Foundation Engineering
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Lecture - 13
Shallow Foundation - Bearing Capacity II

So, in the last class I have discussed about different types of shear failure it is general shear failure, local shear failure and the punching shear failure.

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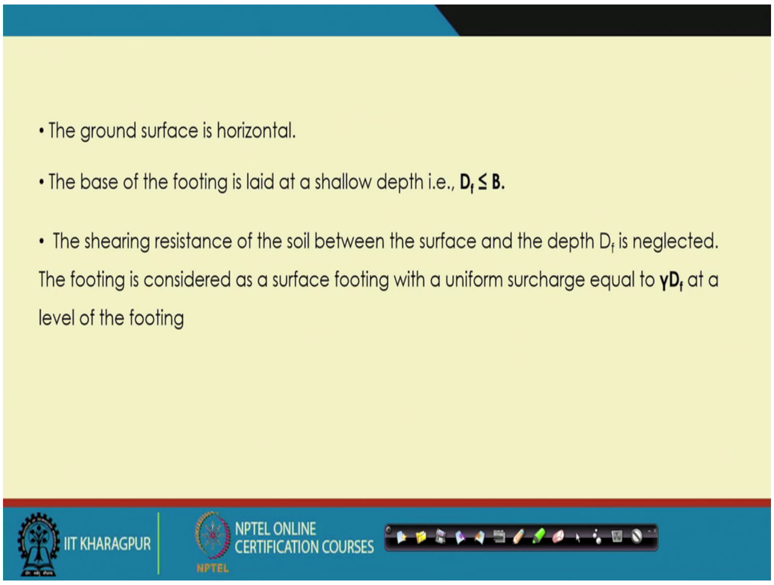

And, then I have discuss about the first bearing capacity theory which was proposed by Terzaghi's and what are the assumptions, that is it is apply applied for the strip footing, then homogeneous soil, 2 D analysis general shear failure, and it is the loading is vertical and the concentric, then it is the surface is horizontal, then it is applicable, if the depth of the footing is less than equal to the width of the footing and all these assumptions I have discussed.

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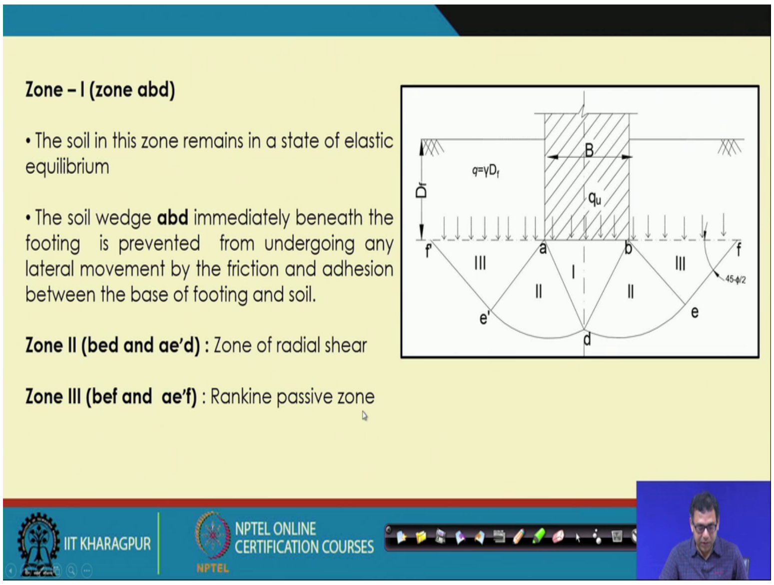

And then there are 3 ; 3 zones how these zones are form? And then what are the 3 zones zone 1 is the elastic equilibrium state of elastic equilibrium zone 2 is this zone of radial shear, and zone 3 is the rankine passive zone.

So, now these are the; what are the forces is acting on this wedge.

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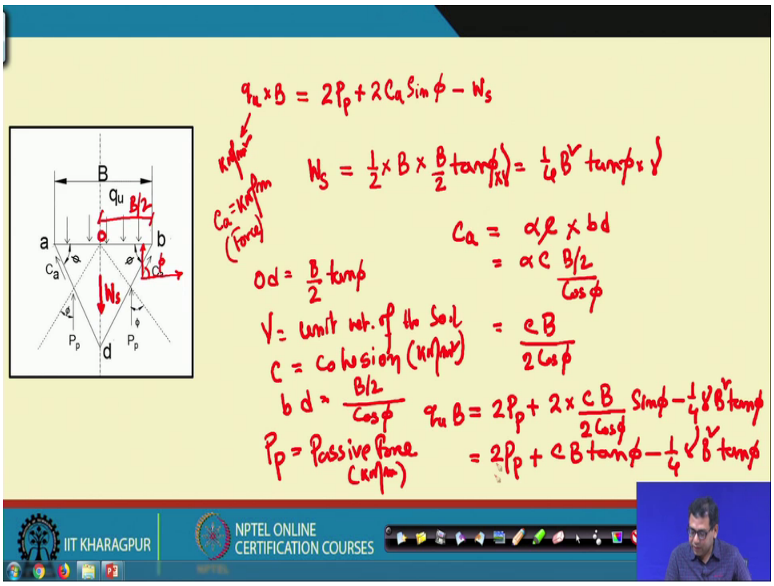

So, now, as I discussed that these are the forces and in addition and to that there will be W s, which is the weight of the soil weight of this wedge. So, now if I now take the equilibrium of the all the forces now if this is the width the first the q u, which is acting in downward direction.

So, q u in to B that will be so, q u is the stress and if we multiply it will be the force. And remember that here it is in 2 D analysis the unit of the force will be kilo Newton per meter, if it is plane strain analysis or it is it is straight footing.

So, the force unit will be kilo Newton per meter stress unit q u unit will be this q u unit will be kilo Newton per meter square, but force will be kilo Newton per meter because it is in first strip footing. So, now, q u into B then the P p is acting P p and this is the into 2 because P p is acting this side P p is acting this side. So, about this is acting in upward direction, but q B is acting in downward direction.

And, because we are now equating all the forces; which is acting in the vertical direction; whether, it is downward or the upward. So, q u b into b P p is the force already and then plus the adhesion is acting so, but if I take the vertical component of the adhesion, because this is the angle phi. So, it has a horizontal component and it has a vertical component ok.

So, adhesion so, if I take the vertical component; so, this will be C a into sine phi ok. So, C a into sine phi and 2, 2 because this 2 is it is in 2 phases. So, C a is the added ship force it is also C a unit is kilo Newton per meter it is the force. Now, this is also acting in upward direction P p is acting on the upward direction, but the W s weight is acting on the downward direction. So, these are the forces is acting in this wedge.

Now, how we can take all our components? So, first we are talking about I will get the W s, W s is the wedge weight of the soil within this wedge a b d. So, we can say this the area of this triangle is half into base base is the B into the height height is this one. So, this value is B by 2. So, this value is B by 2 this angle is phi. So, this will be equal to b by 2 into tan phi; so, height of this or you can say if this is O. So, the O d will be B by 2 into tan phi and that is the height.

So, half into B into B by 2 into tan phi; so, it will be 1 by 4 B square into tan phi ok. And then we have to multiply with the unit weight of the soil; so, gamma is the unit weight of the soil.

So, finally W s we will get now C a. Now, C a is equal to as I mentioned that this is the alpha into the small c, C a is the capital C a small c is small c value is the cohesion. So, small c is the cohesion and it is acting on this side of this wedge. So, C a is the force and alpha C is the cohesion addition, which is acting on this side of the wall and the force means if I want to get the force we have to multiply it with the b d or a d. So, what is the b d? b d value is b d value is this is b by 2 this is 5. So, the b d value will be B by 2 divided by cos phi.

So, we have to multiply with the b d. So, that will give alpha into small c into B by 2 divided by cos phi. Now, as I mentioned this the imaginary wall; so, alpha value will be here 1, because it is basically soil versus soil. So, alpha value if I take 1; so, this will be c B divided by 2 cos phi. Now, if I put all these values here, then I can write that q u B will be equal to 2 P p plus 2 into c B divided by 2 cos phi into sine phi minus 1 4th gamma B square tan phi ok.

And finally, the form will be 2 P p plus c B into tan phi. So, this will be c v tan phi minus 1 by 4 gamma B square into again tan phi. So, this will be 2 P p into c B tan phi into gamma B square into tan phi by 1 4. So, phi is the friction angle, c is the cohesion. So, the c is the cohesion is unit is kilo Newton per meter square.

So, once I get this expression then. So, we have to now here another thing is that P p is the passive force ok. So, this unit is kilo Newton per meter ok. So, will finally, get this expression. So, once I get this expression, then we have to calculate how will simplify this expression and finally, got a common form of expression.

So, now if I take this expression again so, if I write q u B will be equal to 2 P p plus b c tan phi minus 1 by 4 gamma B square into tan phi.

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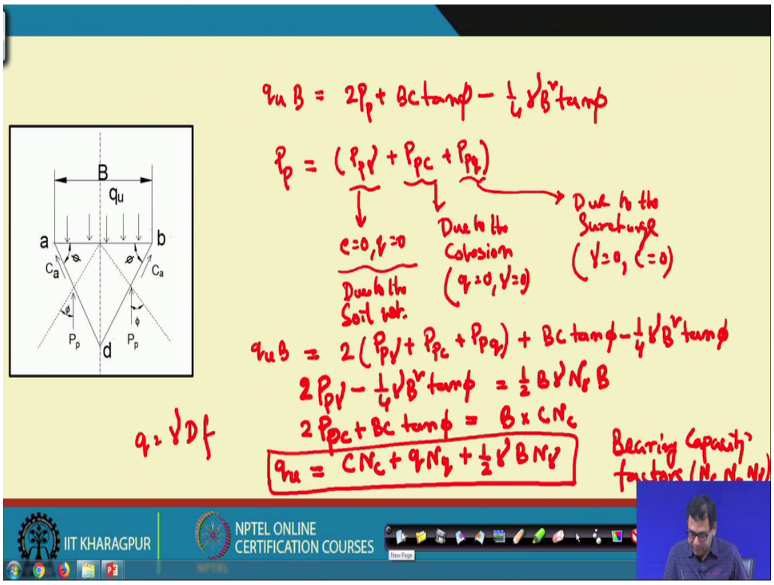

Now, P p is function of 3 factors; that means, P p the passive resistance is coming from for 3 reasons. The first reason is the passive resistance is coming fall for the cohesion of the soil, the second one the passive resistance is coming due to the weight of the soil, and the third reason is the passive resistance is coming due to the surcharge of the that is applied.

Because as I mentioned the soil about the base of the foundation where the contribution is taken as a surcharge, which is acting on the base of the foundation; so, that surcharge will contribute in the passive resistance. So, that mean that is 3 contributions; one is the contribution due to the surcharge, then contribution due to the cohesion of the soil and the contribution due to the weight of the soil.

So, now P p I can write that this P p is this is the P p gamma plus P p c plus P p cube. So, this is the gamma where this is where when we calculate the P p gamma, we consider that c is equal to 0 q is equal to 0; that means, the cohesion is 0 and surcharge is also not applied.

So, this is due to the weight so, this is due to the soil due to the soil weight, and when you can consider this contribution we can consider the assumed that soil is cohesion less and surcharge is nil. And this Ppc this is the contribution due to the cohesion and we when we consider this one we assume that q is equal to 0 and gamma is also equal to 0; that means, the soil is weightless and the surcharge is nil. And the third one this is this is due to the surcharge and when we consider this one we assume that that our soil is weightless and cohesion is also 0.

So, now, these are the 3 contribution that we are writing. So, finally, if I put this contribution in the P p then then the q u B will be equal to 2 p P p gamma plus P p c plus P p p p q ok, plus B c tan phi minus one-fourth gamma B square tan phi. So, now if now finally, after solving these equations so, in the detail solution I am not giving. So, from here, I am just writing the final form of the equation because I have given you the what are the contribution how these things are coming? So, final form of the equation is we can write that from here the 2 all the gamma count component P p gamma 2 P p gamma minus one fourth gamma B square tan phi that is equal to half B gamma N gamma B.

And, we can write that all the cohesion components P p c 2 P p P p c plus B C tan phi is equal to B into C N c. And so, if I put this expression here then finally, the ultimate load carrying capacity q u will be that C N c because this B B will cancel out, C N c plus q N q plus half gamma B N gamma. So, this is the final expression of the ultimate load carrying capacity of the soil so, based on that Terzaghi's.

So, from here to here I am just directly writing, but these are the final things then what is N c N q N gamma. So, these are called bearing capacity factors. So, this is these are the N c N q N gamma. And q what is q this q is equal to gamma into D f. So, that is a surcharge which is acting at the base of the footing that is q.

So; that means, we have the final equation C is the cohesion and q ultimate q u is the ultimate load carrying capacity of the soil and that is equal to C N c q N q plus half gamma B N gamma. Now, on one thing I want to mention because there is 2 gamma is given, one is here also gamma and another gamma is here. So, this is the surcharge. So, remember that if the soil is homogeneous because that is the assumption here. So, in that case all the gammas are same.

So, if the soil is homogeneous doing then it is same gamma to use, but otherwise this gamma represent the soil below the base of footing and this gamma represent the soil above the base of the footing. So, because these are the 2 things I mean 2 different things because this is these so; that means, finally, so, this is the gamma or D f into gamma which is above the footing because this is the about the base of the footing because this is the surcharge.

So, now final form of this expression and getting so, finally, after if I write this expression, then we will get this is the final developed ultimate bearing capacity which is C N c gamma D f depth of foundation N q plus half gamma B N gamma.

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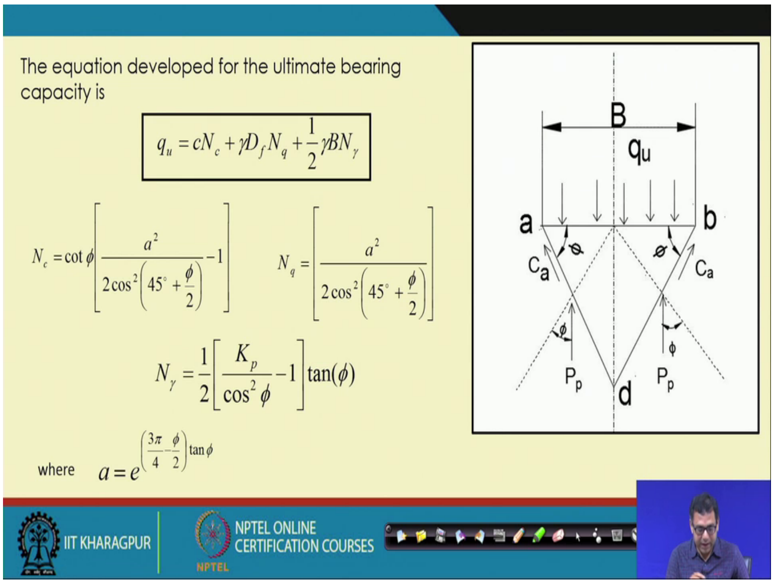

So, these are the expression of these 3 bearing capacity factors are given N c N q and N gamma.

So, now, if we look at this N c N q N gamma so, these are function of phi. So, you can see. So, these are all function of phi. So, here these bearing capacity factors spot that this Terzaghi's these are function of phi and either the equation that finally, will get.

Now from this equation if I know the you need to wake up the soil, if I know the strain parameter; that means, c and phi then if I put this value, if I know the depth of foundation and the width of foundation so; that means, if I know width the foundation and depth of foundation strength parameter and unit weight of the soil, then we can determine what will be the ultimate bearing capacity of the soil or foundation.

Now, so here based on as this N c N q N gamma function of phi.

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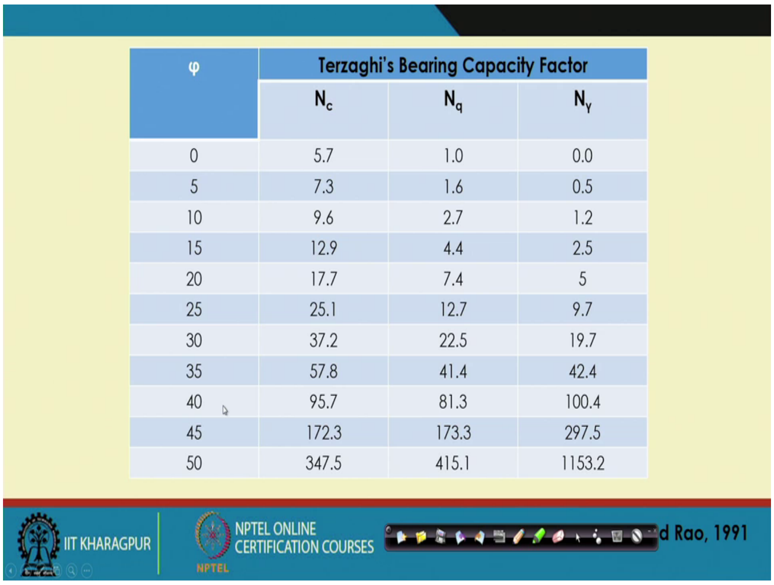

φ	Terzaghi's Bearing Capacity Factor		
	N_c	N_q	N_γ
0	5.7	1.0	0.0
5	7.3	1.6	0.5
10	9.6	2.7	1.2
15	12.9	4.4	2.5
20	17.7	7.4	5
25	25.1	12.7	9.7
30	37.2	22.5	19.7
35	57.8	41.4	42.4
40	95.7	81.3	100.4
45	172.3	173.3	297.5
50	347.5	415.1	1153.2

So, Terzaghi's has given this chart for bearing capacity factors N c N q N gamma for different 5 values ok. These are for different phi values these are in degree phi in degrees from 0 to 50. So, now, if you look at this 0 value, if your phi is equal to 0, then as for the Terzaghi's the N c is 5.7 q N q is 1 and n gamma is 0 ok. And, then these are the factors it is given up to 50 degree for the phi value.

So, this chart table we will use when we calculate the bearing capacity and we will get this N q N c and N gamma value directly from this table.

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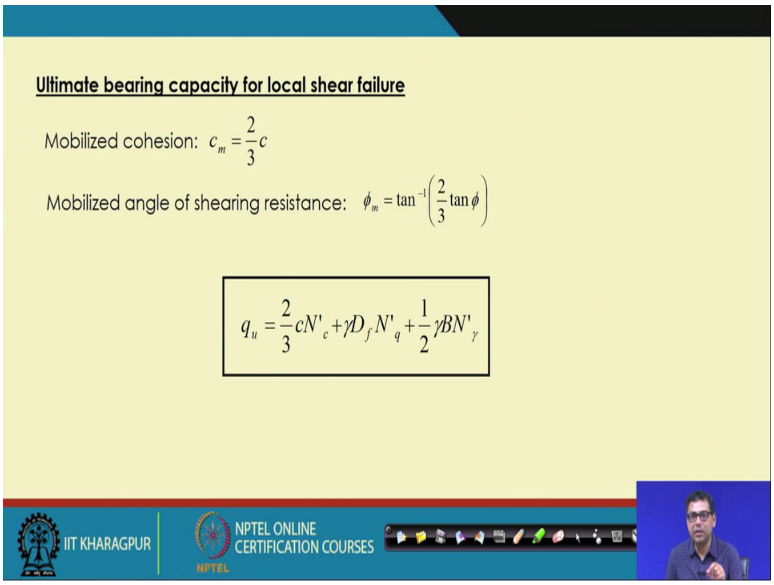

Now, one thing now as I mentioned that this theory, it has several assumptions. The first has a 1 has a 1 of them is that this is applicable for general shear failure. Now, there are other 2 types of failure also, that the local shear failure and the punching shear failure. Because, shear you are not worried about the punching shear failure, because hardly we will place the foundation on a very soft soil ok.

So, that possibility we are neglecting so; that means, your because if the soil is very poor. So, why should I place the foundation over there. So, in that way we can neglect the what will happen in the punching shear failure, but we can place the foundation on the moderately your relatably loose soil or the moderately relatively soft clay. So, in that case the local shear failure will occur. So, now how I will we can use these theory, which is developed for the general shear failure for the local shear failure. Now, Terzaghi's has mentioned that if it is a local shear failure, then the cohesion factor the cohesion value we have to take the 67 percent of the actual c.

So; that means, your c m will be the mobilized 2 third of c and the mobilized shearing resistance phi value will be tan inverse two-third of tan phi. So, what will what will do? We will take phi nu phi which is tan inverse two-third of phi and we will calculate the bearing capacity factor based on this new phi phi (Refer Time: 21:37), that is why here this is N c dash N c dash N q dash and N gamma dash; dash means these bearing capacity factor is for local shear failure. And, it is determined by taking a new phi, which is the 2 third tan inverse 2 third tan of the original phi ok.

So, that is why and then c also you have to take instead of c we have to take two-third of c. So, this is the recommendation for the local shear failure.

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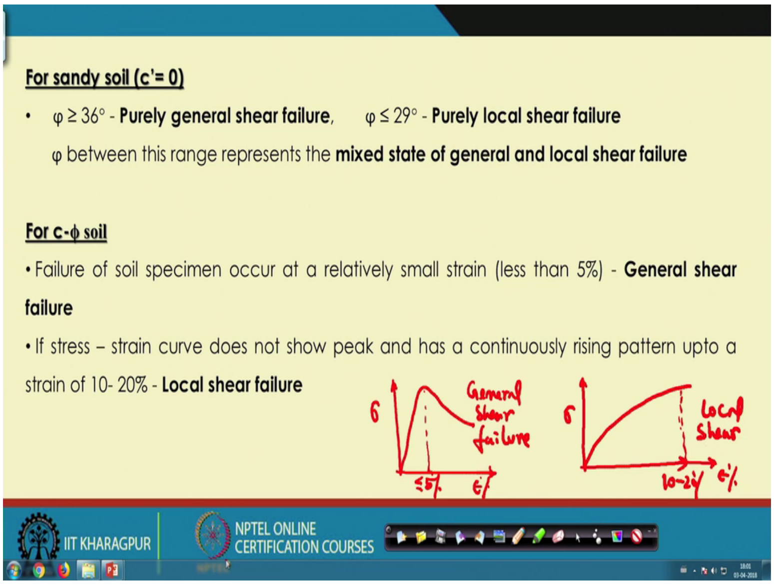

Now, the question is when we will use general shear failure expression and when we will use the local shear failure ex expression. Now, if the soil is sandy soil that in pure sandy soil whole cohesion value is 0. Now here sometimes you will see this c dash c dash is if it is the effective cohesion. So, then water table effect water effect is incorporated.

So, now c dash or c if the it is purely a sandy soil or cohesion less soil, then if phi value is greater than equal to 36 degree. As for the recommendation provided by Terzaghi's, then purely general shear failure will occur. So, if the soil phi value c value is 0 and phi value is greater than equal to 36 degree, then the general shear failure will occur. And, because in that case soil is in dense range if the phi value is greater than equal to 36 degree, and if the soil is phi value is less than equal to 29 degree, then the pure local shear failure will occur.

Now, if soil is in between these range that mean within the other 29 to 36 degree then that is the range of mixed rate of general and local shear failure in that case. So, if I phi is the greater than equal to 36 degree the bearing capacity factor you will use for the corresponding to original phi for our N c N q N gamma and we will put original phi value, not if the it is less than equal 20 9 degree, then will reduce it to a new phi phi n and then will calculate the bearing capacity factor based on that new phi, but if it is in between that then we have 2 linearly interpolate the bearing capacity factors.

So, I will solve our example then I will show how will linearly interpolate this bearing capacity factor, if your phi value is in between this range. Now if, but if the soil is c phi soil, then even if the phi value is less than equal to 29 degree there is no guarantee this although is the local shear failure will occur.

Because, because it is it is not the only sand, some cohesion is there na. So, that is why the strength will not be I mean you cannot say easily that is it is will be the local shear if it is less than 29 degree. Because of the cohesion there may be a general shear failure also. And, then how will identify I will undo identify whether there will be a local shear failure or the general shear failure, but in such case we have to take the load settlement curve.

So, the if it is c phi soil the failure of soil specimen occur at a relatively small a strain less than 5 percent, then the general shear failure occur so; that means, in that case you have to collect the soil sample and if you test it in the lab. And, then when in test it will get the load settlement curve, if soil fails relatively small strain; that means, soil is stiff in that range so; that means, if it is less than 5 percent, then little bit general shear failure my if stress strain curve does not show P and has a continuous rising pattern upto a strain 20 to 10 to 20 percent then the local shear failure will occur.

That means, for example, if we have a shear stress strain curve so; that means, here you will get that there is the particular peak is observed and this soil specimen fail. So, this is the your strain and this is the stress and this is within the 5 percent strain, then this is the here this is the general shear failure, but if the soil stress strain curve is something like this. There is no peak and it is going beyond say 10 to 20 percent of strain, this is the strain, this is the stress beyond 20 to 20 percent strain steel we have this type of pattern, then this will be the local shear failure ok.

So, in the c phi soil not only phi will indicate that whether there will be a general shear failure or the local shear failure, we have to go for the stress strain curve and then we it will indicate whether there will be general shear failure or the local shear failure.

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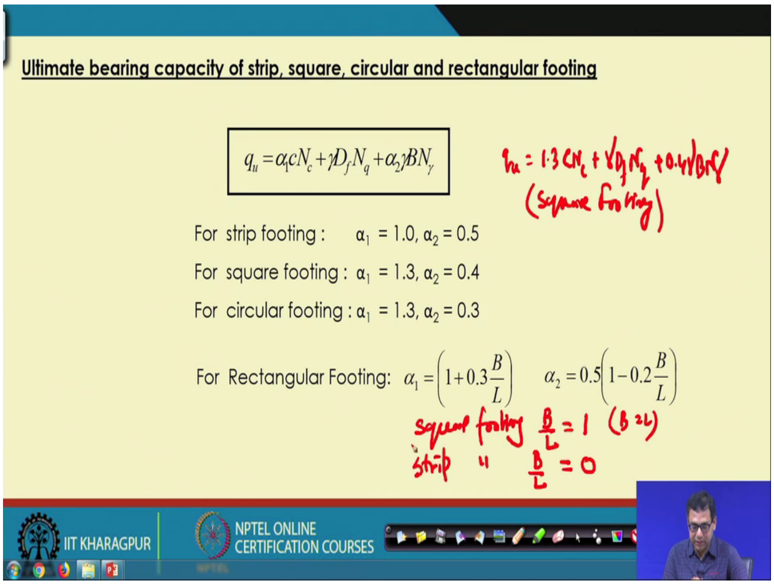

Now, again the Terzaghi's bearing capacity x equation is develop for strip shooting only. So, the original equation is developers strip footing, but later on there is some correction factors are introduced so, that we can use this equation for any kind of foundation, whether this square footing circular footing or rectangular footing. So, this is the correction so; that means, finally, the q u is the alpha 1 and alpha 2 are 2 corrections. So, it is c N c an alpha 2 gamma B N gamma.

So, now for the strip footing this is the original equation develop by terzagis. So, in that case alpha 1 will be 1 and alpha 2 will be the hub or 0.5 that is the case, but for the square footing it is the 1.3 alpha 1 alpha 1 value is 1.3 and alpha 2 is 0.4. So, for the square footing q u e will be 1.3 C N c plus gamma D f N q plus 0.4 gamma B N gamma this is for square footing circular footing alpha 1 is 1.3 and alpha 2 is 0.3. So, these are the corrections for this 3 types of strip footing is a not correction, because it is originally developed for this strip footing.

So, alpha 1 and alpha 2 for circular square footing is recommended like this. Now from these it is extended for the rectangular footing also. So, rectangular footing the alpha 1 is 1 plus 0.3 B by L and alpha 2.5 1 minus 0.2 B by L, where B is the width of the footing s is the length of the footing.

Now, here if it is square footing say, then we will put for the square footing B by L value is equal to 1, because your B is equal to L because it is square footing. So, if you put 1 then it will be 1.3, which is the value for the square footing and it will be 1 minus 0.2. So, 0.8 into 0.5 so, it will be 0.4 which is the value.

Now, if for a strip footing this B by L as the length is very large very very large compared to the width. So, in that case B by L value; that means, is we consider L as the infinite. So, infinite means B by L value is 0. So, in that case if I put this is equal to 0 alpha 1 is equal to 1 and if I put the B by L is equal to 0 then alpha 2 l will be 0.5. So, this way it is extended for the rectangular footing.

So, by using this correction factor now we can you it. So, these are called a shape factors. So, by these safe factors now we can use this expression for any kind of footing.

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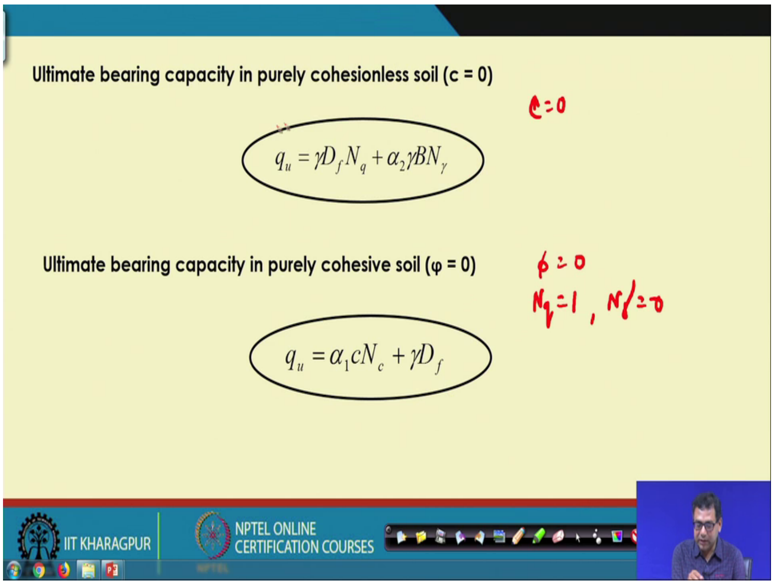

Now if I simplify this equation for ultimate bearing capacity for purely cohesive soil c is equal to 0. So, if c is equal to 0 then as I mentioned that your this N c c is equal to 0 the first term will is vanish. So, that is why the equation will be the second term and the third term.

And, if your phi is equal to 0 then if your phi is equal to 0, then your N q value is equal to 1 and N gamma value is equal to 0. So, already from the chart you can see that. So, that is why the third term will vanish and there will be the first term and the second term in q value is equal to 1.

So, now if it is purely cohesion less soil, then this is the expression simplified form and if it is a purely cohesive soil then this is the expression.

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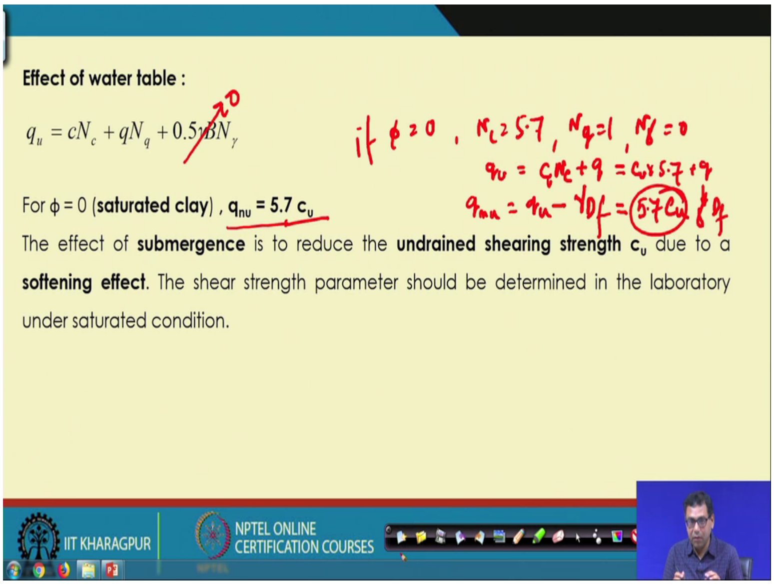

Now, next one is the this is the last effect that I will discuss on the this a equation does water table effect, because or the Terzaghi's expression is developed without considering the water table, but the in the in the feel water table position is very important in the bearing capacity equation. So, now, we have to incorporate the water table expression into the water table effect into this expression. Now, this water table effect so, this is the final expression.

And, now, if our phi is equal to 0, then saturated sets q net ultimate will be 5.7 because if sorry. So, if phi is equal to 0 as I mentioned N c equal to 5.7 equal to 5.7 N q equal to 1 and N gamma is equal to 0 ok.

So, this expression this N gamma will vanish and N cube will be the so, finally, expression q ultimate will be C N c plus q and equal to C u undrained cohesion or C u c into 5.7 plus q. Now this q is equal to d gamma into D f. So, this is the grow ultimate bearing capacity. So, net ultimate bearing capacity q net ultimate bearing capacity will be q ultimate minus gamma D f; because that is the net we have to remove the surcharge. So, that is why the finally, net ultimate expression will be 5 point C u because that is the expression and we are using this net ultimate is 5 point C u C u is retain because here your phi value is 0. So, it is undrained cohesion.

So, as we are taken the phi value is 0 because in the saturated clay. So, this will be 5 point. So, here water table effect. So, in that case if this is the net ultimate because always we are talking about the net ultimate, because ultimate when the allowable bearing capacity it is a definition also we are taking the net ultimate safe load in terms of net so; that means, here you will take so, here. So, water table effect is in the cohesion value, because if it is a saturated clay then water table value will be in the cohesion value, because in that case the shear strength parameter you should determine in the laboratory under saturated condition, because our soil is saturated.

So, here actual condition we have to test it is not in the in the saturated condition we have to test to determine the C u value. So; that means, the effect of submerge submerge or the some effect of the water or the submergence is to reduce the undrained shear strength of the soil so; that means, because of these water effect the C u value will reduce. So, that is why we have to determine the C u value under saturated condition. So, that is the water table effect for the clay soil.

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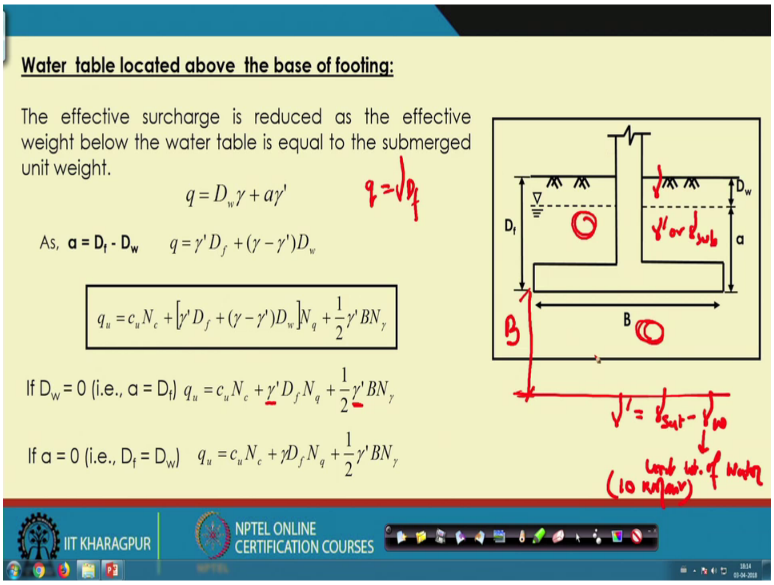

Now, if it is a sandy soil or the C phi soil, then what will happen? Now, in that case so, if we if it is a the this your c phi soil in general c phi soil or the phi soil, then this water table have a have a significant effect. Because, you can see in that case it will that because we have 2 unit weight, now this value will effect this irritated value. Initially you are taking the unit weight of if it is the dry soil, now we have to take the unit weight at different location depending upon the location of the water table because your unit weight will vary ok.

So, now the first case that water table is above the base of the footing ok. So, this is the water table D w is the depth of the water table D f is the depth of the foundation, and the water table is here. Now the as I mentioned the q the surcharge load is the above the base of the foundation soil weight. So, surcharge will be equal to now in that case initially, the if it is a dry soil then surcharge we are writing just D f into or gamma into D f, now why is the dry weight.

But here the unit weight this gamma is the unit weight of here and here the gamma dash or gamma sub. So, as I mentioned if nothing is mentioned in the gamma, if it is only gamma then it is the bulk unit weight and if it is gamma bar or gamma sub then it is submerged unit weight. So, gamma dash is gamma saturated minus gamma w, saturated unit weight minus unit weight of water. So, this is the unit weight of water generally we take 10 kilo Newton per meter square ok.

So, that is why now instead of using this gamma now we have to take the effective overburden pressure. So, effective overburden pressure will be gamma into D w plus a into gamma dash because a is your the zone below the water table. So, that will be the q. So, finally, As your a is equal to D f minus a w. So, we can write this in this form and the finally, we will put this expression here.

So, here now if I 2 cases if D w is equal to 0; that means, it is as the base, then your this gamma will be because if D w is equal to 0, then this will be totally submerged because it is the total soil is submerged. And the second gamma thus this is always submerged whether the D w is 0 or D w is at the base or in between that, because this second gamma or the third term gamma is a soil in this zone and the second term gamma soil in this zone.

So, that is why third term gamma this gamma is always gamma submerged. And if your a is equal to 0 that mean it is at the base then the surcharge of the soil above the base is will be the gamma D f and below the base will be the submerged ok. So, this is for the if the water table above the base of the foundation. And remember that the worst condition we will get if the water table is at the ground level; that means, D w is 0. So, that will give the worst conditions. So, if your that will give you the lowest bearing capacity you can see in both the cases it is submerged.

So, your gamma w will be gamma will be reduced, but as water table decreases a depth of the water table goes down then your bearing capacity will increase. And generally if the water table depth is below the base if it is. So, the if the water table depth is greater than the B from the base of the footing, then we do not consider any effect of the water table so, but if it is within that zone we have to consider the water table.

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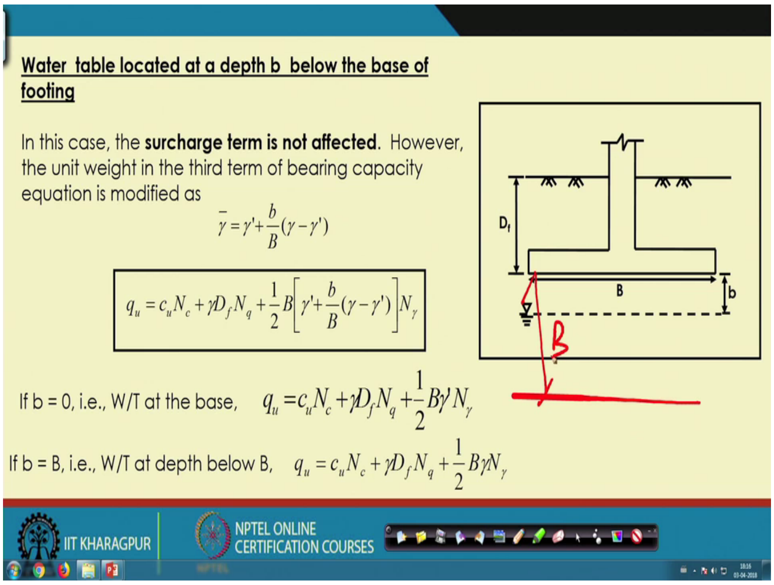

So, last on the next condition is that if the water table is b below the base of the base of the footing. In that case we have to take the gamma bar. So, gamma bar will take in terms of gamma submerge and the gamma bulk and we put.

So, because your first w unit weight first unit weight will be always gamma bulk, because now water table is below the base of the footing. So, this will be always gamma or the gamma bulk, but the second one will change depending upon the position of the water table. And finally, if it is b is 0 that we have the base of the footing then this is will be the gamma sub, because it is total soil below the base of the footing is submerged. Now, if it is beyond the water table is at b then there is no effect.

So, water table effect we will consider up to the depth B below the base of the footing beyond the; it is no water table effect. So, now, these are the all effect I have discussed on the bearing capacity equation that is proposed by the Terzaghi.

So, in the next class I will discuss the others bearing capacity equation proposed by the various researchers and for the different condition and with different correction factors those are applied on those bearing capacity equation.

Thank you.