

**Foundation Engineering**  
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**Lecture – 48**  
**Earth Pressure and Retaining Wall**

So, this class I will discuss the another next earth pressure theory or the second earth pressure theory that is Coulomb's earth pressure theory. In the previous classes I have discuss about the Rankine's Earth Pressure theory and here I will discussed the Coulomb's Earth Pressure theory.

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**Coulomb's earth pressure theory:**

**Assumptions**

- The backfill is dry, cohesionless, homogeneous, isotropic soil. However,  $c-\phi$  soil can also be treated under the coulomb's theory
- The backfill surface is planar and can be inclined
- The back of the wall can be inclined to vertical
- The failure surface is a plane surface which passes through the heel of the wall

The slide includes a diagram of a retaining wall with an inclined backfill surface and a failure surface passing through the heel of the wall. The text 'can be inclined to vertical' is circled in red, and 'heel of the wall' is underlined in red.

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Now, what are the assumptions that the backfill is dry, cohesionless, homogeneous, isotropic. So, these were the same assumptions for the Rankine's theory, but this theory can also be applied for the  $C \phi$  soil under the coulombs theory. Although the Rankine's theory also extended for the  $C \phi$  soil. Then the backfill surface is planar and can be inclined because initially in the Rankine's theory the backfill surface was horizontal, but later on it is been extended for the inclined also. So, but here this theory the backfill surface can be planar or can be inclined it can be horizontal or can be inclined also.

But in the Rankine's theory the our wall or the back of the wall is always vertical, but here in this theory for the Coulomb's theory; the back of the wall can be inclined also, so that back of the wall can be inclined. And another one the failure surface is plane failure

surface or linear failure surface so; that means, if this is the wall as I mentioned the failure surface will be the planar, the linear. The Rankine's also failure surface is linear and it is always passing through the heel of the wall.

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Then the others are next assumptions as that in the Rankine's theory, we saw that that the a direction of the force is parallel to the backfill surface. So, now, that mean it is so; that means, if this is the wall and if it is perfectly horizontal. So, this is the  $P_A$  direction for the Rankine's, but for the Coulomb. So, even if the wall is horizontal so it will act at an angle of  $\delta$ . What is  $\delta$ ? So,  $\delta$  is the friction angle between soil and the wall. So, that is the another difference between the Rankine's and coulombs. In the Rankine's theory it is assumed that wall is smooth perfectly smooth that mean  $\delta$  value is 0. But here the  $\delta$  value that the friction between wall and the soil is also incorporated. So, it is assumed that there is a friction value  $\delta$  and this  $P_A$  will act with an angle  $\delta$  with the with the normal to the wall. So; that means, this line is the normal to the wall and  $P_A$  is acting with the angle  $\delta$  with this normal. That is for the Coulomb's, this is for the Rankine's. If it is inclined backfill suppose this is the inclined backfill.

So, as per the Rankine's so this is the  $P$ ,  $P$  is also acting  $i$  with a with angle  $i$  to the horizontal. So; that means, this as for the Rankine's theory this  $P_A$  is always parallel to the backfill, but a for the inclined also this is making an angle  $\delta$  with the normal to the wall. So, this is Rankine and this one is the Coulomb's and this is also Rankine. So,

these are the difference between the Rankine's and the Coulomb and always this sliding wedge is considered to be rigid body, sliding wedge means as I mentioned that when the value of planar failure surface. So, this wedge so this is the wall this is the wall and this is the failure plane. So, this wedge will slide along this failure plane. So, this wedge is considered to be rigid, this is a rigid body. So, these are the assumptions consider by the coulombs.

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Now, first we will consider for the active case and then we will consider for the passive case. So, this is the wall and as I mention now wall can be inclined also. So, here this wall is making an angle alpha with the horizontal and beta the making an angle beta with the vertical; so alpha will be 90 minus beta. So, this is the wall surface which is no longer vertical which can be inclined also and this is the backfill surface which can be also inclined. And this is the failure wedge, so this is the planar surface this is the planar failure surface.

So, this wedge is rigid body which is sliding along this surface. So, now, if I consider this wedge and What are the forces acting? The forces acting is the weight of this wedge so if we draw the this force diagram. So, this is the weight of the wedge which is acting now this reaction of this soil because this wedge is sliding along this failure surface. So, there would be a reaction force because here there is will be a normal force and in the there would be a this along this line they are be a shear force also. So, that mean a shear stress

this is normal stress shear stress. So, this is the resultant force. So, resultant force is making an angle  $\theta - \phi$ .

So, you can see that this angle is if I draw this line parallel to this one this angle is also  $\theta$ . So, this is making an angle  $\theta - \phi$  so this is the  $\theta - \phi$  with the vertical. So, this angle  $\theta - \phi$  with the vertical so, this R is making an angle  $\theta - \phi$  with the vertical. So, now, what another force another, force is the P A active earth pressure.

So,  $P_a$  is making an angle  $\delta - \phi$  with the vertical.  $\delta - \alpha - \delta$ , so if you draw this line so; that means, this is also  $\alpha$ , so this is also  $\alpha$ . So, your this angle is 90 degree so this total one is also  $\alpha$ . So, this is this angle is  $\alpha$  and this one is the 90 degree. So, we can write this total one is also  $\alpha$ ; this total one is also  $\alpha$ . And this is making an angle with the vertical this is the  $\psi$  is equal to  $\alpha - \delta$ . So, that is why it is  $\alpha - \delta$  angle it is working so; that means, the R is making an angle  $\theta - \phi$  with the vertical and P A is making an angle  $\delta - \alpha - \delta$  with the vertical. So, this is the force diagram all the three forces.

Now, what I will do that actually I want to determine the weight of this wedge. So, before that if I take the force polygon this force polygon. So, I can write that  $P A \sin$  this is  $\theta - \phi$ ;  $\theta - \phi$  that will be equal to W and this angle is 180 degree minus  $\alpha - \delta$  or this will be the minus  $\delta - \theta - \phi$ . So, similarly I can write W divided by  $\sin 180 \text{ degree} - \alpha - \theta + \phi + \delta$ .

So, I can write from this first polygon. So, now, this is the angle so finally, P A will be W into  $\sin \theta - \phi$  180 degree minus  $\alpha - \theta + \phi + \delta$ . So, here only variable is the  $\theta$  other things are known; we know the  $\alpha$  because this is the angle this wall is making that is fixed, your  $\phi$  value is the property that is fixed,  $\delta$  value it is also fixed. So, only variable is the  $\phi$  because depending upon this  $\phi$  this P A value will change. So, now, we have to optimize this P A value. So, how will optimize that and before I going to optimize that by this one we can calculate the weight of the wedge, because this weight of the wedge is required.

So, I will calculate the weight of the wedge. Weight of the wedge will be the half this gamma into A. So, A is the half into the weight of the wedge is equal to half into gamma area is AC into BD. So, this is the weight of this wedge. So, AC will get with these expression and BD will get with these expression. So, these expressions are coming from the geometry. So, you will get that because AC will you will get again from the AC by because this is AC. So, AC divided by sin because this is also alpha. So, I can write from these triangle that AC divided by sin i plus alpha that is equal to AB this is AB and this angle. This angle will be this is your this angle is theta minus i. Divided by sin theta minus i. So, this is theta so this angle will also be this angle will also be theta. The total angle again this is i, so this angle is i so this value these angle is finally, theta minus i.

So, these are form the geometry you can see this geometry. So, finally, this is the expression so AC I can write with this way. And BD also form this triangle this is the BD and this angle BD is the AB is this one and this angle we know this is your this is the alpha and theta. So, this angle will be 180 degree minus alpha plus theta. So, from here I can also get an expression for BD and AB. So, AB also H this is the AB is this one and this is the H this angle is alpha. So, if I know the H if I know the alpha I can get the AB also. AB also H by sin alpha because you know that from here AB H by AB is equal to sin alpha from this triangle if I extend this triangle.

So, this is H is the height by AB is equal to sin alpha. So, I can write AB is equal to H by sin alpha. So, all this value will get from this from this geometry. And then finally, will get the W and this is the expression of the W. So, that expression I will put here. So, I will put this W expression here from, these final expression I will put here. So, we will get a P A expression where except theta all values are fixed. So, only variable is theta so we have to optimize that.

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For maximum value of  $P_A$       If the wall is vertical and smooth, and if the backfill is horizontal,  $i = \delta = 0$  and  $\alpha = 90^\circ$

Active  $\rightarrow$  Maximization  
Passive  $\rightarrow$  Minimization

$\frac{dP_A}{d\theta} = 0$   
 $P_A = \frac{1}{2} \gamma H^2 K_A$

where,  $K_A = \frac{\sin^2(\alpha + \phi)}{\sin^2 \alpha \sin(\alpha - \delta) \left[ 1 + \frac{\sin(\phi + \delta) \sin(\phi - i)}{\sin(\alpha - \delta) \sin(\alpha + i)} \right]^2}$

Or  $K_A = \cos^2(\phi - \delta) \left[ 1 + \frac{\sin(\phi + \delta) \sin(\phi - i)}{\cos(\delta + \theta) \cos(\theta + i)} \right]^2$

$K_A = \frac{1 - \sin \phi}{1 + \sin \phi}$

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So, I will optimize by minimizing these so, making so when we are talking about the active you have to minimize that when you are talking sorry. When you are talking about the active, we have to maximize. When was talking about the passive we have to minimize. So, in the case of active it is minimization and maximization and when the passive it is minimization. So, in case after both the cases you have to make  $dP_A$  by  $d\theta$  and make it 0 because this is the minimization and maximization process and only variable is the theta. So, in remember that in case of active it is maximization, in case of passive it is minimization. In case of active we choose that theta values such that we will get the maximum force and in case of passive we will get the minimum force.

And so finally, after a maximization we will get the expression of  $K_A$  like this. So, this is the expression of  $K_A$  and finally, the force is similar to the Rankine's theory, but only the  $K_A$  expression is different. Now what is the difference in the Rankine's and the coulomb as I mentioned in rankings, the  $i$  value is 0, delta value is 0, and wall is making wall is perfectly vertical. So, alpha value is 90 degree, so if you put this value in this expression you will get the same 1 minus sin phi divided by 1 plus sin phi. So, if you put these values here then you will get this expression. So, it is this is all the things are incorporated in the coulombs theory.

Now, sometimes this  $K_A$  expression is also written in a different form so I am giving that form. So, the  $K_A$  is sometimes written in this form also. This  $K_A$  you can write this

form also and another way  $K_A$  is written  $\cos^2 \phi \sin \delta + \beta \cos^2 \beta \cos \delta + \beta (1 + \sqrt{\sin \phi \sin \delta})$ , then  $\sin \phi \sin \delta$  this is  $\cos \delta + \beta$  then  $\cos^2 \beta$  that is whole square. So, it is written in this form also, either you can use this one or you can use this one so this is for the active.

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**Passive earth pressure (For sand):**

From the force polygon

$$P_p = \frac{W \sin(\theta + \phi)}{(180^\circ - \theta - \phi - \delta - \alpha)}$$

Only variable  $\theta$

$$W = \frac{\gamma H^2}{2 \sin^2 \alpha} \frac{\sin(\alpha + i)}{\sin(\theta - i)}$$

The diagram shows a retaining wall ABC with height H, top B, and bottom A. The wall is inclined at angle i to the vertical. The soil surface is inclined at angle alpha to the horizontal. A force polygon is shown with vectors W (weight), R (reaction), and Pp (passive earth pressure). Angles theta, phi, delta, and alpha are indicated. A force triangle is also shown with angles delta+alpha and theta+phi.

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Similarly, for the passive the similar way we can get the W value, but here it is these angles are changing. Initially for the active this is making alpha minus delta and this is theta minus phi. Now, this angle alpha plus delta this is theta plus phi in case of passive. So, again we will get the P we will get the W the same expression we will put there. And then we will this is we minimize this one so this will be minimization. And then finally, we will get this expression of  $K_A$ .

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For minimum value of  $P_p$

If the wall is vertical and smooth, and if the backfill is horizontal, i.e.  $\delta = 0$  and  $\alpha = 90^\circ$

$$\frac{dP_p}{d\theta} = 0$$

$$P_p = \frac{1}{2} \gamma H^2 K_p$$

where,  $K_p = \frac{\sin^2(\alpha - \phi)}{\sin^2 \alpha \sin(\alpha + \delta) \left[ 1 - \frac{\sin(\phi + \delta) \sin(\phi + i)}{\sin(\alpha + \delta) \sin(\alpha + i)} \right]^2}$

or  $K_p = \frac{\cos^2(\phi + \beta)}{\cos^2 \beta \cos(\delta - \beta) \left[ 1 - \frac{\sin(\phi + \delta) \sin(\phi + i)}{\cos(\delta - \beta) \cos(i - \beta)} \right]^2}$

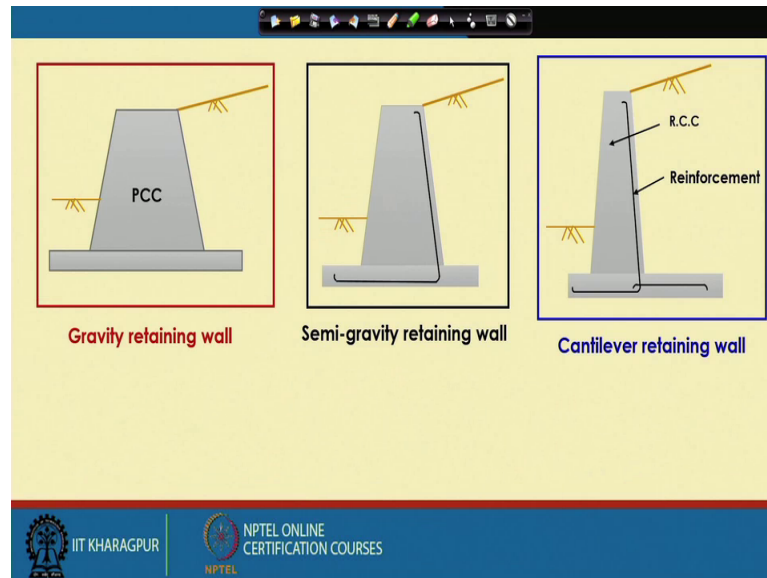
$$K_p = \frac{1 + \sin \phi}{1 - \sin \phi}$$

Now, again if you put this value here will get same the Rankine's K P. And again I am writing this K P can be written an another form. That is cos square phi plus beta this is cos square beta cos delta minus beta 1 minus root over sin phi plus delta sin phi plus i then cos delta minus beta cos i minus beta that is equal to square. So, either you can use this one or this one both are the same expression in different form. So, this way you can use this coulombs theory also to calculate the earth pressure.

So, when I solve the retaining wall problem in that time I will use these coulombs theory and I will show you how you can determine the earth pressure. I when I am not solved the problem for inclined backfill also that I will also solve, when I will designed the retaining wall.



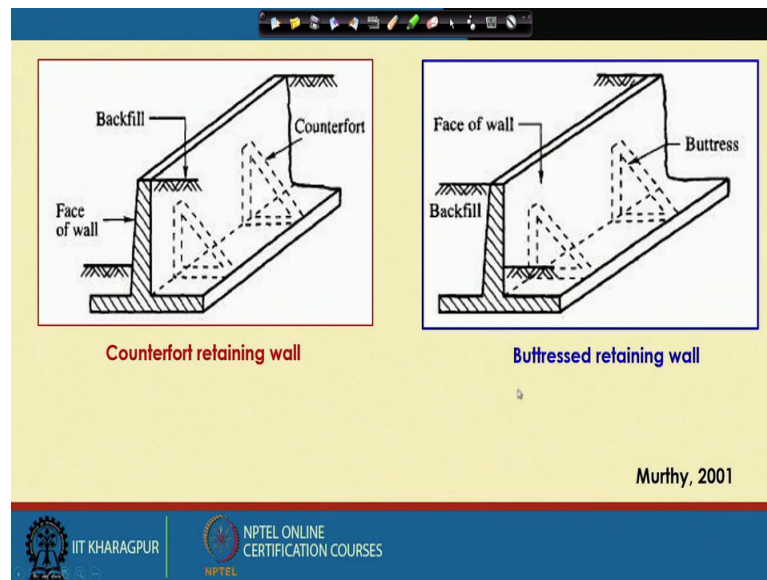
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So, next one I will start that the retaining wall. So, what are the different types of retaining wall and how I will design the retaining wall by the application of these theories? That I have discussed. So, ultimately we have to apply this those theories to design a retaining wall. So, now in this class and the next classes I will also discuss the retaining wall design. And then first you should know what are the different types of retaining wall so retaining walls can be different types. The first one is the gravity retaining wall, next one is the semi gravity retaining wall then, third one is the cantilever retaining wall.

So, what is the difference between these walls? In the semi gravity retaining wall so the no reinforcements are used. So, it is only the wall weight that will give you the resistance. When there is the earth pressure this wall weight is give you the resistant so that is why this dimension is huge. So, that is the gravity retaining wall and then the semi gravity retaining wall the dimension is slightly reduced. We are incorporated small amount of reinforcement and then the cantilever retaining wall the you are using the reinforcement and dimension is significantly reduced. And this, as we go from gravity to the cantilever so you can use them for the higher height of the wall. So, that mean cantilever retaining wall you can use for 6 to 8 meter height.

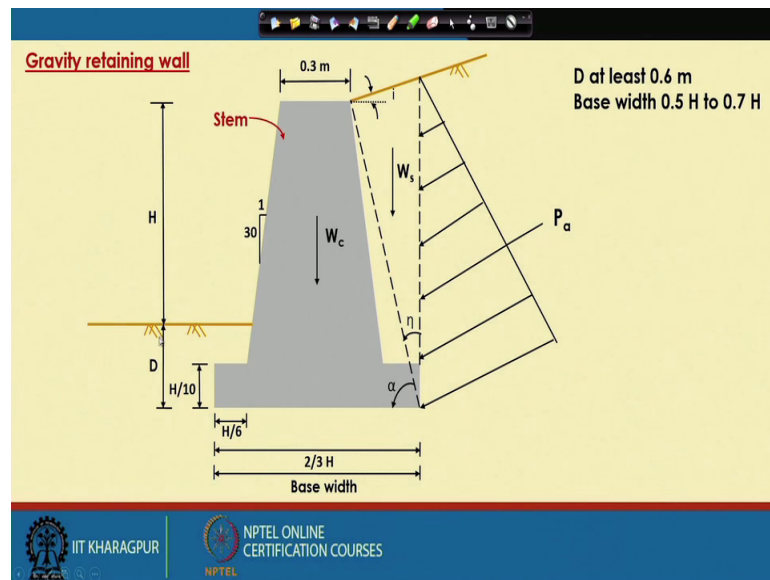
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Now, then if you go for the higher height more than 6 to or the greater height more than 6 to 8 meter then you have to go for the counterfort retaining wall or buttress retaining wall. So, what are the difference between these two. So, here we are providing some counterfort or support. So, if these supports are provided in a backfill side, so this is your backfill side. So, in backfill side and this is the free side. So, if it is providing the backfill side this is in the counterfort retaining wall and counterfort retaining wall. And if you are providing them in a this is the free side then it is called the Buttress retaining wall. So, if this support is the counterfort are provided in the backfill side and this is the free side.

So, these are for the more than 6 to 8 meter height generally we provide these things, but in this course I will concentrate only the design of gravity retaining wall and the cantilever retaining wall. So, I will not discuss about the design of these two types of retaining wall because that is the these two are not I have not included in this course. So, I have included only the gravity retaining wall design and the cantilever retaining wall design. So, I will show you one design of gravity retaining wall and one design of cantilever retaining wall.

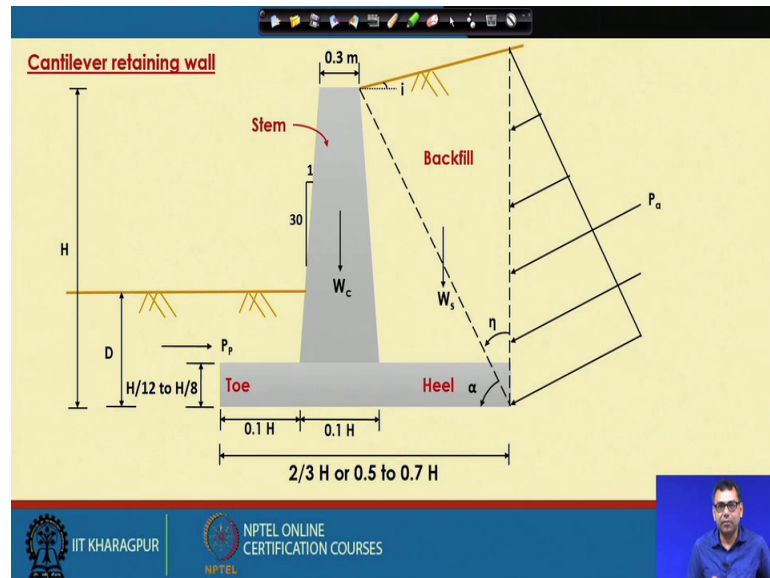
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So, in the next part this is the gravity retaining wall. So, this is the gravity retaining wall and these are the your approximate dimension. So, is given so for the first when you start the design you should have some guidelines. So, these are the guidelines to choose you the dimension for the first trial. So, if this is the height of the wall then  $D$  is the depth of the wall below the existing soil because this is your existing soil, this is the fill soil or backfill this is the free side. So, deep depth at least 0.6 meter depth you have to provide and base to width this width of the wall should be in between 0.5 to 0.7.

But this is the guideline is not the mandatory this should be within this line, but the for the selection of the first dimension you can use these guidelines or it should be or it can be two-third of  $H$ . Now this portion is  $H$  by 6 and this is the generally it should not be your more than 1 by 30 and so. And this width minimum one is 0.3 meter. And this is the wall weight and I will discuss this alpha and this eta value part. What is these two things and this is the  $P_a$  as this  $P_a$  this is the parallel to the backfill. So, that is why it is parallel to the backfill this  $P_a$ .

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Similar for the cantilever retaining wall also these are the guidelines. So, this is the  $D$  and this thickness of this slab is  $H$  by 12 to  $H$  by 8 and this portion is this is the minimum one  $0.1 H$ ,  $0.1 H$  that the range is two third of  $H$  or  $0.5$  to  $0.7 H$ . This is minimum is  $0.3$  meter and  $1$  is to  $30$ . So, this is also backfill is your incline so, it is parallel to the backfill

So these are the all cantilever and the your gravity retaining wall part. Now, I will first discuss that how we can use this we can use this theory. And what are the design checks we will do for different types of wall. So, first we will discuss about we will consider a particular any wall.

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**Checks**

1. No sliding Condition  
 $F.O.S = \frac{R_v/M}{R_{H'}} = \frac{\sum V/M}{\sum H} \geq 1.5 - 2$   
 $\mu = \text{Coefficient of friction between the wall and base soil (tan } \delta)$

2. No overturning Condition  
 $R.O.S = \frac{\sum M_R}{\sum M_o} \geq 1.5 - 2 = \frac{W a}{R (H/3)}$

Equations:  
 $R_v' = W$   
 $R_{H'} = P_a - P_p = P_a$  ( $P_p$  is neglected)  
 $\sum M_{toe} = 0$   
 $R_v' \bar{x} = W a - P_a H/3$   
 $\bar{x} = \frac{W a - P_a H/3}{R_v'}$   
 $= \frac{\sum M_R - \sum M_o}{\sum V}$   
 $e = \frac{b}{2} - \bar{x}$   
 $e > \frac{b}{6}$

Definitions:  
 $\sum M_R = \text{Sum of all resisting Moment about toe}$   
 $\sum M_o = \text{Sum of all overturning Moment about toe}$

So, I am taking one particular wall and then we will discuss what are the checking we will do. So, suppose we have wall like these, and this is the backfill side. So, a height of the wall is say H and this is the free side and this is the foundation below this portion so this is the D, fine. See this is backfill this is the free side and this is the foundation D. So, this portion when this is the backfill side so, while will move this direction. So, there will be P a will act in this direction. So, this Pa is nothing, but H by 3 above the base.

And when this wall will move this direction away from the backfill so this portion will generate a passive force, this will be the passive because in this case wall is moving towards the soil. So, it will be it will give you a passive and this will give you the active. And if this is the toe of the wall and I can write that what are the forces is acting. So, in the wall the weight of the wall is acting so weight of the wall will act. Now, it has a reaction from the base of the wall so this reaction is the R, now it has two components one is R V dash another one is R H dash. So, R V dash is the vertical reaction force and R V, R H dash is the horizontal reaction force.

So, from this figure so my R V dash will be equal to W because W is acting in the downward direction your reaction is acting upward direction. So, and here R V dash is the vertical reaction. So, R V dash will be equal to W similarly R H dash that will be equal to P a minus P p because P a is acting this direction P a is acting opposite direction. But in during the design of retaining wall this P p is neglected.

Because. So, I can write this will be equal to  $P_a$  because if you consider the  $P_p$  it will give you more additional factor of safety. So, that is why you can neglect this  $P_p$  and you can design it considering the  $P_a$ , even if you consider the  $P_a$  it will increase your fact of safety. So, that is why we neglect this  $P_p$  becomes design during the design we consider only the  $P_a$ , but some special cases we also considered  $P_p$  I will discuss those special cases that when you consider the  $P_p$  also. But for this normal design we generally do not consider the  $P_p$  we consider only the  $P_a$  so that is the  $R_H$  dash. Now I can write that summation of the  $M$  with respect to toe is equal to 0. So, this, if this is the weight is acting at a distance of  $a$  from the toe and total width of the retaining wall is  $b$ .

So, this one is  $b/2$  and this one is also  $b/2$  and the reaction is acting say at a distance of  $\bar{x}$  from the 2. And this one is the say  $e$ ,  $e$  is the eccentricity. So, actually your reaction is acting from this point so and you are center is this one. So, there is a eccentricity of this reaction is not acting from the center of the wall or base of the wall. So, it is a eccentricity  $e$   $\bar{x}$  is the reaction that is acting from the toe. So, now, from the toe the moment we can take so, I can write that  $R_v$  dash into  $\bar{x}$ . So, this is  $\bar{x}$  I am taking this is  $R_v$  dash  $R_v$  dash into  $\bar{x}$ ,  $\bar{x}$  is the distance that will be equal to  $W$  into  $a$  minus  $P_a$  into  $H$  by 3. So, I can write that  $\bar{x}$  is equal to  $W a$  minus  $P_a$  into  $H$  by 3 divided by  $R_v$  dash.

So, what is  $W$  into  $a$ ?  $W$  is giving the resistance because your earth pressure is  $P_a$  which is giving the which is acting this direction. So, this weight of the wall is giving the resistance weight of the wall is not allowing this what wall to move so that is a rotate. So, it is giving the resistance. So, that is why this moment due to the  $W$  is the resisting moment. So, we can write this is summation of  $M_R$  minus this is the overturning moment. So, this is the overturning moment and divided by summation of the  $V$ . So, this is the here this  $V$  is  $W$  is only one  $W$  I am talking about, but there can be the summation of vertical forces. So, this is a different vertical forces. So, this is the summation of vertical forces.

So,  $\bar{x}$  is summation of resisting moment minus summation of overturning moment divided by summation of all vertical forces remember that. So, you can say the net summation of net moment divided by the summation of vertical moment so this is very important. So, this  $\bar{x}$  now this  $e$  I can write that  $e$  is equal to because from here to here it is  $b/2$   $b/2$  minus  $\bar{x}$   $b/2$  minus  $\bar{x}$ . So, now, these things is very

important that for any retaining wall design your  $e$  should not be greater than  $b$  by 6. Why that? I will discuss when I will discuss about the different checks.

So, remember that  $b$  should not be greater than  $b$  by 6 so  $e$  limiting value is  $b$  by 6. So, now, what are the checks that we will do for this retaining wall. The first check so we remember that there should be 4 checks. The first one that no sliding condition so what is that; that means, when these forces acting in the lateral direction. So, wall will try to slide so there should not be a sliding; there should not be any sliding condition. So, how we can check the factor of safety; so the factor of safety for the no sliding condition will be  $R V$  dash into  $\mu$  divided by  $R H$  dash or you can write that summation of all vertical forces into  $\mu$  divided by summation of all horizontal forces. And that should not be greater than, that should not be less than 1.5 so this is 1.5 to 2.

So, what is  $\mu$  is the coefficient of friction between the wall and base soil. Generally it is taken as  $\tan \delta$  I remember that this  $\delta$  is the  $\delta$  of base soil so this is the first condition. Now the second condition is that your no overturning condition no overturning condition. So, what is no overturning condition? So, when this wall is giving a lateral stress in this direction. So, wall will try to rotate in this direction. So, that mean it can slide in direction also it can rotate in direction also. So, we have to prevent that rotation also; that means, no overturning condition the factor of safety will be for this condition is equal to summation of  $M R$  divided by summation of  $M O$ . And I can write in another way that that is equal to here for particular this case it will be  $W$  into  $A$  divided by  $P_a$  into  $H$  by 3 for this particular case.

And again that should be greater than equal to 1.5 to 2. So, here also we can write 1.5 to 2. Now, what is  $M R$  is the sum of all resisting moment about toe. And  $M O$  is the sum of all over turning moment about toe. So,  $W$  into  $A$  is the resisting moment here and  $P_a$  into  $H$  by 3 is the overturning moment. So, that is this factor of safety should be greater than 1.5 to 2. So, there are two more checks that we have to do I have discuss two checks one is the no sliding condition and no overturning condition. So, next class I will discuss the two more checks that we have to perform during a design of retaining wall.

Thank you.