critical flow depth for prismatic channels.

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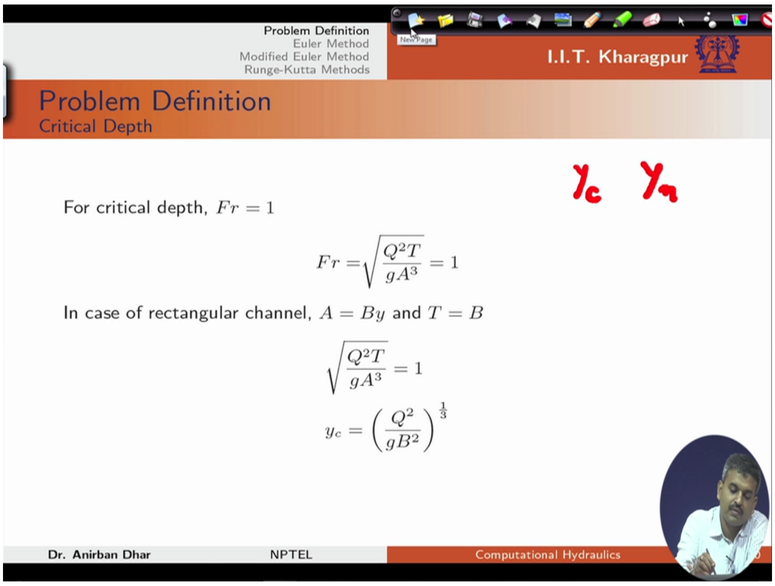

Now in this case Fr we already know as square root of Q square T by gA cube, this is equals to 1. Because in (cui) case of critical flow, Fr should be equal to 1. And in rectangular channel which is a most simplified case, this is b and for flow depth if this is y then we can calculate the area. Area is nothing but B into y. And top width is B only because there is no variation in this sectional parameter over there.

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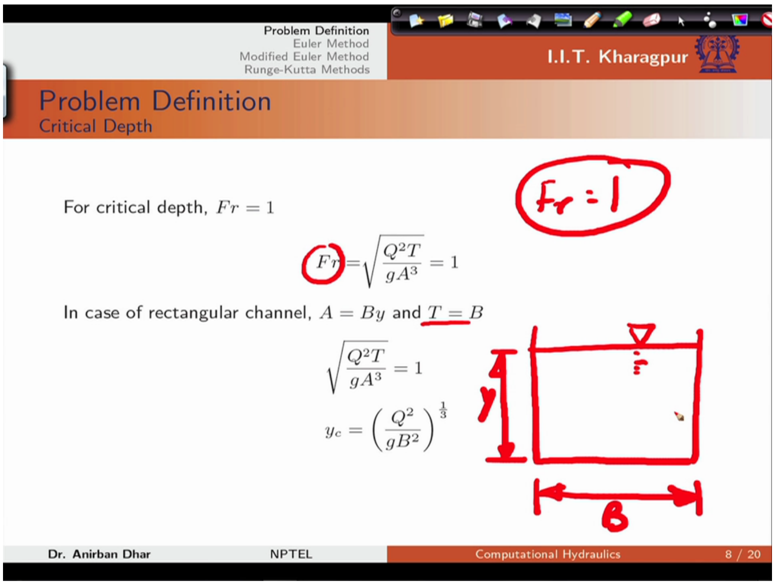

So if this is 1, then we can find out this is Q square, so T is B which is top width of the rectangular section, g and A is By whole cube. So this is nothing but Q square divided by gB square and y cube equals to 1. From there we can get this critical depth expression for rectangular channels.

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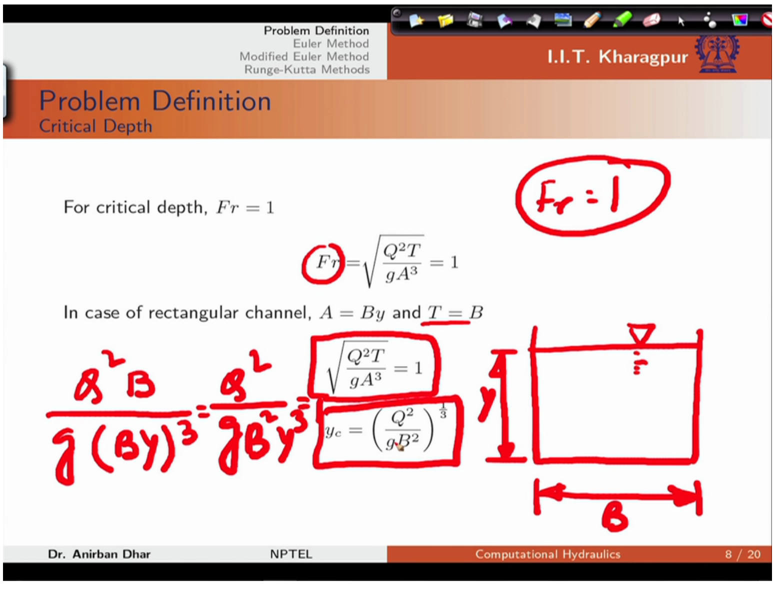

Now if I want to calculate the normal depth, so normal depth can be calculated from Mannings equation. Mannings equation if we write Q equals to 1 by n R to the power 2 3rd S to the power half or S not, specifically in this case into area.

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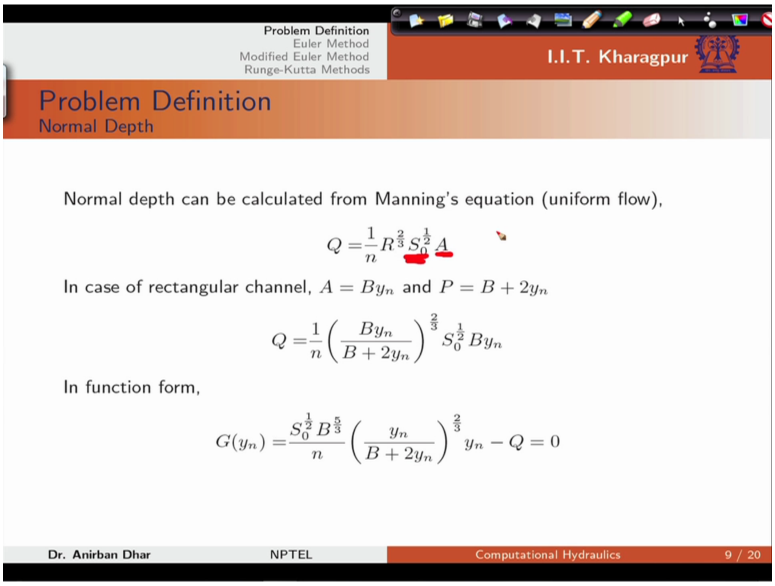

Then for normal depth, B into yn and B plus 2yn. So Q is given, B is given, n is given, S not is given. So only unknown parameter here is yn. So this is one non linear equation. Now we need to solve this equation to get the value of yn or normal depth.

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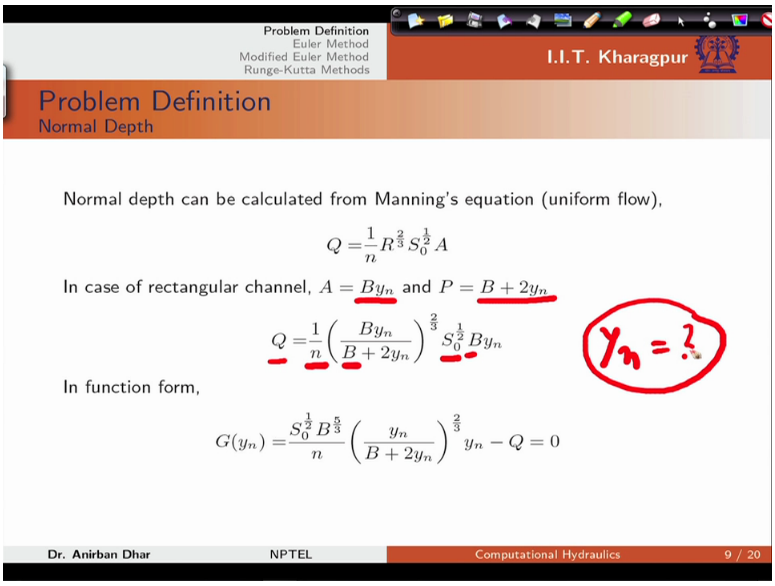

Now what we can do we can form one function which is G yn. G is the function of yn or normal depth of flow. And we can transfer the Q on the right hand side and write it like this and equate it to zero.

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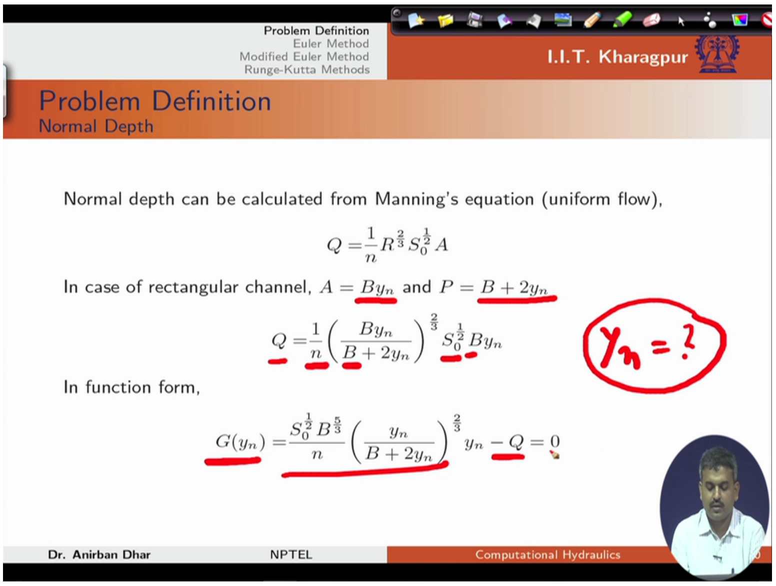

And we already know the Newton Raphson method. In Newton Raphson method let us say we are considering our yn value which is at GP and this is equals to let us say G yn P plus del yn.

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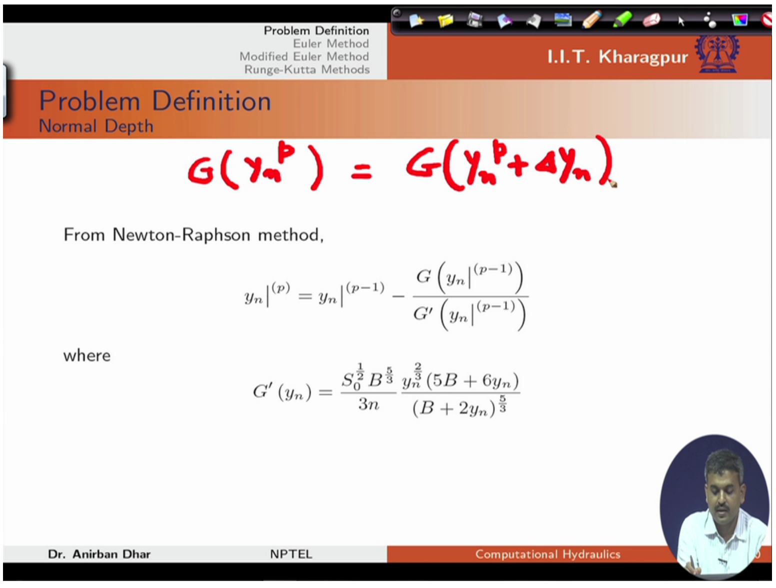

Now in this case there will be another term that is the first order derivative term for this case because we are expanding this term with Taylor series. So if we expand this term obviously we will get one term which is G prime and yn. This is width P minus 1, ultimately this will become P. So this is P minus 1 only and we can multiply this yn.

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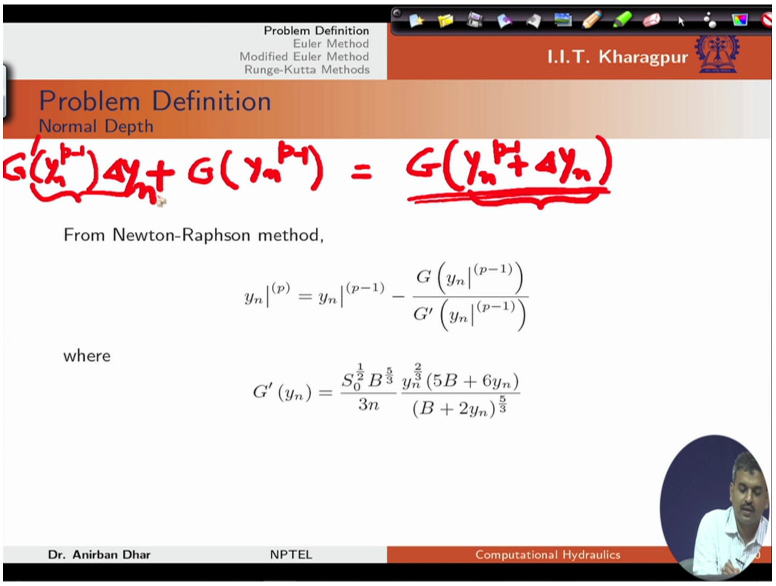

Now at convergence point there should be zero. So what we can do, we can write it as yn equals to minus our G yn P minus 1 divided by G prime yn P minus 1. G prime is nothing but derivative of this function G with respect to yn. Now we can write this one as this equation. So this is nothing but the expanded form of our derived case.

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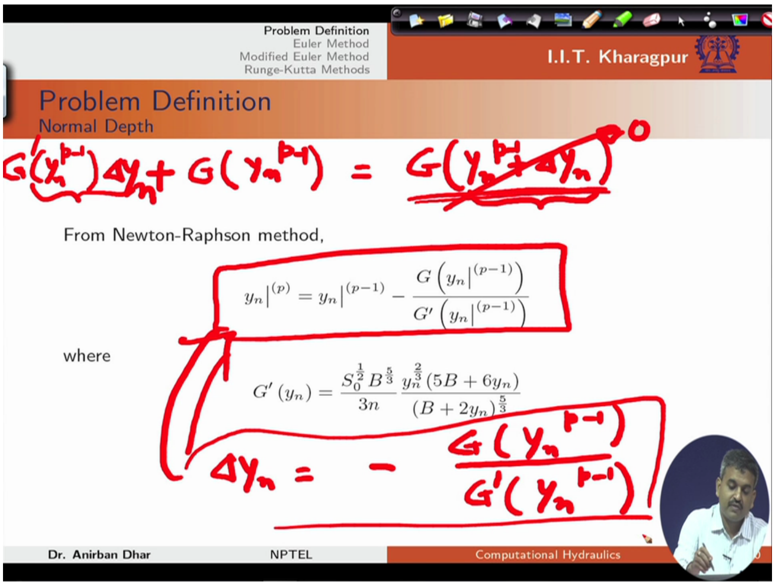

And in case of rectangular channels this G prime can be calculated like this. If we take our non linear expression G and we can differentiate it with respect to, this is G prime yn equals to G yn and this differentiation is with respect to y. We can get this value.

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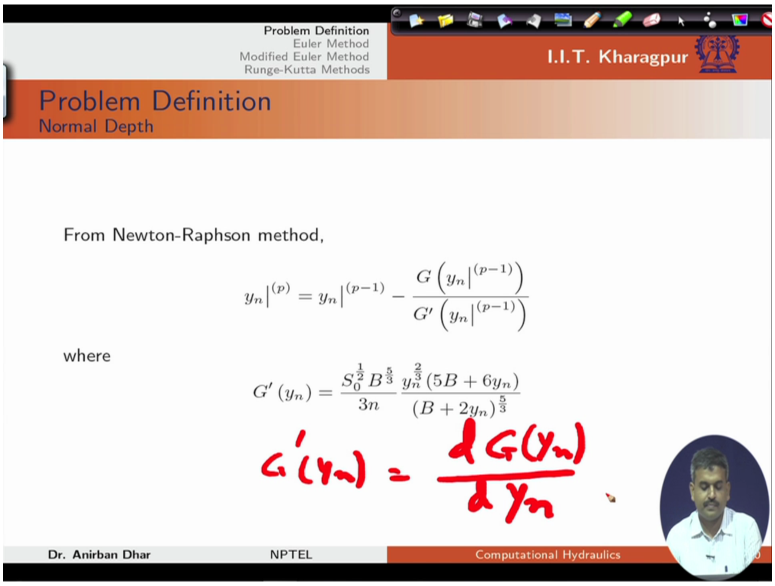

Now possible flow conditions. So now we have information regarding yc yn. Now according to the position of this critical depth line and normal depth line we can classify the flow conditions. If normal depth line is above critical depth line then we can call this condition as mild slope condition.

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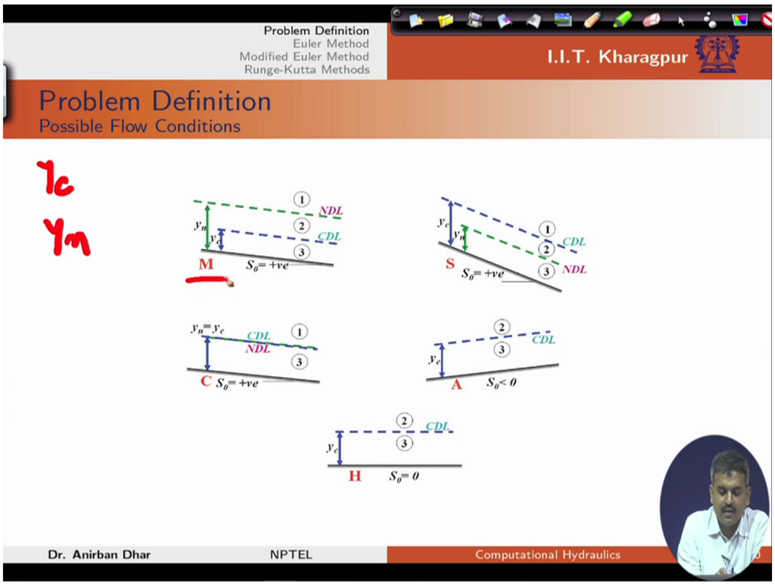

If critical depth line is above normal depth line we can call this condition as steep slope condition. And if normal depth equals to critical depth, in that condition we will call this one as critical slope. If S not less than zero then we call it as adverse slope and for S not equals to zero we have horizontal bed condition or horizontal flow condition.

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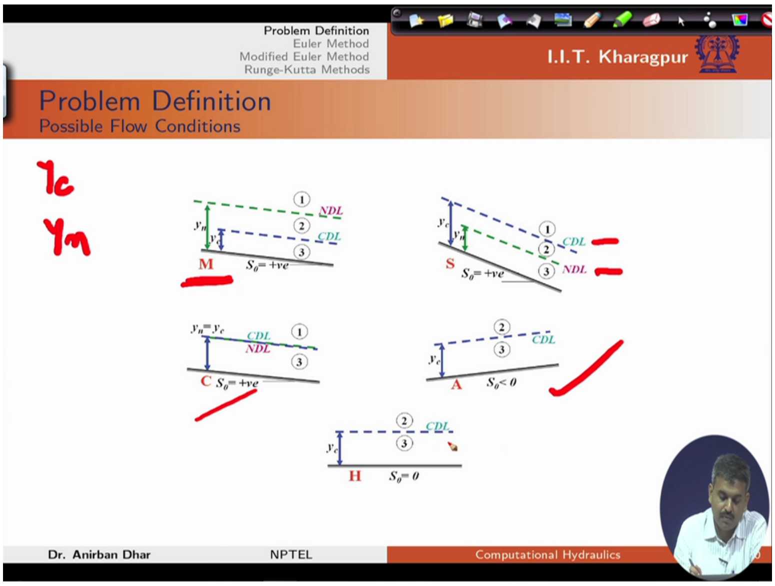

Now in this case we can summarise the thing for mild slope with the symbol M, yn is greater than yc. Steep slope, S, yn less then yc, this is supercritical flow at normal depth. Critical slope, yn equals to yc and horizontal bed slope is not equal to zero and adverse slope we have S not less than zero.

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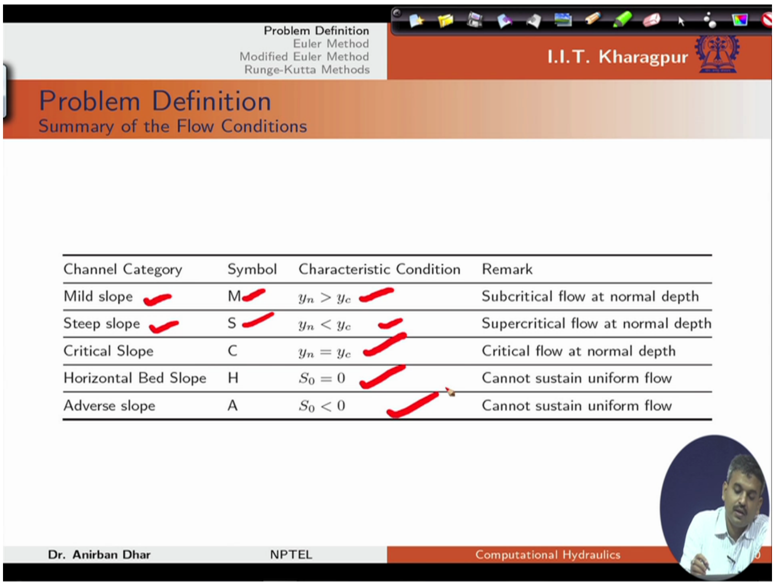

Now from our lecture 7 we can extend the concept of Euler method for solving this gradually varied flow problem. Now for gradually varied flow problem we already know that dy by dx can be represented like x and y.

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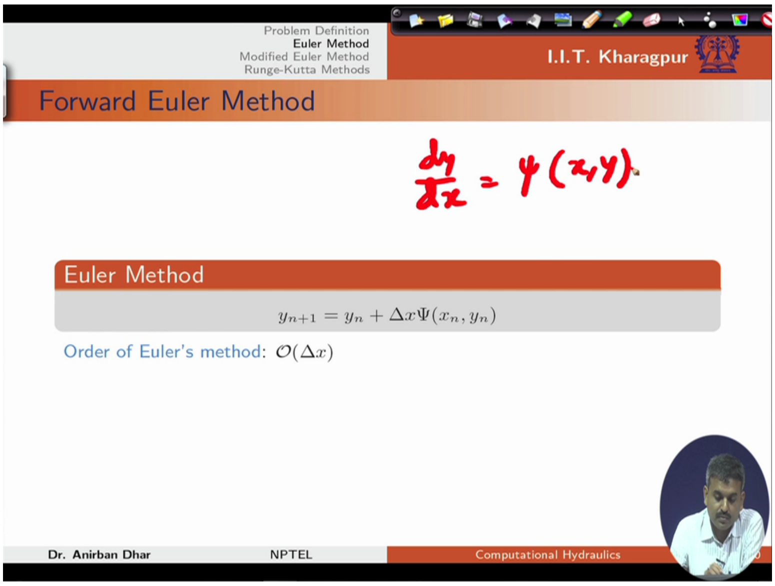

Now if we write this forward Euler method then this is explicit in nature because on the right hand side all terms are known terms. So yn, xn yn, these terms are known terms. So from our problem classification point of view this is explicit in nature.

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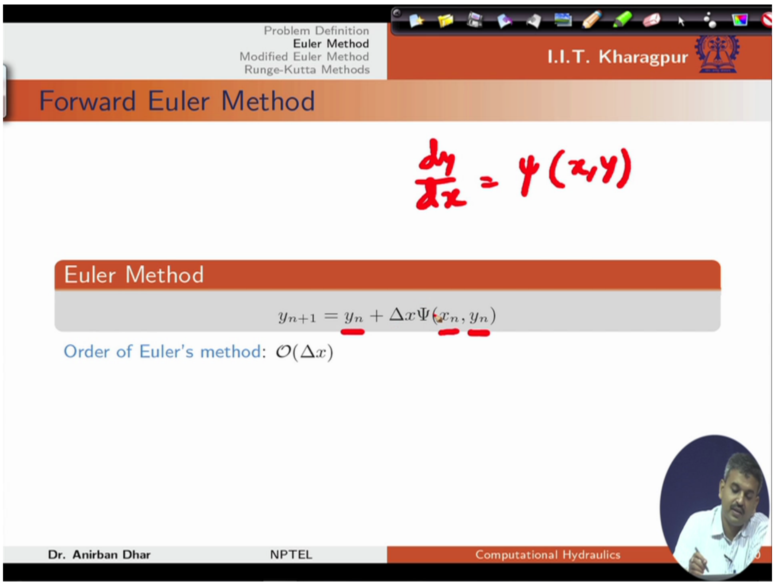

So forward Euler if we open our scilab code then we can see that scilab. Now we can open this GVF forward Euler. Let us say that for given data Q equals to 20 metre cube per second. S not equals to point 0008, n equals to point 015 and bed width is 15 metres, this n is Mannings roughness, g is acceleration due to gravity, Lx is length under consideration. We can consider this length as 200 metres. And y not, this is the initial depth at x is equal to zero we can consider it as point 8.

Now to solve this problem because we have Q, S not, n, B, g. These parameters are available for flow in a rectangular channel.

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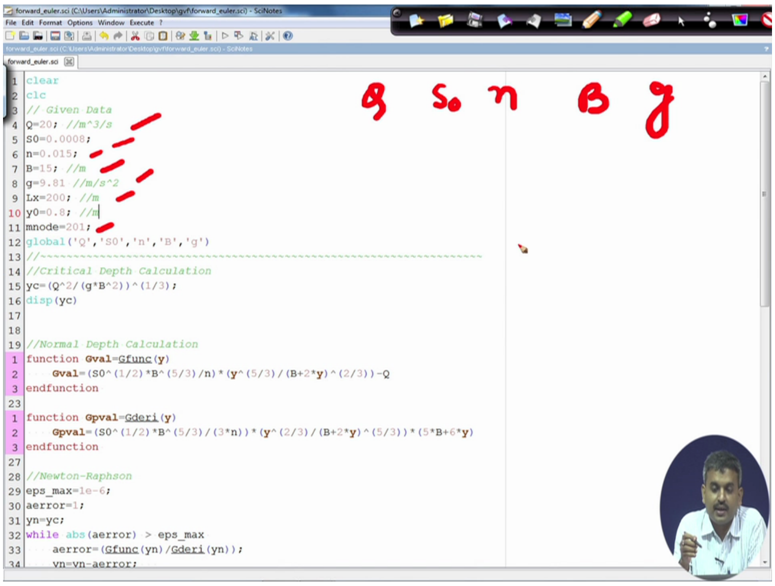

Now if we consider this flow condition with m node that means nodes in the x direction that is m node equals to 201 nodes. We have 200 m length and 200 m nodes are present.

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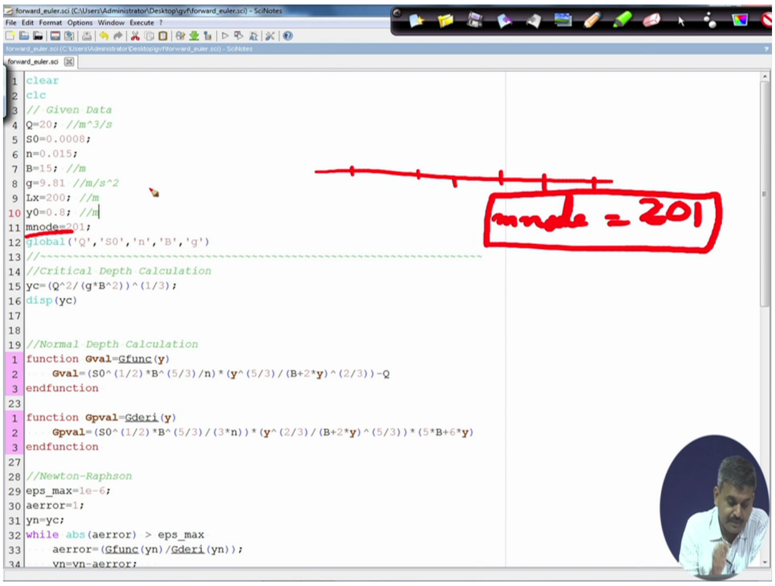

And we are considering that these Q, S not, n, B and g, these are global variables. That means if you write multiple functions we can directly utilise the constant values or constant given data for this case.

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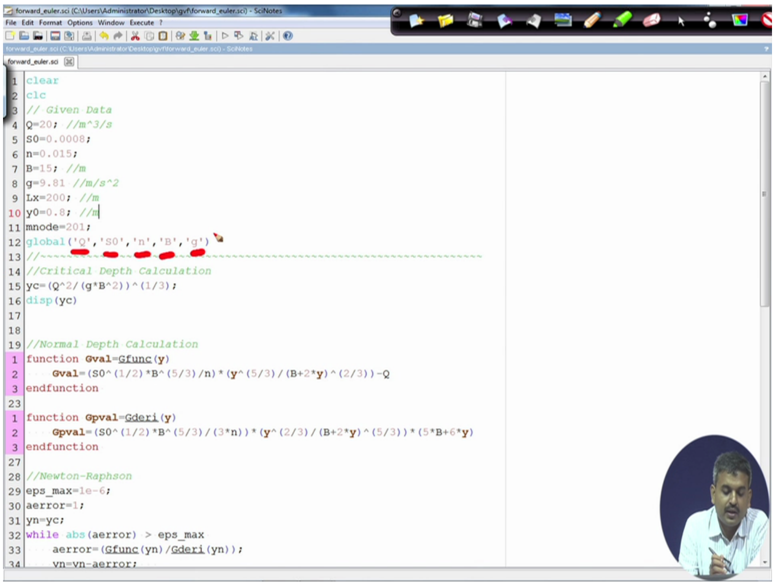

Now to calculate critical depth yc equals to Q square gB square to the power 1 by 3. So that way I have calculated this yc, our critical depth calculation. Now for normal depth calculation we need G yn and G prime yn calculation. So this G n is this function. This is gval equals to gfun c y and this gval is as per our expression, S not to the power half, B to the power 5 by 3 by n.

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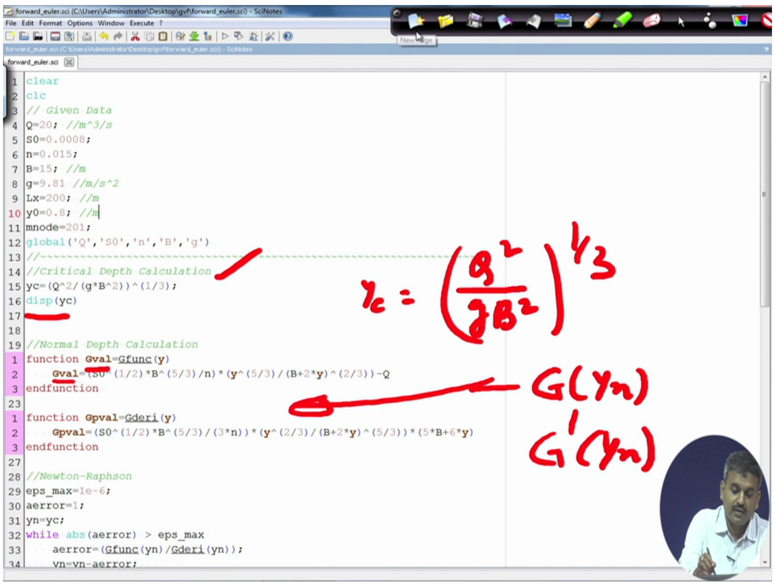

This expression is there. Now if we consider the derivative of G or G prime then this is the G prime that we have calculated.

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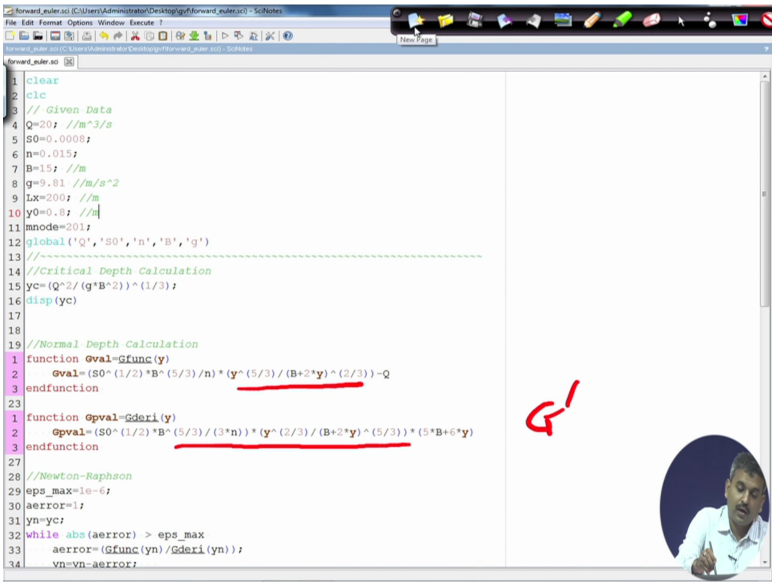

Now after writing these two functions we need to write the basic structure for Newton Raphson so that we can calculate our yn or normal depth. So for Newton Raphson we need that epsilon or error parameter and absolute error let us say this is equals to 1. And yn we are starting from critical depth, whatever critical depth we have calculated we will utilise it as initial guess for our problem. Now with this if we start this iteration loop, now aerror equals to 1 that means absolute of aerror is greater than epsilon max. Whatever is specified here.

Now this error is G function divided by G prime calculated at yn or whatever initial value is there. And we are updating this value with yn equals to yn minus aerror or yn equals to yn minus G by G prime yn. So now from this iteration we will get the normal depth for the problem.

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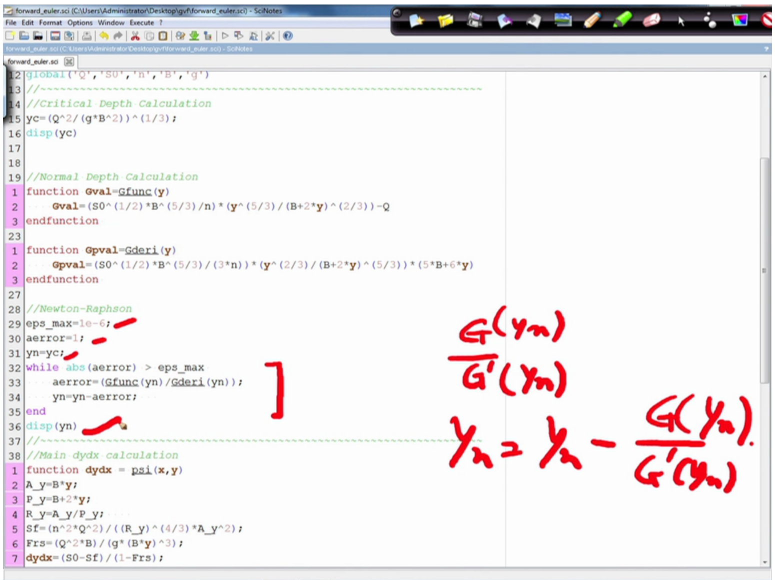

After getting this critical depth and normal depth we can identify the possible flow condition in the channel. So for this problem if I run up to this step then we can see that our critical depth is point 5658 and normal depth is point 8478. So obviously in this case normal depth is greater than our critical depth. So the flow condition is with mild slope condition or M condition.

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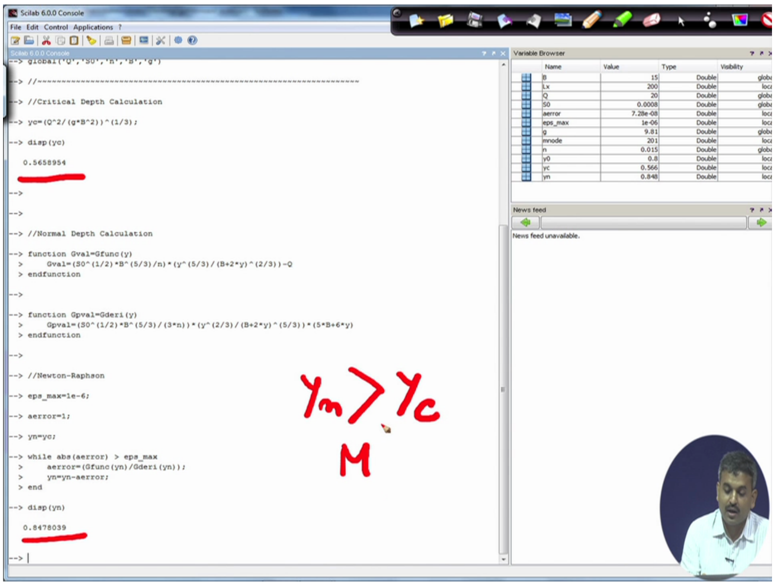

Now in this code next part is our main calculation or dy by dx calculation. In dy by dx calculation this psi is our function, psi xy. We have this Ay which is area, Ty weighted perimeter, so R is A by P and Sf is calculated like this which expression is already there on the slide, n square q square R to the power 4 by 3, A to the power A square and Frs which is Froude number square. So Q square B again this g A cube is there because this is Q square T by gA cube which is Fr square.

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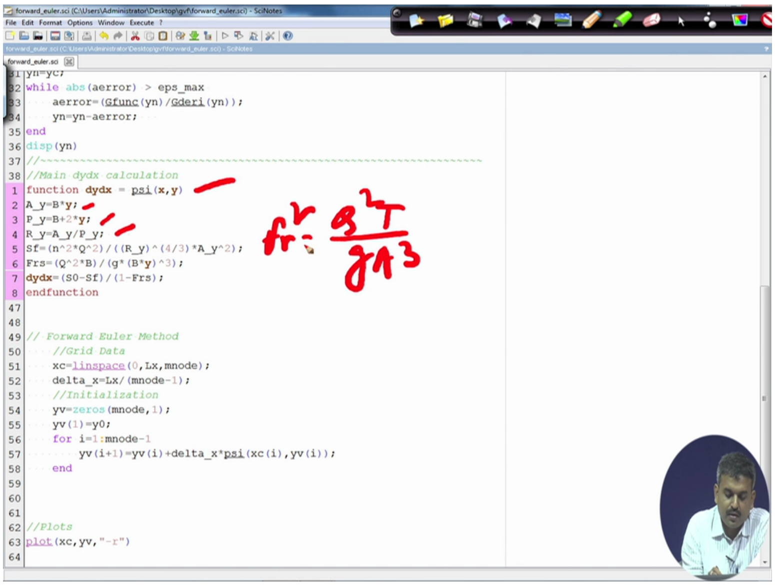

Now from there we can calculate dy by dx. Now if we (uti) call this function we can utilise it in forward Euler method. Forward Euler method let us generate the grid points or x coordinates. So XC is linspace 0 to Lx and m node. M node is the number of nodes in x direction including the end points. And delta x is Lx by m nodes minus 1. So initialization, yv is the y value or flow depth value, zeros m node 1. Yv or 1 equals to y not.

Now in this case if we run this loop from 1 to m node so that if we take 1, this will be yv 2 equals to yv 1 plus delta x and this is psi xc in this case 1, and yv 1. We can calculate the psi value or derivative value. And finally we can get the updated flow depth in the downstream section.

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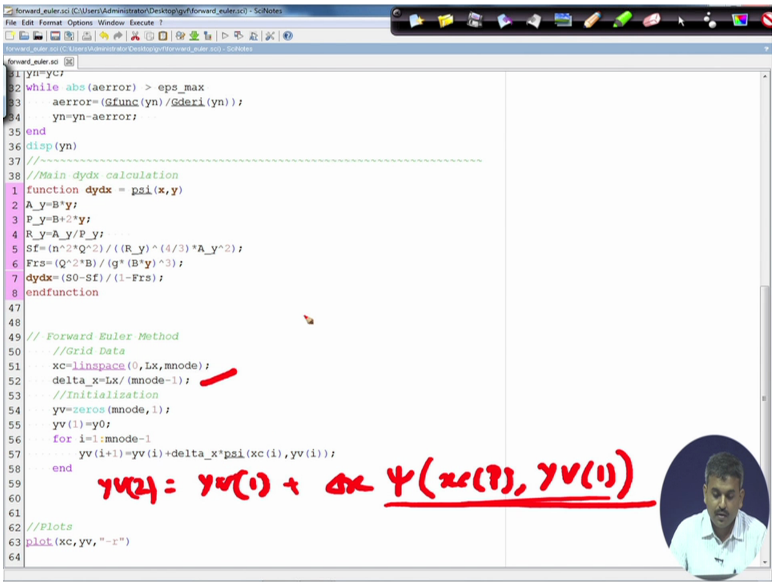

With this we need to plot the results. So plotting first is this xc is the x coordinate, yv is the y coordinate or flow depth value. Now set gca, this line is written so that we can add another line on top of the present plot. So we need to add this normal depth line and critical depth line in the plot. So this is forward Euler method.

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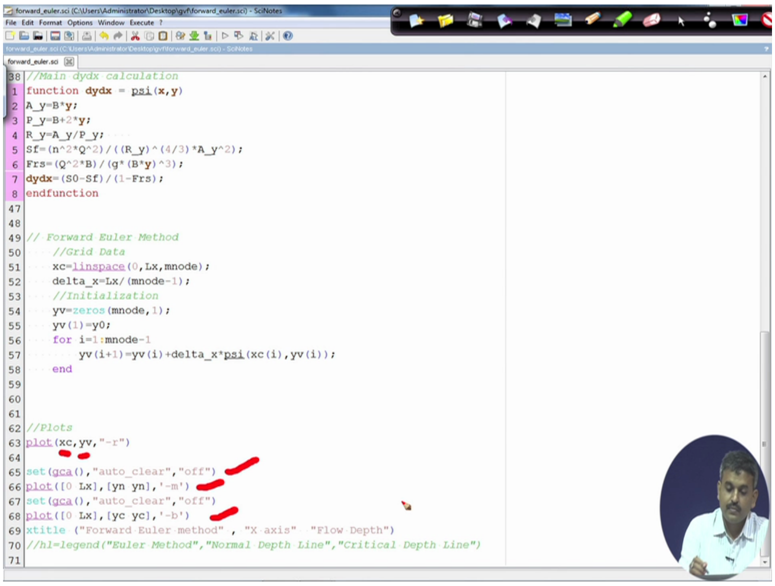

Now let us run this problem. So if you run this problem we can see this top line this is actually normal depth line. The bottom line is critical depth line and our flow condition this is as per mild slope condition and this is zone number 2 as per our classification. So we have M2 profile here in this case.

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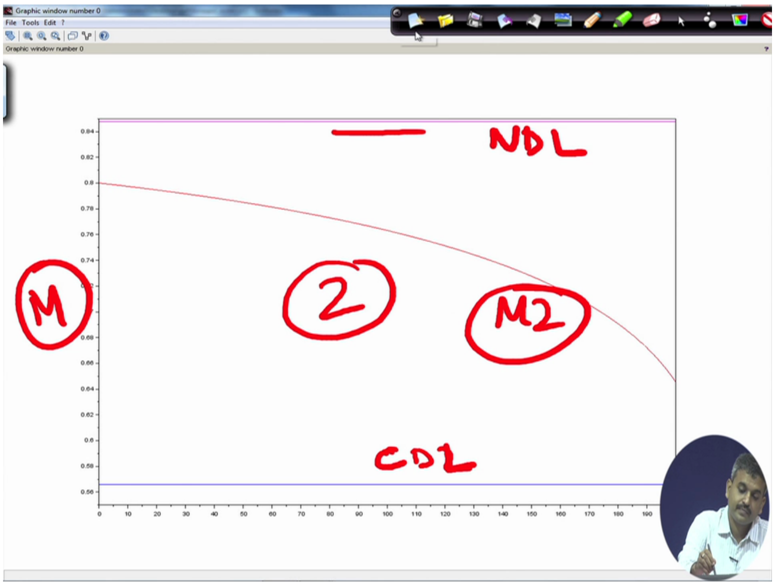

Now we can utilise other methods. in modified Euler method we have this K2. K2 again this is calculated from K1. This K1 is similar to our Euler step but K2 is calculated at half of that or half step.

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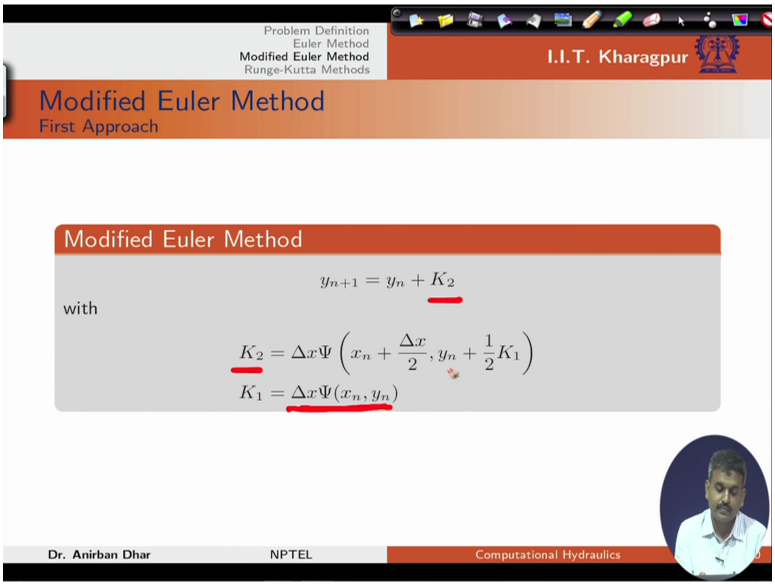

So let us utilise this one for our problem. Now if I open the code corresponding to modified Euler which is first approach. Top portion starting from given data critical depth calculation, normal depth calculation, these are same as given our forward Euler method.

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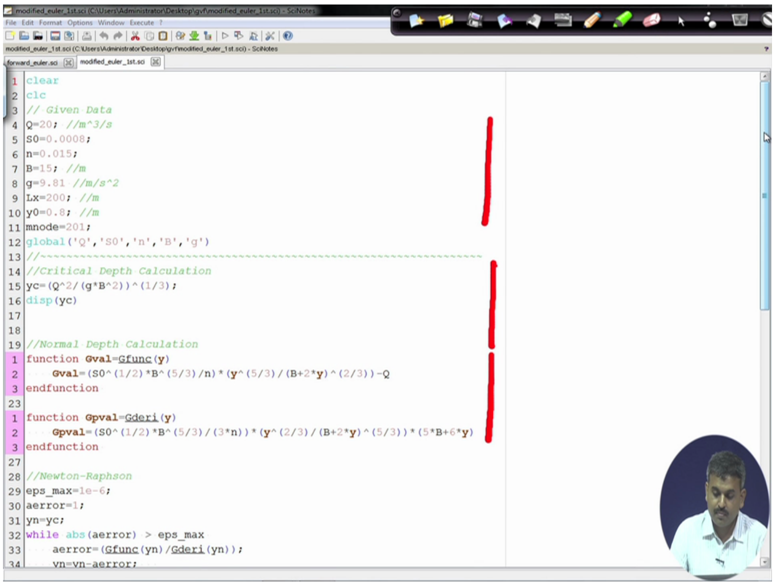

Only change is there in case of these steps. These are internal steps. Now we need to calculate this K1 with xc and yv i and K2 which is calculated at half, this del x by 2 and K1 by 2. And we need to add this K2 with y i so that we can get the final plot.

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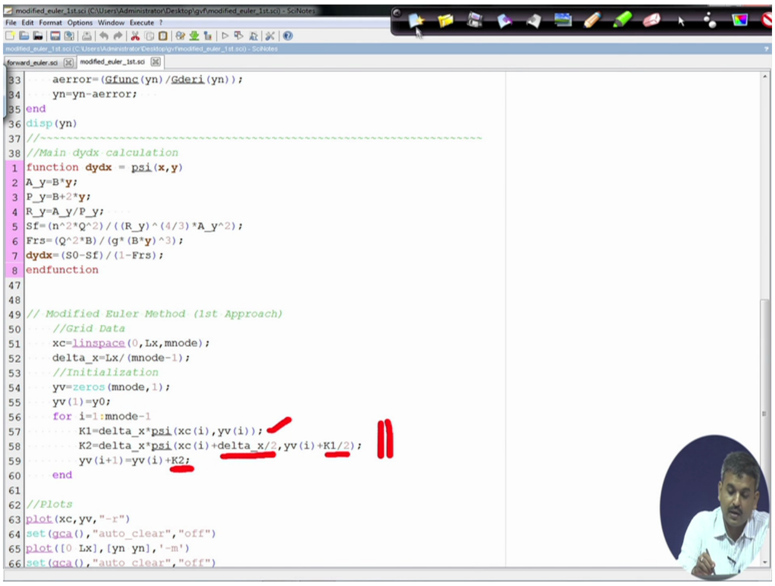

Now if I run this code again we are getting plots which is similar to our old one. Obviously there is some amount of difference in the plot. If we zoom this portion then we can see that some amount of difference is there between these two solutions from forward Euler method and our modified Euler approach which is two step approach in this case.

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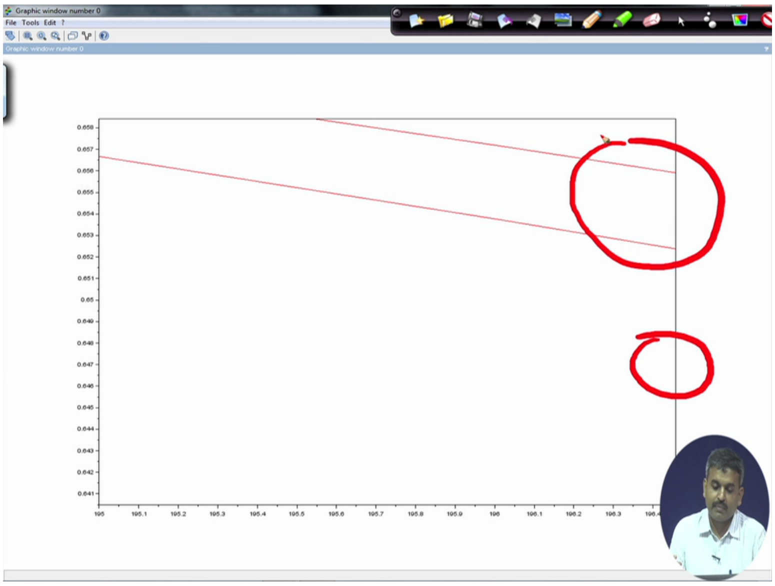

Now again if we apply this second approach which is Euler Cauchy approach and (modi) modified Euler two step method. This is average of K1 and K2.

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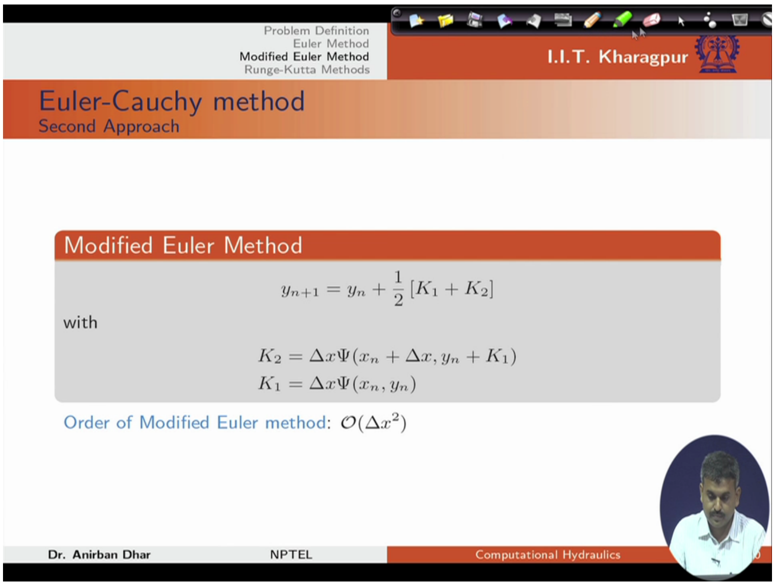

In that case only difference is here, this K1 K2 calculation. This is full step for K2 calculation and K1 is full. In previous case we have used half here. Now in this case we are taking average of K1 and K2.

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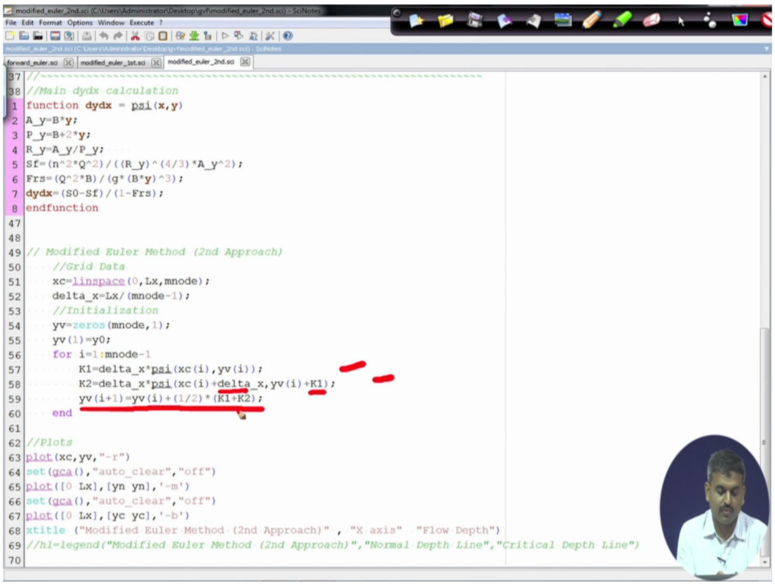

So if I run this code again it is coinciding with one of the cases. But this is not the forward Euler method but the second order approach or modified approach that we have utilised in our previous code. So some amount of difference is there in this case with the forward Euler method but not with the first approach of the modified Euler method.

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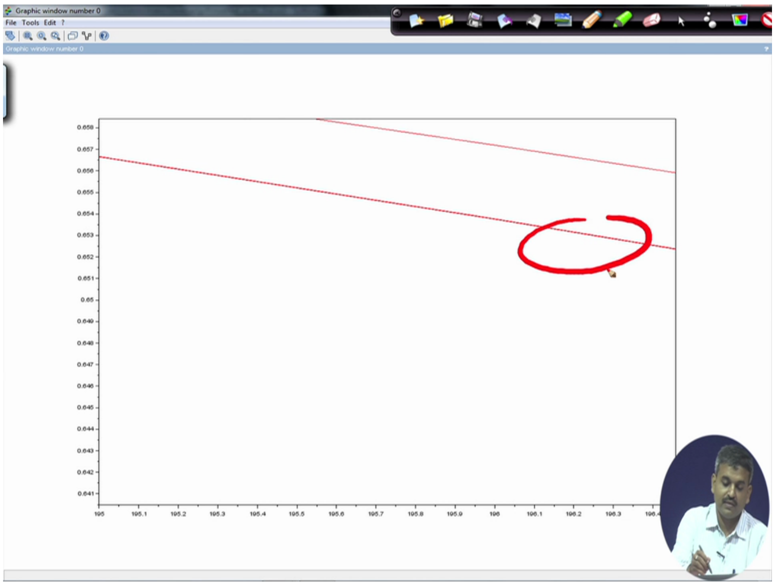

If we utilise our RK2, RK2 again we need to calculate this K1 and K2 as per these expressions.

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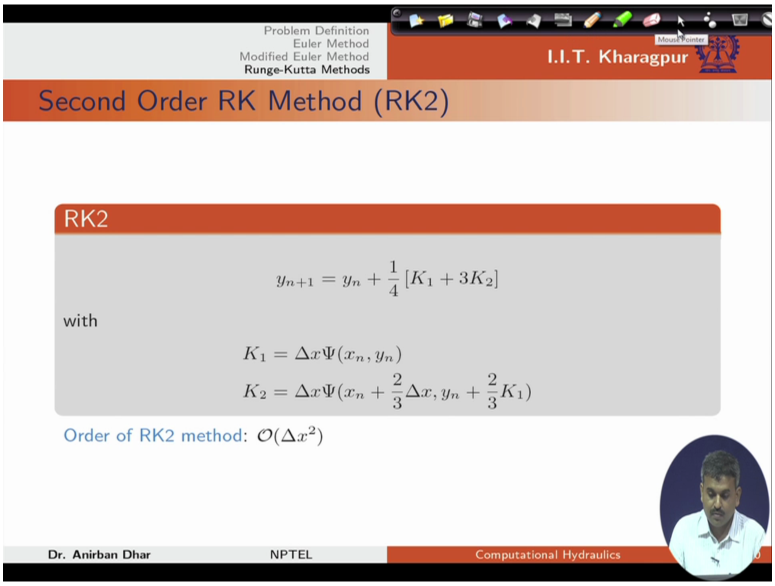

So in that case only change is there for this K2. K1 is similar to our previous one but this 2 3rd delta x and 2 3rd K1 is a different thing. And the final step 1 4th K1 and 3 K2, this is there.

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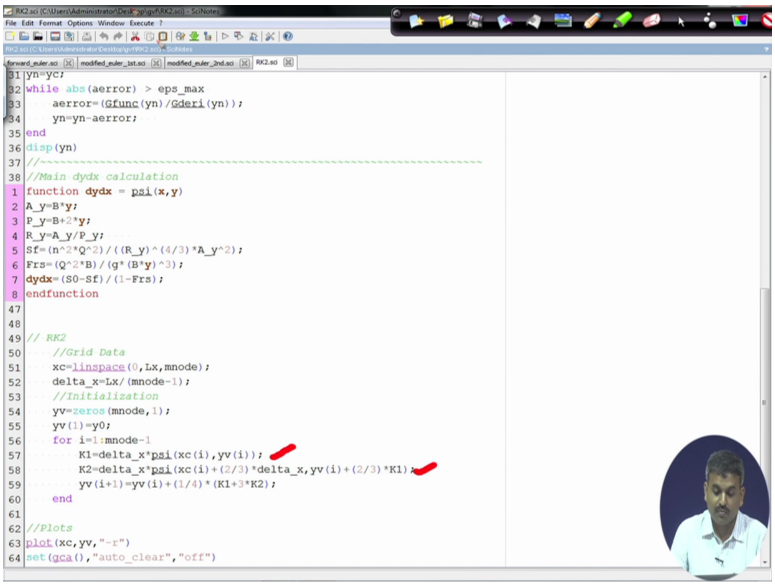

So if we run again this problem with other colour, maybe magenta colour for this one. So again it is coinciding with our previous solution.

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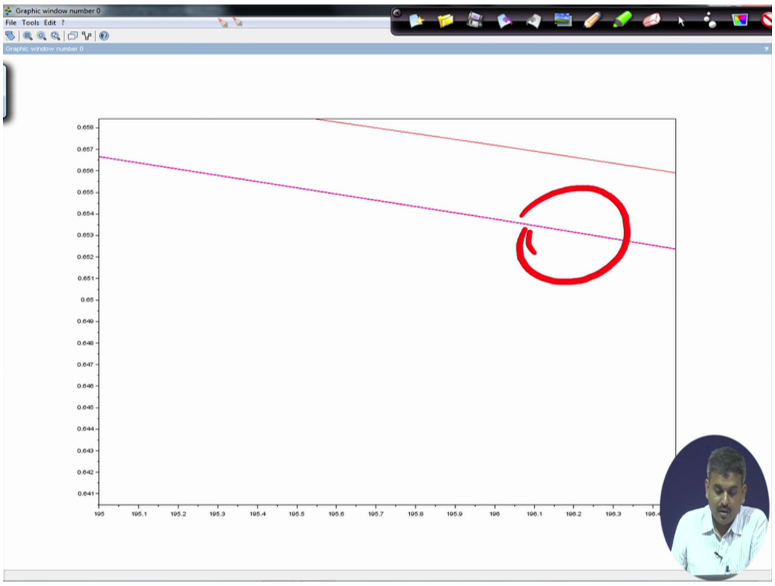

Now in this case we can see that the solutions are converging for our first order and second order methods to a single solution. Now in case of our M2 curve this is decreasing in nature.

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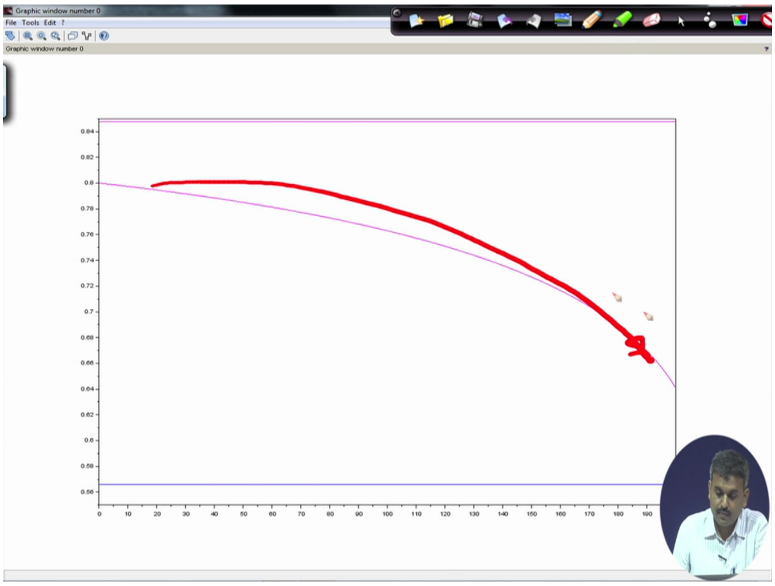

Now if we utilise our RK3 method which is (consi) considering K1, K2, K3 with these expressions.

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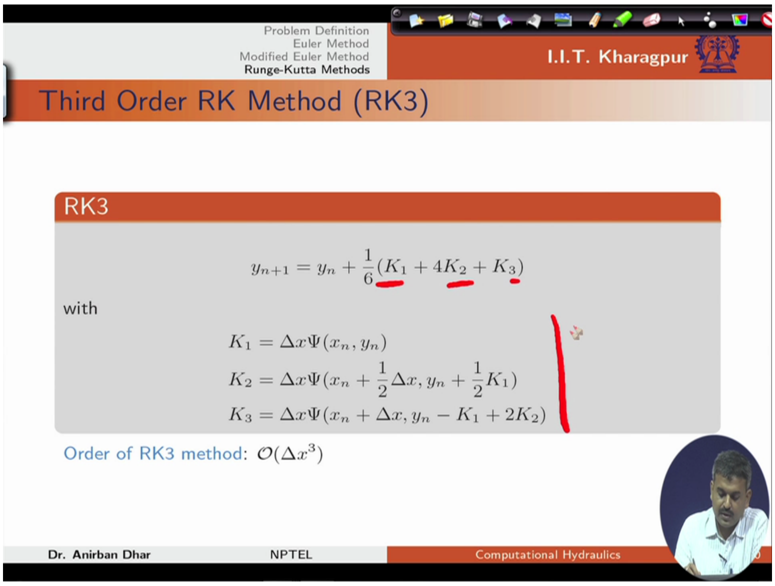

Now we can utilise it for our problem. This is RK3. Now in RK3 other portion, that is similar. Only change is there in terms of K2 and K3 calculation. Now we can take average or weighted average of this K1, K2, K3 slopes and we can get the final outputs.

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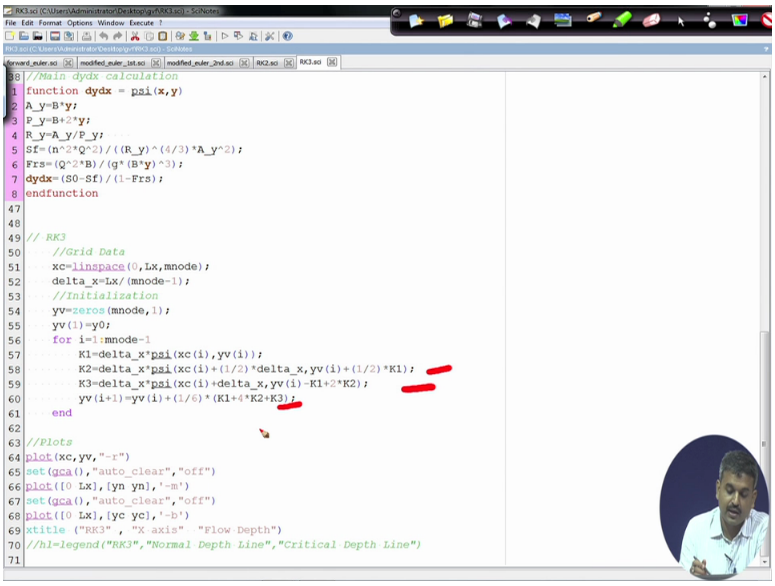

So if we run this code again we will get the solution and solution now same nature.

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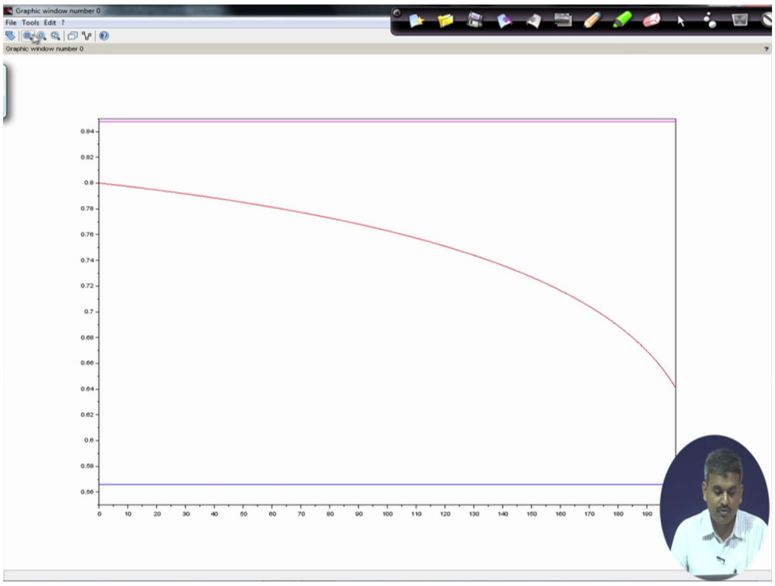

Similarly if I utilise this RK4, so RK4 also we need four terms K1, K2, K3.

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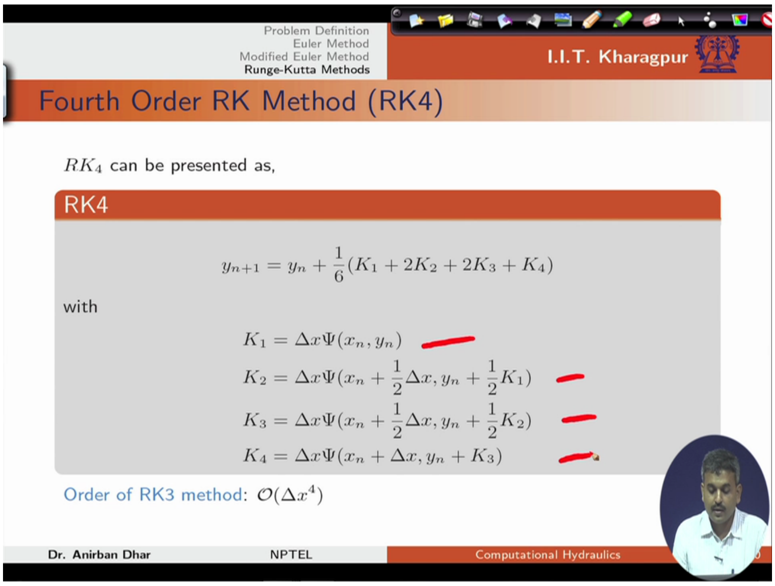

And in RK4 we need to change the program at this internal step. This is K1, this is K2, K3, K4 and again we can take weighted average value here to get the final flow depth value.

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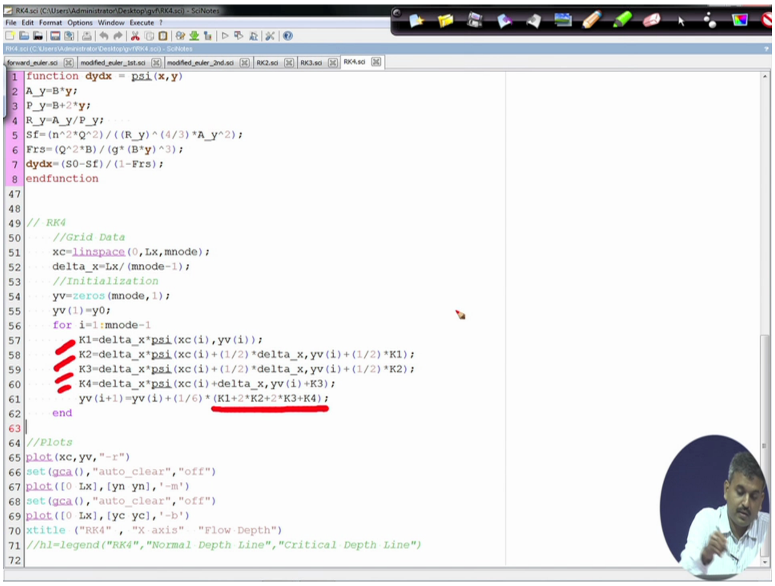

Obviously one thing is clear in this case that whether we are considering a second order, third order methods, always these methods are explicit in nature. If we consider K1, K1 depends on yn. So K2 depends on K1. K1 is again known quantity from our previous step. K3 calculation we need this K2. S2 is coming from here.

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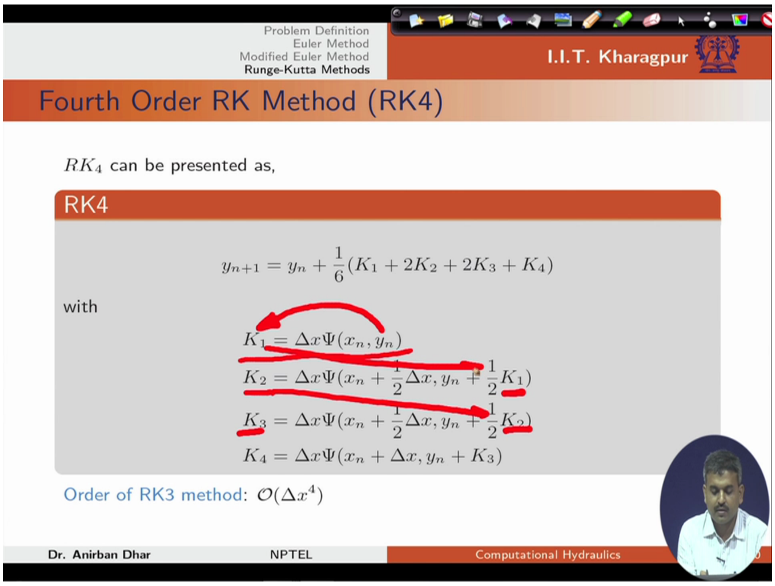

If we see this K1 is coming from here to calculate K2. Again for K3 calculation we are utilising K2. Again for K4 calculation we are utilising K3.

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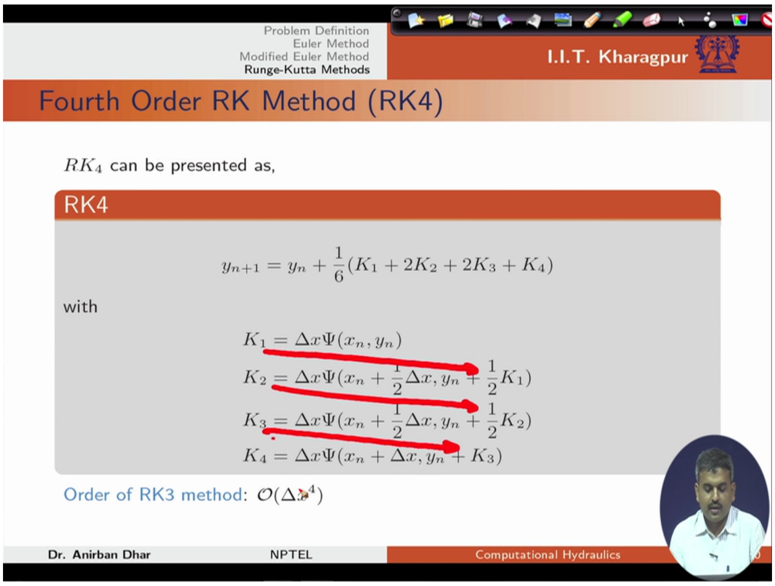

Now all these methods whether of forward Euler, modified Euler first approach, modified Euler second approach, RK2, RK3, RK4, these methods are explicit in nature. So these are the codes that I have written and I will supply this so that you can run these codes and get result and verify whatever I have showed during this lecture class.

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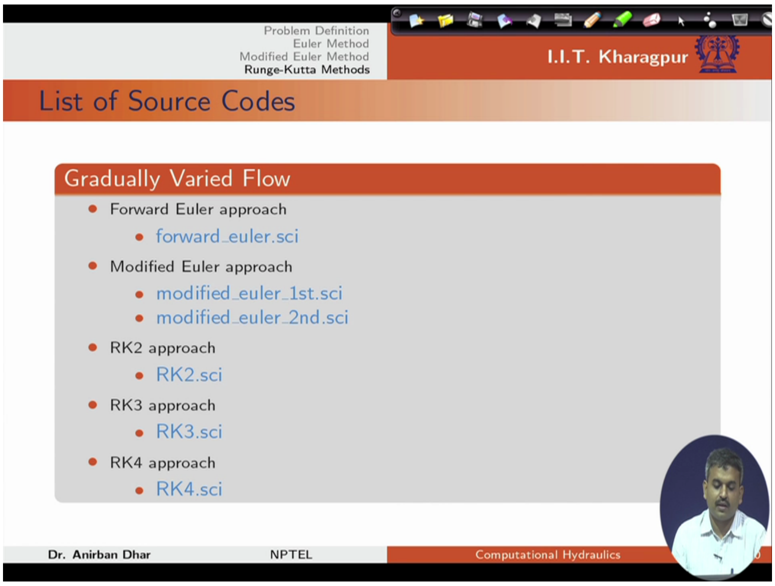

Thank you.