Theory of Elasticity
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Lecture - 13
Constitutive Relation – I (Contd.)

Welcome. This is the module the second lecture for module 3 which is a Constitutive Relation I. So, here actually what I plan to do is to understand physical meaning of this Elastic Moduli. Specifically, I am considering here for the isotropic case because we have not yet done anything anisotropic case. So, Elastic Moduli what is the physical meaning of this? Elastic Moduli, that we see for some of our laboratory test. For instance, the Young's modulus, Poisson's ratio, all those things we have we know all those things from our strength of material knowledge itself, for the basic mechanics knowledge itself.

So, why how we do this thing or how we test these things? Probably all of us know this, but I just wanted to have a relook on the physical meaning of the Elastic Moduli.

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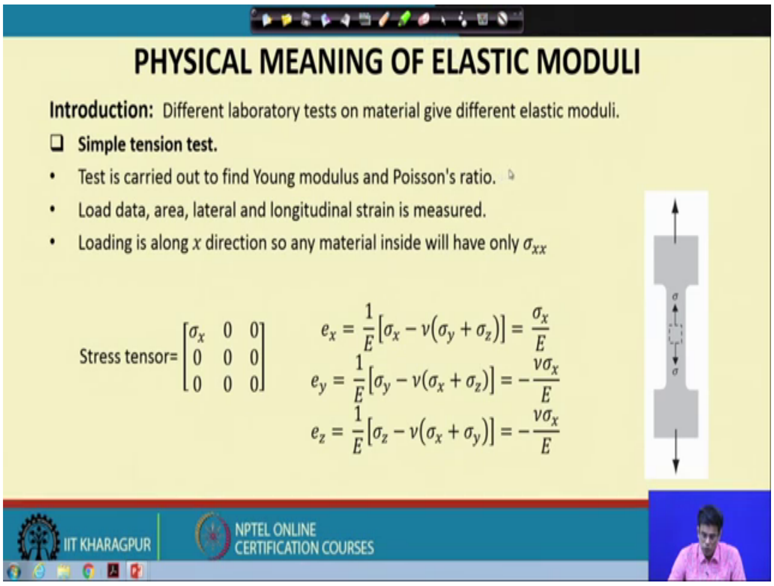

So, to start with, so different laboratory test generally we do on the to find out the Elastic Moduli for the isotropic material. For instance, most of us have done that tensile testing or the uniaxial testing. So, basic objective or for this test is to find out the Young's modulus and then the Poisson's ratio. So, because as we know the isotropic material can be characterized by two material constants, so which is E and nu in the or the Young's modulus and Poisson ratio. So, if we are just simply test a specimen under tension, so which what I mean by under tension is that you apply it inside force this side and apply a tensile force this side. In actual case, in experiments what we do, we fix this bottom and then probably we give a tensile force.

So, now if we consider any element in general here, so it will be under only one directional stress. So, this is the applied stress along this side. Now, if we look carefully or if we know now that what is the stress tensor corresponding to that and if you take this is a x direction, so this stress tensor will be only this. So, this quantity will be a non zero and all other quantities to be 0. Now, once we start the test, so we can take the load data area lateral and longitudinal strain measured. So, that mean the strain along this side, strain along this side, both we can measure with the help of a strain gauges. So, now if from the hooks law, if we assume this is a hooky and material or if it is a linear elastic hooks hooky and material, then we can write it is stress strain relationship which is essentially this. So, this we know from our previous class. So, this is the e x, e y and e z, remember all these things are in void notation.

So, now since if we find out stress tensor, so if I now substitute the values of the stress tensor in this equation, so if you see that sigma y, sigma z everything is 0 and so, finally, e x become sigma x e and e y becomes minus nu sigma x by e and e z becomes minus nu sigma x by e. Now, probably when you didn't learn multidimensional hooks law or generalized kind of hooks law, then probably you have one dimensional hooks law is the stress is proportional to strain which you can see directly from this equation. Now, it is not only that because right now, we are considering this material as a 3 D material. So, it will have it is Poisson's effect in other two direction. So, which will essentially represented in terms of strain is this which will be dependent on the applied stress sigma x or the stress sigma x.

Now, if we want to calculate what is these modulus, so how should I approach?

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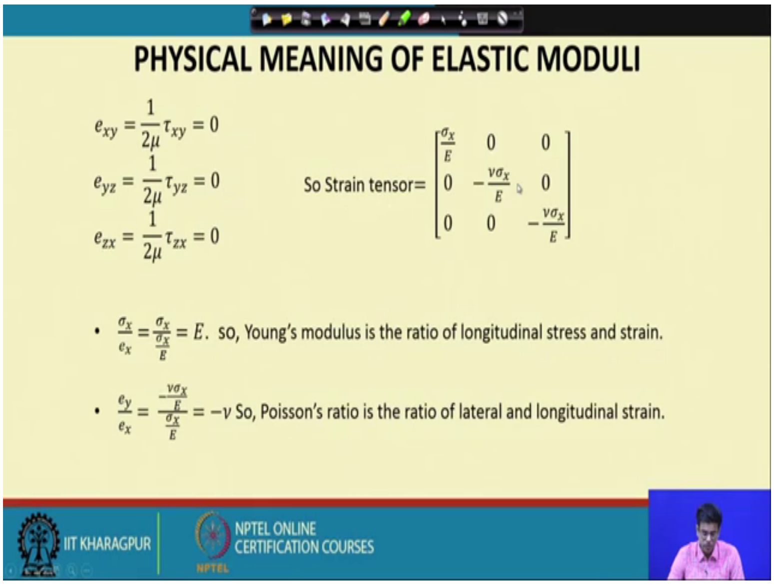

Now, if we for the other cases of strains that is our sheared strain because we assume that there is no sheared strain in this no shear strain or no imperfection in the material or the material geometry or the specimen geometry, there is no imperfectual, something very unrealistic though, but we assume if we assume this or if you believe this because in any material, you will naturally observed imperfection and any geometry will have it is any geometry will have it is imperfection and any natural material will have it is in homogeneity.

So, if we discovered those issues and then the for instance if we want to do it in the lab so, you have to really apply the load very uniformly and then the base where you fix the specimen that has to be properly fixed so that there is no this thing. So, all those conditions if we discovered all those experimental situation, then if we assume that all these things are rightly or perfectly done, then this specimen would not experience any sheared strain; so, which will be which is evident from this. So now, considering this that if we write the strain tensor strain tensor is essentially the diagonal tensor, contrary to this if you if you have seen stress tensor, stress tensor is only the first (Refer Time: 07:15) with the first element is non zero, other elements are 0.

So, now, so,, so this becomes very easy for now we can find out in the elastic modulus from this. So, which is essentially sigma x by e x and so, stress by strain. So, essentially which is the Young's module e and similarly the lateral if we take the ratio of the lateral strain that is the e y n x lateral strain means the transfer strain by longitudinal strain e y by e x, then I get the Poisson's ratio.

Now, you see these Poisson's ratio can also be obtained from the other directional strain that is e y e z by e x because we assume that that is perfect material and perfect geometry. So, this way, this is how we calculate the stress is Young's modulus and Poisson's ratio. So, it becomes clear now, that Young's modulus is essentially the slope of the stress strain curve and similarly the Poisson's ratio is essentially the slope of the two strains. So, this is when we do a simple tension.

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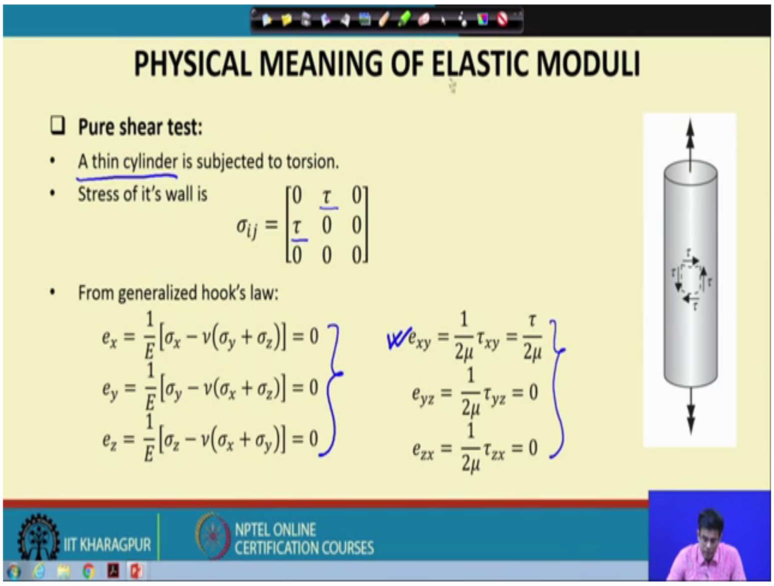

But when we do a pure sheared test; for instance if there is a cylinder which is again we assume all those perfect cylinder, perfectly aligned axis and give a torsion or a if we give a torsion pure kind of torsion where there is no other stresses are generated.

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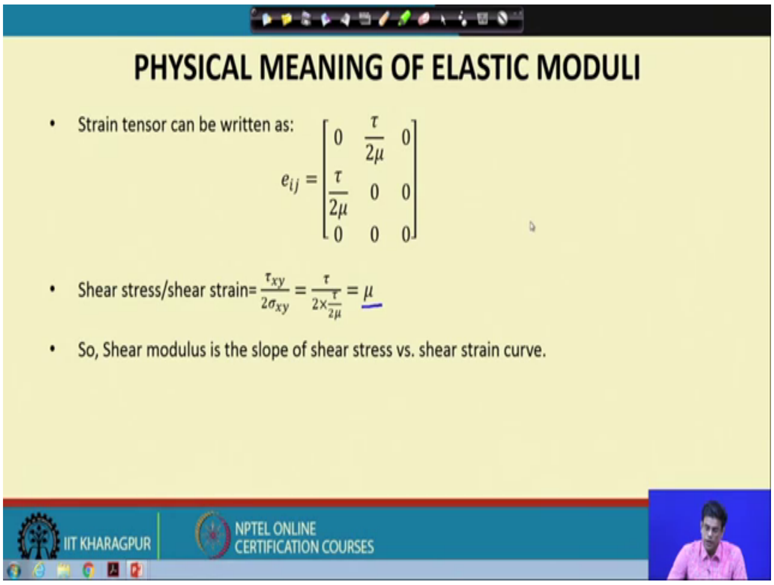

So, then the surface of this cylinder will experience only shear stress. So, this experiment also probably we know from our strength of material knowledge. So, if we write again it is stress tensor. So, it becomes only tau becomes 0 non zero another quantities becomes 0. Again, this is a thin cylinder. So, all those assumptions are I am taking that it is correct. Now, from a generalized hooks law I can write it is strain and stresses. So, if you write the all longitude and stresses as strains, so which implies that all longitude stress strain should be 0 and only the sheared strain corresponding to x y is non zero which is essentially tau x y by 2 mu is essentially the shear modulus. So, it is essentially tau by 2 mu. So, one component of the shear strain is non zero.

Now, if we see the strain tensor similar to the stress tensor, strain sensor will also have the non zero component that is epsilon x, y and or e x y or e y x. Now, if we take the sheared strain by sheared stress by shear strain, we obtain the shear modulus mu. So, again we can comment from that the shear modulus is essentially slope of the shear stress versus shear strain curves. So, this is how we determine the shear modulus in the lab, so in or in the laboratory.

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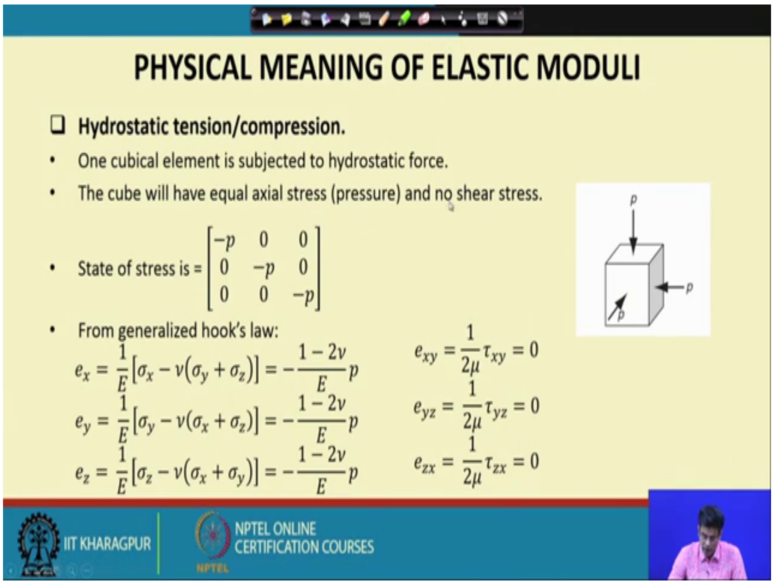

So, now, another test probably not very common is that hydrostatic test. So, Hydrostatic tension or compression, so, if we now take it simple rectangular block and then compress from it is all side, so cubicle one cubicle element is subjected to all from all side, its completion is give it.

Now, if there is the geometry is perfect and then cube will have equal axial stress and no shear stress; so, all these loading application and geometry is very perfect. So, keep always in mind that these are not physical situation, but we assume and our assumption within careful experiment is very well match with the experimental result and theoretical results. So, now, if we take all sides as compression which is minus p and then find out the strains e x, e y, e z and e x y, e y z and e z z, it turns out that only longitudinal strain are non zero, but shear strains are 0. So, now if we want to further process this thing what we can obtain is the strain tensor.

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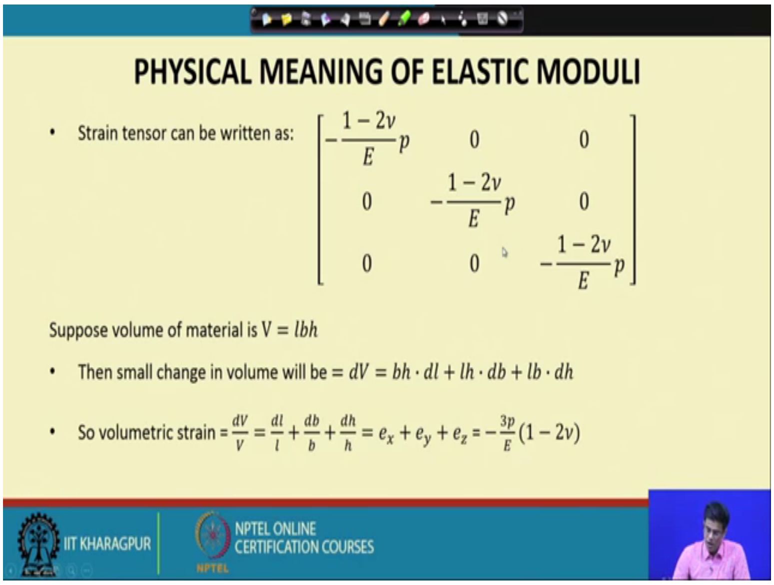

So, the strain tensor will look like this which is coming from the generalized hooks. Now, if we have this cube element is volume v, then we can simply write it is volume is l b h.

Now, if we want to change, if I want to compute the change of this volume d v, so I have to take derivative with respect to or the increment with respect to each of l b and h. So, which I did, so, b h, d l, l h, d b and l b d h. So, now, if I want to compute the volumetric strain d v by v which is d l by l d v by v d h by h which is essentially e x, e y, e z and which is essentially this quantity. So, you see this is the sum of all those who all this strains right this, this, this. So, it is the trace of the stress tensor, now a strain tensor. Now, that means, volumetric strain is essentially the sum of the strain tensor, right.

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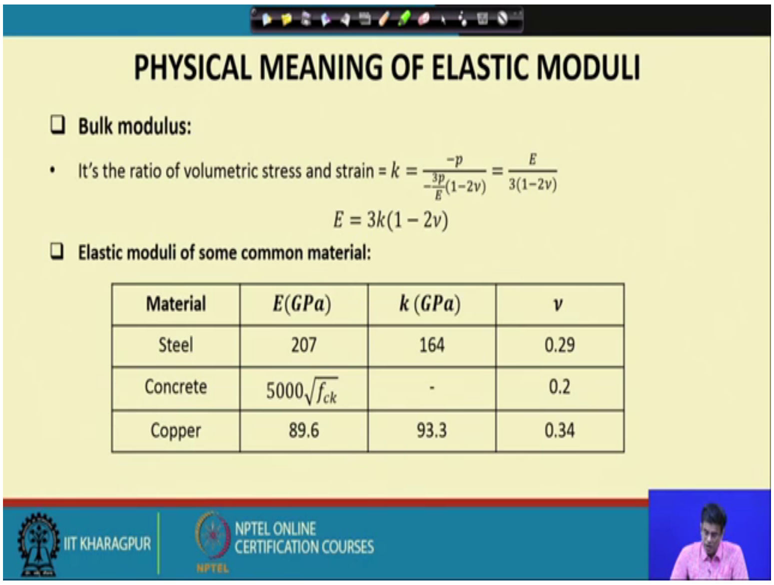

So, if I now define the volumetric stress versus volumetric strain, so which I can simply volumetric stress is p and then if I divide with the volumetric strain, then I can write this I can write this quantity and this quantity seems to be constant and this is known as the bulk modulus of the system.

So, what we learn today? We will learn shear modulus, how to find out shear modulus and how to find out bulk modulus of the system. So, you see all this shear modulus even from your strength of material knowledge, the shear modulus mu is we write the 2 into sorry e by 2 into 1 plus nu. So and bulk modulus also, we write e into e by 3 into 1 minus 2 nu. So, you see these quantities are not independent quantities why, because all can be express with the help of e and nu. So, this is important thing to consider because we are considering the isotropic material and isotropic material will have only two independent constant. So, except that this shear modulus and bulk modulus that can also be treated as independent even; that means, that e and nu can be expressed in terms of k and mu.

So, now there are some common materials which will have for instance steel, the Young's modulus is 207 approximately GPa and if you take the Poisson's ratio near about 0.3. So, bulk modulus will come like this. So, similarly concrete and copper, these can be found from the experiment. Now, if you one important point to remember here is that we are only considering the isotropic material, right and homogeneous material, so let us see some numerical examples.

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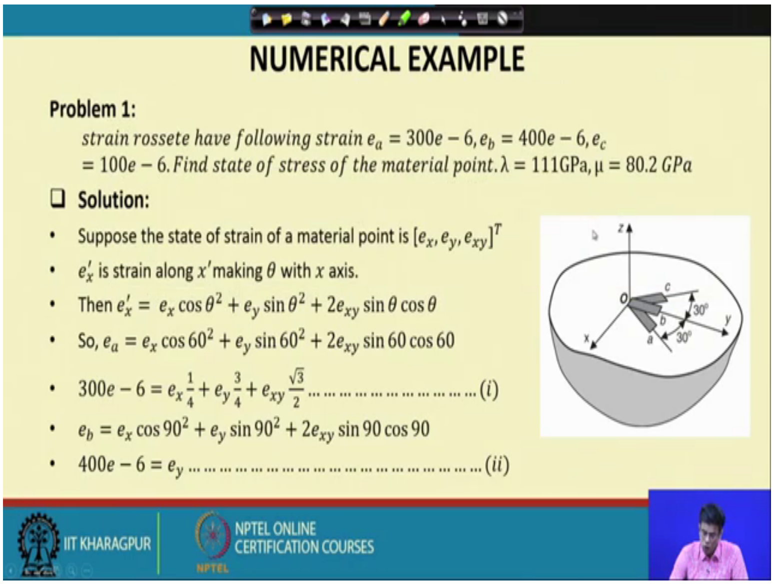

So, suppose there is a strain rossete, strain rossete is essentially a set of strain gauges which is placed with a certain orientation to get the linearly independent strain measures.

So, here there is a three strain gauges apart with the 30 degree angle. Now, the strain measure rossete a, b and c measured strains are 300 into e to the power minus 6. So, this has to be the e to the power minus 6. So, this is essentially 300 into e to the power minus 6, right. So, or they actually this is 10 to the power minus 6. So now, these are these are strain rossete and then find the stress of a material point state of stress of a material point, any material point at that point o. So, now, given lambda or the lame constants first lame constant and second lame constant mu is given.

Now, since this is a planer problem, so I assume that only e x, e y and epsilon e x y will be there now, but these e x, e y, e x y, we have to transfer me according to the direction of the strain because direction is not parallel or the oriented along x, y, z axis. So, any x dash which is making theta with x axis from our knowledge of strain transformation we know e x, e x dash is e x cos square theta e y sin square theta and 2 e x y sin theta cos theta. So, these are from our strength of material knowledge. So, so in case of e a which is cos 60 degrees sin 60 degree and then 2 e x y is a cos sin 60 sin cos 60 and finally, it becomes this equation. So, similarly for e b also, we calculate this.

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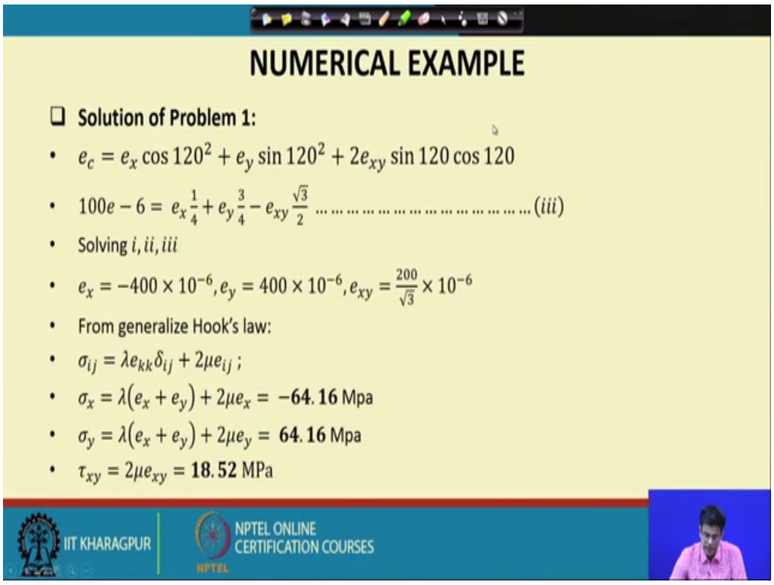

Now, once we are done with this, similarly we calculate e c and then solving this equation, we calculate e x, e y and e x y, so we calculate all strains; now, once we know all strains, we can just substitute in the Hooks law which is essentially sigma i j is lambda stress of e plus 2 mu of e. So, this is a tensorial form, probably we have discussed in the last class.

So, lambda trace of e into I plus 2 mu e that. So, trace of easiest scalar. So, that is why it is represented by a free index is e k k. Now, once we are we know this strains, we can just substitute these strains here which is epsilon k k. So, I say 2 D problem, so, I have to do only epsilon x and epsilon y and then only e i j which is epsilon x. So, I can find out sigma x. So, similarly sigma y, I can find out which comes out 64.16. So, tau x y is again 18.5 52 MPa. So, this is a simple problem which uses the basic Hook's law and compute the stress state. Another basic problem that problem we have solve in strength of material course of the solid mechanics course, if the displacement function is given then how to compute strain and stress.

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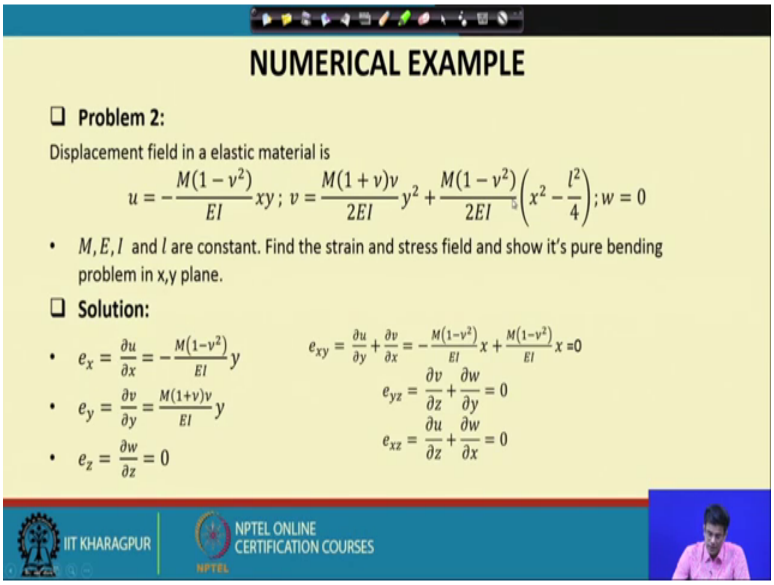

So, the problem is some this displacement functions are giving u, v w and it is ask to show that this is a state of pure bending; that means, pure bending means there will be only bending stresses and no other stresses will be there. So, in a x, y plane, so if you look carefully what is the bending the stress in x, y plane, so sigma axis non zero has to be one of the stress has to be non zero. So, and M E I L are constant. So, if you now use the stress strain or the a strain displacement relation which you have already learning module 2. So, if I use those relation, so, if e x is del u del x and e y del v del y e z del w del z. So, if I just substitute these expression or this discussion functions here and then compute the strains, similar to the longitudinal strains, I can compute the shear strains. So, which is essentially which is essentially comes to be 0 for the e x y and e y z and e x z are 0. So, from this displacement function, we compute the all strains.

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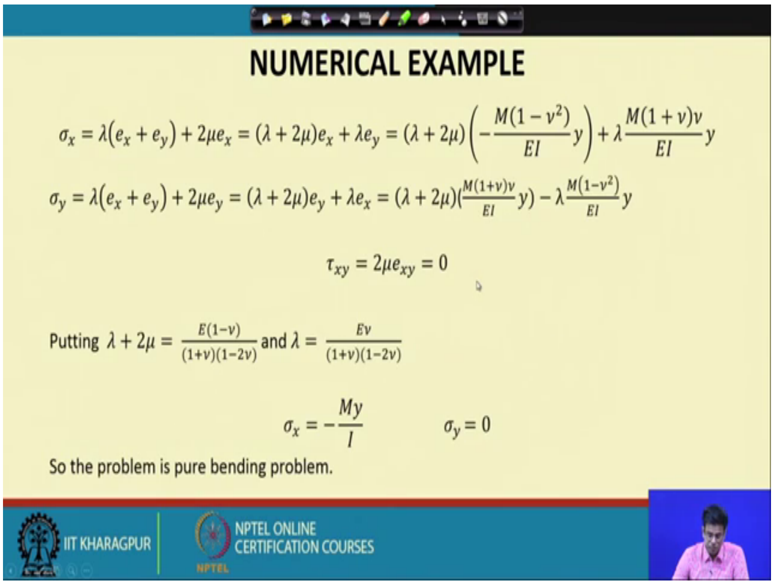

Now, once we know the strains similar to the previous problem, I use the Hooks law. So, which is the lambda stress of e plus 2 mu e and then if I substitute the component wise, this becomes the sigma x and if I now becomes the compute the sigma y and then tau x y.

So, now putting this lambda plus mu which is the relation between the mu and lambda, mu and lambda is a first lame constant and mu is the shear modulus. So, if I sum it and represent it we Young's modulus and Poisson's ratio, then the first lame constant is transferred to be this and then we can say that this is a pure bending because sigma x is only non zero. So, with this, it can be say the problem is pure bending problem. So, today I stop here and in the next class, will learn the strain energy function.

Thank you.