Theory of Elasticity
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Lecture – 25
Formulation of Boundary Value Problems (Contd.)

Hello everyone. This is the 4th lecture of this week. Let us continue our discussion on Formulation of Boundary Value Problems. Today we will discuss problem formulation. You see, you have already discussed what is equilibrium equations, we also discuss what is compatibility relations and then constitutive relations which are the 3 important pillars for elasticity. And, then now we have already arranged all the tools requires, to formulate any boundary value problems.

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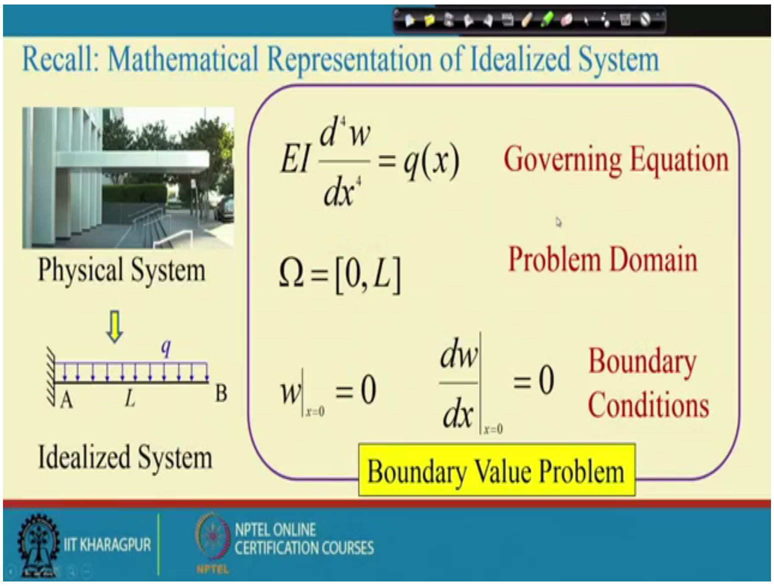

Now, you recall in one of the classes we introduced boundary value problem like this, it is just an example, but important thing here is you take any boundary value problem there are 3 important parts in that problem definition or description of the problem. One is the domain means what is your problem domain and, then here in this case for instance the problem domain is the length of this beam. And, then after that we have the governing equation which is defined on this domain. And, then the equations, the conditions specified at the boundary, there are 3 important part of any boundary value problem.

Now, let us first and the before coming onto the governing equation, let us understand what are the different kinds of domain we can have. Is there something that we cannot have is there restriction on the domain? And, then we also see what are the different kinds of boundary conditions we can have and, then we can we can start writing the derived in the governing equation ok.

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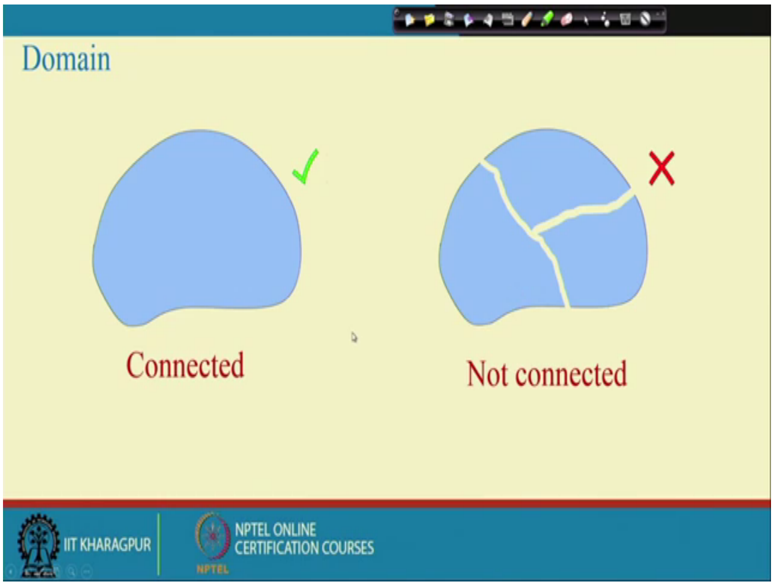

Now, you see in last class we discussed what is the concept of continuum right and we also said that all the problems that we discussed here, that will be dealing with in elasticity, in theory of elasticity be continent nature needs to be preserved it has to be a continuum.

Now, for instance any domain this is a connected domain or it is connected space or connected set in the different contexts it is same, but the essence remain same. And, this is not connected we have it is we have several pieces together which a disjoint pieces to together. Now, what we have all the problems irrespective of the problems we deal with in all the problems domain has to be connected ok. We cannot have a domain which is like this, in this the theory that we are going to develop.

Now, you may say you see suppose you have take you remember the chock example that we gave in the last class we have a chock. And then we apply some load on the chock brakes into pieces. Now, suppose if I want to this is a physical process. Suppose I want to study the breaking of a chock and that is my physical process that is the physical phenomena that I want to study. Does that mean in this problem what is happening? You have a continuum, a single piece of chock and then if we apply the load chock brakes into pieces where, if you combine together cannot be coined as a continuum. Does that mean that we suppose now, we want to study these physical process. We can study this physical process, but the equations that we are going to develop here and, through that equation we cannot do that, we have a separate set of equation separate set of mechanics required to study the evolution of the chock from one from a single chock to several broken pieces, but that will not be dealing with we will not be dealing with in this theory in this course. So, all the domain will be connected, domain connected set.

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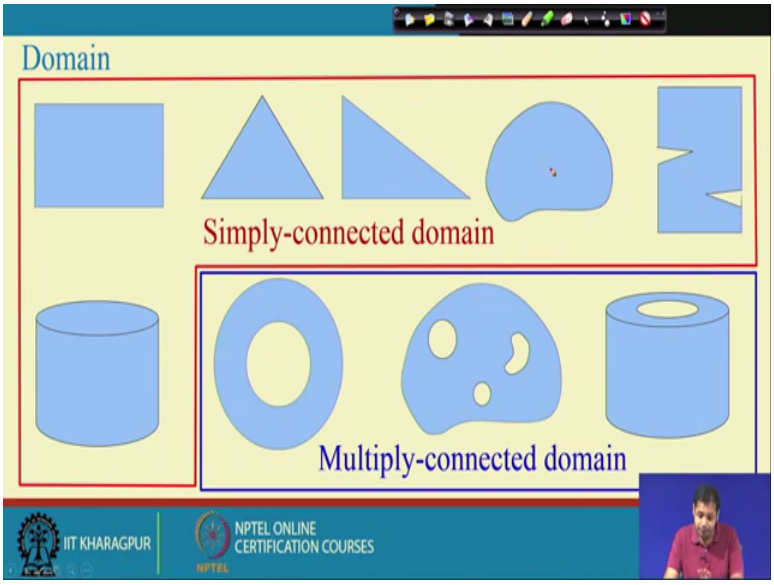

Now, even in connected set, now there are you see some of the examples. In all the example, this all the spaces or the domains are connected domain, but there are some differences and the difference is one of the suppose if you take a closed curve in this domain ok. And, assume this close and this close curve you can shrink this closed curve eventually to a point. And, this in this entire curve belongs to this domain and it can be you can shrink it to a point which also belongs to this domain.

And, you can do it for this domain you can do it for this domain you can do it for this domain all these domain. You can do it for this domain in 3 D as well, but whereas you see if you take a closed curve like this it you shrink it to a point, but if you take another curve for instance, if you take a curve like this which belongs to the entire loop belongs to this domain, but when you shrink it will be it may be somewhere here which is which does not belong to this domain.

In this case also if you take a if you take if you takes curve like this and then this belongs to this domain, but then it cannot be when you shrink it is point now it can it is it is so, it does not belong to this domain, and similarly for these hollow cylinder as well.

Now, why I am saying that? What is the importance of this? What is the importance of highlighting this difference? Ok. Before, that these kind of domain is called they are all connected domain, but these kind of domain are called simply connected domain. Whereas, this kind of domain are called multiply connected domain.

Now, you remember when we derive compatibility equation then one point I mentioned, that the compatibility relation with derive so, far that is fine, but we have not really we have not really made any commons, whether that condition is necessary and sufficient condition to have the displacement field uniquely, or single value displacement field we have not proved that yet.

And, then we said that we will come to this point once again talk about complex variable approach. You see we will see in complex variable approach that compatibility equation is a necessary condition that is fine, but it is sufficient condition only for simply connected domain. If the domain is multiple connected, it does not automatically satisfy the single value in the uniqueness of uniqueness of the displacement field.

And, we have to we have to do something we can prove that uniqueness in some other way, but it is not automatic the way it is very straight forward for the case of connect simply cannot be connected domain, it may not be the case for multiply connected domain.

I am mentioning this term just for the completeness, but as far as the theory that we are going to develop, now and the problems that we are going to come across we will take one thing for granted. That compatibility equation is necessary and sufficient condition, if something satiin this strain. And, since satisfy the compatibility condition, then when integrate it the displacement that you get that is unique we will take it for granted for the time being.

And, we will come to this point during the complex variable approach. So, these all domains are all the domains shown here all are valid domain we can we will see the some of the examples, where the if domains simply connected some of the examples and multiply connected domain. In all the cases we will apply the same we will be using the same set of equations ok.

Now, once we have some discussion on the domain. Once we have understood, what are the domains? That we can have a valid we can have the domains are valid for these equations. Let us move on and then see the different boundary conditions ok.

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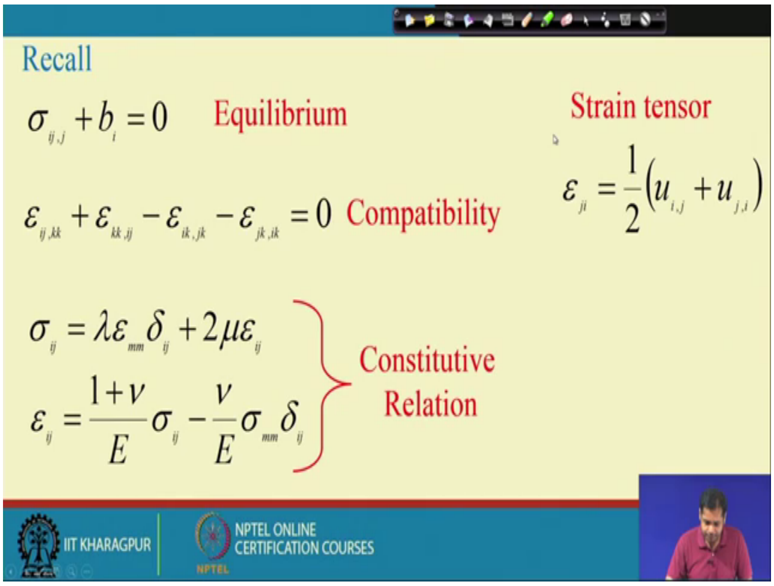

There are 3 parts domain boundary conditions and the equations, let us see the boundary conditions ok.

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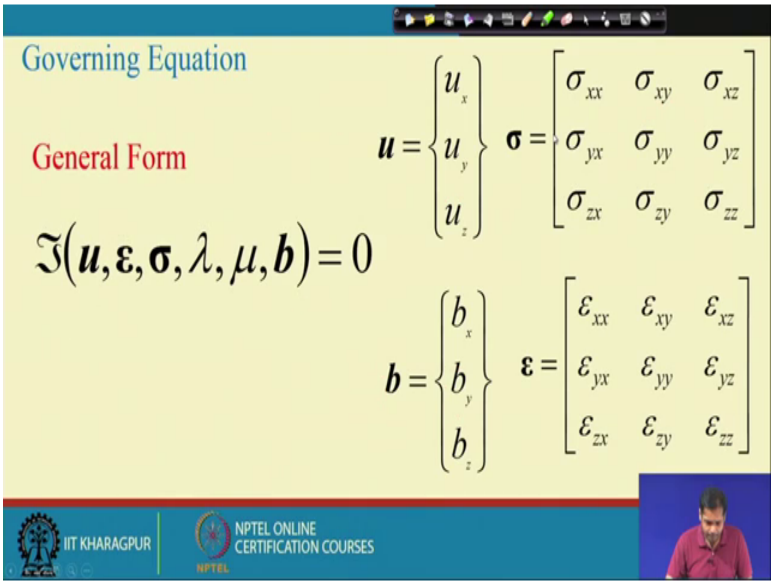

Before, that you see, let us understand the boundary condition here.

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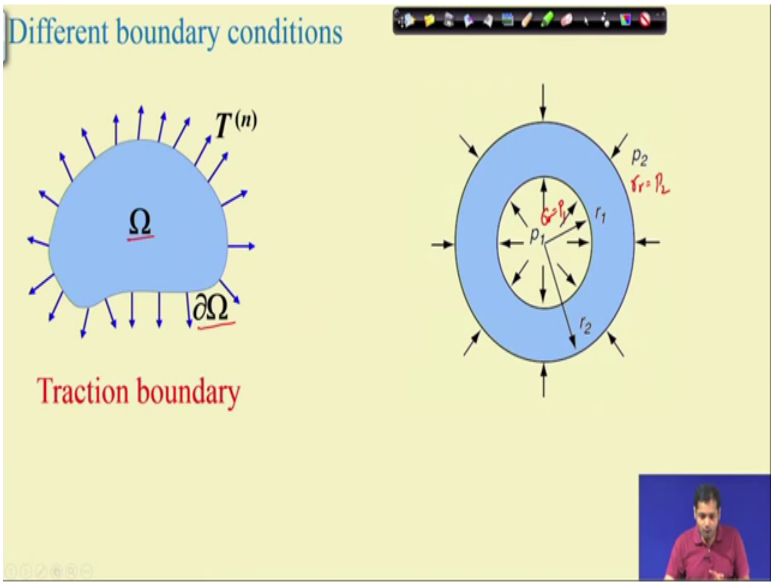

Suppose, we have a bounded we have a domain which is the boundary is domain is omega and, the boundary is a boundaries del omega ok.

Now, what are the different kinds of boundary conditions we can have? We can have only traction boundary one of the tractions has specified at the boundary has boundary conditions. And, in that case the only boundary condition that we have is the traction boundary condition.

If you have to give you some example for instance if you take, this example will address it is example of a of a of a pressure vessel. Suppose you have we have a vessel like this and we have internal pressure we have some out external pressure as well. So, this has 2 boundaries one is the inner circle and outer circle and both the circles we have the boundary condition specified.

Here the boundary condition is the stresses radial stresses is p 1 here sigma r is sigma r is p 1, here also sigma r is p 2 these are the boundary condition traction boundary conditions.

Now, we can also have another boundary condition.

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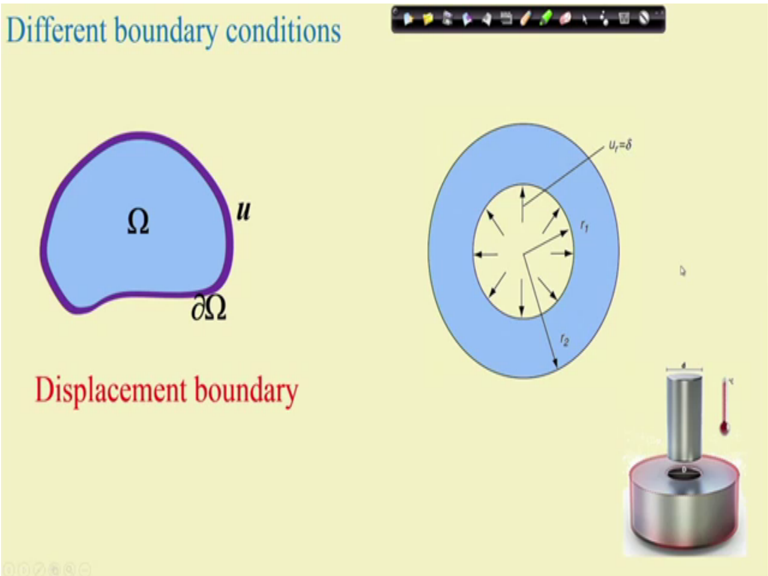

Suppose take the same domain once again. And, we can the boundary condition specified in the boundary is the displacement. And, this one in the boundary conditions at displacement boundary condition. So, one example is this is an example of a shrink field process, where suppose you have a solid cylinder has to be has to be inserted in a in a in a in a hole.

And, then when you try to insert in a hole it creates a displacement boundary at the inner hole like this ok. We will also solve this example with using the theory that we are going to be developed. So, in this case what are the boundary conditions you have is you have specified boundaries this is a specified boundary yeah. Here, the radial directions this boundary is delta.

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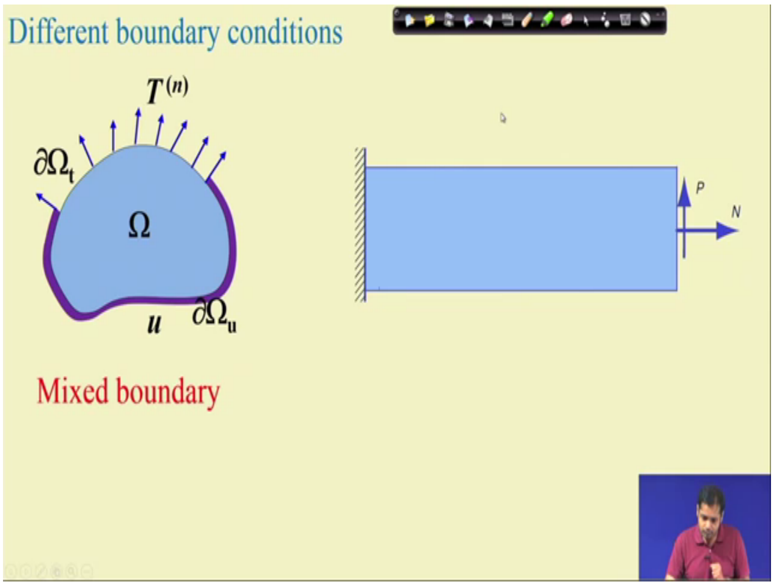

Now, we can have another kind of boundary conditions where, the some part is traction specified in the in the some part the displacements are specified and that boundary that kind of problem is called the mixed boundary problem.

Now so, example of that is take a for instance a cantilever beam, where which is subjected to some normal load and some shear load at the tip. Now, what are the boundary conditions we have you have your displacement boundary conditions, where this all the displacements at 0. And, then you have stress traction boundary condition traction specified at this point. So, this is an example of mixed boundary condition.

Now, now when we when we applied these equations, then probably the how to write with boundary conditions? And how to use different boundary conditions differently those things we discussed in subsequent classes ok. Now, once we have understood what is what are the domains and then what are the possible boundary conditions?

Now, we are ready to derive the equation. Like the compatibility condition, we will explicit we will derive this equation for 2 decades. And when you when we used for the 3 D case 3 D problems, we will be using initial notation to derive this equation ok.

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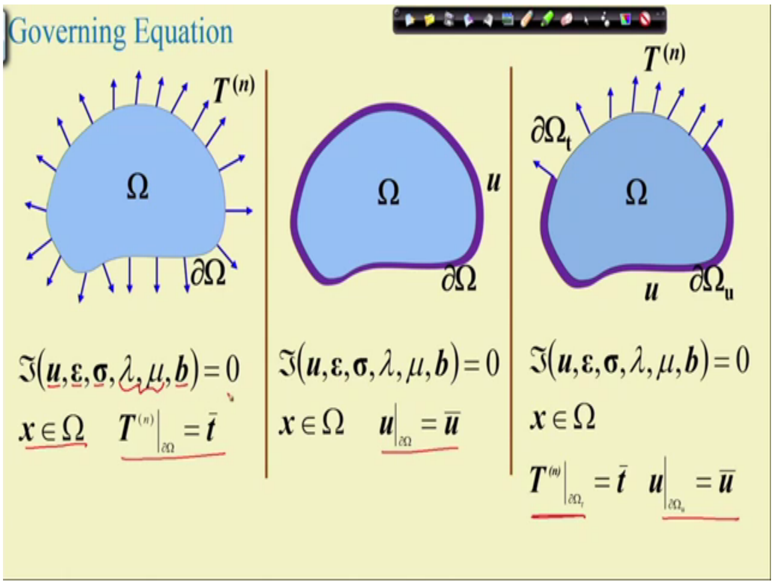

Now, you see so, take these 3 kind of problem. Now, we can have a general form is like this. Suppose general form of this equation will be what are the things we have we have the equation is essentially is a function is writing a function, which is the function of u. Then the strain field, then the stress field, it is also a function of the material property for isotropic material linear isotropic material 2 constants we have the lambda and mu and then we if we have a body force ok.

And, then this is the equation general form of the equation. And, then the domain is omega and this is subjected to the attraction boundary condition, the boundary condition specified of the traction, this is for traction problem. Now, if it is displacement boundary conditions, then again the same equations you have to write, but now the boundary conditions are specified as displacement. And, where in the case of mixed boundary condition, mixed condition we have we have both boundary your displacement boundary condition, as displacement boundary condition as well as traction boundary condition.

Now, at this point if we take a very general framework, very general equation for instance this is a very general form of this equation, suppose if I go back to the previous slides, yes. Now, this is the very general form general equation that we are going to derive in a specific case. Now, we see what are the relations? What are the unknown we have in the general form, we will eventually see it is not that many unknown we have some of the information we can put in and reduce the number of unknown, but in a very general form.

How many relations we have? First let us let us see how many unknowns we have, we have say for displacement we have 3 unknown it is a 3 displacement field in the 3 direction. And, then we have this total 6 strain components we have these are the strain component. And, then out of the these 3 normal strain and 3 shear strain and then similarly we have 6 stress components 3 normal stresses and 3 shear stresses.

So, total 15 unknown unknowns we have right, when we have to solve a problem suppose you take a any example any problem which is subjected to some kind of load. And, when you analyze the problem eventually what it means? Eventually, it means and you have to find out what are the internal stresses, stress at a given point.

We have to find out what are the strains at a given point? And, we also find out what are the displacements at a given point ok. Displacements and strain are related to each other we will come to this point.

But these are the 3 important t is a 3 information that we need as a outcome of analysis right. What are the relations we are available is available to us, we have say the relations available to us is you go back, we have equilibrium equations this is the equilibrium equations we have 3 equilibrium equations.

And, then we have constitutive relations we have 6 consecutive relation. And, then we have either you consider this relation of this relation. We have 3 relations it gives you how these strains are related to displacement we have 6 such relations. So, total how many relation are available 6 plus 6 plus 5th total 15 relations are available. And, how many unknowns we have we have 15 unknown.

But, as I said it is a very general form, but then some of the relations are we can we can reduce the number of unknown reduce the number of relations, because some of the things are dependent on each other. We will we will come to this point. Now so, again go back to this classification of different problems. We have based on the boundary conditions then you can have 3 kinds of problem. One is the traction problem where the only reflection boundary conditions are specified.

Body forces are known, what we have to find out we have to find out these strain all the parameter. And, the interior of the domain and then this displacement boundary problem and this is the mixed boundary, mixed boundary problem ok. This is the classification of the problem depending on the boundary conditions.

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Now, now coming to the formulation of the problem formulation of the problem is writing the governing equations. Now, these equations can be writ10 in 2 ways; one is the one is the equations are written in the form of stresses or the equations can also be written in the form of displacements ok.

So, based on how we are representing these equations? We have 2 formulations; one formulation is called stress formulation and another formulation is called displacement bond displacement formulation. Today, what we do is we will what is stress formulation? And, the displacement formulation will between the next class ok.

So, let us see what we have this is a same slide just now we have these are the information's we have ok. Equilibrium conditions, compatibility condition, constitutive relation and the definition of strain that we have. So, we will do the all the steps explicitly for 2 D problem, 2 dimension problem and then when we say 2 dimension problem please note one thing, we have not really formally introduced when I say 2 dimensional problem what does actually mean.

Because, you remember in many times we mention that there is nothing like 2 dimensional problem all the problems are 3 dimensional problem, but for our is there are some class of problems which can be idolize the 2 dimensional problem. And, that idealization we have not discussed so, far will be doing in the next class, but for the time being when I say the 2 dimension problem, it mean that we have the stress component epsilon x epsilon y and epsilon xy, corresponding stress components are epsilon sigma xx sigma yy and sigma xy, and then we have 2 displacements condition u and v ok.

Now so, for 2 dimension problem then what we have is?

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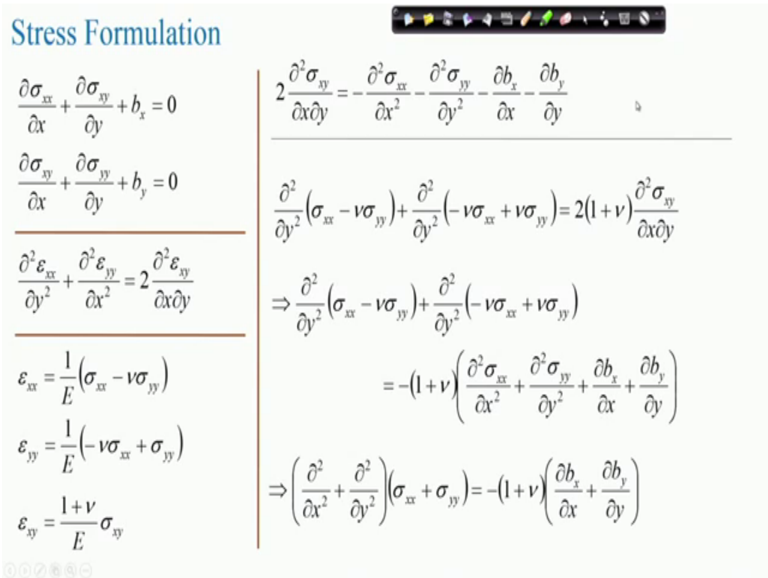

We have these are the equilibrium equations, the same equilibrium equations, can be can we can have for 2 dimensional cases this. Now, then we have the compatibility condition. If you remember last class we derive these equation and then there is there is a correction please this should be y y this should be y y ok. Now, then we have thing this then we have this thing constitutive relation ok.

Let us see how this thing to be formed? Ok. First what we do if, first we first suppose this is equation number 1 and equation number 2 the governing equation. Let us differentiate these equation with respect to x and differentiate with these equation with respect to y.

Now, if you do that then these equation becomes what these equation becomes del 2 sigma xx del x 2 and then plus del 2 sigma xy del x del y plus del bx bx and by bx and by are the body forces in x and y directions. So, this is the first equation. And, if we differentiate these with respect to y, then we have del 2 sigma x y. And, then del x del y then plus del 2 sigma y y del y 2 plus del b y del y this is equal to 0.

So, let us this is 1 a and 1 b 2 or a 2 a. Now, if we add them then what we have is we have this equation. So, this and this can be this and this gives you this and rest of the thing if you take outside we have this equation. So, this is the equation we have from the equilibrium equation.

So, essentially what we did here is we have combine we have equilibrium equations and here these 2 equilibrium equations are combined and we have just one equation. So, these equation essentially give you these 2 equation. Now, once we have this form of equilibrium equation. Now, let us put compatibility condition and the contusive relation into this. So, then so, this is the equation we have ok.

Now, let us what we do is it? Let us let us leave it this equation for the time being the equilibrium equations for the time being, this is equilibrium equation ok. Now let us now come to substitute this constitutive relation into these compatibility equation.

So, essentially when we say governing equation governing equation carries all these information. The governing equation which means the mathematical representation of that process that problem, should carry all the information all the information means the equilibrium conditions, the compatibility conditions, and also the constitutive relations.

. So, we have the equilibrium conditions here. Now substitute this constitutive relation into this compatibility relate compatibility conditions. And, if you do that what equations we get is this? Ok. You can try parallelly. So, we have this equation just direct substitution of this we get this.

Now, this equation gives you what this equation is essentially? So, as I said this equation is the equilibrium equation, this and this equation is these 2 equation. What we do is now is we combine these 2 equation. So, if we combine, how do we combine? Now substitute this terms from this, so, this if I place here, what we have is this we have this ok.

Now so, this we have a now then if you do that is do some manipulation. And then finally, please do not take this expression for granted as I said and probably I will be repeating this again and again, not for this do not take any derivations. Even, if it is given in book do not take any derivation for granted there they may be correct, but until and unless you do this exercise yourself, until and unless you derive that yourself things will not be clear.

So, whatever things are you can see here please drive it and then convince yourself. And, then why if you do some this manipulation and eventually you will get this expression.

This is now this is the now you see what is this expression? These expression this is one expression this equation carries all this information. These carries information about if this is satisfied there are 2 things to be observed here. These equations these equation is written in purely in terms of stresses, you see sigma x and sigma y not only that this equation carries all this information;

information about the equilibrium condition, information about the compatibility condition and information about the constitutive relations right. Now, if you have space we satisfy the equation, it may you can say that satisfy all these expressions as well, all these conditions as well ok.

Now, next let us do this exercise this is for 2 dimensional case.

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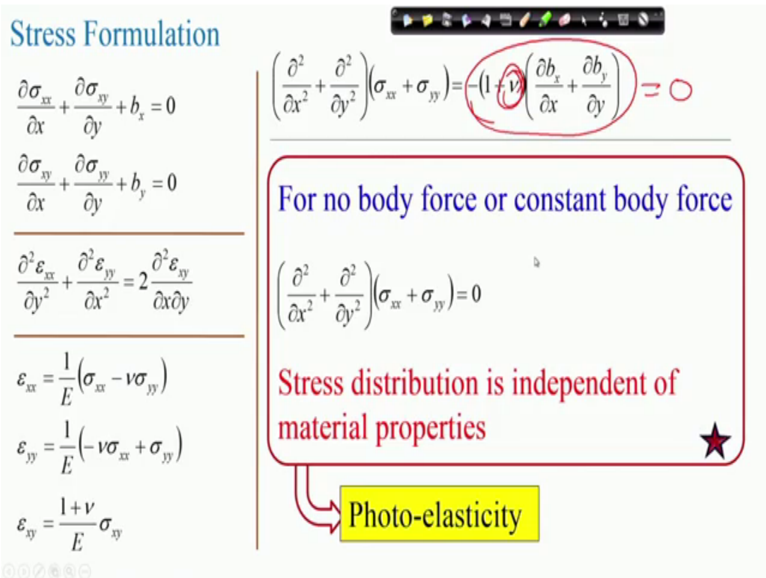

Let us do this exercise for 3 dimensions. And 3 dimension as I said we will be doing it for ok. Just before that one very important observation. You see this is the governing this is the equation just now we derived. Now, suppose you have a problem where the body force is 0.

For instance, you take any example, where the body force say gravity is a body force. So, gravity you do not consider the gravity. So, the body force is 0 or you have a problem where the body force is constant ok, it does not depend on say why it is just constant? Means either bx and by this bx and by either they are 0 or they are constant. If, they are 0 or they are constant, then what will happened then either bx is equal to 0 or by is equal to 0, if they are or bx is equal to sum constant and b y is equal to sum constant.

But, in both the cases what will happen this term this del bx del x term becomes 0 and del b y del y term that becomes 0. If these are become 0 the right hand side this entire thing become 0. So, what happens? If your body force term is if there is nobody force in the constant body force, then what will happen? Then these term become 0 and then equation eventually become this ok. So, this term become the entire term become 0 this term become this.

Now, this is a very important equation. Looking at this equation we will see how to solve this equation and all these things that is that is absolutely fine, but this equation also tells us a very important thing and it is. So, important that it takes you to a to one section of elasticity all one variant of elasticity, we will just shortly that is we will shortly see that.

You see what this equation tells you? This equation tells us, that if there is no body force or the constant body force. This is the governing equation you mind it. Then this equation if you solve this equation, what we get we get stresses right? If there is no body force and no or the constant body force, these equation is independent of material property you see, when they was a body it was nu was there nu was there, which is a Poisson’s ratio, which is the material property.

Now, when you have when you have no body forces constant body force, it is independent of material property. It means that if you have body force, on the constant body force, the stresses they do not depend on the material property and this is a very important observation reason.

Now, these tells you the stress distribution is independent of material properties this, because the equation that is solve to get stress that equation does not carry information about the material properties. And, these observations will important this observation will take you in another variant or section of elasticity, which is called photo elasticity. And, how this equation this observation is exploited or use in photo elasticity, we will discuss that in the week when we talk about elasticity and in there we start with this observation ok; that is why it is very important ok.

Now, you recall I means those who have taken those who have taken in structural analysis or even in mechanics when you when you solve trusses. Now, if your truss is determinacy truss of course, if for a determinate structure, when you derive, when you solve the any truss problem and obtain the member forces in different members or for a determinant beam.

When you calculate the bending moment diagram of shear moment shear force diagram, you recall the bending moment diagram, shear force diagram, and also the forces in the member for truss all they did not depend on the properties of the material. There was no young's modulus and Poisson's ratio in the bending moment right.

So, um, but Poisson's ratio and bending moment comes in the case of displacement. It only says that stresses are independent of material property, but when the stresses are used to calculate strain and eventually the displacement then the material property comes in. Because, stresses to strain we need the constitutive relations where the material property comes in. It is this phases which are independent of material property, then we will discuss this things in detail in the case in photo elasticity ok.

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Stress Formulation

$$\varepsilon_{ij,kk} + \varepsilon_{kk,ij} - \varepsilon_{ik,jk} - \varepsilon_{jk,ik} = 0 \qquad \varepsilon_{ij} = \frac{1+\nu}{E}\sigma_{ij} - \frac{\nu}{E}\sigma_{mm}\delta_{ij}$$

$$\epsilon_{ij,kk} = \frac{1+\nu}{E}\sigma_{ij,kk} - \frac{\nu}{E}\sigma_{mm,kk}\delta_{ij}$$

$$\sigma_{ij,kk} + \sigma_{kk,ij} - \sigma_{ik,jk} - \sigma_{jk,ik} = \frac{\nu}{1+\nu}(\sigma_{mm,kk}\delta_{ij} + \sigma_{mm,ij}\delta_{kk} - \sigma_{mm,jk}\delta_{ik} - \sigma_{mm,ik}\delta_{jk})$$

Now, we move on to 3 dimensional case so, we will be doing it just browse to the derivation using the initial notation, but you have to do it all the steps to convince yourself. So, this is the so, we had what are the things we use in the case of 2 dimensional case, equilibrium equations compatibility equation and then the constitutive relations right. Now, this is the constitutive relation if you remember not the if this is the compatibility conditions and this is the constitutive we have equilibrium equation, we will using it shortly.

Now, what we do first is first to do these exercise the exercise that we did 2 exercise we did if in the previous slide. First we equilibrium we combine this equilibrium equation, and the second exercise we substituted this constitutive relation into the into the compatibility condition. The same thing we do here this is the compatibility condition it has substitute this is equilibrium equations.

And, if you do that we will just see what is the first term first term become the first term becomes epsilon ij hm. So, if you substitute this epsilon ij here is the first term become epsilon ij k k not that, yes epsilon ij kk these becomes 1 plus nu by E, then sigma ij differentiation with respect to k k minus nu by E and then sigma mm differentiation with respect to kk and then delta ij which is the Kronecker delta this is the first term.

Similarly, you can have all these term. And, then write this in equation and then you get the compatibility condition has this. So, this is the compatibility condition ok. You can check yourself. Now, once we have the compatibility condition. Now next is so, this is the compatibility condition.

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Stress Formulation

$$\sigma_{ij,kk} + \sigma_{kk,ij} - \sigma_{ik,jk} - \sigma_{jk,ik} = \frac{v}{1+v}\left(\sigma_{mm,kk}\delta_{ij} + \sigma_{mm,ij}\delta_{kk} - \sigma_{mm,jk}\delta_{ik} - \sigma_{mm,ik}\delta_{jk}\right)$$

Equilibrium $\sigma_{ij,j} + b_i = 0$

$$\sigma_{ij,kk} + \frac{1}{1+v}\sigma_{kk,ij} = \frac{v}{1+v}\sigma_{mm,kk}\delta_{ij} - b_{i,j} - b_{j,i}$$

Next, what we do if we substitute the equilibrium condition into this compatibility condition into this expression. And, if we the equilibrium condition that we have this is the written equilibrium condition in initial notation. Now, if we if we substitute this equilibrium condition into this, what we get is these expression we get? Ok.

Now, we have this expression written in initial form.

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Stress Formulation

$$\sigma_{ij,kk} + \frac{1}{1+v}\sigma_{kk,ij} = \frac{v}{1+v}\sigma_{mm,kk}\delta_{ij} - b_{i,j} - b_{j,i}$$

Substituting $i=j$ $\quad \sigma_{ii,kk} = -\dfrac{1+v}{1-v}\, b_{i,i}$

$$\boxed{\sigma_{ij,kk} + \frac{1}{1+v}\sigma_{kk,ij} = -\frac{v}{1-v}\delta_{ij} b_{k,k} - b_{i,j} - b_{j,i}}$$

Beltrami-Michell Compatibility Equations

Now, this is the expression, now what we do is, now we have here sigma mm term sigma mm term and then sigma kk term this will replace and how do we replace that we have to find out what is the expression for sigma mm sigma kk and so on. So, what we can do is in this expression if you substitute i is equal to j i is equal to j we get the expression for sigma sigma ii kk has this.

Now, we substitute this into this expression the sigma sigma mm kk and so on. This, and if you substitute this is the entire thing if you substitute here what you get is finally, this expression you get ok.

Now, again here also you check, if in absence of body force if the body forces are 0 ok. Let us write the final expression then we discuss their. If you in this expression, if you check, if you make the body forces are 0 or constant the entire thing becomes 0 the entire thing become 0. And, then eventually you will show you can show that it is independent we will just the expression we are going to write it shortly; these expression is called Beltrami-Michell Compatibility Equation.

So, this is you will call the compatibility equation when we wrote in terms of strain it was called Saint-Venant compatibility equation and the same equation we it is written when you combined all these compatibility, when you substituted equilibrium and the and the constitutive relations in the compatibility condition. This is the compatibility condition we get and this is known as Beltrami-Michell compatibility condition.

Now, coming back to the previous point that we were discussing in the case of when the constant body force or the 0 body force this is the all the term associated with the body force becomes 0. And, if you make it 0 and then you if you write all this expression, how many expressions we can have here?

You see this is sigma ij, how many questions we can have? This is sigma ij kk is dummy index here, because it is reputation kk is a dummy index here. So, essentially it is it becomes second order tensor, but if you put the symmetry that sigma ij is equal to sigma ji. So, number of components become 6. So, essentially we will be having 6 equation set of equations, if you write xy in place of ij and for every directions, if you write that expression and make the body force is 0, then the 6 equation that you have is this.

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Stress Formulation

$$(1+\nu)\nabla^2\sigma_x + \frac{\partial^2}{\partial x^2}(\sigma_x + \sigma_y + \sigma_z) = 0$$

$$(1+\nu)\nabla^2\sigma_y + \frac{\partial^2}{\partial y^2}(\sigma_x + \sigma_y + \sigma_z) = 0$$

$$(1+\nu)\nabla^2\sigma_z + \frac{\partial^2}{\partial z^2}(\sigma_x + \sigma_y + \sigma_z) = 0$$

$$(1+\nu)\nabla^2\sigma_{xy} + \frac{\partial^2}{\partial x \partial y}(\sigma_x + \sigma_y + \sigma_z) = 0$$

$$(1+\nu)\nabla^2\sigma_{yz} + \frac{\partial^2}{\partial y \partial z}(\sigma_x + \sigma_y + \sigma_z) = 0$$

$$(1+\nu)\nabla^2\sigma_{zx} + \frac{\partial^2}{\partial z \partial x}(\sigma_x + \sigma_y + \sigma_z) = 0$$

And, here also you see that you have the same observation these all these equations are written in terms of stresses, but none of these equations carry information about the material properties. Means, if the body force is 0 this is the distribution of stress equal the that they do not depend on the material property, but again when, but it does not mean that entire the entire behavior of the material is independent of the independent of his property, it is not like that when we when we when the stress is converted into strain their material property comes in ok.

So, this is the equation for a stresses, but again we will see that this is not the equation that we directly solve, next class when you when we discuss different methodology to solve these equations. We will see that how to write this expression, the same expression can be written in different ways. So, the solution can be made easier.

But before that there is one more formulation which is called displacement formulation, where the same set of equations written in terms of displacement, but one thing you will see the stresses how many stresses we have? We have 6 component of stresses and therefore, we have 6 equations here similarly when we write this expression in terms of displacements by intuition we can say that, we have 3 components of displacements displacement in 3 directions. So, displacement equations will be only with 3 equations we derived that in the next class.

So, next class what we do is next class we will first derived what is the displacement formulation quickly? And then we also summarized what are the possible? Once, we have the formulation the governing equation ready then we have we have a full description of the boundary value problems, the domain, the equations and also the boundary conditions.

And, then next we have to see once we have the description once we have described the boundary problems next step is to solution of that equation solution of the mathematical model. What are the available techniques we have what are the advantages and disadvantages of different techniques? And then from there we will select particular technique and then see how that technique can be used to solve different problems using the same set of equations, ok. Then I stop here today see you in the next class.

Thank you.