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Lecture - 43
Earth Pressure – III

So, last class I have discuss the how to calculate the Earth Pressure, if it is on the only soil, soil with surcharge and if the soil is below water table ok. But, all the cases the backfill was horizontal. Now, this class first I will discuss the how we can calculate the Earth Pressure if the backfill is inclined ok.

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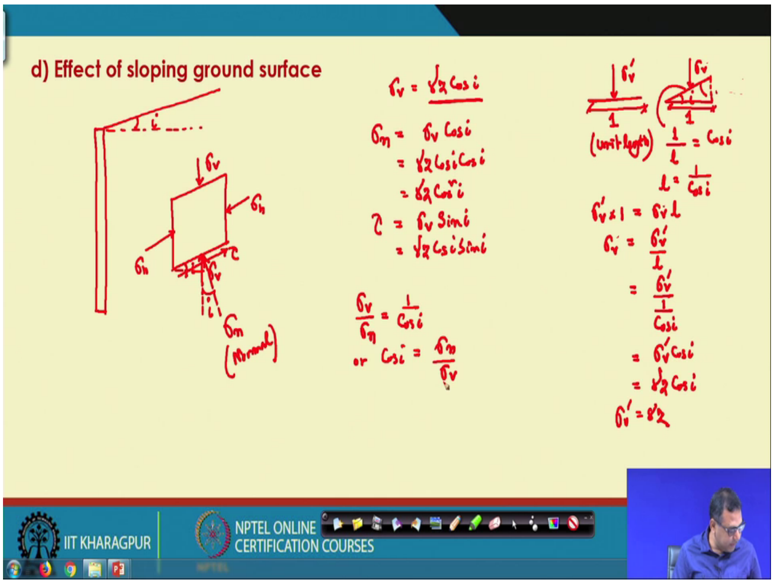

So, we will calculate the effect of sloping ground or inclined backfill because in the previous cases your backfill was horizontal. Suppose if we have a wall and the previous all the cases this is the backfill because, backfill is horizontal perfectly horizontal.

Now, if the backfill is inclined say which is making an angle i with the horizontal ok. Then how we can calculate the earth pressure? So, in that case first if I take a element and this element is also making an angle i with the horizontal. So, and this side this is sigma h or sigma x, this side also it is sigma h or sigma s this is sigma v, this is sigma v ok. Or sometimes you can write that this is your sigma z and instead of sigma h you can write sigma x also ok.

Sigma h means horizontal, sigma v means vertical, sigma x means also horizontal, sigma z means vertical. So, you can use either sigma v or sigma z or you can use sigma h or sigma x. But, in the previous cases I have used sigma x here I am using sigma h ok. So, now this is the element and so, your sigma v, I can write that sigma v is equal to sigma z cos i ok. How I can I am writing that or another one that this angle is i ok. And, if I draw a line which is perpendicular to this plane ok. This is sigma n or sigma normal because, in previous cases your plane where perfectly either horizontal or vertical.

So, sigma v is also a sigma n normal both cases it was normal, but here sigma v which is acting in the vertical direction which is not normal to this plane because, if your plane is perfectly horizontal same then the stress which is acting in vertical direction which is also normal to that plane, but here your plane is an inclined. So, the stress which is acting in the vertical direction is not normal to this plane, the normal to this plane is sigma n ok. So, that is the normal and here I can write the tau also because, initial cases the plane where the principal plane.

Because there was no shear stresses, but now these planes are not principal plane because, principal plane means where the shear stress is 0. But, now this plane is not principal plane here shear stress is existing ok. So that means, there will be a normal stress, there will a shear stress both ok. So, I can write here because if this is the in the vertical direction this is the normal this angle is i.

So, this angle will also be i. So, from here I can write that my sigma n is what? Sigma n this is sigma n, this is sigma v. So, I can write sigma n is equal to sigma v cos i because, this is sigma v, this is sigma n making an angle i. So, this will be sigma v cos i so, your sigma v is equal to gamma into z cos i.

So, this is also cos i so, this will be sigma z cos square i. Now, how sigma v is equal to sigma z cos i? Because, in the previous case when your plane were perfectly horizontal that case we are making that one as the sigma v ok. Now, your plane has changed ok, this is change which is the making an angle i. So, now, if I say that this is the sigma v dash so, for this plane we are applying the same force. So, this plane is also your if this is unit plane or unit distance 1 say. So, if this is 1 so, in this one will be 1 into cos i. So, this plane will be 1 into cos i. If this plane is 1 this is 1 because, this is your 1 and this is i.

So, this plane will be 1 sorry, this plane will be 1 divided by cos i. So, suppose we have a so that means, that the length of this plane will be how much? If this one is 1 unit length or unit length then this one will be 1 divided by this plane say l is the length, that will be equal to cos i. So, l will be 1 divided by cos i ok. Now, we I am applying the same 4 so, I can write sigma v into unit length that is equal to sigma v dash into l. So, I can write sigma v dash is equal to sigma v divided by l.

So, I can write sigma v 1 by cos i so, that is equal to sigma v cos i and sigma v is equal to gamma z. So, this is gamma z cos i. So, this the here the stress which is acting I am making as a sigma v. So, that is sigma v or you can use this is as a sigma v dash and this one is a sigma v. So, whatever it is, but in that case your sigma v dash will be the gamma into z ok. So, or you can write in this way because I have written this is sigma v. So, I am writing in this way, this is sigma v dash, this is sigma.

So, sigma v this is sigma v; sigma v will be sigma v dash is sigma v into l or this is yeah sigma v into l. So, I can write this is sigma v is equal to sigma v dash into l sigma v divided by cos and sigma v was sigma v dash equal to gamma into z, which is acting on the unit length horizontal plane. So, this way you can write that your this stress which is acting on the inclined plane will be sigma z into cos i. So, sigma n normal stress is sigma v cos i and this is gamma into z cos i cos i. So, gamma into z cos square i ok.

So, similarly tau the shear stress which is this is the shear stress direction so, shear stress will be sigma v sin i ok. So, this will be the sigma v sin i. So, sigma v is nothing, but that your gamma z cos i, this is sin i ok. So, now we can get the normal stress and the shear stress and both the values you know this is gamma z cos square i, this is gamma z cos i sin i. Now, if I so now if I draw the Mohr circle for this case.

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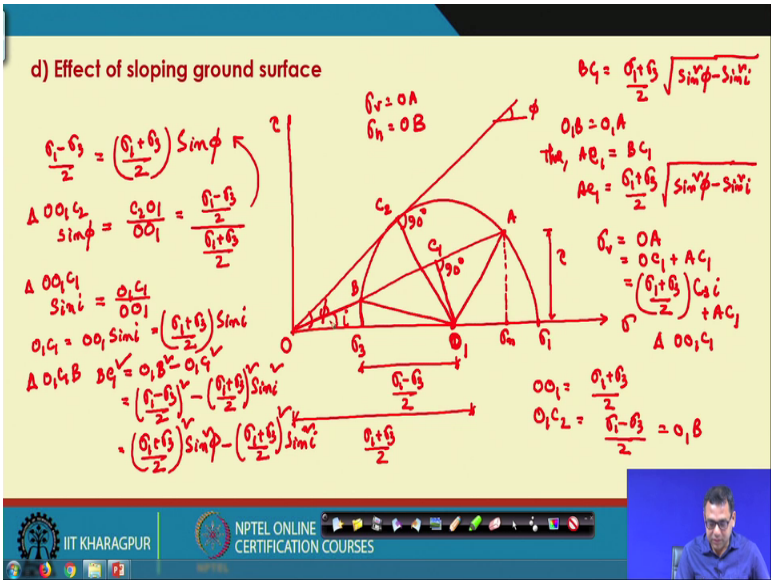

This is sigma and this is the tau again and this is the failure envelop and again we are doing this for only friction soil. So, and if this is the Mohr circle so, this is sigma 1 and this is sigma 3. So, this is for the active case, but these stresses are not the stresses on this plane on this plane because, this is not the principal plane. And, this stress is the principal stresses that we are talking about. So, now we can draw this is the line phi.

Now, we are drawing a line which is making an angle i because, that is the inclination of the backfill. This is the making an angle I, that is the inclination of the backfill ok. So, now this line is passing with a point A and B to the Mohr circle. So that means, we have drawn a Mohr circle, this is the failure envelop and sigma 1 and sigma 2 are the two stresses principal stresses.

But if, but that those two's are for principal plane, but this plane are not these planes are not principal plane because, it is making an angle i with the horizontal ok. So, now if I draw a line i with line making an angle i with the horizontal then A and B are the 2 point, where it touches or it passes through the Mohr circle.

Now, so now, if I consider this A point then I can write this is if I draw a line perpendicular to this point. So, I can write this is as a tau; this as a tau and this one is a sigma n ok. Because, that is the point where it is on the Mohr circle and on the Mohr circle at this plane is the same as the plane that we are representing. This is the same plane ok, this it is also making an angle i. So, in this plane there is there is shear stress and the normal stress. So, here also this is the shear stress and this is the normal stress of this plane or this point ok.

So, now I can write that if I this is the center of the Mohr circle say O 1 or O 1 is the center of the Mohr circle, this is the origin and if I join this 2 point. So, this is say C 2 and definitely it is 90 degree and again I am drawing a line which is perpendicular to A B line. I am drawing a line on the A B, which is perpendicular to the A B and it is passing through O 1. So, this line is also 90 degree ok, I have drawn that. So, again I can write these radius of the circle is sigma 1 minus sigma 3 divided by 2. And, this O 1 from this origin is also sigma 1 plus sigma 3 divided by 2.

So, O O 1 is sigma 1 plus sigma 3 divided by 2. So, this is the construction side of the all the cases. Now, I will again write that from this Mohr circle I have already proved that sigma 1 minus sigma 3 divided by 2 is sigma 1 plus sigma 3 divided by 2 into the sin phi; that I have already proved ok. Now, how we are getting that that if we have a with triangle O O 1 and C 2 triangle, O O 1 and C 2, this triangle O O 1 and C 2; I can write that sin phi is equal to C 2 O 1 divided by O O 1 O O 1 this is the sin phi.

So, in C 2 O 1 is sigma 1 minus sigma 3 divided by 2 and O O 1 is sigma 1 plus sigma 3 divided by 2. Because, this is the O O 1 and O 1 C 2 is nothing, but the radius which is sigma 1 minus sigma 3 divided by 2. Now, this is the case so, from here we will get this equation the sigma 1 minus sigma 3 by 2 is equal to sin phi sigma 1 plus sigma 3 divided by 2. Now, if I take triangle O O 1 C 1 the if this point is C 1, now I am taking this triangle ok. So, I can write that sin i is equal to again this is the perpendicular line O 1 C 1 divided by O O 1 ok. So, I can write O 1 C 1 is equal to O O 1 sin i and O O 1 is equal to sigma 1 plus sigma 3 divided by 2 into sin i ok.

So, now from this triangle O 1 C 1 B now, if I take this triangle. Now, if I take this triangle this O 1 C 1 and B from this triangle O 1 C 1 and B ok. This triangle I can write that B C 1 square B C 1 square because is equal to O 1 B square minus O 1 C 1 square. Because, this is B C 1 square is equal to O 1 B square minus O 1 C 1 square ok. So, here it is O 1 B O 1 B again the radius. So, this is also O 1 B I can write so, O 1 B is the radius. So, I can write sigma 1 minus sigma 3 divided by 2 whole square minus O 1 C 1 square a square O 1 C 1 is equal to sigma 1 plus sigma 3 divided by 2 and sin i.

So, I can write sigma 1 plus sigma 3 divided by 2 square and sin i square ok. Now, sigma 1 minus sigma 3 divided by 2 is sigma 1 plus sigma 3 divided by 2 sin phi so, I am replacing this. So, I am writing here sigma 1 plus sigma 3 divided by 2 whole square sin square phi minus sigma 1 plus sigma 3 divided by 2 whole square sin square i ok. Now, from here I can write that, BC 1 is equal to sigma 1 plus sigma 3 by 2. Because, this is square I am taking the root, then root over sin square phi minus sin square i this is the BC 1 B BC square. So, will be BC 1 square will be, if you take sigma 1 plus sigma 3 by 2 whole square common.

So, this will be sin square phi minus sin square i. So, I am taking the root so, I this will give you the value. And similarly, again that your BC C 1 and again these if I join this line AC 1 are both same because, this is the same value. Because, this is the equal because these are radius, this is the common side between the 2 triangle and this is 90 degree this angle of 90 degree. So, this angle 90 degree, this angle 90 degree, this is the common side. This O's B O 1 is equal to your O 1 B is equal to O 1 A and this is 90 degree. So, this we can write that AC 1 is equal to BC 1 thus; AC 1 is equal to BC 1. So, I can write AC 1 is also sigma 1 plus sigma 3 divided by 2 sin square phi minus sin square i ok.

Now, we will go for the next part that here sigma v is how much; from here in this line sigma v now I am taking sigma v. So, sigma v is what? Sigma v is O A. How I am getting that? Because, this is your sigma n, this is i so, sigma n is this one and this is i. So, this will be definitely the sigma v because, I have already proved that sigma n. So, I can write that sigma v divided by sigma n is equal to 1 by cos i or I can write that cos i is equal to sigma n divided by sigma v.

So, now if you look at this one this is sigma n and this is the i. So, definitely this will be O A will be the sigma v ok. Because, that is you know that if I draw this one this is sigma n so, this will be the sigma v and this is the tau. So, this is the sigma v is O A. So, I can write OA is nothing, but OC 1 OC 1 plus AC 1 ok; OA is OA is OC 1 plus AC 1. Now, what is OC? OC 1 OC 1 I have already got that OC 1 is equal to this is equal to this is the cos. So, I can write OC 1 is sigma 1 plus sigma 3 divided by 2 into cos i because, this is the OC 1.

How I am getting this? Because, if you take this triangle if I take this triangle O O 1 C 1 so, this is a i, this is 90 degree. So, this value is O O 1 is sigma 1 plus sigma 3 by 2 and I am getting this OC 1. So, that will be sigma 1 plus sigma 3 divided by 2 into cos i from this triangle. So, this is the cos i plus AC 1 and AC 1 is this value.

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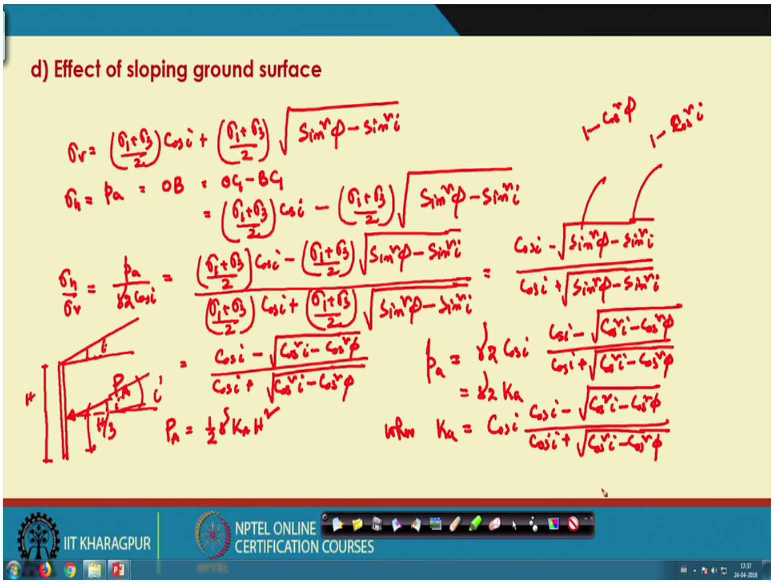

Now finally, if I write that sigma v is equal to sigma 1 plus sigma 3 divided by 2 cos i plus sigma 1 plus sigma 3 divided by 2 root over sin square phi minus sin square i ok. And, here sigma h is equal to P a ok, this is the horizontal stress and it is the active case. And, the sigma h value is from this Mohr circle we can write sigma h value is how much. Sigma h value is, if this is the sigma n and then sigma h value will be OC 1 minus BC 1 ok. So, sigma h value will be this one will be the sigma h value.

So, this is the sigma h value that OC 1 minus O BC 1. So that means, this is the sigma h value because, we got this is total one sigma v value. So, I can write that sigma v is equal to OA, I have written that. Now, this is the sigma h is the this the stress which is the lateral stress and which is smaller than the sigma v. So, we can write that sigma h is equal to OB ok, sigma v is equal to OA and sigma h is equal to OB because, this is the smaller stress. So, now I can write that sigma is equal to sigma a is equal to O B OB and OB is nothing, but OC 1 minus BC 1 ok.

So, I can write OB is equal to OC 1 minus BC 1 and again the OC 1 is sigma 1 plus sigma 3 divided by 2 into cos i, I have already written. This is minus BC 1 which is sigma 1 plus sigma 3 divided by 2 root over sin square phi minus sin square i. So, I can write that finally, sigma h divided by sigma v which is sigma h is p a, this is sigma h is p a. And, sigma v, I have written sigma v is equal to gamma into z into cos i. So, I will write sigma v is equal to gamma into z into cos I, that is equal to again I can write sigma h is in this form.

So, this is I can write sigma h in this form also. So, this is sigma 1 plus sigma 3 divided by 2 into cos i minus sigma 1 plus sigma 3 divided by 2 root over sin square phi minus sin square i. And, sigma v also I can write that sigma v sigma 1 plus sigma 3 divided by 2 cos i plus sigma 1 plus sigma 3 divided by 2 root over sin square phi minus sin square i. So, I can write sigma h as a p a and I can write sigma h also this expression ok. And, I can write sigma v is equal to gamma z into cos i, also I can write sigma v with this expression; so I have written that. So, now if sigma 1 plus sigma 3 by 2 we can cancel ok.

So, I can write this is cos i minus root over sin square phi minus sin square i divided by cos i plus root over sin square phi minus sin square i ok. Now, here all the terms are in cos so, we can represent them in terms of cos also. Now, if I convert this sin to cos so, I then I can write cos i minus root over 1 minus cos square phi minus 1 plus sin square phi a sin square i. So, finally I can write this is this is this sin square phi, I can write 1 1 minus cos square phi and this one I can write 1 minus cos square i. So, this is 1 1 will cancel and this will be cos square i minus cos square phi. Similarly, this is cos i plus root over cos square i minus cos square phi.

So, finally the p a active pressure is gamma into z cos i cos i minus root over cos square i minus cos square phi; this is cos i plus root over cos square i minus cos square phi. So, I can write gamma z into K a ok. So, where this K a is again the coefficient of active earth pressure and K a I can write cos i cos i minus root over cos square i minus cos square phi divided by cos i plus root over cos square i minus cos square.

So, this is the inclined one and now finally, the earth pressure for the inclined surface this is the wall. So, this will act parallel to the backfill surface. So, this is finally, the P A and the P A will be again we can write that P A is equal to half into gamma into K A into H square. So, this is the H or this is the H is the height of the wall.

And, later on I will show you that H calculation also we can change depending upon where I am applying this force P A, but here I am applying this force P A. So, then this will be the H so that means, H square. So, here K a remember that this expression. And, again this is acting at the height of H by 3 from the base of the wall ok. So, but that means, the P A is here acting at an angle of i with the horizontal ok.

Previous case P A is totally horizontal because, the backfill was horizontal. Now backfill is inclined so, in this case P A will always act parallel to the backfill surface. If backfill is horizontal P A is also horizontal, if backfill is inclined P A is also inclined with that with that angle of inclination with the horizontal ok.

So that means, here P A is acting with an angle i with the horizontal because, this is also i this is backfill surface otherwise, this is this is the expression. So, similarly for the K p it will be the opposite. It will be sin i plus and this will be this is cos i plus, this will be the cos i minus. And, this will also change because you can you can derive that expression also. Because, in that case your sigma 3 sigma 1 that thing will change that I have discuss in the last class. So, this is the inclined this is the earth pressure of a inclined backfill according to the Rankine's theory.

Because, this is the Rankine's theory means extended because, original Rankine's theory it is for the horizontal backfill, but it is the inclined backfill which is been extended. So, next class I will discuss about that the how I will determine the earth pressure if the soil is a c phi soil. Here we have consider all the soil for c is equal to 0, only the phi value is there. Now, in the next class we will discuss what will happen if the soil as both c and phi.

Thank you.